ASME PTB-4-2013

ASME Section VIII – Division 1 Example Problem Manual

ASME

SETTING THE STANDARD

PTB-4-2013

ASME Section VIII - Division 1 Example Problem Manual

James C. Sowinski, P.E.

David A. Osage, P.E.

The Equity Engineering Group, Inc.

Date of Issuance: May 24, 2013

The American Society of Mechanical Engineers

Two Park Avenue, New York, NY 10016-5990

TABLE OF CONTENTS

FOREWORD

This document is the second edition of the ASME Section VIII – Division 1 example problem manual. The purpose of this second edition is to update the example problems to keep current with the changes incorporated into the 2013 edition of the ASME B&PV Code, Section VIII, Division 1. The example problems included in the first edition of the manual were based on the contents of the 2010 edition of the B&PV Code. In 2011, ASME transitioned to a two year publishing cycle for the B&PV Code without the release of addenda. The release of the 2011 addenda to the 2010 edition was the last addenda published by ASME and numerous changes to the Code were since adopted.

This second edition of the example manual includes two new sections covering examples for tube–to–tubesheet welds and required markings of pressure vessel nameplates. Known corrections to design equations and results have also been made in this second edition. Additionally, some formatting modifications were made to facilitate better use of the example manual, as applicable.

This document is the Division 1 example problem manual. In this manual, example problems are solved using both the Division 1 and Division 2 rules. When the design rule is the same, the example problem is solved using the Division 2 rules with the Division 1 allowable stress and weld joint efficiency. With this approach, users of Division 1 will become familiar and adept at using Division 2, and this will also provide a significant training benefit to the Division 1 user in that Division 2 has been designed as the home for the common rules initiative being undertaken by the ASME Section VIII Committee.

In 2007, ASME released a new version of the ASME B&PV Code, Section VIII, Division 2. This new version of Division 2 incorporated the latest technologies to enhance competitiveness and is structured in a way to make it more user-friendly for both users and the committees that maintain it. In addition to updating many of the design-by-analysis technologies, the design-by-rule technologies, many adopted from the Division 1 rules, were modernized. ASME has issued *ASME Section VIII – Division 2 Criteria and Commentary, PTB-1-2009* that provides background and insight into design-by-analysis and design-by-rule technologies.

The ASME Section VIII Committee is currently undertaking an effort to review and identify common rules contained in the Section VIII Division 1, Division 2, and Division 3 B&PV Codes. In this context, common rules are defined as those rules in the Section VIII, Division 1, Division 2, and Division 3 Codes that are identical and difficult to maintain because they are computationally or editorially complex, or they require frequent updating because of the introduction of new technologies. Common rules typically occur in the design-by-rule and design-by-analysis parts of the code; but also exist in material, fabrication, and examination requirements. A plan has been developed to coordinate common rules with the following objectives.

- Common rules in the Section VIII Division 1, 2, and 3 codes should be identical and updated at the same time to ensure consistency.
- Common rules will be identified and published in a single document and referenced by other documents to; promote user-friendliness, minimize volunteer time on maintenance activities, and increase volunteer time for incorporation of new technologies to keep the Section VIII codes competitive and to facilitate publication.
- Core rules for basic vessel design such as wall thickness for shells and formed heads, nozzle design, etc. will be maintained in Division 1; although different from Division 2 these rules are time-proven and should remain in Division1 because they provide sufficient design requirements for many vessels.
- ASME Section VIII Committee recognizes that Division 2 is the most technically advanced and best organized for referencing from the other Divisions and recommends that, with the exception of overpressure protection requirements, common rules identified by the committee shall reside in Division 2 and be referenced from Division 1 and Division 3, as applicable.

As a starting point for the common rules initiative, the ASME Section VIII Committee has developed Code Case 2695 to permit the use of some the design-by-rule procedures in Division 2 to be used for Division 1 construction.

As part of the common rules initiative, the ASME Section VIII Committee is working with ASME ST-LLC to create separate example problem manuals for each Division. These manuals will contain problem examples that illustrate the proper use of code rules in design.

ACKNOWLEDGEMENTS

We wish to acknowledge the review performed by the following members of the BPV VIII Committee: Gabriel Aurioles, Anne Chaudouet, Michael Clark, Maan Jawad, Scott Mayeux, Ramsey Mahadeen, Urey Miller, Clyde Neely, Frank Richter, and Jay Vattappilly.

We would also like to commend the efforts of Allison Bradfield, Jeffrey Gifford, and Tiffany Shaughnessy for their documentation control and preparation skills in the publication of this manual.

PART 1

GENERAL REQUIREMENTS

1.1 Introduction

ASME B&PV Code, Section VIII, Division 1 contains mandatory requirements, specific prohibitions, and non-mandatory guidance for the design, materials, fabrication, examination, inspection, testing, and certification of pressure vessels and their associated pressure relief devices.

1.2 Scope

Example problems illustrating the use of the design-by-rule methods in ASME B&PV Code, Section VIII, Division 1 are provided in this document. Example problems are provided for most of the calculation procedures in either SI or US Customary units.

1.3 Definitions

The following definitions are used in this manual.

VIII-1 – ASME B&PV Code, Section VIII, Division 1, 2013

VIII-2 – ASME B&PV Code, Section VIII, Division 2, 2013

1.4 Organization and Use

An introduction to the example problems in this document is described in Part 2 of this document. The remaining Parts of this document contain the example problems. All paragraph references without a code designation, i.e. VIII-1 or VIII-2, see Definitions, are to the ASME B&PV Code, Section VIII, Division 1, 2013 [1].

The example problems in this manual follow the design by rule methods in ASME B&PV Code, Section VIII, Division 1. Many of the example problems are also solved using ASME B&PV Code, Section VIII, Division 2 design-by-rule procedures contained in Part 4 of this Code using the allowable stress from VIII-1. In addition, where the design rules are the same, the VIII-2 format has been used in this example problem manual because of the user-friendliness of these rules.

1.5 Comparison of VIII-1 and VIII-2 Design Rules

Since many of the design rules in VIII-2 were developed using the principles of VIII-1, it is recommended that users of this manual obtain a copy of ASME PTB-1-2013 [2] that contains the VIII-2 criteria and commentary on the technical background to these rules. A comparison of the design-by-rule procedures in VIII-2 compared with VIII-1 is shown in Table E1.1.

1.6 ASME Code Case 2695

In recognition of the similarities and the use of the latest technology in developing the design-by-rule part of VIII-2, ASME has issued Code Case 2695 that permits the use of VIII-2 design rules with VIII-1 allowable stresses with some limitations. Code Case 2695 is shown in Table E1.2.

1.7 References

1. ASME B&PV Code, Section VIII, Division 1, *Rules for Construction of Pressure Vessels*, 2013, ASME, New York, New York, 2013.
2. ASME B&PV Code, Section VIII, Division 2, *Rules for Construction of Pressure Vessels – Alternative Rules*, 2013, ASME, New York, New York, 2013.
3. Osage, D., *ASME Section VIII – Division 2 Criteria and Commentary*, PTB-1-2013, ASME, New York, New York, 2013.

1.8 Tables

Table E1.1 – Comparison of Design Rules Between VIII-2 and VIII-1

Paragraph in Section VIII, Division 2	Comments Pertaining to Section VIII, Division 1
4.1	General Requirements, harmonized with VIII-1, i.e. MAWP introduced, etc.
4.2	Design Rules for Welded Joints, a restrictive subset of rules in VIII-1, UG & UW
4.3	Design Rules for Shells Under Pressure, mostly new technology
4.4	Design Rules for Shells Under External Pressure and Allowable Compressive Stresses, almost identical to CC2286 with exception of stiffening ring requirements at cone-to-cylinder junctions
4.5	Design Rules for Shells Openings in Shells and Heads, new technology
4.6	Design Rules for Flat Heads, identical to UG-34
4.7	Design Rules for Spherically Dished Bolted Covers, identical to Appendix 1-6 and Appendix 14 except Soehern's stress analysis method for Type 6D Heads is included
4.8	Design Rules for Quick Actuating (Quick Opening) Closures, identical to UG-35.2
4.9	Design Rules for Braced and Stayed Surfaces, a restrictive subset of rules in paragraph UG-47(a)
4.10	Design Rules for Ligaments, identical to paragraph UG-53
4.11	Design Rules for Jacketed Vessels, a more restrictive subset of rules in Appendix 9
4.12	Design Rules for Non-circular vessels, identical to Appendix 13 but re-written for clarity
4.13	Design Rules for Layered Vessels, identical to Part ULW
4.14	Evaluation of Vessels Outside of Tolerance, new technology per API 579-1/ASME FFS-1
4.15	Design Rules for Supports and Attachments, new for VIII-2 using existing technology
4.16	Design Rules for Flanged Joints, almost identical to Appendix 2
4.17	Design Rules for Clamped Connections, identical to Appendix 24
4.18	Design Rules for Shell and Tube Heat Exchangers, identical to Part UHX
4.19	Design Rules for Bellows Expansion Joints, identical to Appendix 26

Notes:

1. During the VIII-2 re-write project, an effort was made to harmonize the design-by-rule requirements in VIII-2 with VIII-1. AS shown in this table, based on this effort, the design rules in VIII-2 and VIII-1 are either identical or represent a more restrictive subset of the design rules in VIII-1.
2. In the comparison of code rules in presented in this table, the term identical is used but is difficult to achieve and maintain because of coordination of ballot items on VIII-1 and VIII-2. There may be slight differences, but the objective is to make the design rules identical. The restrictive subset of the rules in VIII-1 was introduced in VIII-2 mainly in the area of weld details. In general, it was thought by the committee the full penetration welds should be used in most of the construction details of a VIII-2 vessel.

Table E1.2 – ASME BPV Code Case 2695

Code Case 2695

Allowing Section VIII, Division 2 Design Rules to Be Used for Section VIII, Division 1 Pressure Vessel Section VIII, Divisions 1 and 2

Inquiry: Under what conditions may the design-by-rule requirements in Part 4 of Section VIII, Division 2 be used to design the components for a Section VIII, Division 1 pressure vessel?

Reply: It is the opinion of the Committee that the design-by-rule requirements in Part 4 of Section VIII, Division 2 may be used to design the components for a Section VIII, Division 1 pressure vessel, provided the following conditions are met:

a) The allowable design tensile stress shall be in accordance with UG-23 of Section VIII, Division 1.

b) The weld joint efficiency shall be established in accordance with UW-11 and UW-12 of Section VIII, Division 1.

c) Material impact test exemptions shall be in accordance with the rules of Section VIII, Division 1.

d) If the thickness of a shell section or formed head is determined using Section VIII, Division 2 design rules, the following requirements apply:

    1) For design of nozzles, any nozzle and its reinforcement attached to that shell section or formed head shall be designed in accordance with Section VIII, Division 2.

    2) For conical transitions, each of the shell elements comprising the junction and the junction itself shall be designed in accordance with Section VIII, Division 2.

    3) For material impact test exemptions, the required thickness used in the coincident ratio defined in Section VIII, Division 1 shall be calculated in accordance with Section VIII, Division 2.

e) The fatigue analysis screening in accordance with Part 4, paragraph 4.1.1.4 of Section VIII, Division 2 is not required. However, it may be used when required by UG-22 of Section VIII, Division 1.

f) The provisions shown in Part 4 of Section VIII, Division 2 to establish the design thickness and/or configuration using the design-by-analysis procedures of Part 5 of Section VIII, Division 2 are not permitted.

g) The Design Loads and Load Case Combinations specified in Part 4, paragraph 4.1.5.3 of Section VIII, Division 2 are not required.

h) The primary stress check specified in Part 4, paragraph 4.1.6 of Section VIII, Division 2 is not required.

i) Weld Joint details shall be in accordance with Part 4, paragraph 4.2 of Section VIII, Division 2 with the exclusion of Category E welds.

j) The fabrication tolerances specified in Part 4, paragraph 4.3 and 4.4 of Section VIII, Division 2 shall be satisfied. The provision of evaluation of vessels outside of tolerance per Part 4, paragraph 4.14 of Section VIII, Division 2 is not permitted.

k) The vessel and vessel components designed using these rules shall be noted on the Manufacturer's Data Report.

l) All other requirements for construction shall comply with Section VIII, Division 1.

m) This Case number shall be shown on the Manufacturer's Data Report.

PART 2

EXAMPLE PROBLEM DESCRIPTIONS

2.1 General

Example problems are provided for;

- Part 3 – Materials Requirements
- Part 4 – Design By Rule Requirements parts in Section VIII, Division 1
- Part 5 – Design By Analysis
- Part 6 – Fabrication Requirements
- Part 7 – Examination Requirements
- Part 8 – Pressure Testing Requirements

A summary of the example problems provided is contained in Table of Contents.

2.2 Example Problem Format

In all of the example problems, with the exception of tubesheet design rules in paragraph 4.18, the code equations are shown with symbols and with substituted numerical values to fully illustrate the use of the code rules. Because of the complexity of the tubesheet rules, only the results for each step in the calculation producer is shown.

If the design rules in VIII-1 are the same as those in VIII-2, the example problems are typically solved using the procedures given in VIII-2 because of the structured format of the rules, i.e. a step-by-step procedure is provided. When this is done, the paragraphs containing rules are shown for both VIII-1 and VIII-2.

2.3 Calculation Precision

The calculation precision used in the example problems is intended for demonstration proposes only; any intended precision is not implied. In general, the calculation precision should be equivalent to that obtained by computer implementation, rounding of calculations should only be done on the final results.

PART 3

MATERIALS REQUIREMENTS

3.1 Commentary on Rules to Establish the Minimum Design Metal Temperature (MDMT)

Requirements for low temperature operation for vessels and vessel parts constructed of carbon and low alloy steels are provided in paragraphs UCS-66, UCS-67 and UCS-68. The organization of the requirements is as follows:

a) Paragraph UCS-66 – provides rules for exemption of impact test requirements for carbon and low alloy steel base material listed in Part UCS.
b) Paragraph UCS-67 – provides rules for exemption of impact test requirements for welding procedures.
c) Paragraph UCS-68 – provides supplemental design rules for carbon and low alloy steels with regard to Weld Joint Categories, Joint Types, post weld heat treatment requirements, and allowable stress values.

Paragraph UCS-66(a) provides impact test exemption rules based on a combination of material specification, governing thickness, and required MDMT using exemption curves. The rules are applicable to individual components and welded assemblies comprised of two or more components with a governing thickness. Welded, nonwelded, and cast components are covered with limitation of the exemption rules based on thickness.

Paragraph UCS-66(b) provides for an additional reduction of temperature for impact test exemption based on a temperature reduction curve and a coincident ratio defined simply as the required thickness to the nominal thickness. The coincident ratio can also be applied to pressure and or stress.

The following logic diagrams, shown in Figure E3.1.1, Figure E3.1.2, and Figure E3.1.3, were developed to help provide guidance to the user/designated agent/Manufacturer for determining the impact test exemption rules of paragraphs UCS-66(a) and UCS-66(b).

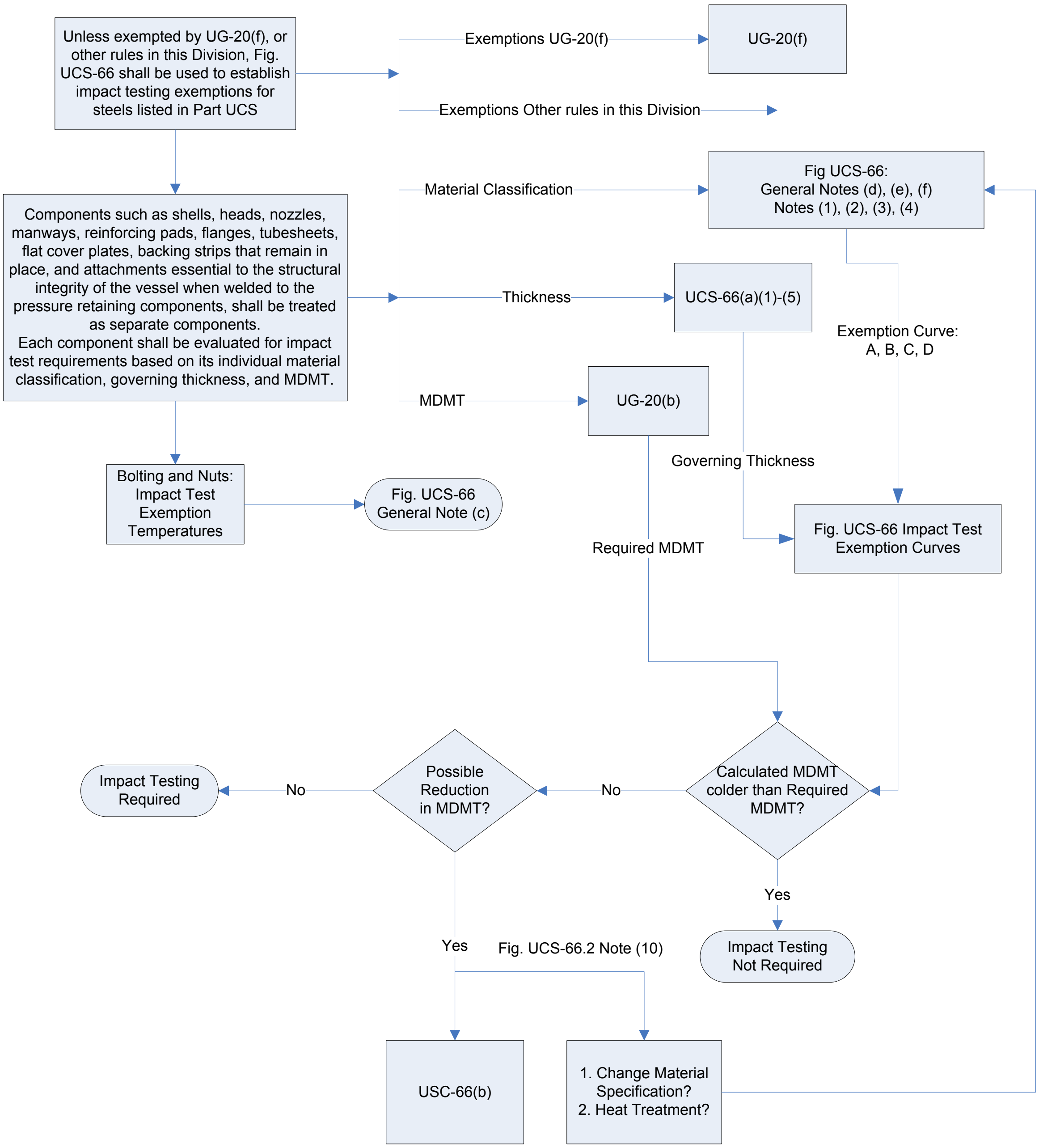


Figure E3.1.1 – Logic Diagram for UCS-66(a)

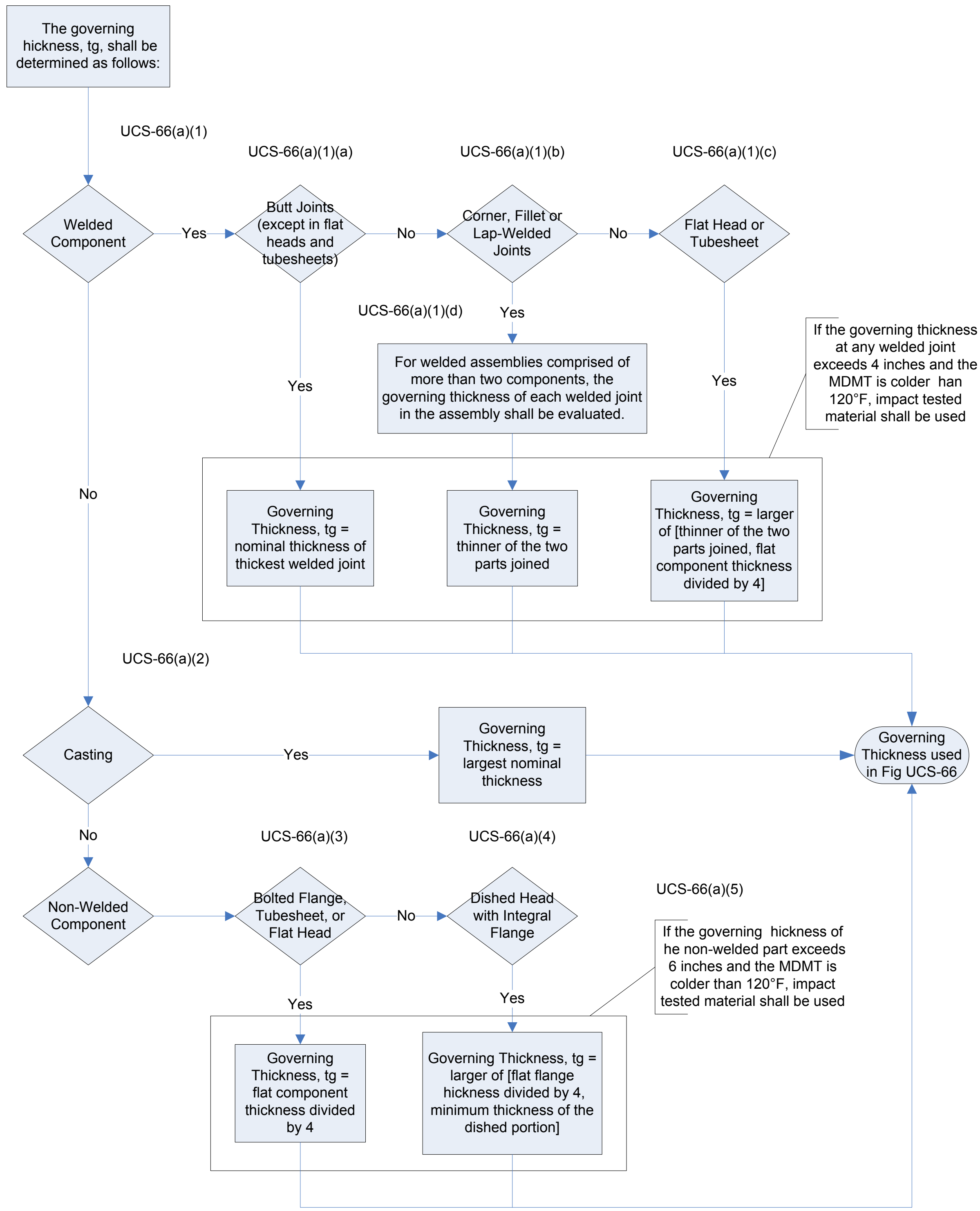


Figure E3.1.2 – Logic Diagram for UCS-66(a)(1)-(5)

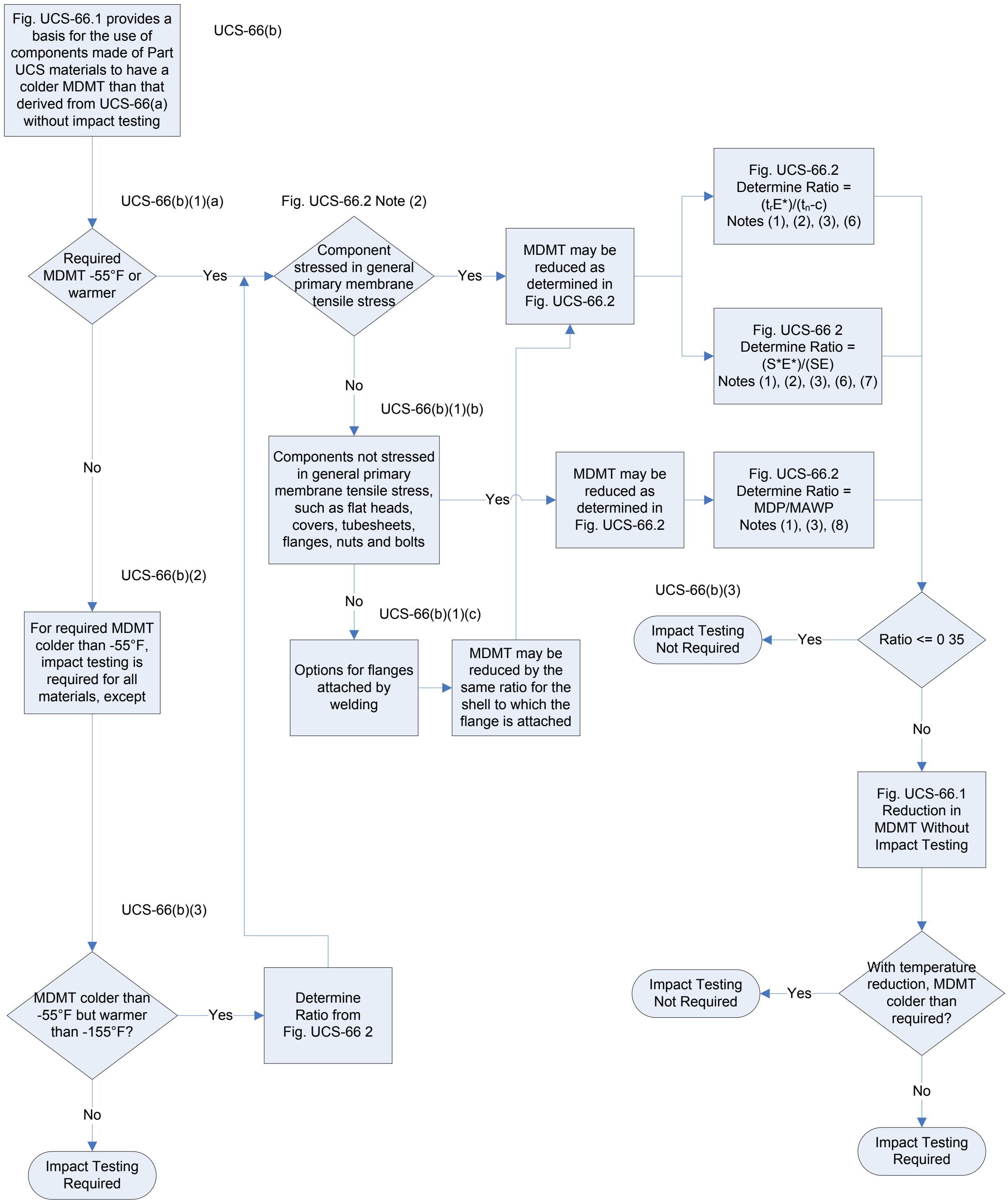


Figure E3.1.3 – Logic Diagram for UCS-66(b)

3.2 Example E3.1 – Use of MDMT Exemptions Curves

Determine if Impact Testing is required for the proposed shell section. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Nominal Thickness = $1.8125\ in$
- PWHT = Yes
- MDMT = $-20°F$
- Corrosion Allowance = $0.125\ in$

In accordance with paragraph UCS-66(a), the procedure that is used to establish impact testing exemptions is shown below.

Paragraph UCS-66(a): unless exempted by the rules of UG-20(f) or other rules of this Division, Fig. UCS-66 shall be used to establish impact testing exemptions for steels listed in Part UCS. When Fig. UCS-66 is used, impact testing is required for a combination of minimum design metal temperature (MDMT) and thickness which is below the curve assigned to the subject material. If a MDMT and thickness combination is on or above the curve, impact testing is not required by the rules of this Division.

a) STEP 1 – From the Notes of Fig. UCS-66, the appropriate impact test exemption curve for the material specification $SA-516,\ Grade\ 70,\ Normalized$ is designated a Curve D material.

b) STEP 2 – The governing thickness to be used in Fig. UCS-66 is determined from paragraph UCS-66(a)(1) through (a)(5) based upon if the component under consideration is a welded part, casting, flat non-welded part, or a dished non-welded part. In this example, the cylindrical shell is a welded part attached by a butt joint and the governing thickness is equal to the nominal thickness of the thickest welded joint, see Fig. UCS-66.3.

$$t_g = 1.8125\ in$$

c) STEP 3 – The required MDMT is determined from paragraph UG-20(b) and is stated in the vessel data above as $-20°F$.

d) STEP 4 – Interpreting the value of MDMT from Fig. UCS-66 is performed as follows. Enter the figure along the abscissa with a governing thickness of $t_g = 1.8125\ in$ and project upward until an intersection with the Curve D material is achieved. Project this point left to the ordinate and interpret the MDMT. This results in an approximate value of $MDMT = -7°F$. Another approach to determine the MDMT with more consistency can be achieved by using the tabular values found in Table UCS-66. Linear interpolation between thicknesses shown in the table is permitted. For a $t_g = 1.8125\ in$ and a Curve D material the following value for MDMT is determined.

$$MDMT = -7°F$$

Since the calculated MDMT of $-7°F$ is warmer than the required MDMT of $-20°F$, impact testing is required using only the rules in paragraph UCS-66(a). However, impact testing may still be avoided by applying the rules of paragraph UCS-66(b) and other noted impact test exemptions referenced in paragraph UCS-66.

Additionally, paragraph UCS-68(c) permits a $30°F$ reduction in impact testing exemption temperature from that determined in Fig. UCS-66 if the component is subject to a postweld heat treatment (PWHT) when not otherwise a requirement of this Division. Although the vessel under consideration in this example was subject to PWHT, it was done so because the nominal thickness was in excess of that permitted without PWHT per paragraph UCS-56. Therefore, the $30°F$ reduction in impact testing temperature is not permitted.

3.3 Example E3.2 – Use of MDMT Exemption Curves with Stress Reduction

Determine if impact testing is required for the proposed shell section in E3.1. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $356\ psi\ @\ 300°F$
- Inside Diameter = $150\ in$
- Nominal Thickness = $1.8125\ in$
- PWHT = Yes
- MDMT = $-20°F$
- Weld Joint Efficiency = 1.0
- Corrosion Allowance = $0.125\ in$
- Allowable Stress at Ambient Temperature = = $20000\ psi$
- Allowable Stress at Design Temperature = = $20000\ psi$

In accordance with paragraph UCS-66(b), the procedure that is used to determine the exemption from impact testing based on a coincident thickness ratio is shown below.

Paragraph UCS-66(b): when the coincident ratio defined in Fig. UCS-66.1 is less than one, Fig UCS-66.1 provides a basis for the use of components made of Part UCS material to have a colder MDMT than that derived from paragraph UCS-66(a) without impact testing.

Paragraph UCS-66(b)(1)(a): for such components, and for a required MDMT of $-55°F$ and warmer, the MDMT without impact testing determined in paragraph UCS-66(a) for the given material and thickness may be reduced as determined from Fig. UCS-66.2. If the resulting temperature is colder than the required MDMT, impact testing of the material is not required.

a) STEP 1 – The appropriate impact test exemption curve for the material specification $SA-516,\ Grade\ 70,\ Normalized$ from the Notes of Fig. UCS-66, was found to be Curve D.

b) STEP 2 – The governing thickness t_g to be used in Fig UCS-66, for the welded part under consideration, was found to be $t_g = 1.8125\ in$.

c) STEP 3 – The required MDMT is determined from paragraph UG-20(b) and is stated in the vessel data above as $-20°F$.

d) STEP 4 – Interpreting the value of MDMT from Fig. UCS-66/Table UCS-66, $MDMT = -7°F$.

e) STEP 5 – Based on the design loading conditions at the MDMT, determine the ratio, R_{ts}, using the thickness basis from Fig UCS-66.2.

$$R_{ts} = \frac{t_r E^*}{t_n - CA}$$

Where, t_r is the required thickness of the cylindrical shell at the specified MDMT of $-20°F$, using paragraph UG-27(c)(1).

$$t_r = \frac{PR}{SE - 0.6P} = \frac{356(75.125)}{\left(20000(1.0) - 0.6(356)\right)} = 1.3517$$

$$where,$$

$$R = \frac{D}{2} + Corrosion\ \ Allowance = \frac{150.0}{2} + 0.125 = 75.125\ in$$

The variables E^*, t_n, and CA are defined as follows:

$$E^* = \max\left[E,\ 0.80\right] = \max\left[1.0, 0.8\right] = 1.0 \qquad \rightarrow Fig.\ UCS-66.2,\ Note\ 3$$

$$t_n = 1.8125\ in$$

$$CA = 0.125\ in$$

Therefore,

$$R_{ts} = \frac{t_r E*}{t_n - CA} = \frac{1.3517(1.0)}{1.8125 - 0.125} = 0.8010$$

f) STEP 6 – Interpreting the value of the temperature reduction, T_R from Fig. UCS-66.1 is performed as follows. Enter the figure along the ordinate with a value of $R_{ts} = 0.8010$, project horizontally until an intersection with the provided curve is achieved. Project this point downward to the abscissa and interpret T_R. This results in an approximate value of $T_R = 20°F$.

g) STEP 7 – The final adjusted value of the MDMT is determined as follows.

$$MDMT = MDMT_{STEP3} - T_R = -7°F - 20°F = -27°F$$

Since the final value of MDMT is colder than the proposed MDMT, impact testing is not required.

3.4 Example E3.3 – Determine the MDMT for a Nozzle-to-Shell Welded Assembly

Determine if impact testing is required for the proposed nozzle assembly comprised of a shell and integrally reinforced nozzle. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined. The nozzle parameters used in the design procedure is shown in Figure E3.3.1.

<u>Vessel Data:</u>

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $356\ psi\ @\ 300°F$

- Inside Diameter = $150\ in$
- Nominal Thickness = $1.8125\ in$
- PWHT = Yes
- MDMT = $-20°F$
- Weld Joint Efficiency = 1.0
- Corrosion Allowance = $0.125\ in$
- Allowable Stress at Ambient Temperature = = $20000\ psi$
- Allowable Stress at Design Temperature = = $20000\ psi$

Nozzle:

- Material = $SA-105$
- Outside Diameter = $25.5\ in$
- Thickness = $4.75\ in$
- Allowable Stress at Ambient Temperature = = $20000\ psi$
- Allowable Stress at Design Temperature = = $20000\ psi$

The nozzle is inserted through the shell, i.e. set–in type nozzle.

In accordance with paragraph UCS-66(a)(1)(d), the procedure that is used to establish the governing thickness, t_g, is shown below.

Paragraph UCS-66(a)(1)(d): for welded assemblies comprised of more than two components (e.g., nozzle-to shell joint with reinforcing pad), the governing thickness and permissible minimum design metal temperature of each of the individual welded joints of the assembly shall be determined, and the warmest of the minimum design metal temperatures shall be used as the permissible minimum design metal temperature of the welded assembly. See Fig. UCS-66.3 Sketch (g) and Figure E3.3.1 of this example.

a) STEP 1 – The appropriate impact test exemption curve for the cylindrical shell material specification $SA-516,\ Grade\ 70,\ Norm.$ from the Notes of Fig. UCS-66, was found to be Curve D. Similarly, the appropriate impact test exemption curve for the integrally reinforced nozzle material specification $SA-105$ from the Notes of Fig. UCS-66, was found to be Curve B.

b) STEP 2 – The governing thickness of the full penetration corner joint, t_{g1} to be used in Fig UCS-66, for the welded joint under consideration, was determined per Fig. UCS-66.3 Sketch (g).

$$t_{g1} = \min\left[t_A,\ t_C\right] = \min\left[1.8125,\ 4.75\right] = 1.8125\ in$$

Where,

$$t_A = Shell\ thickness,\ 1.8125\ in$$
$$t_C = Nozzle\ thickness,\ 4.75\ in$$

c) STEP 3 – The required MDMT is determined from paragraph UG-20(b) and is stated in the vessel data above as $-20°F$.

d) STEP 4 – Interpreting the value of MDMT from Fig. UCS-66 for the welded joint requires that both the shell and nozzle material be evaluated and the warmest minimum design metal temperature shall be used for the assembly. The procedure is performed as follows.

For the cylindrical shell material, $SA-516,\ Grade\ 70,\ Normalized$: Enter the figure along the abscissa with a governing thickness of $t_g = 1.8125\ in$ and project upward until an intersection with the Curve D material is achieved. Project this point left to the ordinate and interpret the MDMT. This results in an approximate value of $MDMT = -7°F$. Another approach to determine the MDMT with more consistency can be achieved by using the tabular values found in Table UCS-66. Linear interpolation between thicknesses shown in the table is permitted. For a $t_g = 1.8125\ in$ and a Curve D material the following value for MDMT is determined.

$$MDMT_{curve\,D} = -7°F$$

For the nozzle material, $SA-105$: Enter the figure along the abscissa with a governing thickness of $t_g = 1.8125\ in$ and project upward until an intersection with the Curve B material is achieved. Project this point left to the ordinate and interpret the MDMT. This results in an approximate value of $MDMT = 59°F$. Similarly, a more accurate value for MDMT can be achieved by using the tabular values found in Table UCS-66. Linear interpolation between thicknesses shown in the table is permitted. For a $t_g = 1.8125\ in$ and a Curve B material the following value for MDMT is determined.

$$MDMT_{curve\ B} = 59°F$$

Therefore, the nozzle assembly minimum design metal temperature is determined as follows.

$$MDMT_{assembly} = Warmest\left[MDMT_{curve\ D}, MDMT_{curve\ B}\right]$$
$$MDMT_{assembly} = Warmest\left[-7,\ 59\right]$$
$$MDMT_{assembly} = 59°F$$

Applying paragraph UCS-66(b): when the coincident ratio defined in Fig UCS-66.1 is less than one, Fig UCS-66.1 provides a basis for the use of components made of Part UCS material to have a colder MDMT than that derived from paragraph UCS-66(a) without impact testing.

e) STEP 5 – Based on the design loading conditions at the MDMT, determine the ratio, R_{ts}, using the thickness basis from Fig UCS-66.2.

Commentary: VIII-1 does not provide explicit guidance as to which component of a welded assembly shall R_{ts} be based upon. This example provides one possible method of satisfying the requirement and is consistent with ASME Interpretation VIII-1-01-37.

For a welded assembly, the value of R_{ts} is calculated based upon the assembly's component with the governing thickness. In this example the governing thickness of the assembly was based on the cylindrical shell.

$$R_{ts} = \frac{t_r E^*}{t_n - CA}$$

Where, t_r is the required thickness of the cylindrical shell at the specified MDMT of $-20°F$, using paragraph UG-27(c)(1).

$$t_r = \frac{PR}{SE-0.6P} = \frac{356(75.125)}{(20000(1.0)-0.6(356))} = 1.3517$$

$$where,$$

$$R = \frac{D}{2} + Corrosion\ \ Allowance = \frac{150.0}{2} + 0.125 = 75.125\ in$$

The variables E^*, t_n, and CA are defined as follows:

$$E^* = \max[E,\ 0.80] = \max[1.0, 0.8] = 1.0 \qquad \to Fig.\ UCS-66.2,\ Note\ 3$$

$$t_n = 1.8125\ in$$

$$CA = 0.125\ in$$

Therefore,

$$R_{ts} = \frac{t_r E*}{t_n - CA} = \frac{1.3517(1.0)}{1.8125-0.125} = 0.8010$$

f) STEP 6 – Interpreting the value of the temperature reduction, T_R from Fig. UCS-66.1 is performed as follows. Enter the figure along the ordinate with a value of $R_{ts} = 0.8010$, project horizontally until an intersection with the provided curve is achieved. Project this point downward to the abscissa and interpret T_R. This results in an approximate value of $T_R = 20°F$

g) STEP 7 – The final adjusted MDMT of the assembly is determined as follows.

$$MDMT_{assembly} = MDMT_{STEP4} - T_R = 59°F - 20°F = 39°F$$

Since the final adjusted MDMT of the assembly is warmer than the proposed MDMT, impact testing of the nozzle forging is required.

An MDMT colder than the determined in this example would be possible if the nozzle forging were fabricated from a material specification that includes the provisions of impact testing, such as SA-350. See UCS-66(g) and General Note (c) of Fig. UG-84.

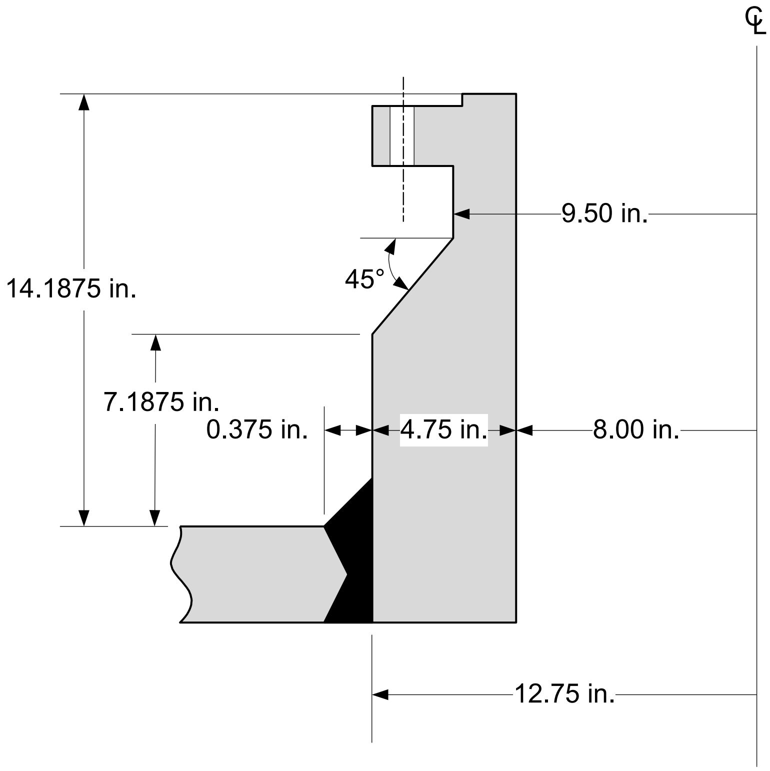


Figure E3.3.1 – Nozzle-Shell Detail

PART 4

DESIGN BY RULE REQUIREMENTS

4.1 General Requirements

4.1.1 Example E4.1.1 – Review of General Requirements for a Vessel Design

a) General Requirements

An engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 1 (VIII-1). With the adoption of ASME Code Case 2695, which permits the use of ASME B&PV Code, Section VIII, Division 2 (VIII-2) Part 4 design rules to be used for VIII-1 pressure vessels, the VIII-2 Code is being considered in an attempt to take advantage of the updated design rules. The vessel in question is to be constructed of carbon steel with a specified corrosion allowance and a design pressure and temperature of 1650 psig at 300°F. As part of developing the design specification, the following items need to be evaluated.

b) Introduction

The scope of VIII-1 has been established to identify the components and parameters considered in formulating the rules given in VIII-1 as presented in U-1(a) through U-1(j). The user of the vessel shall establish the design requirements for pressure vessels, taking into consideration factors associated with normal operation, startup and shutdown, and abnormal conditions which may become a governing design consideration in accordance with U-2(a).

1) The design temperature shall be established in accordance with UG-20.
2) The design pressure shall be established in accordance with UG-21.
3) The design loads shall be established in accordance with UG-22.

c) Material Requirements

General material requirements as well as specific requirements based on product form and process service shall be in accordance with UG-4 through UG-15.

d) Minimum Thickness Requirements

Based on product form and process service, the parts of the vessel must meet the minimum thickness requirements presented in UG-16.

e) Corrosion Allowance in Design Equations

The equations used in the design-by-rule procedures of VIII-1 are performed in a corroded condition. The term corrosion allowance is representative of loss of metal due to corrosion, erosion, mechanical abrasion, or other environmental effects, see UG-25.

f) Design Basis

1) The pressure used in the design of a vessel component together with the coincident design metal temperature must be specified. Where applicable, the pressure resulting from static head shall be included in addition to the specified design pressure, see UG-21.
2) The specified design temperature shall not be less than the mean metal temperature expected coincidentally with the corresponding maximum pressure, see UG-20.
3) A minimum design metal temperature shall be determined and shall consider the coldest

operating temperature, operational upsets, auto refrigeration, atmospheric temperature, and any other source of cooling.

4) All applicable loads shall be considered in the design to determine the minimum required wall thickness for a vessel part, see UG-22.

g) Design Allowable Stress

Specifications for all materials of construction and allowable design stresses are determined in accordance with UG-23.

4.1.2 Example E4.1.2 – Required Wall Thickness of a Hemispherical Head

Determine the required thickness for a hemispherical head at the bottom of a vertical vessel considering the following design conditions. All Category A joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $1650\ psig@300°F$
- Liquid Head = $60\ ft$
- Liquid Specific Gravity = 0.89
- Inside Diameter = $96.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0

The design pressure used to establish the wall thickness must be adjusted for the liquid head in accordance with paragraph UG-21.

$$Design\ Pressure = Specified\ Design\ Pressure + \gamma gh$$

$$Design\ Pressure = 1650 + \frac{0.89(62.4)(60)}{144} = 1673.140\ psig$$

Section VIII, Division 1 Solution

In accordance with UG-32(f), determine the required thickness of the bottom hemispherical head.

$$t = \frac{PL}{2SE - 0.2P}$$

$$L = \frac{96.0 + 2(Corrosion\ Allowance)}{2} = \frac{96.0 + 2(0.125)}{2} = 48.125\ in$$

$$t = \frac{1673.14(48.125)}{2(20000)(1.0) - 0.2(1673.14)} = 2.03\ in$$

$$t = 2.03 + Corrosion\ Allowance = 2.03 + 0.125 = 2.155\ in$$

The required thickness of the bottom head is $2.155\ in$

Section VIII, Division 2 Solution Using VIII-1 Allowable Stresses

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the bottom hemispherical head. Similarly, the design pressure used to establish the wall thickness must be adjusted for the liquid head in accordance with Part 4, paragraph 4.1.5.2.a as shown above.

$$t=\frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right]-1\right)$$

$$D=96.0+2\left(Corrosion\ \ Allowance\right)=96.0+2\left(0.125\right)=96.25\ in$$

$$t=\frac{96.25}{2}\left(\exp\left[\frac{0.5\left(1673.14\right)}{20000\left(1.0\right)}\right]-1\right)=2.0557\ in$$

$$t=1.8313+Corrosion\ \ Allowance=2.0557+0.125=2.1807\ in$$

The required thickness of the bottom head is $2.1807\ in$.

4.2 Welded Joints

4.2.1 Example E4.2.1 – Nondestructive Examination Requirement for Vessel Design

An engineer is tasked with preparing the design specification for a new pressure vessel that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 1 (VIII-1). Based on the process service description, anticipated design data, materials of construction, and welding process, the engineer verifies that full radiography is not required in accordance with paragraph UW-11(a) and spot radiography in accordance with paragraph UW-11(b) would be adequate. However, the savings in cost for reduced examination may be offset by the increase in materials and fabrication costs. The designer conducts a comparison for a cylindrical shell to aid in the decision for NDE requirements.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $725\ psig\ @300°F$
- Inside Diameter = $60.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$

Section VIII, Division 1 Solution

For Full RT Examination, consider the requirements for a Category A Type 1 weld in a cylindrical shell. The required wall thickness in accordance with UG-27(c)(1) is computed as shown below.

$$t=\frac{PR}{SE-0.6P}$$

$$R=\frac{60.0+2(Corrosion\ \ Allowance)}{2}=\frac{60.0+2(0.125)}{2}=30.125\ in$$

$$t=\frac{725(30.125)}{20000(1.0)-0.6(725)}=1.1163\ in$$

$$t=1.1163+Corrosion\ \ Allowance=1.1163+0.125=1.2413\ in$$

Alternatively, for Spot RT Examination, the required wall thickness for a Category A Type 1 weld in accordance with UG-27(c)(1) is computed as shown below.

$$t=\frac{PR}{SE-0.6P}$$

$$R=\frac{60.0+2(Corrosion\ \ Allowance)}{2}=\frac{60.0+2(0.125)}{2}=30.125\ in$$

$$t=\frac{725(30.125)}{20000(0.85)-0.6(725)}=1.3185\ in$$

$$t=1.3185+Corrosion\ \ Allowance=1.3185+0.125=1.4435\ in$$

Full RT Examination when compared to Spot RT Examination results in an approximate 14% reduction in wall thickness. Cost savings for this reduction in wall thickness will include less material and less welding, and these reductions may offset the increased examination costs. Other potential savings may include reduced shipping and reduced foundation costs.

Although the process service description, anticipated design data, materials of construction, and welding process, did not require that full radiography be performed, it should be noted that because the calculation using Spot RT Examination produces a required thickness of $1.4435\ in$, Full RT Examination would become mandatory per Table UCS-57.

Commentary – This example is intended to look only at one specific Category A weld seam and provide a comparison of the differences in vessel wall thickness and examination requirements. Other examination options are available (see paragraph UW-11(a)(5)(b)) but require examination of intersecting category B weld seams, which is outside the scope of this example problem.

Section VIII, Division 2 Solution Using VIII-1 Allowable Stresses

For Full RT Examination, consider the requirements for a Category A Type 1 weld in a cylindrical shell. The required wall thickness in accordance with paragraph 4.3.3 is computed as shown below.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right] - 1\right)$$

$$D = 60.0 + 2(Corrosion\ \ Allowance) = 60.0 + 2(0.125) = 60.25\ in$$

$$t = \frac{60.25}{2}\left(\exp\left[\frac{725}{20000(1.0)}\right] - 1\right) = 1.1121\ in$$

$$t = 1.1121 + Corrosion\ \ Allowance = 1.1121 + 0.125 = 1.2371\ in$$

Alternatively, for Spot RT Examination, the required wall thickness for a Category A Type 1 weld in accordance with paragraph 4.3.3 is computed as shown below.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right] - 1\right)$$

$$D = 60.0 + 2(Corrosion\ \ Allowance) = 60.0 + 2(0.125) = 60.25\ in$$

$$t = \frac{60.25}{2}\left(\exp\left[\frac{725}{20000(0.85)}\right] - 1\right) = 1.3125\ in$$

$$t = 1.3125 + Corrosion\ \ Allowance = 1.3125 + 0.125 = 1.4375\ in$$

Similarly, Full RT Examination when compared to Spot RT Examination results in an approximate 14% reduction in wall thickness.

4.2.2 Example E4.2.2 – Nozzle Detail and Weld Sizing

Determine the required fillet weld size and inside corner radius of a set-in type nozzle as shown in Figure UW-16.1(d), (Figure E4.2.2). The vessel and nozzle were designed such that their nominal thicknesses were established as follows.

Vessel Data:

- Cylinder Thickness = $0.625\ inches$
- Nozzle Diameter = $NPS\ 10$
- Nozzle Thickness = $Schedule\ XS \rightarrow 0.500\ inches$
- Corrosion Allowance = $0.125\ inches$

Adjust variables for corrosion.

$$t_s = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$
$$t_n = 0.500 - Corrosion\ \ Allowance = 0.500 - 0.125 = 0.375\ in$$

Section VIII, Division 1 Solution

The minimum fillet weld throat dimension, t_c, is calculated as follows.

$$t_{\min} = \min\left[t_n,\ t_s,\ 19\,mm\ (0.75\ in)\ \right] = \min\left[0.375,\ 0.500,\ 19\,mm\ (0.75\ in)\ \right] = 0.375\ in$$
$$t_c \geq \min\left[0.7t_{\min},\ 6\,mm\ (0.25\ in)\right]$$
$$t_c \geq \min\left[0.70(0.375), 0.25\right]$$
$$t_c \geq 0.25\ in$$

The resulting fillet weld leg size is determined as, $\dfrac{t_c}{0.7} = 0.357\ in$. Therefore, a fillet weld leg size of $0.375\ in$ would be acceptable.

Note: VIII-1 does not provide acceptance criterion for the inside corner radius of the exposed nozzle edge, see Fig. UW-16.1(d), (Figure E4.2.2). However, paragraphs UG-76(b) and (c) reference; end of nozzles or manhole necks which are to remain unwelded in the completed vessel may be cut by methods that produce a smooth finish, and exposed inside edges shall be chamfered or rounded.

Section VIII, Division 2 Solution

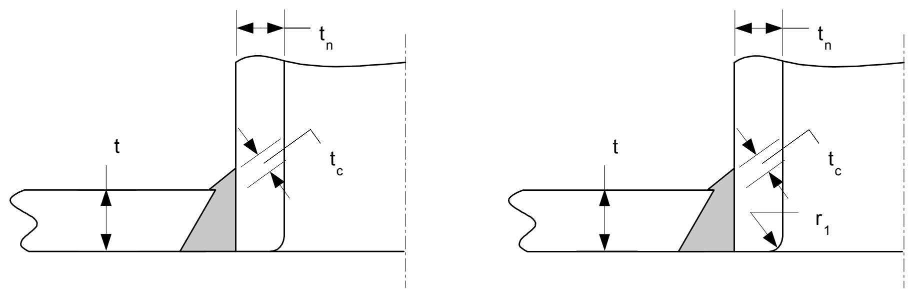
The reference sketch per VIII-2 is found in Table 4.2.10, Detail 4, (Figure E4.2.2). The minimum fillet weld throat dimension, t_c, is calculated as follows.

$$t_c \geq \min\left[0.7t_n,\ 6\,mm\ (0.25\ in)\right]$$
$$t_n = 0.500 - Corrosion\ \ Allowance = 0.5 - 0.125 = 0.375\ in$$
$$t_c \geq \min\left[0.7(0.375), 0.25\right]$$
$$t_c \geq 0.25\ in$$

The resulting fillet weld leg size is determined as, $\dfrac{t_c}{0.7} = 0.357\ in$. Therefore, a fillet weld leg size of $0.375\ in$ would be acceptable.

The minimum inside corner radius, r_1, is calculated as follows.

$$0.125t \le r_1 \le 0.5t$$
$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$
$$0.125(0.500) \le r_1 \le 0.5(0.500)$$
$$0.0625 \le r_1 \le 0.250\ in$$



VIII-1, Fig. UW-16.1(d) VIII-2, Table 4.2.10, Detail 4

Figure E4.2.2 – Weld Details

4.2.3 Example E4.2.3 – Nozzle Detail with Reinforcement Pad and Weld Sizing

Determine the required fillet weld sizes and inside corner radius of a set-in type nozzle with added reinforcement pad as shown in Figure UW-16.1(q), (Figure E4.2.3). The vessel and nozzle were designed such that their nominal thicknesses were established as follows.

Vessel Data:

- Cylinder Thickness = $0.625\ inches$
- Nozzle Diameter = $NPS\ 10$
- Nozzle Thickness = $Schedule\ \ XS \rightarrow 0.500\ inches$
- Reinforcement Pad Thickness = $0.625\ inches$
- Corrosion Allowance = $0.125\ inches$

Adjust variables for corrosion.

$$t_s = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$
$$t_n = 0.500 - Corrosion\ \ Allowance = 0.500 - 0.125 = 0.375\ in$$
$$t_e = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.0 = 0.625\ in$$

Note: The corrosion allowance specified is for internal corrosion, not external corrosion. Therefore, the reinforcement pad corrosion allowance is set equal to zero.

Section VIII, Division 1 Solution

The minimum fillet weld throat dimension, t_c, is calculated as follows.

$$t_{\min} = \min\left[t_n,\, t_e,\, 19\,mm\ (0.75\ in)\ \right] = \min\left[0.375,\ 0.625,\ 19\,mm\ (0.75\ in)\ \right] = 0.375\ in$$
$$t_c \geq \min\left[0.7t_{\min},\ 6\,mm\ (0.25\ in)\right]$$
$$t_c \geq \min\left[0.7(0.375), 0.25\right]$$
$$t_c \geq 0.25\ in$$

The resulting fillet weld leg size is determined as, $\dfrac{t_c}{0.7} = 0.357\ in$. Therefore, a fillet weld leg size of $0.375\ in$ would be acceptable.

The minimum fillet weld throat at the outer edge of the reinforcement padis calculated as follows.

$$throat = 0.5t_{\min} = 0.5\min[t_s,\ t_e,\ 19\,mm\left(0.75\ in\right)]$$
$$throat = 0.5\min[0.500,\ 0.625,\ 19\,mm\left(0.75\ in\right)]$$
$$throat = 0.25\ in$$

The resulting fillet weld leg size is determined as, $\dfrac{throat}{0.7} = 0.357\ in$. Therefore, a fillet weld leg size of $0.375\ in$ would be acceptable.

The minimum partial penetration weld for the reinforcement pad, t_w is calculated as follows.

$$t_w = 0.7t_{\min} = 0.7\min[t_n,\ t_e,\ 19\,mm\left(0.75\ in\right)]$$
$$t_w = 0.7\min[0.375,\ 0.625,\ 19\,mm\left(0.75\ in\right)]$$
$$t_w = 0.2625\ in$$

The minimum partial penetration weld for the vessel, t_w is calculated as follows.

$$t_w = 0.7t_{\min} = 0.7\min[t_s,\ t_n,\ 19\,mm\left(0.75\ in\right)]$$
$$t_w = 0.7\min[0.500,\ 0.375,\ 19\,mm\left(0.75\ in\right)]$$
$$t_w = 0.2625\ in$$

Note: VIII-1 does not provide acceptance criterion for the inside corner radius of the exposed nozzle edge, see Fig. UW-16.1(q), (Figure E4.2.2). However, paragraphs UG-76(b) and (c) reference; end of nozzles or manhole necks which are to remain unwelded in the completed vessel may be cut by methods that produce a smooth finish, and exposed inside edges shall be chamfered or rounded.

Section VIII, Division 2 Solution

The reference sketch per VIII-2 is found in Table 4.2.11, Detail 2. The minimum fillet weld throat dimension, t_c, is calculated as follows.

$$t_c \geq \min\left[0.7t_n, 6\,mm\ (0.25\ in)\right]$$
$$t_n = 0.500 - Corrosion\ \ Allowance = 0.500 - 0.125 = 0.375\ in$$
$$t_c \geq \min\left[0.7(0.375), 0.25\right]$$
$$t_c \geq 0.25\ in$$

The resulting fillet weld leg size is determined as, $\dfrac{t_c}{0.7} = 0.357\ in$. Therefore, a fillet weld leg size of $0.375\ in$ would be acceptable.

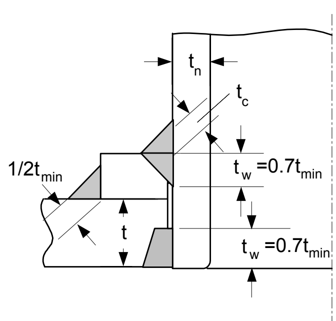
The minimum fillet weld throat dimension, t_{f1}, is calculated as follows.

$$t_{f1} \geq \min\left[0.6t_e,\ 0.6t\right]$$
$$t_e = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.0 = 0.625\ in$$
$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$
$$t_{f1} \geq \min\left[0.6(0.625), 0.6(0.500)\right]$$
$$t_{f1} \geq 0.300\ in$$

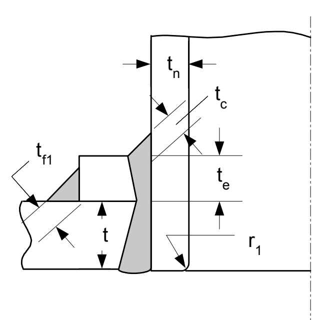
The resulting fillet weld leg size is determined as, $\dfrac{t_{f1}}{0.7} = 0.429\ in$. Therefore, a fillet weld leg size of $0.4375\ in$ would be acceptable.

The minimum inside corner radius, r_1, is calculated as follows.

$$0.125t \leq r_1 \leq 0.5t$$
$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$
$$0.125(0.500) \leq r_1 \leq 0.5(0.500)$$
$$0.0625 \leq r_1 \leq 0.250\ in$$



VIII-1, Fig. UW-16.1(q)



VIII-2, Table 4.2.11, Detail 2

Figure E4.2.3 – Weld Details

4.3 Internal Design Pressure

4.3.1 Example E4.3.1 – Cylindrical Shell

Determine the required thickness for a cylindrical shell considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psig@300°F$
- Inside Diameter = $90.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0

Determine the inside radius and adjust for corrosion allowance.

$$D = 90.0 + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 90.25$$

$$R = \frac{D}{2} = \frac{90.25\ in}{2} = 45.125\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-27(c) – The minimum thickness or maximum allowable working pressure of cylindrical shells shall be the greater thickness or lesser pressure as given by the following.

UG-27(c)(1), Circumferential Stress.

$$t_c = \frac{PR}{SE-0.6P} = \frac{356(45.125)}{20000(1.0)-0.6(356)} = 0.8119\ in$$

$$t_c = 0.8119 + Corrosion\ \ Allowance = 0.8119 + 0.125 = 0.9369\ in$$

UG-27(c)(2), Longitudinal Stress.

$$t_l = \frac{PR}{2SE+0.4P} = \frac{356(45.125)}{2(20000)(1.0)+0.4(356)} = 0.4002\ in$$

$$t_l = 0.4002 + Corrosion\ \ Allowance = 0.4002 + 0.125 = 0.5252\ in$$

Therefore, the required thickness is determined as follows.

$$t = \max[t_c,\ t_l] = \max[0.9369,\ 0.5252] = 0.9369\ in$$

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.3.3.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{90.25}{2}\left(\exp\left[\frac{356}{20000(1.0)}\right]-1\right) = 0.8104\ in$$

$$t = 0.8104 + Corrosion\ \ Allowance = 0.8104 + 0.125 = 0.9354\ in$$

The required thickness is $0.9354\ in$

4.3.2 Example E4.3.2 – Conical Shell

Determine the required thickness for a conical shell considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psig@300°F$
- Inside Diameter (Large End) = $150.0\ in$
- Inside Diameter (Small End) = $90.0\ in$
- Length of Conical Section = $78.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0

Adjust for corrosion allowance and determine the cone angle.

$$D_L = 150.0 + 2(Corrosion\ \ Allowance) = 150.0 + 2(0.125) = 150.25\ in$$

$$D_s = 90.0 + 2(Corrosion\ \ Allowance) = 150.0 + 2(0.125) = 90.25\ in$$

$$L_C = 78.0$$

$$\alpha = \arctan\left[\frac{0.5(D_L - D_S)}{L_C}\right] = \arctan\left[\frac{0.5(150.25-90.25)}{78.0}\right] = 21.0375\ deg$$

<u>Section VIII, Division 1 Solution</u>

Evaluate per UG-32(g) using the large end diameter of the conical shell.

$$t = \frac{PD}{2\cos[\alpha](SE-0.6P)} = \frac{356(150.25)}{2\cos[21.0375]\left(20000(1.0)-0.6(356)\right)} = 1.4482\ in$$

$$t = 1.4482 + Corrosion\ \ Allowance = 1.4482 + 0.125 = 1.5732\ in$$

The required thickness is $1.5732\ in$

<u>Section VIII, Division 2 Solution with VIII-1 Allowable Stresses</u>

Evaluate per VIII-2, paragraph 4.3.4 using the large end diameter of the conical shell.

$$t = \frac{D}{2\cos[\alpha]}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{150.25}{2\cos[21.0375]}\left(\exp\left[\frac{356}{20000(1.0)}\right]-1\right) = 1.4455\ in$$

$$t = 1.4455 + Corrosion\ \ Allowance = 1.4455 + 0.125 = 1.5705\ in$$

The required thickness is $1.5705\ in$

4.3.3 Example E4.3.3 – Spherical Shell

Determine the required thickness for a spherical shell considering the following design conditions. All Category A joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-542,\ Type\ D,\ Class\ 4a$
- Design Conditions = $2080\ psig @ 850°F$
- Inside Diameter = $149.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $21000\ psi$
- Weld Joint Efficiency = 1.0

Section VIII, Division 1 Solution

Evaluate per UG-32(f).

$$L = \frac{D}{2} = \frac{149}{2} = 74.5\ in$$

$$t = \frac{PL}{2SE-0.2P} = \frac{2080(74.5)}{2(21000)(1.0)-0.2(2080)} = 3.7264\ in$$

The required thickness is $3.7264\ in$

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.3.5.

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right]-1\right) = \frac{149.0}{2}\left(\exp\left[\frac{0.5(2080)}{21000(1.0)}\right]-1\right) = 3.7824\ in$$

The required thickness is $3.7824\ in$

4.3.4 Example E4.3.4 – Torispherical Head

Determine the maximum allowable working pressure (MAWP) for the proposed seamless torispherical head. The Category B joint joining the head to the shell is a Type 1 butt weld and has been 100% radiographically examined.

Vessel Data:

- Material = $SA-387,\ Grade\ 11,\ Class\ 1$
- Design Temperature = $650°F$
- Inside Diameter = $72.0\ in$
- Crown Radius = $72.0\ in$
- Knuckle Radius = $4.375\ in$
- Thickness = $0.625\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $17100\ psi$
- Weld Joint Efficiency = 1.0
- Modulus of Elasticity at Design Temperature = $26.55E+06\ psi$
- Yield Strength at Design Temperature = $26900\ psi$

Adjust for corrosion allowance

$$D = 72.0 + 2(Corrosion\ \ Allowance) = 72.0 + 2(0.125) = 72.25\ in$$
$$L = 72.0 + Corrosion\ \ Allowance = 72.0 + 0.125 = 72.125\ in$$
$$r = 4.375 + Corrosion\ \ Allowance = 4.375 + 0.125 = 4.5\ in$$
$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.5\ in$$

Section VIII, Division 1 Solution

Evaluate per Mandatory Appendix 1-4(d). Note, the rules of UG-32(e) can also be used to evaluate torispherical heads. However, the rules contained in this paragraph are only applicable for a specific geometry, i.e. the knuckle radius is 6% of the inside crown radius, and the inside crown radius equals the outside diameter of the skirt. Additionally, if the ratio $t_s / L \geq 0.002$, is not satisfied, the rules of Mandatory Appendix 1-4(f) shall also be met.

$$M = 0.25\left(3 + \sqrt{\frac{L}{r}}\right) = 0.25\left(3 + \sqrt{\frac{72.125}{4.5}}\right) = 1.7509$$

$$P = \frac{2SEt}{LM + 0.2t} = \frac{2(17100)(1.0)(0.5)}{72.125(1.7509) + 0.2(0.5)} = 135.3023\ psi$$

$$Note: \left\{\frac{t}{L} = \frac{0.5}{72.125} = 0.0069\right\} > 0.002,\quad therefore\ the\ rules\ of\ 1\text{-}4(f)\ are\ not\ required$$

The MAWP is 135 psi.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.3.5.

a) STEP 1 – Determine, D, assume values for $L,\ r$ and t (known variables from above).

$$D = 72.25\ in$$
$$L = 72.125\ in$$
$$r = 4.5\ in$$
$$t = 0.5\ in$$

b) STEP 2 – Compute the head L/D, r/D, and L/t ratios and determine if the following equations are satisfied.

$$0.7 \le \left\{ \frac{L}{D} = \frac{72.125}{72.25} = 0.9983 \right\} \le 1.0 \qquad True$$

$$\left\{ \frac{r}{D} = \frac{4.5}{72.25} = 0.0623 \right\} \ge 0.06 \qquad True$$

$$20 \le \left\{ \frac{L}{t} = \frac{72.125}{0.5} = 144.25 \right\} \le 2000 \qquad True$$

c) STEP 3 – Calculate the geometric constants β_{th}, ϕ_{th}, R_{th}

$$\beta_{th} = \arccos\left[\frac{0.5D - r}{L - r}\right] = \arccos\left[\frac{0.5(72.25) - 4.5}{72.125 - 4.5}\right] = 1.0842\ rad$$

$$\phi_{th} = \frac{\sqrt{Lt}}{r} = \frac{\sqrt{72.125(0.5)}}{4.5} = 1.3345\ rad$$

Since $\phi_{th} \ge \beta_{th}$, calculate R_{th} as follows:

$$R_{th} = 0.5D = 0.5(72.25) = 36.125\ in$$

d) STEP 4 – Compute the coefficients C_1 and C_2

Since $\frac{r}{D} = 0.0623 \le 0.08$, calculate C_1 and C_2 as follows:

$$C_1 = 9.31\left(\frac{r}{D}\right) - 0.086 = 9.31(0.0623) - 0.086 = 0.4940$$
$$C_2 = 1.25$$

e) STEP 5 – Calculate the value of internal pressure expected to produce elastic buckling of the knuckle, P_{eth}.

$$P_{eth} = \frac{C_1 E_T t^2}{C_2 R_{th}\left(\frac{R_{th}}{2} - r\right)} = \frac{(0.4940)(26.55E+06)(0.5)^2}{1.25(36.125)\left(\frac{36.125}{2} - 4.5\right)} = 5353.9445\ psi$$

f) STEP 6 – Calculate the value of internal pressure that will result in a maximum stress in the knuckle equal to the material yield strength, P_y.

$$P_y = \frac{C_3 t}{C_2 R_{th}\left(\frac{R_{th}}{2r} - 1\right)}$$

Since the allowable stress at design temperature is governed by time-independent properties, is C_3 the material yield strength at the design temperature, or $C_3 = S_y$.

$$P_y = \frac{26900(0.5)}{1.25(36.125)\left(\frac{36.125}{2(4.5)} - 1\right)} = 98.8274 \ psi$$

g) STEP 7 – Calculate the value of internal pressure expected to result in a buckling failure of the knuckle, P_{ck}.

Calculate variable G:

$$G = \frac{P_{eth}}{P_y} = \frac{5353.9445}{98.8274} = 54.1747$$

Since $G > 1.0$, calculate P_{ck} as follows:

$$P_{ck} = \left(\frac{0.77508G - 0.20354G^2 + 0.019274G^3}{1 + 0.19014G - 0.089534G^2 + 0.0093965G^3}\right)(P_y)$$

$$P_{ck} = \left(\frac{0.77508(54.1747) - 0.20354(54.1747)^2 + 0.019274(54.1747)^3}{1 + 0.19014(54.1747) - 0.089534(54.1747)^2 + 0.0093965(54.1747)^3}\right)(98.8274)$$

$$P_{ck} = 199.5671 \, psi$$

h) STEP 8 – Calculate the allowable pressure based on a buckling failure of the knuckle, P_{ak}.

$$P_{ak} = \frac{P_{ck}}{1.5} = \frac{199.5671}{1.5} = 133.0447 \ psi$$

i) STEP 9 – Calculate the allowable pressure based on rupture of the crown, P_{ac}.

$$P_{ac} = \frac{2SE}{\frac{L}{t} + 0.5} = \frac{2(17100)(1.0)}{\frac{72.125}{0.5} + 0.5} = 236.2694 \ psi$$

j) STEP 10 – Calculate the maximum allowable internal pressure, P_a.

$$P_a = \min\left[P_{ak}, \, P_{ac}\right] = \min\left[133.0447, \, 236.2694\right] = 133.0 \ psi$$

k) STEP 11 – If the allowable internal pressure computed from STEP 10 is greater than or equal to the design pressure, then the design is complete. If the allowable internal pressure computed from STEP 10 is less than the design pressure, then increase the head thickness and repeat steps 2 through 10.

The MAWP is $133.0\ psi$

4.3.5 Example E4.3.5 – Elliptical Head

Determine the maximum allowable working pressure (MAWP) for the proposed seamless 2:1 elliptical head. The Category B joint joining the head to the shell is a Type 1 butt weld and has been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Temperature = $300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength at Design Temperature = $33600\ psi$

Determine the elliptical head diameter to height ratio, k, and adjust for corrosion allowance.

$$k = \frac{D}{2h} = \frac{90.0}{2(22.5)} = 2.0$$

$$D = 90.0 + 2(Corrosion\ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$L = 81.0 + Corrosion\ Allowance = 81.0 + 0.125 = 81.125\ in$$

$$r = 15.3 + Corrosion\ Allowance = 15.3 + 0.125 = 15.425\ in$$

$$t = 1.125 - Corrosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$

Section VIII, Division 1 Solution

Evaluate per Mandatory Appendix 1-4(c). Note, the rules of UG-32(d) can also be used to evaluate ellipsoidal heads. However, the rules contained in this paragraph are only applicable for a specific geometry, i.e. half the minor axis (inside depth of head minus the skirt) equals one–fourth of the inside diameter of the head skirt. Additionally, if the ratio $t_s / L \geq 0.002$, is not satisfied, the rules of Mandatory Appendix 1-4(f) shall also be met.

$$K = \frac{1}{6}\left(2+\left[\frac{D}{2h}\right]^2\right) = \frac{1}{6}\left(2+\left[\frac{90.0}{2(22.5)}\right]^2\right) = 1.0,\ Note: Base\ on\ uncorroded\ dimensions$$

$$P = \frac{2SEt}{KD+0.2t} = \frac{2(20000)(1.0)(1.0)}{1.0(90.25)+0.2(1.0)} = 442.2333\ psi$$

$$Note:\ \left\{\frac{t}{L} = \frac{1.0}{81.125} = 0.0123\right\} > 0.002,\ therefore\ the\ rules\ of\ 1\text{-}4(f)\ are\ not\ required$$

The MAWP is 442 psi.

<u>Section VIII, Division 2 Solution with VIII-1 Allowable Stresses</u>

Evaluate per VIII-2, paragraph 4.3.7 and paragraph 4.3.6.

Verify that the elliptical head diameter to height ratio, k, is within the established limits, permitting the use of the rules of VIII-2, paragraph 4.3.7.

$$1.7 \le \{k = 2\} \le 2.2 \qquad True$$

Determine the variables r and L using the uncorroded inside diameter, D.

$$r = D\left(\frac{0.5}{k} - 0.08\right) = 90.0\left(\frac{0.5}{2.0} - 0.08\right) = 15.3\ in$$

$$L = D(0.44k + 0.02) = 90.0(0.44(2.0) + 0.02) = 81.0\ in$$

Proceed with the design following the steps outlined in VIII-2, paragraph 4.3.6.

a) STEP 1 – Determine, D, assume values for L, r and t. (determined from paragraph 4.3.7)

$$D = 90.25\ in$$
$$L = 81.125\ in$$
$$r = 15.425\ in$$
$$t = 1.0\ in$$

b) STEP 2 – Compute the head L/D, r/D, and L/t ratios and determine if the following equations are satisfied.

$$0.7 \le \left\{\frac{L}{D} = \frac{81.125}{90.25} = 0.8989\right\} \le 1.0 \qquad True$$

$$\left\{\frac{r}{D} = \frac{15.425}{90.25} = 0.1709\right\} \ge 0.06 \qquad True$$

$$20 \le \left\{\frac{L}{t} = \frac{81.125}{1.000} = 81.125\right\} \le 2000 \qquad True$$

c) STEP 3 – Calculate the geometric constants β_{th}, ϕ_{th}, R_{th}

$$\beta_{th} = \arccos\left[\frac{0.5D - r}{L - r}\right] = \arccos\left[\frac{0.5(90.25) - 15.425}{81.125 - 15.425}\right] = 1.1017\ rad$$

$$\phi_{th} = \frac{\sqrt{Lt}}{r} = \frac{\sqrt{81.125(1.0)}}{15.425} = 0.5839\ rad$$

Since $\phi_{th} < \beta_{th}$, calculate R_{th} as follows:

$$R_{th} = \frac{0.5D - r}{\cos\left[\beta_{th} - \phi_{th}\right]} + r = \frac{0.5(90.25) - 15.425}{\cos\left[1.1017 - 0.5839\right]} + 15.425 = 49.6057\ in$$

d) STEP 4 – Compute the coefficients C_1 and C_2

Since $\frac{r}{D} = 0.1709 > 0.08$, calculate C_1 and C_2 as follows:

$$C_1 = 0.692\left(\frac{r}{D}\right) + 0.605 = 0.692(0.1709) + 0.605 = 0.7233$$

$$C_2 = 1.46 - 2.6\left(\frac{r}{D}\right) = 1.46 - 2.6(0.1709) = 1.0157$$

e) STEP 5 – Calculate the value of internal pressure expected to produce elastic buckling of the knuckle, P_{eth}.

$$P_{eth} = \frac{C_1 E_T t^2}{C_2 R_{th}\left(\frac{R_{th}}{2} - r\right)} = \frac{(0.7233)(28.3E+06)(1.0)^2}{1.0157(49.6057)\left(\frac{49.6057}{2} - 15.425\right)} = 43321.6096\ psi$$

f) STEP 6 – Calculate the value of internal pressure that will result in a maximum stress in the knuckle equal to the material yield strength, P_y.

$$P_y = \frac{C_3 t}{C_2 R_{th}\left(\frac{R_{th}}{2r} - 1\right)}$$

Since the allowable stress at design temperature is governed by time-independent properties, is C_3 the material yield strength at the design temperature, or $C_3 = S_y$.

$$P_y = \frac{33600(1.0)}{1.0157(49.6057)\left(\frac{49.6057}{2(15.425)} - 1\right)} = 1096.8927\ psi$$

g) STEP 7 – Calculate the value of internal pressure expected to result in a buckling failure of the knuckle, P_{ck}.

Calculate variable G:

$$G = \frac{P_{eth}}{P_y} = \frac{43321.6096}{1096.8927} = 39.4948$$

Since $G > 1.0$, calculate P_{ck} as follows:

$$P_{ck} = \left(\frac{0.77508G - 0.20354G^2 + 0.019274G^3}{1 + 0.19014G - 0.089534G^2 + 0.0093965G^3} \right) (P_y)$$

$$P_{ck} = \left(\frac{0.77508(39.4948) - 0.20354(39.4948)^2 + 0.019274(39.4948)^3}{1 + 0.19014(39.4948) - 0.089534(39.4948)^2 + 0.0093965(39.4948)^3} \right) (1096.8927)$$

$$P_{ck} = 2206.1634\ psi$$

h) STEP 8 – Calculate the allowable pressure based on a buckling failure of the knuckle, P_{ak}.

$$P_{ak} = \frac{P_{ck}}{1.5} = \frac{2206.1634}{1.5} = 1470.8\ psi$$

i) STEP 9 – Calculate the allowable pressure based on rupture of the crown, P_{ac}.

$$P_{ac} = \frac{2SE}{\frac{L}{t} + 0.5} = \frac{2(20000)(1.0)}{\frac{81.125}{1.0} + 0.5} = 490.0459\ psi$$

j) STEP 10 – Calculate the maximum allowable internal pressure, P_a.

$$P_a = \min[P_{ak}, P_{ac}] = \min[1470.8, 490.0459] = 490.0\ psi$$

k) STEP 11 – If the allowable internal pressure computed from STEP 10 is greater than or equal to the design pressure, then the design is complete. If the allowable internal pressure computed from STEP 10 is less than the design pressure, then increase the head thickness and repeat STEPs 2 through 10.

The MAWP is $490\ psi$

4.3.6 Example E4.3.6 – Combined Loadings and Allowable Stresses

Determine the maximum tensile stress of the proposed cylindrical shell section considering the following design conditions and specified applied loadings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psig @ 300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$

- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0
- Axial Force = $-66152.5\ lbs$
- Net Section Bending Moment = $3.048E+06\ in-lbs$
- Torsional Moment = $0.0\ in-lbs$

Adjust variables for corrosion and determine outside dimensions.

$$D = 90.0 + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$R = \frac{D}{2} = 45.125\ in$$

$$t = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

$$D_o = 90.0 + 2(Uncorroded\ \ Thickness) = 90.0 + 2(1.125) = 92.25\ in$$

Section VIII, Division 1 Solution

VIII-1 does not provide rules on the loadings to be considered in the design of a vessel. However, UG-22 requires consideration of such loadings and the provisions of U-2(g) apply. This example provides one possible method of satisfying U-2(g); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

This example uses VIII-2, paragraph 4.1.5.3 which provides specific requirements to account for both loads and load case combinations used in the design of a vessel. These loads and load case combinations (Table 4.1.1 and Table 4.1.2 of VIII-2, respectively) are shown in this example problem in Table E4.3.6.1 for reference.

Additionally, VIII-1 does not provide a procedure for the calculation of combined stresses. Paragraph 4.3.10.2, in VIII-2, does provide a procedure and this procedure is used in this example problem with modifications to address specific requirements of VIII-1.

In accordance with VIII-2, paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.3.6.2 and Table E4.3.6.3, Load Case 5 is determined to be the governing load case. The pressure, net section axial force, and bending moment at the location of interest for Load Case 5 are:

$$0.9P + P_s = 320.4\ \ psi$$

$$F_5 = -66152.5\ lbs$$

$$M_5 = 3048000\ in-lbs$$

a) STEP 1 – Calculate the membrane stress for the cylindrical shell. Note that the circumferential membrane stress, $\sigma_{\theta m}$, is determined based on the equations in UG-27(c)(1) and the exact strength of materials solution for the longitudinal membrane stress, σ_{sm}, is used in place of the approximate solution provided in UG-27(c)(2). The shear stress is computed based on the known strength of materials solution.

   Note: θ is defined as the angle measured around the circumference from the direction of the applied bending moment to the point under consideration. For this example problem

$\theta = 0.0\ deg$ to maximize the bending stress.

$$\sigma_{\theta m} = \frac{1}{E}\left(\frac{PR}{t} + 0.6P\right) = \frac{1}{1.0}\left(\frac{320.4(45.125)}{1.0} + 0.6(320.4)\right) = 14650.29\ psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\frac{PD^2}{D_o^2 - D^2} + \frac{4F}{\pi\left(D_o^2 - D^2\right)} \pm \frac{32MD_o\cos[\theta]}{\pi\left(D_o^4 - D^4\right)}\right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(\frac{320.4(90.25)^2}{(92.25)^2 - (90.25)^2} + \frac{4(-66152.5)}{\pi\left[(92.25)^2 - (90.25)^2\right]} \pm \frac{32(3048000)(92.25)\cos[0.0]}{\pi\left[(92.25)^4 - (90.25)^4\right]}\right)$$

$$\sigma_{sm} = \begin{Bmatrix} 7149.8028 + (-230.7616) + 471.1299 = 7390.1711\ psi \\ 7149.8028 + (-230.7616) - 471.1299 = 6447.9113\ psi \end{Bmatrix}$$

$$\tau = \frac{16M_tD_o}{\pi\left(D_o^4 - D^4\right)} = \frac{16(0.0)(92.25)}{\pi\left[(92.25)^4 - (90.25)^4\right]} = 0.0\ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2}\right)$$

$$\sigma_1 = \begin{Bmatrix} 0.5\left(14650.29 + 7390.1711 + \sqrt{(14650.29 - 7390.1711)^2 + 4(0.0)^2}\right) = 14650.29\ psi \\ 0.5\left(14650.29 + 6447.9113 + \sqrt{(14650.29 - 6447.9113)^2 + 4(0.0)^2}\right) = 14650.29\ psi \end{Bmatrix}$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2}\right)$$

$$\sigma_2 = \begin{Bmatrix} 0.5\left(14650.29 + 7390.1711 - \sqrt{(14650.29 - 7390.1711)^2 + 4(0.0)^2}\right) \\ 0.5\left(14650.29 + 6447.9113 - \sqrt{(14650.29 - 6447.9113)^2 + 4(0.0)^2}\right) \end{Bmatrix}$$

$$\sigma_2 = \begin{Bmatrix} 7390.1711\ psi \\ 6447.9113\ psi \end{Bmatrix}$$

$$\sigma_3 = \sigma_r = -0.5P = -0.5(320.4) = -160.2\ psi$$

c) STEP 3 – At any point on the shell, the following limit shall be satisfied.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2\right]^{0.5} \le S$$

$$\sigma_e = \begin{Bmatrix} \dfrac{1}{\sqrt{2}}\left[\begin{array}{l}\left(14650.29-7390.1711\right)^2+ \\ \left(7390.1711-\left(-160.2\right)\right)^2+\left(\left(-160.2\right)-14650.29\right)^2\end{array}\right]^{0.5} = 12827.0816\ psi \\ \dfrac{1}{\sqrt{2}}\left[\begin{array}{l}\left(14650.29-6447.9113\right)^2+ \\ \left(6447.9113-\left(-160.2\right)\right)^2+\left(\left(-160.2\right)-14650.29\right)^2\end{array}\right]^{0.5} = 12851.0071\ psi \end{Bmatrix}$$

$$\begin{Bmatrix}\sigma_e = 12827.0816\ psi \\ \sigma_e = 12851.0071\ psi\end{Bmatrix} \le \left\{S = 20000\ psi\right\} \qquad True$$

Note that VIII-2 use an equivalent stress for the acceptance criterion. A combined stress calculation in VIII-1 would be based on the maximum principal stress; therefore,

$$\max\left[\sigma_1,\ \sigma_2,\ \sigma_3\right] \le S$$

$$\left\{\max\left[14650.3, 7390.2, \left|-160.0\right|\right] = 14650.3\ psi\right\} \le \left\{S = 20000\ psi\right\}\ True$$

Since the maximum tensile principal stress is less than the acceptance criteria, the shell section is adequately designed.

d) STEP 4 – For cylindrical and conical shells, if the meridional stress, σ_{sm} is compressive, then Equation (4.3.45) shall be satisfied where F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2 with $\lambda = 0.15$.

Note that this step in VIII-2 is based on Code Case 2286. Step 4 is not necessary in this example because the meridional stress, σ_{sm} , is tensile.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.3.10.

The loads transmitted to the cylindrical shell are given in the Table E4.3.6.2. Note that this table is given in terms of the load parameters shown in VIII-2, Table 4.1.1 and Table 4.1.2 (Table E4.3.6.1 of this example). As shown in Table E4.3.6.1, the acceptance criteria is that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition.

In accordance with VIII-2, paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.3.6.3, Load Case 5 is determined to be the governing load case. The pressure, net section axial force, and bending moment at the location of interest for Load Case 5 are:

$$P = 320.4\ psi$$

$$F_5 = -66152.5\ lbs$$

$$M_5 = 3048000\ in-lbs$$

Determine applicability of the rules of VIII-2, paragraph 4.3.10 based on satisfaction of the following requirements.

The section of interest is at least $2.5\sqrt{Rt}$ away from any major structural discontinuity.

$$2.5\sqrt{Rt} = 2.5\sqrt{(45.125)(1.0)} = 16.7938\ in$$

Shear force is not applicable.

The shell R/t ratio is greater than 3.0, or:

$$\left\{R/t = \frac{45.125}{1.0} = 45.125\right\} > 3.0 \qquad\qquad True$$

a) STEP 1 – Calculate the membrane stress for the cylindrical shell. Note that the maximum bending stress occurs at $\theta = 0.0\ deg$.

$$\sigma_{\theta m} = \frac{PD}{E(D_o - D)} = \frac{320.4(90.25)}{1.0(92.25-90.25)} = 14458.05\ psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\frac{PD^2}{D_o^2 - D^2} + \frac{4F}{\pi(D_o^2 - D^2)} \pm \frac{32MD_o\cos[\theta]}{\pi(D_o^4 - D^4)}\right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(\begin{array}{l}\dfrac{320.4(90.25)^2}{(92.25)^2-(90.25)^2} + \dfrac{4(-66152.5)}{\pi\left[(92.25)^2-(90.25)^2\right]} \pm \\ \dfrac{32(3048000)(92.25)\cos[0.0]}{\pi\left[(92.25)^4-(90.25)^4\right]}\end{array}\right)$$

$$\sigma_{sm} = \begin{Bmatrix} 7149.8028 + (-230.7616) + 471.1299 = 7390.1711\ psi \\ 7149.8028 + (-230.7616) - 471.1299 = 6447.9113\ psi \end{Bmatrix}$$

$$\tau = \frac{16M_tD_o}{\pi(D_o^4 - D^4)} = \frac{16(0.0)(92.25)}{\pi\left[(92.25)^4-(90.25)^4\right]} = 0.0\ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{(\sigma_{\theta m} - \sigma_{sm})^2 + 4\tau^2}\right)$$

$$\sigma_1 = \begin{Bmatrix} 0.5\left(14458.05 + 7390.1711 + \sqrt{(14458.05 - 7390.1711)^2 + 4(0.0)^2}\right) = 14458.05\ psi \\ 0.5\left(14458.05 + 6447.9113 + \sqrt{(14458.05 - 6447.9113)^2 + 4(0.0)^2}\right) = 14458.05\ psi \end{Bmatrix}$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2} \right)$$

$$\sigma_2 = \begin{Bmatrix} 0.5\left(14458.05 + 7390.1711 - \sqrt{\left(14458.05 - 7390.1711\right)^2 + 4\left(0.0\right)^2} \right) \\ 0.5\left(14458.05 + 6447.9113 - \sqrt{\left(14458.05 - 6447.9113\right)^2 + 4\left(0.0\right)^2} \right) \end{Bmatrix}$$

$$\sigma_2 = \begin{Bmatrix} 7390.1711\ psi \\ 6447.9113\ psi \end{Bmatrix}$$

$$\sigma_3 = \sigma_r = -0.5P = -0.5\left(320.4\right) = -160.2\ psi$$

c) STEP 3 – At any point on the shell, the following limit shall be satisfied.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]^{0.5} \le S$$

$$\sigma_e = \begin{Bmatrix} \dfrac{1}{\sqrt{2}}\begin{bmatrix} \left(14458.05 - 7390.1711\right)^2 + \left(7390.1711 - \left(-160.2\right)\right)^2 \\ + \left(\left(-160.2\right) - 14458.05\right)^2 \end{bmatrix}^{0.5} = 12662.1\ psi \\ \dfrac{1}{\sqrt{2}}\begin{bmatrix} \left(14458.05 - 6447.9113\right)^2 + \left(6447.9113 - \left(-160.2\right)\right)^2 \\ + \left(\left(-160.2\right) - 14458.05\right)^2 \end{bmatrix}^{0.5} = 12679.2\ psi \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_e = 12662.1\ psi \\ \sigma_e = 12679.2\ psi \end{Bmatrix} \le \left\{ S = 20000\ psi \right\} \qquad True$$

Since the equivalent stress is less than the acceptance criteria, the shell section is adequately designed.

d) STEP 4 – For cylindrical and conical shells, if the meridional stress, σ_{sm} is compressive, then Equation (4.3.45) shall be satisfied where F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2 with $\lambda = 0.15$.

Note that this step in VIII-2 is based on Code Case 2286. Step 4 is not necessary in this example because the meridional stress, σ_{sm}, is tensile.

Table E4.3.6.1: Design Loads and Load Combinations from VIII-2

Table 4.1.1 – Design Loads

Design Load Parameter	Description
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g. catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g. skirts, lugs, saddles, and legs), and appurtenances (e.g. platforms, ladders, etc.) • Weight of vessel contents under operating and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping
L	• Appurtenance Live loading • Effects of fluid flow
E	Earthquake loads (see ASCE 7 for the specific definition of the earthquake load, as applicable)
W	Wind Loads
S	Snow Loads
F	Loads due to Deflagration

Table 4.1.2 – Design Load Combinations

Design Load Combination (1)	General Primary Membrane Allowable Stress (2)
$P+P_s+D$	S
$P+P_s+D+L$	S
$P+P_s+D+S$	S
$0.9P+P_s+D+0.75L+0.75S$	S
$0.9P+P_s+D+\left(W\,or\,0.7E\right)$	S
$0.9P+P_S+D+0.75\left(W\,or\,0.7E\right)+0.75L+0.75S$	S
$0.6D+\left(W\,or\,0.7E\right)$ (3)	S
P_s+D+F	See Annex 4.D

Notes
1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) S is the allowable stress for the load case combination (see paragraph 4.1.5.3.c)
3) This load combination addresses an overturning condition. If anchorage is included in the design, consideration of this load combination is not required.

Table E4.3.6.2: Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	$P = 356.0$
P_s	Static head from liquid or bulk materials (e.g. catalyst)	$P_s = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -66152.5\ lbs$ $D_M = 0.0\ in-lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = 0.0\ lbs$ $L_M = 0.0\ in-lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 0.0\ in-lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 3.048E+06\ in-lbs$
S	Snow Loads	$S_F = 0.0\ lbs$ $S_M = 0.0\ in-lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in-lbs$

Based on these loads, the shell is required to be designed for the load case combinations shown in Table E4.3.6.3. Note that this table is given in terms of the load combinations shown in VIII-2, Table 4.1.2 (Table E4.3.6.1 of this example).

Table E4.3.6.3 – Load Case Combination at the Location of Interest

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P+P_s+D$	$P+P_s=356.0\ psi$ $F_1=-66152.5\ lbs$ $M_1=0.0\ in-lbs$	S
2	$P+P_s+D+L$	$P+P_s=356.0\ psi$ $F_2=-66152.5\ lbs$ $M_2=0.0\ in-lbs$	S
3	$P+P_s+D+S$	$P+P_s=356.0\ psi$ $F_3=-66152.5\ lbs$ $M_3=0.0\ in-lbs$	S
4	$0.9P+P_s+D+0.75L+0.75S$	$0.9P+P_s=320.4\ psi$ $F_4=-66152.5\ lbs$ $M_4=0.0\ in-lbs$	S
5	$0.9P+P_s+D+(W\ or\ 0.7E)$	$0.9P+P_s=320.4\ psi$ $F_5=-66152.5\ lbs$ $M_5=3048000\ in-lbs$	S
6	$\begin{pmatrix}0.9P+P_S+D+0.75(W\ or\ 0.7E)+\\ 0.75L+0.75S\end{pmatrix}$	$0.9P+P_s=320.4\ psi$ $F_6=-66152.5\ lbs$ $M_6=2286000\ in-lbs$	S
7	$0.6D+(W\ or\ 0.7E)$ Anchorage is included in the design. Therefore, consideration of this load combination is not required.	$F_7=-39691.5\ lbs$ $M_7=3048000\ in-lbs$	S
8	P_s+D+F	$P_s=0.0\ psi$ $F_8=-66152.5\ lbs$ $M_8=0.0\ in-lbs$	See Annex 4.D

4.3.7 Example E4.3.7 – Conical Transitions Without a Knuckle

Determine if the proposed large and small end cylinder-to-cone transitions are adequately designed considering the following design conditions and applied forces and moments. Evaluate the stresses in the cylinder and cone at both the large and small end junction.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $356\ psig\ @300°F$
- Inside Radius (Large End) = $75.0\ in$
- Thickness (Large End) = $1.8125\ in$
- Cylinder Length (Large End) = $60.0\ in$
- Inside Radius (Small End) = $45.0\ in$
- Thickness (Small End) = $1.125\ in$
- Cylinder Length (Small End) = $48.0\ in$
- Thickness (Conical Section) = $1.9375\ in$
- Length of Conical Section = $78.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle (See Figure E4.3.7) = $21.0375\ deg$
- Axial Force (Large End) = $-99167\ lbs$
- Net Section Bending Moment (Large End) = $5.406E+06\ in-lbs$
- Axial Force (Small End) = $-78104\ lbs$
- Net Section Bending Moment (Small End) = $4.301E+06\ in-lbs$

Adjust variables for corrosion.

$$R_L = 75.0 + Corrosion\ \ Allowance = 75.0 + 0.125 = 75.125\ in$$
$$R_S = 45.0 + Corrosion\ \ Allowance = 45.0 + 0.125 = 45.125\ in$$
$$t_L = 1.8125 - Corrosion\ \ Allowance = 1.8125 - 0.125 = 1.6875\ in$$
$$t_S = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$
$$t_C = 1.9375 - Corrosion\ \ Allowance = 1.9375 - 0.125 = 1.8125\ in$$
$$\alpha = 21.0375\ deg$$

Section VIII, Division 1 Solution

Rules for conical reducer sections subject to internal pressure are covered in Appendix 1-5. Rules are provided for the design of reinforcement, if needed, at the cone-to-cylinder junctions for conical reducer sections and conical head where all the elements have a common axis and the half-apex angle satisfies $\alpha \le 30\ deg$.

Large End

Cylindrical shell thickness per UG-27(c)(1) and conical shell thickness per UG-32(g)

$$t = \frac{PR}{SE - 0.6P} = \frac{356(75.125)}{20000(1.0) - 0.6(356)} = 1.3517\ in$$

$$t = 1.3517 + Corrosion\ \ Allowance = 1.3517 + 0.125 = 1.4767\ in$$

$$t_r = \frac{PD}{2\cos[\alpha](SE - 0.6P)} = \frac{356(2(75.125))}{2\cos[21.0375](20000(1.0) - 0.6(356))} = 1.4482\ in$$

$$t_r = 1.4482 + Corrosion\ \ Allowance = 1.4482 + 0.125 = 1.5732\ in$$

In accordance with Appendix 1-5(c)(1), when a cylinder having a minimum length of $2.0\sqrt{R_L t_s}$ is attached to the large end of the cone, determine the ratio $P / S_s E_1$ and the corresponding Δ per Table 1-5.1 at the large end cylinder. Note: if a cylinder is not present or does not meet the minimum length requirement, Δ is not calculated.

$$\left\{Cylinder\ \ Length = 60.0\ in\right\} \geq \left\{2.0\sqrt{R_L t_s} = 2.0\sqrt{(75.125)(1.6875)} = 22.5187\ in\right\} \quad True$$

$$\frac{P}{S_s E_1} = \frac{356}{20000(1.0)} = 0.0178$$

$$\frac{P}{S_s E_1} = 0.0178 > 0.009; \quad therefore,\ \Delta = 30\ deg\ \ per\ \ Note\ (1)$$

In accordance with Appendix 1-5(d), reinforcement shall be provided at the large end of the cone when required by the following:

Appendix 1-5(d)(1), for cones attached to a cylinder having a minimum length of $2.0\sqrt{R_L t_s}$, reinforcement shall be provided at the junction of the cone with the large cylinder for conical heads and reducers without knuckles when the value of Δ obtained from Table 1-5.1, using the appropriate ratio $P / S_s E_1$, is less than α.

$$\{\Delta = 30\} \geq \{\alpha = 21.0375\}; \ \ reinforcement\ \ is\ \ not\ \ required\ at\ the\ \ large\ \ end$$

Appendix 1-5(c)(3) revisited, since reinforcement is not required at the large end, $k = 1.0$.

Small End

Cylindrical shell thickness per UG-27(c)(1) and conical shell thickness per UG-32(g)

$$t = \frac{PR}{SE - 0.6P} = \frac{356(45.125)}{20000(1.0) - 0.6(356)} = 0.8119\ in$$

$$t = 0.8119 + Corrosion\ \ Allowance = 0.8119 + 0.125 = 0.9369\ in$$

$$t_r = \frac{PD}{2\cos[\alpha](SE - 0.6P)} = \frac{356(2(45.125))}{2\cos[21.0375](20000(1.0) - 0.6(356))} = 0.8699\ in$$

$$t_r = 0.8699 + Corrosion\ \ Allowance = 0.8699 + 0.125 = 0.9949\ in$$

In accordance with Appendix 1-5(c)(2), when a cylinder having a minimum length of $1.4\sqrt{R_S t_s}$ is attached to the small end of the cone, determine the ratio $P/S_s E_1$ and the corresponding Δ per Table 1-5.2 at the small end cylinder. Note: if a cylinder is not present or does not meet the minimum length requirement, Δ is not calculated.

$$\{Cylinder\ \ Length = 48.0\ in\} \geq \left\{1.4\sqrt{R_S t_s} = 1.4\sqrt{(45.125)(1.0)} = 9.4045\ in\right\} \qquad True$$

$$\frac{P}{S_s E_1} = \frac{356}{20000(1.0)} = 0.0178$$

$$\frac{P}{S_s E_1} = 0.0178; \qquad therefore, \begin{Bmatrix} x_1 = 0.010 \\ \Delta_1 = 9.0\ deg \end{Bmatrix} \leq x \leq \begin{Bmatrix} x_2 = 0.02 \\ \Delta_2 = 12.5\ deg \end{Bmatrix}$$

$$\Delta = \left(\frac{x - x_1}{x_2 - x_1}\right)(\Delta_2 - \Delta_1) + \Delta_1 = \left(\frac{0.0178 - 0.010}{0.020 - 0.010}\right)(12.5 - 9) + 9 = 11.73\ deg$$

In accordance with Appendix 1-5(e), reinforcement shall be provided at the small end of the cone when required by the following:

Appendix 1-5(e)(1), for cones attached to a cylinder having a minimum length of $1.4\sqrt{R_S t_s}$, reinforcement shall be provided at the junction of the conical shell of a reducer without a flare and the small cylinder when the value of Δ obtained from Table 1-5.2, using the appropriate ratio $P/S_s E_1$, is less than α.

$$\{\Delta = 11.73\} < \{\alpha = 21.0375\};\ reinforcement\ \ is\ \ required\ \ at\ \ the\ \ small\ \ end$$

Appendix 1-5(c)(3) revisited, since reinforcement is required at the small end, determine the value k. Assuming the reinforcement will be place on the cylinder, if required;

$$k = \frac{y}{S_r E_r} = \frac{20000}{20000(1.0)} = 1.0$$

$$where,$$

$$y = S_s E_s = 20000(1.0) = 20000$$

In accordance with Appendix 1-5(e)(1), the required area of reinforcement, A_{rs}, shall be at least equal to that indicated by the following equation when Q_s is in tension. At the small end of the cone-to-cylinder juncture, the $PR_s/2$ term is in tension. When f_2 is in compression and the quantity is larger than the $PR_s/2$ term, the design shall be in accordance with U–2(g).

$$A_{rs} = \frac{kQ_sR_s}{S_sE_1}\left(1-\frac{\Delta}{\alpha}\right)\tan[\alpha]$$

$$A_{rs} = \frac{1.0(8429.1122)(45.125)}{20000(1.0)}\left(1-\frac{11.73}{21.0375}\right)\tan[21.0375] = 3.2362\ in^2$$

Where,

$$Q_s = \frac{PR_s}{2} + f_2 = \begin{Bmatrix} \dfrac{356(45.125)}{2} + 396.8622 = 8429.1122 \dfrac{lbs}{in\ of\ cir} \\ \dfrac{356(45.125)}{2} + (-947.8060) = 7084.4440 \dfrac{lbs}{in\ of\ cir} \end{Bmatrix}$$

$$Q_s = 8429.1122 \frac{lbs}{in\ of\ cir} \qquad Use\ the\ maximum\ positive\ value$$

And,

$$f_2 = \frac{F_S}{2\pi R_S} \pm \frac{M_S}{\pi R_S^2} = \begin{Bmatrix} \dfrac{-78104}{2\pi(45.125)} + \dfrac{4.301E+06}{\pi(45.125)^2} = +396.8622 \dfrac{lbs}{in\ of\ cir} \\ \dfrac{-78104}{2\pi(45.125)} - \dfrac{4.301E+06}{\pi(45.125)^2} = -947.8060 \dfrac{lbs}{in\ of\ cir} \end{Bmatrix}$$

The effective area of reinforcement can be determined in accordance with the following:

$$A_{es} = 0.78\sqrt{R_st_s}\left[(t_s-t) + \frac{(t_c-t_r)}{\cos[\alpha]}\right]$$

$$A_{es} = 0.78\sqrt{45.125(1.0)}\left[(1.0-0.8119) + \frac{(1.8125-0.8699)}{\cos[21.0375]}\right] = 6.2772\ in^2$$

The effective area of available reinforcement due to the excess thickness in the cylindrical shell and conical shell, A_{es}, exceeds the required reinforcement, A_{rs}.

$$\left\{A_{es} = 6.2772\ in^2\right\} \ge \left\{A_{rs} = 3.2362\ in^2\right\} \qquad True$$

If this was not true, reinforcement would need to be added to the cylindrical or conical shell using a thick insert plate or reinforcing ring. Any additional area of reinforcement which is required shall be situated within a distance of $\sqrt{R_st_s}$ from the junction, and the centroid of the added area shall be within a distance of $0.25\sqrt{R_st_s}$ from the junction. In addition, note that in the above solution, the next section axial force and next section bending moment are included as an equivalent axial load per unit circumference.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.3.11.

Per VIII-2, paragraph 4.3.11.3, the length of the conical shell, measured parallel to the surface of the cone, shall be equal to or greater than the following:

$$L_C \geq 2.0\sqrt{\frac{R_L t_C}{\cos[\alpha]}} + 1.4\sqrt{\frac{R_S t_C}{\cos[\alpha]}}$$

$$2.0\sqrt{\frac{75.125(1.8125)}{\cos[21.0375]}} + 1.4\sqrt{\frac{45.125(1.8125)}{\cos[21.0375]}} = 37.2624 \ in$$

$$L_C = 78.0 \geq 37.2624 \qquad True$$

Evaluate the Large End cylinder-to-cone junction per VIII-2, paragraph 4.3.11.4.

a) STEP 1 – Compute the large end cylinder thickness, t_L, using VIII-2, paragraph 4.3.3., (as specified in design conditions).

$$t_L = 1.6875 \ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the large end using VIII-2, paragraph 4.3.4., (as specified in design conditions).

$$\alpha = 21.0375 \ deg$$

$$t_C = 1.8125 \ in$$

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all of these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with VIII-2, Part 5. In the calculations, if $0 \ deg < \alpha \leq 10 \ deg$, then use $\alpha = 10 \ deg$.

$$20 \leq \left\{ \frac{R_L}{t_L} = \frac{75.125}{1.6875} = 44.5185 \right\} \leq 500 \qquad True$$

$$1 \leq \left\{ \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741 \right\} \leq 2 \qquad True$$

$$\{\alpha = 21.0375 \ deg\} \leq \{60 \ deg\} \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force, F_L. Calculate the equivalent line load, X_L, using the specified net section axial force, F_L, and bending moment, M_L.

$$X_L = \frac{F_L}{2\pi R_L} \pm \frac{M_L}{\pi R_L^2} = \begin{Bmatrix} \dfrac{-99167}{2\pi(75.125)} + \dfrac{5406000}{\pi(75.125)^2} = 94.8111\ \dfrac{lbs}{in} \\ \dfrac{-99167}{2\pi(75.125)} - \dfrac{5406000}{\pi(75.125)^2} = -514.9886\ \dfrac{lbs}{in} \end{Bmatrix}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per VIII-2, Table 4.3.3 and Table 4.3.4, respectively. For calculated values of n other than those presented in VIII-2, Table 4.3.3 and Table 4.3.4, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741$$

$$H = \sqrt{\frac{R_L}{t_L}} = \sqrt{\frac{75.125}{1.6875}} = 6.6722$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in VIII-2, Table 4.3.3 and Table 4.3.4 is required. The results of the interpolation are summarized with the following values for C_i.

Equation Coefficients C_i	VIII-2, Table 4.3.3		VIII-2, Table 4.3.4	
	Pressure Applied Junction Moment Resultant M_{sN}	Pressure Applied Junction Shear Force Resultant Q_N	Equivalent Line Load Junction Moment Resultant M_{sN}	Equivalent Line Load Junction Shear Force Resultant Q_N
1	-3.079751	-1.962370	-5.706141	-4.878520
2	3.662099	2.375540	0.004705	0.006808
3	0.788301	0.582454	0.474988	-0.018569
4	-0.226515	-0.107299	-0.213112	-0.197037
5	-0.080019	-0.103635	2.241065	2.033876
6	0.049314	0.151522	0.000025	-0.000085
7	0.026266	0.010704	0.002759	-0.000109
8	-0.015486	-0.018356	-0.001786	-0.004071
9	0.001773	0.006551	-0.214046	-0.208830
10	-0.007868	-0.021739	0.000065	-0.000781
11	---	---	-0.106223	0.004724

For the applied pressure case both M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\begin{bmatrix} C_1 + C_2 \ln[H] + C_3 \ln[B] + C_4 (\ln[H])^2 + C_5 (\ln[B])^2 + \\ C_6 \ln[H]\ln[B] + C_7 (\ln[H])^3 + C_8 (\ln[B])^3 + \\ C_9 \ln[H](\ln[B])^2 + C_{10} (\ln[H])^2 \ln[B] \end{bmatrix}$$

This results in

$$M_{sN} = -\exp\left[\begin{array}{l} -3.079751 + 3.662099 \cdot \ln[6.6722] + 0.788301 \cdot \ln[0.3846] + \\ (-0.226515)(\ln[6.6722])^2 + (-0.080019)(\ln[0.3846])^2 + \\ 0.049314 \cdot \ln[6.6722] \cdot \ln[0.3846] + \\ 0.026266(\ln[6.6722])^3 + (-0.015486)(\ln[0.3846])^3 + \\ 0.001773 \cdot \ln[6.6722] \cdot (\ln[0.3846])^2 + \\ (-0.007868)(\ln[6.6722])^2 \cdot \ln[0.3846] \end{array}\right] = -10.6148$$

$$Q_N = -\exp\left[\begin{array}{l} -1.962370 + 2.375540 \cdot \ln[6.6722] + 0.582454 \cdot \ln[0.3846] + \\ (-0.107299)(\ln[6.6722])^2 + (-0.103635)(\ln[0.3846])^2 + \\ 0.151522 \cdot \ln[6.6722] \cdot \ln[0.3846] + \\ 0.010704(\ln[6.6722])^3 + (-0.018356)(\ln[0.3846])^3 + \\ 0.006551 \cdot \ln[6.6722] \cdot (\ln[0.3846])^2 + \\ (-0.021739)(\ln[6.6722])^2 \cdot \ln[0.3846] \end{array}\right] = -4.0925$$

For the Equivalent Line Load case, M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\left[\frac{\left(\begin{array}{l} C_1 + C_3 \ln[H^2] + C_5 \ln[\alpha] + C_7 (\ln[H^2])^2 + \\ C_9 (\ln[\alpha])^2 + C_{11} \ln[H^2] \ln[\alpha] \end{array}\right)}{\left(\begin{array}{l} 1 + C_2 \ln[H^2] + C_4 \ln[\alpha] + C_6 (\ln[H^2])^2 + \\ C_8 (\ln[\alpha])^2 + C_{10} \ln[H^2] \ln[\alpha] \end{array}\right)}\right]$$

This results in

$$M_{sN} = -\exp\left[\frac{\begin{pmatrix} -5.706141 + 0.474988 \cdot \ln\left[6.6722^2\right] + \\ 2.241065 \cdot \ln\left[21.0375\right] + 0.002759\left(\ln\left[6.6722^2\right]\right)^2 + \\ (-0.214046)\left(\ln\left[21.0375\right]\right)^2 + \\ (-0.106223)\ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{pmatrix}}{\begin{pmatrix} 1 + 0.004705 \cdot \ln\left[6.6722^2\right] + (-0.213112)\ln\left[21.0375\right] + \\ 0.000025\left(\ln\left[6.6722^2\right]\right)^2 + (-0.001786)\left(\ln\left[21.0375\right]\right)^2 + \\ 0.000065 \cdot \ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{pmatrix}}\right] = -0.4912$$

$$Q_{N} = -\exp\left[\frac{\begin{pmatrix} -4.878520 + (-0.018569)\ln\left[6.6722^2\right] + \\ 2.033876 \cdot \ln\left[21.0375\right] + (-0.000109)\left(\ln\left[6.6722^2\right]\right)^2 + \\ (-0.208830)\left(\ln\left[21.0375\right]\right)^2 + \\ 0.004724 \cdot \ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{pmatrix}}{\begin{pmatrix} 1 + 0.006808 \cdot \ln\left[6.6722^2\right] + (-0.197037)\ln\left[21.0375\right] + \\ (-0.000085)\left(\ln\left[6.6722^2\right]\right)^2 + (-0.004071)\left(\ln\left[21.0375\right]\right)^2 + \\ (-0.000781)\ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{pmatrix}}\right] = -0.1845$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\begin{aligned} &\textit{Internal Pressure}: && M_{sN} = -10.6148, && Q_N = -4.0925 \\ &\textit{Equivalent Line Load}: && M_{sN} = -0.4912, && Q_N = -0.1845 \end{aligned}$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in VIII-2, Table 4.3.1 for the Large End Junction.

Evaluate the Cylinder at the Large End:

Stress Resultant Calculations:

$$M_{sP} = Pt_L^2 M_{sN} = 356(1.6875)^2(-10.6148) = -10760.9194 \ \frac{in-lbs}{in}$$

$$M_{sX} = X_L t_L M_{sN} = \begin{cases} 94.8111(1.6875)(-0.4912) = -78.5889 \ \frac{in-lbs}{in} \\ -514.9886(1.6875)(-0.4912) = 426.8741 \ \frac{in-lbs}{in} \end{cases}$$

$$M_s = M_{sP} + M_{sX} = \begin{cases} -10760.9194 + (-78.5889) = -10839.5083 \ \frac{in-lbs}{in} \\ -10760.9194 + 426.8741 = -10334.0453 \ \frac{in-lbs}{in} \end{cases}$$

$$Q_P = Pt_L Q_N = 356(1.6875)(-4.0925) = -2458.5694 \ \frac{lbs}{in}$$

$$Q_X = X_L Q_N = \begin{cases} 94.8111(-0.1845) = -17.4926 \ \frac{lbs}{in} \\ -514.9886(-0.1845) = 95.0154 \ \frac{lbs}{in} \end{cases}$$

$$Q = Q_P + Q_X = \begin{cases} -2458.5694 + (-17.4926) = -2476.0620 \ \frac{lbs}{in} \\ -2458.5694 + 95.0154 = -2363.5540 \ \frac{lbs}{in} \end{cases}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_L^2 t_L^2}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(75.125)^2(1.6875)^2}\right]^{0.25} = 0.1142 \ in^{-1}$$

$$N_s = \frac{PR_L}{2} + X_L = \begin{cases} \frac{356(75.125)}{2} + 94.8111 = 13467.0611 \ \frac{lbs}{in} \\ \frac{356(75.125)}{2} + (-514.9886) = 12857.2614 \ \frac{lbs}{in} \end{cases}$$

$$N_\theta = PR_L + 2\beta_{cy}R_L\left(-M_s\beta_{cy} + Q\right)$$

$$N_\theta = \begin{cases} 356(75.125) + 2(0.1142)(75.125)\left(-(-10839.5083)(0.1142) + (-2476.0620)\right) \\ 356(75.125) + 2(0.1142)(75.125)\left(-(-10334.0453)(0.1142) + (-2363.553)\right) \end{cases}$$

$$N_\theta = \begin{cases} 5498.9524 \ \dfrac{lbs}{in} \\ 6438.9685 \ \dfrac{lbs}{in} \end{cases}$$

$$K_{pc} = 1.0$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_s}{t_L} = \begin{cases} \dfrac{13467.0611}{1.6875} = 7980.4807 \ psi \\ \dfrac{12857.2614}{1.6875} = 7619.1179 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_s}{t_L^2 K_{pc}} = \begin{cases} \dfrac{6(-10839.5083)}{(1.6875)^2(1.0)} = -22838.7994 \ psi \\ \dfrac{6(-10334.0453)}{(1.6875)^2(1.0)} = -21773.7909 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_L} = \begin{cases} \dfrac{5498.9524}{1.6875} = 3258.6385 \ psi \\ \dfrac{6438.9685}{1.6875} = 3815.6850 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_L^2 K_{pc}} = \begin{cases} \dfrac{6(0.3)(-10839.5083)}{(1.6875)^2(1.0)} = -6851.6398 \ psi \\ \dfrac{6(0.3)(-10334.0453)}{(1.6875)^2(1.0)} = -6532.1373 \ psi \end{cases}$$

Check Acceptance Criteria:

$$\left\{\begin{array}{l}\sigma_{sm} = 7980.4807 \; psi \\ \sigma_{sm} = 7619.1179 \; psi\end{array}\right\} \le \left\{1.5S = 1.5(20000) = 30000 \;\; psi\right\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{sm} + \sigma_{sb} = 7980.4807 + (-22838.7994) = |-14858.3 \;\; psi| \\ \sigma_{sm} - \sigma_{sb} = 7980.4807 - (-22838.7994) = 30819.3 \;\; psi \\ \sigma_{sm} + \sigma_{sb} = 7619.1179 + (-21773.7909) = |-14154.7 \;\; psi| \\ \sigma_{sm} - \sigma_{sb} = 7619.1179 - (-21773.7909) = 29392.9 \;\; psi\end{array}\right\} \le \left\{S_{PS} = 60000 \;\; psi\right\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m} = 3258.6385 \;\; psi \\ \sigma_{\theta m} = 3815.6850 \;\; psi\end{array}\right\} \le \left\{1.5S = 1.5(20000) = 30000 \;\; psi\right\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m} + \sigma_{\theta b} = 3258.6385 + (-6851.6398) = |-3593.0 \;\; psi| \\ \sigma_{\theta m} - \sigma_{\theta b} = 3258.6385 - (-6851.6398) = 10110.3 \; psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 3815.6850 + (-6532.1373) = |-2716.5 \;\; psi| \\ \sigma_{\theta m} - \sigma_{\theta b} = 3815.6850 - (-6532.1373) = 10347.8 \; psi\end{array}\right\} \le \left\{S_{PS} = 60000 \;\; psi\right\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cylinder at the cylinder-to-cone junction at the large end is adequately designed.

<u>Evaluate the Cone at the Large End:</u>

Stress Resultant Calculations - as determined above:

$$M_{csP} = M_{sP} = -10760.9194 \; \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \left\{\begin{array}{l}-78.5889 \; \dfrac{in-lbs}{in} \\ 426.8741 \; \dfrac{in-lbs}{in}\end{array}\right\}$$

$$M_{cs} = M_{csP} + M_{csX} = \left\{\begin{array}{l}-10760.9194 + (-78.5889) = -10839.5083 \; \dfrac{in-lbs}{in} \\ -10760.9194 + 426.8741 = -10334.0453 \; \dfrac{in-lbs}{in}\end{array}\right\}$$

$$Q_c = Q\cos[\alpha] + N_s \sin[\alpha]$$

$$Q_c = \left\{\begin{array}{l}((-2476.0620)\cos[21.0375] + (13467.0611)\sin[21.0375]) = 2523.3690 \; \dfrac{lbs}{in} \\ ((-2363.5540)\cos[21.0375] + (12857.2614)\sin[21.0375]) = 2409.4726 \; \dfrac{lbs}{in}\end{array}\right\}$$

$$R_C = \frac{R_L}{\cos[\alpha]} = \frac{75.125}{\cos[21.0375]} = 80.4900\ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_C^2 t_C^2}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(80.4900)^2(1.8125)^2}\right]^{0.25} = 0.1064\ in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{Bmatrix} (13467.0611)\cos[21.0375] - (-2476.0620)\sin[21.0375] = 13458.2772\ \dfrac{lbs}{in} \\ (12857.2614)\cos[21.0375] - (-2363.5540)\sin[21.0375] = 12848.7353\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_{c\theta} = \frac{PR_L}{\cos[\alpha]} + 2\beta_{co}R_C\left(-M_{cs}\beta_{co} - Q_c\right)$$

$$N_{c\theta} = \begin{Bmatrix} \dfrac{356(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(-10839.5083)(0.1064) - 2523.3690\right) \\ \dfrac{356(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(-10334.0453)(0.1064) - 2409.4726\right) \end{Bmatrix}$$

$$N_{c\theta} = \begin{Bmatrix} 5187.9337\ \dfrac{lbs}{in} \\ 6217.6021\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{cpc} = 1.0$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{Bmatrix} \dfrac{13458.2772}{1.8125} = 7425.2564 \; psi \\ \dfrac{12848.7353}{1.8125} = 7088.9574 \; psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(-10839.5083)}{(1.8125)^2(1.0)} = -19797.2470 \; psi \\ \dfrac{6(-10334.0453)}{(1.8125)^2(1.0)} = -18874.0708 \; psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{Bmatrix} \dfrac{5187.9337}{1.8125} = 2862.3082 \; psi \\ \dfrac{6217.6021}{1.8125} = 3430.4012 \; psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(0.3)(-10839.5083)}{(1.8125)^2(1.0)} = -5939.1741 \; psi \\ \dfrac{6(0.3)(-10334.0453)}{(1.8125)^2(1.0)} = -5662.2213 \; psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\begin{Bmatrix} \sigma_{sm} = 7425.2564 \; psi \\ \sigma_{sm} = 7088.9574 \; psi \end{Bmatrix} \le \left\{ 1.5S = 1.5(20000) = 30000 \; psi \right\} \qquad True$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = 7425.2564 + (-19797.2470) = |-12371.9906 \; psi| \\ \sigma_{sm} - \sigma_{sb} = 7425.2564 - (-19797.2470) = 27222.5034 \; psi \\ \sigma_{sm} + \sigma_{sb} = 7088.9574 + (-18874.0708) = |-11785.1 \; psi| \\ \sigma_{sm} - \sigma_{sb} = 7088.9574 - (-18874.0708) = 25963.0 \; psi \end{Bmatrix} \le \left\{ S_{PS} = 60000 \; psi \right\} True$$

$$\begin{Bmatrix} \sigma_{\theta m} = 2862.3082 \; psi \\ \sigma_{\theta m} = 3430.4012 \; psi \end{Bmatrix} \le \left\{ 1.5S = 1.5(20000) = 30000 \; psi \right\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = 2862.3082 + (-5939.1741) = |-3076.8659 \; psi| \\ \sigma_{\theta m} - \sigma_{\theta b} = 2862.3082 - (-5939.1741) = 8801.4823 \; psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 3430.4012 + (-5662.2213) = |-2231.8 \; psi| \\ \sigma_{\theta m} - \sigma_{\theta b} = 3430.4012 - (-5662.2213) = 9092.6 \; psi \end{Bmatrix} \le \left\{ S_{PS} = 60000 \; psi \right\} \quad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cone at the cylinder-to-cone junction at the large end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore the design is complete.

Evaluate the Small End cylinder-to-cone junction per VIII-2, paragraph 4.3.11.5.

a) STEP 1 – Compute the small end cylinder thickness, t_S, using VIII-2, paragraph 4.3.3., (as specified in design conditions).

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the small end using VIII-2, paragraph 4.3.4., (as specified in design conditions

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all of these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with VIII-2, Part 5. In the calculations, if $0° < \alpha \le 10°$, then use $\alpha = 10°$.

$$20 \le \left(\frac{R_S}{t_S} = \frac{45.125}{1.0} = 45.125 \right) \le 500 \qquad \textit{True}$$

$$1 \le \left(\frac{t_C}{t_S} = \frac{1.8125}{1.0} = 1.8125 \right) \le 2 \qquad \textit{True}$$

$$\{\alpha = 21.0375\ deg\} \le \{60\ deg\} \qquad \textit{True}$$

d) STEP 4 – Calculate the equivalent line load, X_S, given the net section axial force, F_S, and bending moment, M_S, applied at the conical transition.

$$X_S = \frac{F_S}{2\pi R_S} \pm \frac{M_S}{\pi R_S^2} = \begin{Bmatrix} \dfrac{-78104.2}{2\pi(45.125)} + \dfrac{4301000}{\pi(45.125)^2} = 396.8622\ \dfrac{lbs}{in} \\ \dfrac{-78104.2}{2\pi(45.125)} - \dfrac{4301000}{\pi(45.125)^2} = -947.8060\ \dfrac{lbs}{in} \end{Bmatrix}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per VIII-2, Table 4.3.5 and Table 4.3.6, respectively. For calculated values of n other than those presented in VIII-2, Table 4.3.5 and Table 4.3.6, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_S} = \frac{1.8125}{1.000} = 1.8125$$

$$H = \sqrt{\frac{R_S}{t_S}} = \sqrt{\frac{45.125}{1.000}} = 6.7175$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in VIII-2, Table 4.3.5 and Table 4.3.6 is required. The results of the interpolation is summarized with the following values for C_i.

Equation Coefficients C_i	VIII-2, Table 4.3.5		VIII-2, Table 4.3.6	
	Pressure Applied Junction Moment Resultant M_{sN}	Pressure Applied Junction Shear Force Resultant Q_N	Equivalent Line Load Junction Moment Resultant M_{sN}	Equivalent Line Load Junction Shear Force Resultant Q_N
1	-15.144683	0.569891	0.006792	-0.408044
2	3.036812	-0.000027	0.000290	0.021200
3	6.460714	0.008431	-0.000928	-0.325518
4	-0.155909	0.002690	0.121611	-0.003988
5	-1.462862	-0.002884	0.010581	-0.111262
6	-0.369444	0.000000	-0.000011	0.002204
7	0.007742	-0.000005	-0.000008	0.000255
8	0.143191	-0.000117	0.005957	-0.014431
9	0.040944	-0.000087	0.001310	0.000820
10	0.007178	0.000001	0.000186	0.000106
11	---	-0.003778	0.194433	---

For the applied pressure case M_{sN} is calculated using the following equation

$$M_{sN} = \exp\left[\begin{array}{l} C_1 + C_2 \ln\left[H^2\right] + C_3 \ln\left[\alpha\right] + C_4\left(\ln\left[H^2\right]\right)^2 + C_5\left(\ln\left[\alpha\right]\right)^2 + \\ C_6 \ln\left[H^2\right]\ln\left[\alpha\right] + C_7\left(\ln\left[H^2\right]\right)^3 + C_8\left(\ln\left[\alpha\right]\right)^3 + \\ C_9 \ln\left[H^2\right]\left(\ln\left[\alpha\right]\right)^2 + C_{10}\left(\ln\left[H^2\right]\right)^2 \ln\left[\alpha\right] \end{array}\right]$$

This results in

$$M_{sN}=\exp\left[\begin{array}{l}-15.144683+3.036812\cdot\ln\left[6.7175^2\right]+6.460714\cdot\ln\left[21.0375\right]+\\(-0.155909)\left(\ln\left[6.7175^2\right]\right)^2+(-1.462862)\left(\ln\left[21.0375\right]\right)^2+\\(-0.369444)\ln\left[6.7175^2\right]\cdot\ln\left[21.0375\right]+\\0.007742\left(\ln\left[6.7175^2\right]\right)^3+0.143191\left(\ln\left[21.0375\right]\right)^3+\\0.040944\cdot\ln\left[6.7175^2\right]\cdot\left(\ln\left[21.0375\right]\right)^2+\\0.007178\left(\ln\left[6.7175^2\right]\right)^2\cdot\ln\left[21.0375\right]\end{array}\right]=9.2135$$

For the applied pressure case Q_N is calculated using the following equation

$$Q_N=\left(\frac{C_1+C_3H^2+C_5\alpha+C_7H^4+C_9\alpha^2+C_{11}H^2\alpha}{1+C_2H^2+C_4\alpha+C_6H^4+C_8\alpha^2+C_{10}H^2\alpha}\right)$$

This results in

$$Q_N=\left(\frac{\left(\begin{array}{l}0.569891+0.008431(6.7175)^2+(-0.002884)(21.0375)+\\(-0.000005)(6.7175)^4+(-0.000087)(21.0375)^2+\\(-0.003778)(6.7175)^2(21.0375)\end{array}\right)}{\left(\begin{array}{l}1+(-0.000027)(6.7175)^2+0.002690(21.0375)+\\0.000000(6.7175)^4+(-0.000117)(21.0375)^2+\\0.000001(6.7175)^2(21.0375)\end{array}\right)}\right)=-2.7333$$

For the Equivalent Line Load case, M_{sN} is calculated using the following equation

$$M_{sN}=\left(\frac{C_1+C_3H+C_5B+C_7H^2+C_9B^2+C_{11}HB}{1+C_2H+C_4B+C_6H^2+C_8B^2+C_{10}HB}\right)$$

This results in

$$M_{sN}=\left(\frac{\begin{pmatrix}0.006792+(-0.000928)(6.7175)+0.010581(0.3846)+\\(-0.000008)(6.7175)^2+0.001310(0.3846)^2+\\0.194433(6.7175)(0.3846)\end{pmatrix}}{\begin{pmatrix}1+0.000290(6.7175)+0.121611(0.3846)+\\(-0.000011)(6.7175)^2+0.005957(0.3846)^2+\\0.000186(6.7175)(0.3846)\end{pmatrix}}\right)=0.4828$$

For the Equivalent Line Load case, Q_N is calculated using the following equation

$$Q_N=\begin{pmatrix}C_1+C_2\ln[H]+C_3\ln[B]+C_4(\ln[H])^2+C_5(\ln[B])^2+C_6\ln[H]\ln[B]+\\C_7(\ln[H])^3+C_8(\ln[B])^3+C_9\ln[H](\ln[B])^2+C_{10}(\ln[H])^2\ln[B]\end{pmatrix}$$

This results in

$$Q_N=\begin{pmatrix}-0.408044+0.021200\cdot\ln[6.7175]+(-0.325518)\ln[0.3846]+\\(-0.003988)(\ln[6.7175])^2+(-0.111262)(\ln[0.3846])^2+\\0.002204\cdot\ln[6.7175]\cdot\ln[0.3846]+0.000255\cdot(\ln[6.7175])^3+\\(-0.014431)(\ln[0.3846])^3+0.000820\cdot\ln[6.7175]\cdot(\ln[0.3846])^2+\\0.000106(\ln[6.7175])^2\cdot\ln[0.3846]\end{pmatrix}=-0.1613$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\begin{array}{lll}\textit{Internal Pressure}: & M_{sN}=9.2135, & Q_N=-2.7333\\\textit{Equivalent Line Load}: & M_{sN}=0.4828, & Q_N=-0.1613\end{array}$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in VIII-2, Table 4.3.2 for the Small End Junction.

Evaluate the Cylinder at the Small End:

Stress Resultant Calculations:

$$M_{sP} = Pt_S^2 M_{sN} = 356(1.000)^2(9.2135) = 3280.0060\ \frac{in-lbs}{in}$$

$$M_{sX} = X_S t_S M_{sN} = \begin{Bmatrix} 396.8622(1.0000)(0.4828) = -191.6051\ \dfrac{in-lbs}{in} \\ -947.8060(1.0000)(0.4828) = -457.6007\ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$M_s = M_{sP} + M_{sX} = \begin{Bmatrix} 3280.0060 + (191.6051) = 3471.6111\ \dfrac{in-lbs}{in} \\ 3280.0060 + (-457.6007) = 2822.4053\ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$Q_P = Pt_S Q_N = 356(1.0000)(-2.7333) = -973.0548\ \frac{lbs}{in}$$

$$Q_X = X_S Q_N = \begin{Bmatrix} 396.8622(-0.1613) = -64.0139\ \dfrac{lbs}{in} \\ -947.8060(-0.1613) = 152.8811\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$Q = Q_P + Q_X = \begin{Bmatrix} -973.0548 + (-64.0139) = -1037.0687\ \dfrac{lbs}{in} \\ -973.0548 + 152.8811 = -820.1737\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_S^2 t_S^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(45.1250)^2(1.000)^2}\right]^{0.25} = 0.1914\ in^{-1}$$

$$N_s = \frac{PR_S}{2} + X_S = \begin{Bmatrix} \dfrac{356(45.125)}{2} + 396.8622 = 8429.1122\ \dfrac{lbs}{in} \\ \dfrac{356(45.125)}{2} + (-947.8060) = 7084.4440\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_\theta = PR_S + 2\beta_{cy}R_S\left(-M_s\beta_{cy} - Q\right)$$

$$N_\theta = \begin{Bmatrix} 356(45.125)+2(0.1914)(45.125)\left(-(3471.6111)(0.1914)-(-1037.0687)\right) \\ 356(45.125)+2(0.1914)(45.125)\left(-(2822.4053)(0.1914)-(-820.1737)\right) \end{Bmatrix}$$

$$N_\theta = \begin{Bmatrix} 22500.7769\ \dfrac{lbs}{in} \\ 20900.5790\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_s}{t_S} = \begin{Bmatrix} \dfrac{8429.1122}{1.0000} = 8429.1122\ psi \\ \dfrac{7084.4440}{1.0000} = 7084.4440\ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(3471.6111)}{(1.0000)^2(1.0)} = 20829.6666\ psi \\ \dfrac{6(2822.4053)}{(1.0000)^2(1.0)} = 16934.4318\ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_S} = \begin{Bmatrix} \dfrac{22500.7769}{1.0000} = 22500.7769\ psi \\ \dfrac{20900.5790}{1.0000} = 20900.5790\ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(0.3)(3471.6111)}{(1.0000)^2(1.0)} = 6248.8999\ psi \\ \dfrac{6(0.3)(2822.4053)}{(1.0000)^2(1.0)} = 5080.3295\ psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\begin{Bmatrix} \sigma_{sm} = 8429.1122 \ psi \\ \sigma_{sm} = 7084.4440 \ psi \end{Bmatrix} \le \{1.5S = 1.5(20000) = 30000 \ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = 8429.1122 + (20829.6666) = 29258.8 \ psi \\ \sigma_{sm} - \sigma_{sb} = 8429.1122 - (20829.6666) = |-12400.6 \ psi| \\ \sigma_{sm} + \sigma_{sb} = 7084.4440 + (16934.4318) = 24018.9 \ psi \\ \sigma_{sm} - \sigma_{sb} = 7084.4440 - (16934.4318) = |-9850.0 \ psi| \end{Bmatrix} \le \{S_{PS} = 60000 \ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = 22500.7769 \\ \sigma_{\theta m} = 20900.5790 \end{Bmatrix} \le \{1.5S = 1.5(20000) = 30000 \ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = 22500.7769 + (6248.8999) = 28749.7 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 22500.7769 - (6248.8999) = 16251.9 \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 20900.5790 + (5080.3295) = 25981.0 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 20900.5790 - (5080.3295) = 15820.2 \ psi \end{Bmatrix} \le \{S_{PS} = 60000 \ psi\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cylinder at the cylinder-to-cone junction at the small end is adequately designed.

<u>Evaluate the Cone at the Small End:</u>

Stress Resultant Calculations:

$$M_{csP} = M_{sP} = 3280.0060 \ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \begin{Bmatrix} 191.6051 \ \dfrac{in-lbs}{in} \\ -457.6007 \ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$M_{cs} = M_{csP} + M_{csX} = \begin{Bmatrix} 3280.0060 + 191.6051 = 3471.6111 \ \dfrac{in-lbs}{in} \\ 3280.0060 + (-457.6007) = 2822.4053 \ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$Q_c = Q\cos[\alpha] + N_s \sin[\alpha]$$

$$Q_c = \begin{Bmatrix} -1037.0687\cos[21.0375] + 8429.1122\sin[21.0375] = 2057.9298 \ \dfrac{lbs}{in} \\ -820.1737\cos[21.0375] + 7084.4440\sin[21.0375] = 1777.6603 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$R_c = \frac{R_c}{\cos[\alpha]} = \frac{45.1250}{\cos[21.0375]} = 48.3476\ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_c^{\ 2} t_c^{\ 2}}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(48.3476)^2(1.8125)^2}\right]^{0.25} = 0.1373\ in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{Bmatrix} \left(8429.1122\cos[21.0375] - (-1037.0687)\sin[21.0375]\right) = 8239.5612\ \dfrac{lbs}{in} \\ \left(7084.4440\cos[21.0375] - (-820.1737)\sin[21.0375]\right) = 6906.6602\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_{c\theta} = \frac{PR_s}{\cos[\alpha]} + 2\beta_{co}R_c\left(-M_{cs}\beta_{co} + Q_c\right)$$

$$N_{c\theta} = \begin{Bmatrix} \dfrac{356(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(3471.6111)(0.1373) + 2057.9298\right) \\ \dfrac{356(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(2822.4053)(0.1373) + 1777.6603\right) \end{Bmatrix}$$

$$N_{c\theta} = \begin{Bmatrix} 38205.1749\ \dfrac{lbs}{in} \\ 35667.6380\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{cpc} = 1.0$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{cases} \dfrac{8239.5612}{1.8125} = 4545.9648 \ psi \\ \dfrac{6906.6602}{1.8125} = 3810.5711 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{cases} \dfrac{6(3471.6111)}{(1.8125)^2(1.0)} = 6340.5406 \ psi \\ \dfrac{6(2822.4053)}{(1.8125)^2(1.0)} = 5154.8330 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{cases} \dfrac{38205.1749}{1.8125} = 21078.7172 \ psi \\ \dfrac{35667.6380}{1.8125} = 19678.6968 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{cases} \dfrac{6(0.3)(3471.6111)}{(1.8125)^2(1.0)} = 1902.1622 \ psi \\ \dfrac{6(0.3)(2822.4053)}{(1.8125)^2(1.0)} = 1546.4499 \ psi \end{cases}$$

Check Acceptable Criteria:

$$\begin{Bmatrix} \sigma_{sm} = 4545.9648 \ psi \\ \sigma_{sm} = 3810.5711 \ psi \end{Bmatrix} \le \{1.5S = 1.5(20000) = 30000 \ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = 4545.9648 + (6340.5406) = 10886.5 \ psi \\ \sigma_{sm} - \sigma_{sb} = 4545.9648 - (6340.5406) = |-1794.6 \ psi| \\ \sigma_{sm} + \sigma_{sb} = 3810.5711 + (5154.8330) = 8965.4 \ psi \\ \sigma_{sm} - \sigma_{sb} = 3810.5711 - (5154.8330) = |-1344.3 \ psi| \end{Bmatrix} \le \{S_{PS} = 60000 \ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = 21078.7172 \\ \sigma_{\theta m} = 19678.6968 \end{Bmatrix} \le \{1.5S = 1.5(20000) = 30000 \ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = 21078.7172 + (1902.1622) = 22980.9 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 21078.7172 - (1902.1622) = 19176.6 \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 19678.6968 + (1546.4499) = 21225.1 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 19678.6968 - (1546.4499) = 18132.2 \ psi \end{Bmatrix} \le \{S_{PS} = 60000 \ psi\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cone at the cylinder-to-cone junction at the small end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore the design is complete.

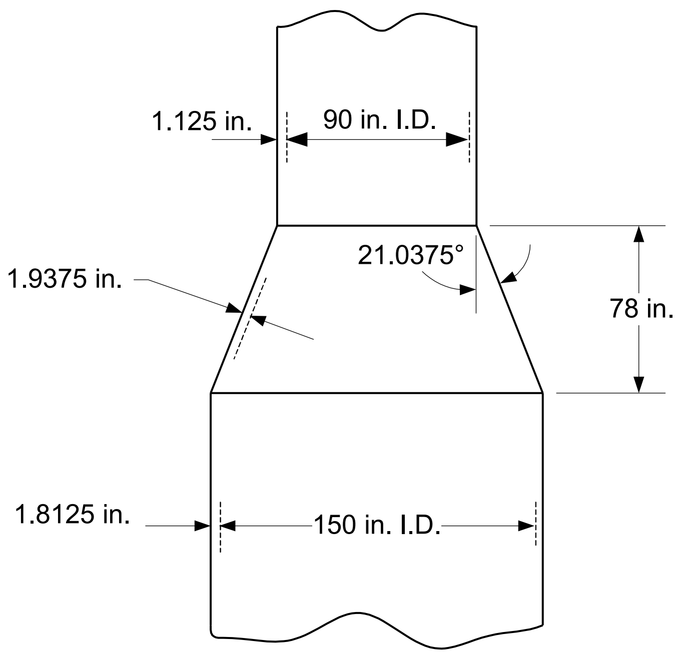


Figure E4.3.7 – Conical Transition

4.3.8 Example E4.3.8 - Conical Transitions with a Knuckle

Determine if the proposed design for the large end of a cylinder-to-cone junction with a knuckle is adequately designed considering the following design conditions and applied forces and moments, see Figure E4.3.8 for details.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Normalized$
- Design Conditions = $280\ psig@300°F$
- Inside Diameter (Large End) = $120.0\ in$
- Inside Radius (Large End) = $60.0\ in$
- Knuckle Radius = $10.0\ in$
- Large End Thickness = $1.0\ in$
- Cone Thickness = $1.0\ in$
- Knuckle Thickness = $1.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0

- One-Half Apex Angle = $30.0 \; deg$
- Axial Force (Large End) = $-10000 \; lbs$
- Net Section Bending Moment (Large End) = $2.0E+06 \; in-lbs$

Section VIII, Division 1 Solution

VIII-1 does not provide rules for the required thickness of toriconical heads and section subject to pressure and supplemental loadings. However, UG-22 requires consideration of such loadings and the provisions of U-2(g) apply. This example provides one possible method of satisfying U-2(g); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

This example accounts for the specified net-section axial force, F_L, and bending moment, M_L, applied to the conical transition at the location of the knuckle by calculating an effective pressure, P_L. The effective pressure from the applied loading and the specified design pressure are summed to determine the equivalent pressure, P_e to be used in the procedure, $P_e = P + P_L$.

Evaluate per UG-32(h). The required thickness of the conical portion of a toriconical head or section, in which the knuckle radius is neither less than 6% of the outside diameter of the head skirt nor less than three times the knuckle thickness, shall be determined by UG-32(g) and substituting D_i for D. The required thickness of the knuckle shall be determined using Mandatory Appendix 1-4(d) with a modified value of L.

$$D_i = D - 2r\left(1-\cos[\alpha]\right) = 120.0 - 2(10.0)\left(1-\cos[30.0]\right) = 117.3205 \; in$$

$$L = \frac{D_i}{2\cos[\alpha]} = \frac{117.3205}{2\cos[30.0]} = 67.7350 \; in$$

The equivalent design pressure is computed as follows.

$$P_e = P + P_L = \begin{Bmatrix} 280.0 + (5.3828) = 285.3828 \; psi \\ 280.0 + (-7.2329) = 272.7671 \; psi \end{Bmatrix} \quad \textit{Use the maximum positive value}$$

Where,

$$P_L = \frac{4f_L}{D_i} = \begin{Bmatrix} \dfrac{4(157.8771)}{117.3205} = 5.3828 \; psi \\ \dfrac{4(-212.1404)}{117.3205} = -7.2329 \; psi \end{Bmatrix}$$

$$f_L = \frac{F_L}{\pi D_i} \pm \frac{4M_L}{\pi D_i^2} = \begin{Bmatrix} \dfrac{-10000}{\pi(117.3205)} + \dfrac{4(2.0E+06)}{\pi(117.3205)^2} = 157.8771 \; \dfrac{lbs}{in} \\ \dfrac{-10000}{\pi(117.3205)} - \dfrac{4(2.0E+06)}{\pi(117.3205)^2} = -212.1404 \; \dfrac{lbs}{in} \end{Bmatrix}$$

Determine the required thickness of the knuckle per UG-32(h) and Mandatory Appendix 1-4(d) using the equivalent design pressure.

$$M = 0.25\left(3+\sqrt{\frac{L}{r}}\right) = 0.25\left(3+\sqrt{\frac{67.7350}{10.0}}\right) = 1.4006$$

$$t_k = \frac{P_e LM}{2SE - 0.2P_e} = \frac{285.3828(67.7350)(1.4006)}{2(20000)(1.0) - 0.2(285.3828)} = 0.6778\ in$$

The required knuckle thickness is less than the design thickness; therefore, the knuckle is adequately designed for the internal pressure and applied forces and moments.

Determine the required thickness of the cone at the knuckle-to-cone intersection at the large end using UG-32(g) using the equivalent design pressure.

$$t_c = \frac{P_e D_i}{2\cos[\alpha](SE - 0.6P_e)} = \frac{285.3828(117.3205)}{2\cos[30.0](20000(1.0) - 0.6(285.3828))} = 0.9749\ in$$

The required cone thickness is less than the design thickness; therefore, the cone is adequately designed for the internal pressure and applied forces and moments.

<u>Section VIII, Division 2 Solution with VIII-1 Allowable Stresses</u>

Evaluate per VIII-2, paragraph 4.3.12.

a) STEP 1 – Compute the large end cylinder thickness, t_L, using VIII-2, paragraph 4.3.3.

$$t_L = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right] - 1\right) = \frac{120.0}{2}\left(\exp\left[\frac{280.0}{20000.0}\right] - 1\right) = 0.8459\ in$$

As specified in the design conditions,

$$t_L = 1.0\ in$$

Since the required thickness is less than the design thickness, the cylinder is adequately designed for internal pressure.

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the large end using VIII-2, paragraph 4.3.4.

$$\alpha = 30.0\ deg$$

$$t_C = \frac{D}{2\cos[\alpha]}\left(\exp\left[\frac{P}{SE}\right] - 1\right) = \frac{120.0}{2\cos[30.0]}\left(\exp\left[\frac{280.0}{20000.0}\right] - 1\right) = 0.9768\ in$$

As specified in the design conditions,

$$t_C = 1.0\ in$$

Since the required thickness is less than the design thickness, the cone is adequately designed for internal pressure.

c) STEP 3 – Proportion the transition geometry by assuming a value for the knuckle radius, r_k, and knuckle thickness, t_k, such that the following equations are satisfied. If all of these equations cannot be satisfied, the cylinder-to-cone junction shall be designed in accordance with VIII-2, Part 5.

$$\{t_k = 1.0\ in\} \geq \{t_L = 1.0\ in\} \qquad True$$

$$\{r_k = 10.0\ in\} > \{3t_k = 3.0\ in\} \qquad True$$

$$\left\{\frac{r_k}{R_L} = \frac{10.0}{60.0} = 0.1667\right\} > \{0.03\} \qquad True$$

$$\{\alpha = 30\ deg\} \leq \{60\ deg\} \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition at the location of the knuckle. The thrust load due to pressure shall not be included as part of the axial force, F_L.

$$F_L = -10000\ lbs$$

$$M_L = 2.0E+06\ in-lbs$$

e) STEP 5 – Compute the stresses in the knuckle at the junction using the equations in VIII-2, Table 4.3.7.

Determine if the knuckle is considered to be compact or non-compact.

$$\alpha r_k < 2K_m\left(\left\{R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5} + r_k\right\}t_k\right)^{0.5}$$

$$\{0.5236(10.0)\} = \left\{2(0.7)\left(\left\{50.0\left((0.5236)^{-1}\tan[0.5236]\right)^{0.5} + 10\right\}1\right)^{0.5}\right\}$$

$$\{5.2360\ in\} < \{11.0683\ in\} \qquad True$$

Where,

$$K_m = 0.7$$

$$\alpha = \frac{30.0}{180}\pi = 0.5236\ rad$$

$$R_k = R_L - r_k = 60.0 - 10.0 = 50.0\ in$$

Therefore, analyze the knuckle junction as a compact knuckle.

Stress Calculations: Determine the hoop and axial membrane stresses at the knuckle:

$$\sigma_{\theta m} = \frac{PK_m\left(R_L\sqrt{R_L t_L} + L_k\sqrt{L_k t_C}\right) + \alpha\left(PL_{1k}r_k - 0.5P_e L_{1k}^2\right)}{K_m\left(t_L\sqrt{R_L t_L} + t_C\sqrt{L_k t_C}\right)\alpha t_k r_k}$$

$$\sigma_{sm} = \frac{P_e L_{1k}}{2t_k}$$

Where,

$$L_{1k} = R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5} + r_k = 50.0\left((0.5236)^{-1}\tan[0.5236]\right)^{0.5} + 10.0 = 62.5038\ in$$

$$L_k = \frac{R_k}{\cos[\alpha]} + r_k = \frac{50.0}{\cos[0.5236]} + 10.0 = 67.7351\ in$$

$$P_e = P + \frac{F_L}{\pi L_{1k}^2 \cos^2\left[\frac{\alpha}{2}\right]} \pm \frac{2M_L}{\pi L_{1k}^3 \cos^3\left[\frac{\alpha}{2}\right]}$$

$$P_e = \begin{Bmatrix} 280 + \dfrac{-10000.0}{\pi(62.5038)^2 \cos^2\left[\dfrac{0.5236}{2}\right]} + \dfrac{2(2.0E+06)}{\pi(62.5038)^3 \cos^3\left[\dfrac{0.5236}{2}\right]} \\ 280 + \dfrac{-10000.0}{\pi(62.5038)^2 \cos^2\left[\dfrac{0.5236}{2}\right]} - \dfrac{2(2.0E+06)}{\pi(62.5038)^3 \cos^3\left[\dfrac{0.5236}{2}\right]} \end{Bmatrix}$$

$$P_e = \begin{Bmatrix} 284.9125\ psi \\ 273.3410\ psi \end{Bmatrix}$$

Therefore,

$$\sigma_{\theta m} = \begin{Bmatrix} \dfrac{\left(\begin{array}{l} 280(0.7)\left(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}\right) + \\ 0.5236\left(280(62.5038)(10.0) - 0.5(284.9125)(62.5038)^2\right) \end{array} \right)}{0.7\left(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}\right) + 0.5236(1.0)(10.0)} = 35.8767\ psi \\ \dfrac{\left(\begin{array}{l} 280(0.7)\left(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}\right) + \\ 0.5236\left(280(62.5038)(10.0) - 0.5(273.3410)(62.5038)^2\right) \end{array} \right)}{0.7\left(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}\right) + 0.5236(1.0)(10.0)} = 756.6825\ psi \end{Bmatrix}$$

And,

$$\sigma_{sm} = \begin{Bmatrix} \dfrac{P_e L_{1k}}{2t_k} = \dfrac{284.9125(62.5038)}{2(1.0)} = 8904.0570\ psi \\ \dfrac{P_e L_{1k}}{2t_k} = \dfrac{273.3410(62.5038)}{2(1.0)} = 8542.4256\ psi \end{Bmatrix}$$

Check Acceptable Criteria:

$$\begin{Bmatrix} \sigma_{\theta m} = 35.9\ psi \\ \sigma_{\theta m} = 756.7\ psi \end{Bmatrix} \le \{S = 20000\ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{sm} = 8904.1\ psi \\ \sigma_{sm} = 8542.4\ psi \end{Bmatrix} \le \{S = 20000\ psi\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ in the knuckle are both tensile, the condition of local buckling need not be considered. Therefore, the knuckle at the cylinder-to-cone junction at the large end is adequately designed.

f) STEP 6 – The stress acceptance criterion in STEP 5 is satisfied for the knuckle. Therefore, the design is complete.

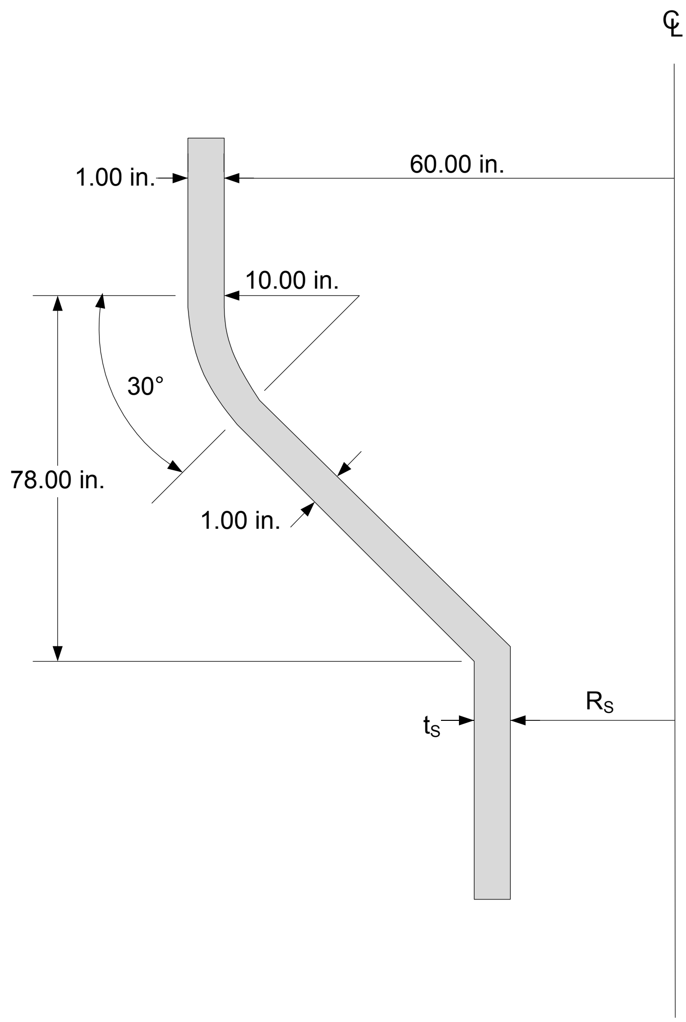


Figure E4.3.8 – Knuckle Detail

4.4 Shells Under External Pressure and Allowable Compressive Stresses

4.4.1 Example E4.4.1 - Cylindrical Shell

Determine the maximum allowable external pressure (MAEP) for a cylindrical shell considering the following design conditions.

Vessel Data:

- Material = $SA-516, \ Grade\ 70, \ Norm.$
- Design Temperature = $300°F$
- Inside Diameter = $90.0 \ in$
- Thickness = $1.125 \ in$
- Corrosion Allowance = $0.125 \ in$
- Unsupported Length = $636.0 \ in$
- Modulus of Elasticity at Design Temperature = $28.3E+06 \ psi$
- Yield Strength = $33600 \ psi$

Section VIII, Division 1 Solution

Evaluate per paragraph UG-28(c).

a) STEP 1 – UG-28(c)(1), Cylinders having $D_o/t \geq 10$

$$\left\{\frac{D_o}{t} = \frac{92.25}{1} = 92.25\right\} \geq 10 \qquad True$$

Where,

$$D_o = D + 2(Uncorroded\ Thickness) = 90.0 + 2(1.125) = 92.25 \ in$$
$$t = t - Corrosion\ Allowance = 1.125 - 0.125 = 1.0 \ in$$
$$L = 636.0 \ in$$

Assume a value for t and determine the ratios L/D_o and D_o/t.

$$\frac{L}{D_o} = \frac{636.0}{92.25} = 6.8943$$
$$\frac{D_o}{t} = 92.25$$

b) STEP 2 – Enter Fig. G in Subpart 3 of Section II, Part D at the value of L/D_o determined in STEP 1. For values of $L/D_o > 50$, enter the chart at a values of $L/D_o = 50$. For values of $L/D_o < 0.05$, enter the chart at a values of $L/D_o = 0.05$.

c) STEP 3 – Move horizontally to the line for the value of D_o/t determined in STEP 1. Interpolation may be made for intermediate values of D_o/t; extrapolation is not permitted. From this point of intersection move vertically downward to determine the value of factor A.

$$A = 0.00019$$

d) STEP 4 – Using the value of A calculated in STEP 3, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 7.

Per Section II Part D, Table 1A, a material specification of $SA-516-70N$ is assigned an External Pressure Chart No. CS-2.

e) STEP 5 – From the intersection obtained in Step 4, move horizontally to the right and read the value of factor B.

$$B = 2700$$

f) STEP 6 – Using this value of B, calculate the value of the maximum allowable external working pressure P_a using the following formula:

$$P_a = \frac{4B}{3\left(\dfrac{D_o}{t}\right)} = \frac{4(2700)}{3\left(\dfrac{92.25}{1.0}\right)} = 39.0 \ psi$$

g) STEP 7 – For values of A falling to the left of the applicable material/temperature line, the value of P_a can be calculated using the following formula:

$$P_a = \frac{2AE}{3\left(\dfrac{D_o}{t}\right)} \qquad \textit{Not required}$$

h) STEP 8 – Compare the calculated value of P_a obtained in STEPS 6 or 7 with P. If P_a is smaller than P, select a larger value of t and repeat the design procedure until a value of P_a is obtained that is equal to or greater than P.

The allowable external pressure is $P_a = 39.0 \ psi$

<u>Section VIII, Division 2 Solution</u>

Evaluate per VIII-2, paragraph 4.4.5.

a) STEP 1 – Assume an initial thickness, t, and unsupported length, L.

$$t = t - \textit{Corrosion Allowance} = 1.125 - 0.125 = 1.0 \ in$$

$$L = 636.0 \ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{1.6C_h E_y t}{D_o} = \frac{1.6(0.0092)(28.3\text{E}{+}06)(1.0)}{92.25} = 4515.7290 \ psi$$

Where,

$$D_o = D + 2(Uncorroded\ Thickness) = 90.0 + 2(1.125) = 92.25\ in$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{\left(\frac{92.25}{2}\right)1.0}} = 93.6459$$

$$2\left(\frac{D_o}{t}\right)^{0.94} = 2\left(\frac{92.25}{1.0}\right)^{0.94} = 140.6366$$

Since $13 < M_x < 2\left(\frac{D_o}{t}\right)^{0.94}$, calculate C_h as follows:

$$C_h = 1.12 M_x^{-1.058} = 1.12(93.6459)^{-1.058} = 0.0092$$

c) STEP 3 – Calculate the predicted buckling stress, F_{ic}.

$$\frac{F_{he}}{S_y} = \frac{4515.7290}{33600.0} = 0.1344$$

Since $\frac{F_{he}}{S_y} \le 0.552$, calculate F_{ic} as follows:

$$F_{ic} = F_{he} = 4515.7290\ psi$$

d) STEP 4 – Calculate the value of design factor, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0\ psi$$

Since $F_{ic} \le 0.55S_y$, calculate FS as follows:

$$FS = 2.0$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{D_o}\right) = 2(2257.8645)\left(\frac{1.0}{92.25}\right) = 48.9\ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{4515.7290}{2.0} = 2257.8645\ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness or reduce the unsupported length of the shell (i.e. by the addition of a stiffening rings) and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The allowable external pressure is $P_a = 48.9\ psi$

Combined Loadings – cylindrical shells subject to external pressure and other loadings shall satisfy the requirements of VIII-2, paragraph 4.4.12. In this example problem, the cylindrical shell is only subject to external pressure.

4.4.2 Example E4.4.2 - Conical Shell

Determine the maximum allowable external pressure (MAEP) for a conical shell considering the following design conditions.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Temperature = $300°F$
- Inside Diameter (Large End) = $150.0\ in$
- Thickness (Large End) = $1.8125\ in$
- Inside Diameter (Small End) = $90.0\ in$
- Thickness (Small End) = $1.125\ in$
- Thickness (Conical Section) = $1.9375\ in$
- Axial Cone Length = $78.0\ in$
- One-Half Apex Angle = $21.0375\ deg$
- Corrosion Allowance = $0.125\ in$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$

Section VIII, Division 1 Solution

Evaluate per paragraph UG-33(f): Conical heads and Sections. When the cone-to-cylinder junction is not a line of support, the required thickness of a conical head or section under pressure on the convex side, either seamless or of built-up construction with butt joints shall not be less than the minimum required thickness of the adjacent cylindrical shell and, when a knuckle is not provided, the reinforcement requirement of Appendix 1-8 shall be satisfied. When the cone-to-cylinder junction is a line of support the required thickness shall be determined in accordance with the following procedure.

For this example, it is assumed that the cone-to-cylinder junction is a line of support. However, the supplemental checks on reinforcement and moment of inertia per Appendix 1-8 are not performed.

a) STEP 1 – UG-33(f)(1), When $\alpha \le 60\ deg$ and cones having $D_L / t_e \ge 10$

$$\left\{\frac{D_L}{t_e} = \frac{153.625}{1.6917} = 90.8110\right\} \ge 10 \qquad True$$

Where,

$$D_L = Inside\ Diameter + 2(Uncorroded\ Thickness) = 150.0 + 2(1.8125) = 153.625\ in$$

$$D_S = Inside\ Diameter + 2(Uncorroded\ Thickness) = 90.0 + 2(1.125) = 92.25\ in$$

$$t = t - Corrosion\ Allowance = 1.9375 - 0.125 = 1.8125\ in$$

$$t_e = t\cos[\alpha] = 1.8125 \cdot \cos[21.0375] = 1.6917\ in$$

$$L_e = \frac{L_c}{2}\left(1 + \frac{D_S}{D_L}\right) = \frac{78.0}{2}\left(1 + \frac{92.25}{153.625}\right) = 62.4\ in \qquad See\ UG-33(g), Fig.\ UG-33.1(a)$$

Assume a value for t_e and determine the ratios L_e/D_L and D_L/t_e.

$$\frac{L_e}{D_L} = \frac{62.4}{153.625} = 0.4061$$

$$\frac{D_L}{t_e} = 90.8110$$

b) STEP 2 – Enter Fig. G in Subpart 3 of Section II, Part D at the value of L/D_o equivalent to the value of L_e/D_L determined in STEP 1. For values of $L_e/D_L > 50$, enter the chart at a values of $L_e/D_L = 50$.

c) STEP 3 – Move horizontally to the line for the value of D_o/t equivalent to the value of D_L/t_e determined in STEP 1. Interpolation may be made for intermediate values of D_L/t_e; extrapolation is not permitted. From this point of intersection move vertically downward to determine the value of factor A.

$$A = 0.0045$$

d) STEP 4 – Using the value of A calculated in STEP 3, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 7.

Per Section II Part D, Table 1A, a material specification of $SA-516-70N$ is assigned an External Pressure Chart No. CS-2.

e) STEP 5 – From the intersection obtained in Step 4, move horizontally to the right and read the value of factor B.

$$B = 17000$$

f) STEP 6 – Using this value of B, calculate the value of the maximum allowable external working pressure P_a using the following formula:

$$P_a = \frac{4B}{3\left(\frac{D_L}{t_e}\right)} = \frac{4(17000)}{3\left(\frac{153.625}{1.6917}\right)} = 249.6\ psi$$

g) STEP 7 – For values of A falling to the left of the applicable material/temperature line, the value of P_a can be calculated using the following formula:

$$P_a = \frac{2AE}{3\left(\frac{D_L}{t_e}\right)} \qquad \textit{Not required}$$

h) STEP 8 – Compare the calculated value of P_a obtained in STEPS 6 or 7 with P. If P_a is smaller than P, select a larger value of t and repeat the design procedure until a value of P_a is obtained that is equal to or greater than P.

The allowable external pressure is $P_a = 249.6\ psi$

Section VIII, Division 2 Solution

Evaluate per VIII-2, paragraph 4.4.6. and 4.4.5.

The required thickness of a conical shell subjected to external pressure loading shall be determined using the equations for a cylinder by making the following substitutions:

a) The value of t_c is substituted for t in the equations in paragraph 4.4.5.

$$t_c = t = 1.9375 - Corrosion\ Allowance = 1.9375 - 0.125 = 1.8125\ in$$

b) For offset cones, the cone angle, α, shall satisfy the requirements of paragraph 4.3.4.

The conical shell in this example problem is not of the offset type. Therefore, no additional requirements are necessary.

c) The value of $0.5(D_L + D_S)/\cos[\alpha]$ is substituted for D_o in the equations in VIII-2, paragraph 4.4.5, (concentric cone design with common center line per VIII-2, Figure 4.4.7 Sketch (a)).

$$D_o = \frac{0.5(D_L + D_s)}{\cos[\alpha]} = \frac{0.5\left[\left(150.0 + 2(1.8125)\right) + \left(90.0 + 2(1.125)\right)\right]}{\cos[21.0375]} = 131.7170\ in$$

d) The value of $L_{ce}/\cos[\alpha]$ is substituted for L in the equations in VIII-2, paragraph 4.4.5 where L_{ce} is determined as shown below. For Sketches (a) and (e) in VIII-2, Figure 4.4.7:

$$L_{ce} = L_c$$

$$L = \frac{L_{ce}}{\cos[\alpha]} = \frac{78.0}{\cos[21.0375]} = 83.5703\ in$$

e) Note that the half-apex angle of a conical transition can be computed knowing the shell geometry with the following equations. These equations were developed with the assumption that the conical transition contains a cone section, knuckle, or flare. If the transition does not contain a knuckle or flare, the radii of these components should be set to zero when computing the half-apex angle (see VIII-2, Figure 4.4.7).

$$If\ (R_L - r_k) \geq (R_S + r_f):$$

$$\alpha = \beta + \phi = 0.3672 - 0 = 0.3672\ rad = 21.0375\ deg$$

$$\beta = \arctan\left[\frac{(R_L - r_k) - (R_S + r_f)}{L_c}\right] = \arctan\left[\frac{(75.0 - 0) - (45.0 + 0)}{78.0}\right] = 0.3672\ rad$$

$$\phi = \arcsin\left[\frac{(r_f + r_k)\cos[\beta]}{L_c}\right] = \arcsin\left[\frac{(0.0 + 0.0)\cos[0.3672]}{78.0}\right] = 0.0\ rad$$

Proceed with the design following the steps outlined in VIII-2, paragraph 4.4.5.

a) STEP 1 – Assume an initial thickness, t, and unsupported length, L (see VIII-2, Figures 4.4.1 and 4.4.2).

$$t = 1.8125\ in$$

$$L = 83.5703\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{1.6C_h E_y t}{D_o} = \frac{1.6(0.1301)(28.3E+06)(1.8125)}{131.7170} = 81062.4824\ psi$$

Where,

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{83.5703}{\sqrt{\left(\frac{131.7170}{2.0}\right)1.8125}} = 7.6490$$

$$2\left(\frac{D_o}{t}\right)^{0.94} = 2\left(\frac{131.7170}{1.0}\right)^{0.94} = 112.3859$$

Since $1.5 < M_x < 13$, calculate C_h as follows:

$$C_h = \frac{0.92}{M_x - 0.579} = \frac{0.92}{7.6490 - 0.579} = 0.1301$$

c) STEP 3 – Calculate the predicted buckling stress, F_{ic}.

$$\frac{F_{he}}{S_y} = \frac{81062.4824}{33600.0} = 2.4126$$

Since $0.552 < \left(\frac{F_{he}}{S_y}\right) < 2.439$, calculate F_{ic} as follows:

$$F_{ic} = 0.7S_y\left(\frac{F_{he}}{S_y}\right)^{0.4} = 0.7(33600.0)\left(\frac{81062.4824}{33600.0}\right)^{0.4} = 33452.5760\ psi$$

d) STEP 4 – Calculate the value of design factor, FS, per VIII-2, paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0 \ psi$$

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{33452.5760}{33600.0}\right) = 1.6693$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{D_o}\right) = 2(20039.8826)\left(\frac{1.8125}{131.7170}\right) = 551.5 \ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{33452.5760}{1.6693} = 20039.8826 \ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness or reduce the unsupported length of the shell (i.e. by the addition of a stiffening rings) and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 551.5 \ psi$

Combined Loadings – conical shells subject to external pressure and other loadings shall satisfy the requirements of VIII-2, paragraph 4.4.12. In this example problem, the conical shell is only subject to external pressure.

4.4.3 Example E4.4.3 - Spherical Shell and Hemispherical Head

Determine the maximum allowable external pressure (MAEP) for a hemispherical head considering the following design conditions.

<u>Vessel Data:</u>

- Material = $SA-542, \ Type \ D, \ Class \ 4a$
- Design Temperature = $350°F$
- Inside Diameter = $149.0 \ in$
- Thickness = $2.8125 \ in$
- Corrosion Allowance = $0.0 \ in$
- Modulus of Elasticity at Design Temperature = $29.1E+06 \ psi$
- Yield Strength = $58000 \ psi$

<u>Section VIII, Division 1 Solution</u>

Evaluate per paragraph UG-28(d). As noted in paragraph UG-33(c), the required thickness of a hemispherical head having pressure on the convex side shall be determined in the same manner as outlined in paragraph UG-28(d) for determining the thickness for a spherical shell.

a) STEP 1 – UG-28(d)(1), Assume a value for t and calculate the value of factor A using the following formula:

$$A = \frac{0.125}{\left(\frac{R_o}{t}\right)} = \frac{0.125}{\left(\frac{77.3125}{2.8125}\right)} = 0.00455$$

Where,

$$R_o = \frac{D + 2(Uncorroded\ Thickness)}{2} = \frac{149.0 + 2(2.8125)}{2} = 77.3125\ in$$

$$t = t - Corrosion\ Allowance = 2.8125 - 0.0 = 2.8125\ in$$

b) STEP 2 – Using the value of A calculated in STEP 1, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 5.

Per Section II Part D, Table 1A, a material specification of $SA-542\ Type\ D\ Cl.\ 4a$ is assigned an External Pressure Chart No. CS-2.

c) STEP 3 – From the intersection obtained in Step 2, move horizontally to the right and read the value of factor B.

$$B = 15700$$

d) STEP 4 – Using the value of B obtained in STEP 3, calculate the value of the maximum allowable external working pressure P_a using the following formula:

$$P_a = \frac{B}{\left(\frac{R_o}{t}\right)} = \frac{15700}{\left(\frac{77.3125}{2.8125}\right)} = 571.1\ psi$$

e) STEP 5 – For values of A falling to the left of the applicable material/temperature line, the value of P_a can be calculated using the following formula:

$$P_a = \frac{0.0625E}{\left(\frac{R_o}{t}\right)^2} \qquad Not\ required$$

f) STEP 6 – Compare the calculated value of P_a obtained in STEPS 4 or 5 with P. If P_a is smaller than P, select a larger value of t and repeat the design procedure until a value of P_a is obtained that is equal to or greater than P.

The allowable external pressure is $P_a = 571.1\ psi$

Section VIII, Division 2 Solution

Evaluate per VIII-2, paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the spherical shell.

$$t = 2.8125\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075E_y\left(\frac{t}{R_o}\right) = 0.075(29.1E+06)\left(\frac{2.8125}{\frac{149.0}{2}+2.8125}\right) = 79395.7154\ psi$$

c) STEP 3 – Calculate the predicted buckling stress, F_{ic}.

$$\frac{F_{he}}{S_y} = \frac{79395.7154}{58000.0} = 1.3689$$

Since $0.55 < \left(\frac{F_{he}}{S_y}\right) < 1.6$, calculate F_{ic} as follows:

$$F_{ic} = 0.18F_{he} + 0.45S_y = 0.18(79395.7154) + 0.45(58000.0) = 40391.2288\ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per VIII-2, paragraph 4.4.2.

$$0.55S_y = 0.55(58000.0) = 31900.0\ psi$$

Since $0.55S_y < F_{ic} < S_y$, calculate the FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{40391.2288}{58000.0}\right) = 1.8910$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(21359.7191)\left(\frac{2.8125}{\frac{149.0}{2}+2.8125}\right) = 1554.1\ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{40391.2288}{1.8910} = 21359.7191\ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 1554.1\ psi$

Combined Loadings – spherical shells and hemispherical heads subject to external pressure and other loadings shall satisfy the requirements of VIII-2, paragraph 4.4.12. In this example problem, the torispherical head is only subject to external pressure.

4.4.4 Example E4.4.4 - Torispherical Head

Determine the maximum allowable external pressure (MAEP) for a torispherical head considering the following design conditions.

Vessel Data:

- Material = $SA-387,\ Grade\ 11,\ Class\ 1$
- Design Temperature = $650°F$
- Inside Diameter = $72.0\ in$
- Crown Radius = $72.0\ in$
- Knuckle Radius = $4.375\ in$
- Thickness = $0.625\ in$
- Corrosion Allowance = $0.125\ in$
- Modulus of Elasticity at Design Temperature = $26.55E+06\ psi$
- Yield Strength at Design Temperature = $26900\ psi$

Section VIII, Division 1 Solution

Per UG-33(a)(1), the required thickness for a torispherical head having pressure on the convex side shall be the greater of: (a) the thickness computed by the procedure given in UG-32 for heads with pressure on the concave side using a design pressure 1.67 times the design pressure on the convex side, assuming joint efficiency $E=1.00$ for all cases; or (b) the thickness as computed by paragraph UG-33(e). In determining the MAEP on the convex side of a torispherical head, reverse the procedures in UG-33(a)(1)(a) and (a)(1)(b) and use the smaller of the pressures obtained.

The rules of UG-32(e) can be used to evaluate torispherical heads, however, the rules contained in this paragraph are only applicable for a specific geometry, i.e. the knuckle radius is 6% of the inside crown radius and the inside crown radius equals the outside diameter of the skirt. Additionally, if the ratio $t_s/L \geq 0.002$ is not satisfied, the rules of Mandatory Appendix 1-4(f) shall also be met. As an alternative, the general procedure for evaluating a torispherical head can be performed using the rules of Mandatory Appendix 1-4(d).

Evaluate per Mandatory Appendix 1-4(d) and determine the MAEP. As shown in Example Problem, E4.3.4, the torispherical head has an $MAEP=135.3\ psi$.

As noted in paragraph UG-33(e), the required thickness of a torispherical head having pressure on the convex side shall be determined in the same manner as outlined in paragraph UG-28(d) for determining the thickness for a spherical shell, with the appropriate value of R_o.

a) STEP 1 – UG-28(d)(1), Assume a value for t and calculate the value of factor A using the following formula:

$$A=\frac{0.125}{\left(\frac{R_o}{t}\right)}=\frac{0.125}{\left(\frac{72.625}{0.500}\right)}=0.00086$$

$$R_o = Inside\ Crown\ Radius + Uncorroded\ Thickness = 72.0+0.625=72.625\ in$$

$$t = t - Corrosion\ Allowance = 0.625-0.125=0.500\ in$$

b) STEP 2 – Using the value of A calculated in STEP 1, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 5.

Per Section II Part D, Table 1A, a material specification of $SA-387-11, \ Class\ 1$ is assigned an External Pressure Chart No. CS-2.

c) STEP 3 – From the intersection obtained in Step 2, move horizontally to the right and read the value of factor B.

$$B = 8100$$

d) STEP 4 – Using the value of B obtained in STEP 3, calculate the value of the maximum allowable external working pressure P_a using the following formula:

$$P_a = \frac{B}{\left(\dfrac{R_o}{t}\right)} = \frac{8100}{\left(\dfrac{72.625}{0.500}\right)} = 55.8\ psi$$

e) STEP 5 – For values of A falling to the left of the applicable material/temperature line, the value of P_a can be calculated using the following formula:

$$P_a = \frac{0.0625E}{\left(\dfrac{R_o}{t}\right)^2} \qquad Not\ required$$

f) STEP 6 – Compare the calculated value of P_a obtained in STEPS 4 or 5 with P. If P_a is smaller than P, select a larger value of t and repeat the design procedure until a value of P_a is obtained that is equal to or greater than P.

The allowable external pressure is $P_a = 55.8\ psi$.

Therefore, in accordance with UG-33(a)(1),

$$MAEP = \min\left[135.3,\ 55.8\right] = 55.8\ psi$$

<u>Section VIII, Division 2 Solution</u>

Evaluate per VIII-2, paragraph 4.4.8 and 4.4.7.

The required thickness of a torispherical head subjected to external pressure loading shall be determined using the equations for a spherical shell in VIII-2, paragraph 4.4.7 by substituting the outside crown radius for R_o.

$$R_o = 72.0 + 0.625 = 72.625\ in$$

Restrictions on Torispherical Head Geometry – the restriction of VIII-2 paragraph 4.3.6 shall apply. See VIII-2 paragraph 4.3.6.1.b and STEP 2 of E4.3.4.

Torispherical heads With Different Dome and Knuckle Thickness – heads with this configuration shall be designed in accordance with VIII-2, Part 5. In this example problem, the dome and knuckle thickness are the same.

Proceed with the design following the steps outlined in VIII-2, paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the torispherical head.

$$t = 0.625 - Corrosion\ Allowance = 0.625 - 0.125 = 0.500\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075E_y\left(\frac{t}{R_o}\right) = 0.075(26.55E+06)\left(\frac{0.500}{72.625}\right) = 13709.1222\ psi$$

c) STEP 3 – Calculate the predicted buckling stress, F_{ic}.

$$\frac{F_{he}}{S_y} = \frac{13709.1222}{26900.0} = 0.5096$$

Since $\dfrac{F_{he}}{S_y} \le 0.55$, calculate F_{ic} as follows:

$$F_{ic} = F_{he} = 13709.1222\ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per VIII-2, paragraph 4.4.2.

$$0.55S_y = 0.55(26900.0) = 14795.0\ psi$$

Since $F_{ic} \le 0.55S_y$, calculate the FS as follows:

$$FS = 2.0$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(6854.5611)\left(\frac{0.500}{72.625}\right) = 94.4\ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{13709.1222}{2.0} = 6854.5611\ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 94.4\ psi$

Combined Loadings – torispherical heads subject to external pressure and other loadings shall satisfy the requirements of VIII-2, paragraph 4.4.12. In this example problem, the torispherical head is only subject to external pressure.

4.4.5 Example E4.4.5 - Elliptical Head

Determine the maximum allowable external pressure (MAEP) for a 2:1 elliptical head considering the following design conditions.

<u>Vessel Data:</u>

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Temperature = $300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$

<u>Section VIII, Division 1 Solution</u>

Per UG-33(a)(1), the required thickness for an ellipsoidal head having pressure on the convex side shall be the greater of: (a) the thickness computed by the procedure given in UG-32 for heads with pressure on the concave side using a design pressure 1.67 times the design pressure on the convex side, assuming joint efficiency $E=1.00$ for all cases; or (b) the thickness as computed by paragraph UG-33(d). In determining the MAEP on the convex side of an ellipsoidal head, reverse the procedures in UG-33(a)(1)(a) and (a)(1)(b) and use the smaller of the pressures obtained.

The rules of UG-32(d) can be used to evaluate ellipsoidal heads, however, the rules contained in this paragraph are only applicable for a specific geometry, i.e. half the minor axis (inside depth of head minus the skirt) equals one-fourth of the inside diameter of the head skirt. Additionally, if the ratio $t_s/L \geq 0.002$ is not satisfied, the rules of Mandatory Appendix 1-4(f) shall also be met. As an alternative, the general procedure for evaluating an ellipsoidal head can be performed using the rules of Mandatory Appendix 1-4(c).

Evaluate per Mandatory Appendix 1-4(c) and determine the MAEP. As shown in Example Problem, E4.3.5, the ellipsoidal head has an $MAEP=442.2\ psi$.

As noted in paragraph UG-33(d), the required thickness of an ellipsoidal head having pressure on the convex side shall be determined in the same manner as outlined in paragraph UG-28(d) for determining the thickness for a spherical shell, with the appropriate value of R_o.

a) STEP 1 – UG-28(d)(1), Assume a value for t and calculate the value of factor A using the following formula:

$$A=\frac{0.125}{\left(\frac{R_o}{t}\right)}=\frac{0.125}{\left(\frac{83.025}{1.0}\right)}=0.00151$$

$$R_o = K_o D_o = K_o\left(D + 2\left(Uncorroded\ Thickness\right)\right) = 0.9\left(90.0 + 2\left(1.125\right)\right) = 83.025\ in$$

$where,$

$K_o\ is\ taken\ from\ Table\ UG-33.1\ for\ a\ 2:1\ ellipse$

$t = t - Corrosion\ Allowance = 1.125 - 0.125 = 1.0\ in$

b) STEP 2 – Using the value of A calculated in STEP 1, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 5.

Per Section II Part D, Table 1A, a material specification of $SA-516-70N$ is assigned an External Pressure Chart No. CS-2.

c) STEP 3 – From the intersection obtained in Step 2, move horizontally to the right and read the value of factor B.

$$B = 13800$$

d) STEP 4 – Using the value of B obtained in STEP 3, calculate the value of the maximum allowable external working pressure P_a using the following formula:

$$P_a = \frac{B}{\left(\frac{R_o}{t}\right)} = \frac{13800}{\left(\frac{83.025}{1.0}\right)} = 166.2\ psi$$

e) STEP 5 – For values of A falling to the left of the applicable material/temperature line, the value of P_a can be calculated using the following formula:

$$P_a = \frac{0.0625E}{\left(\frac{R_o}{t}\right)^2} \qquad Not\ required$$

f) STEP 6 – Compare the calculated value of P_a obtained in STEPS 4 or 5 with P. If P_a is smaller than P, select a larger value of t and repeat the design procedure until a value of P_a is obtained that is equal to or greater than P.

The allowable external pressure is $P_a = 166.2\ psi$.

Therefore, in accordance with UG-33(a)(1),

$$MAEP = \min\left[442.2, 166.2\right] = 166.2\ psi$$

<u>Section VIII, Division 2 Solution</u>

Evaluate per VIII-2, paragraph 4.4.9 and 4.4.7.

The required thickness of an elliptical head subjected to external pressure loading shall be determined using the equations for a spherical shell in VIII-2, paragraph 4.4.7 by substituting $K_o D_o$ for R_o where K_o is given by the following equation.

$$K_o = 0.25346 + 0.13995\left(\frac{D_o}{2h_o}\right) + 0.12238\left(\frac{D_o}{2h_o}\right)^2 - 0.015297\left(\frac{D_o}{2h_o}\right)^3$$

$$K_o = \left(\begin{array}{l} 0.25346 + 0.13995\left(\dfrac{92.25}{2(23.0625)}\right) + 0.12238\left(\dfrac{92.25}{2(23.0625)}\right)^2 - \\ 0.015297\left(\dfrac{92.25}{2(23.0625)}\right)^3 \end{array}\right) = 0.9005$$

$$D_o = 90.0 + 2(1.125) = 92.25$$

$$h_o = \left(\frac{D_o}{4}\right) = \frac{92.25}{4} = 23.0625\ in$$

Therefore,

$$R_o = K_o D_o = 0.9005(92.25) = 83.0711\ in$$

Proceed with the design following the steps outlined in VIII-2, paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the spherical shell.

$$t = 1.125 - Corrosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075E_y\left(\frac{t}{R_o}\right) = 0.075(28.3E+06)\left(\frac{1.0}{83.0711}\right) = 25550.4020\ psi$$

c) STEP 3 – Calculate the predicted buckling stress, F_{ic}.

$$\frac{F_{he}}{S_y} = \frac{25550.4020}{33600} = 0.7604$$

Since $0.55 < \left(\frac{F_{he}}{S_y}\right) \le 1.6$, calculate F_{ic} as follows:

$$F_{ic} = 0.18F_{he} + 0.45S_y$$

$$F_{ic} = 0.18(25550.4020) + 0.45(33600.0) = 19719.0724\ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per VIII-2, paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0\ psi$$

Since $0.55S_y < F_{ic} < S_y$, calculate the FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{19719.0724}{33600.0}\right) = 1.9721$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(9999.0226)\left(\frac{1.0}{83.0711}\right) = 240.7 \ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{19719.0724}{1.9721} = 9999.0226 \ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 240.7\,psi$

Combined Loadings – ellipsoidal heads subject to external pressure and other loadings shall satisfy the requirements of VIII-2, paragraph 4.4.12. In this example problem, the ellipsoidal head is only subject to external pressure.

4.4.6 Example E4.4.6 - Combined Loadings and Allowable Compressive Stresses

Determine the allowable compressive stresses of the proposed cylindrical shell section considering the following design conditions and specified applied loadings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

<u>Vessel Data:</u>

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $-14.7\ psig\ @\ 300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Unsupported Length = $636.0\ in$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$
- Applied Axial Force = $-66152.5\ lbs$
- Applied Net Section Bending Moment = $3.048E+06\ in-lbs$
- Applied Shear Force = $11257.6\ lbs$

Adjust variables for corrosion and determine outside dimensions.

$$D = 90.0 + 2(Corrosion\ Allowance) = 90.0 + 2\left(0.125\right) = 90.25\ in$$
$$R = 0.5D = 0.5\left(90.25\right) = 45.125\ in$$
$$t = 1.125 - Corosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$
$$D_o = 90.0 + 2(Uncorroded\ \ Thickness) = 90.0 + 2\left(1.125\right) = 92.25\ in$$
$$R_o = 0.5D_o = 0.5\left(92.25\right) = 46.125\ in$$

Section VIII, Division 1 Solution

VIII-1 does not provide rules on the loadings to be considered in the design of a vessel. However, UG-22 requires consideration of such loadings and the provisions of U-2(g) apply. This example provides one possible method of satisfying U-2(g); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

This example uses VIII-2, paragraph 4.1 which provides specific requirements to account for both loads and load case combinations used in the design of a vessel. These loads and load case combinations (Table 4.1.1 and Table 4.1.2 of VIII-2, respectively) are shown in this example problem in Table E4.4.6.1 for reference.

Additionally, VIII-1 does not provide a procedure for the calculation of combined stresses. Paragraph 4.3.10.2, in VIII-2, does provide a procedure and this procedure is used in this example problem with modifications to address specific requirements of VIII-1.

In accordance with VIII-2, paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.4.6.2 and Table E4.4.6.3, Load Case 5 is determined to be the governing load case. The pressure, net section axial force, bending moment, and radial shear force at the location of interest for Load Case 5 are:

$$0.9P + P_s = -13.2\ \ psi$$
$$F_5 = -66152.5\ lbs$$
$$M_5 = 3048000\ in - lbs$$
$$V_5 = 11257.6\ lbs \rightarrow \quad The\ radial\ shear\ force\ is\ not\ addressed\ in\ VIII-1$$
$$and\ is\ not\ included\ in\ this\ example.$$

a) STEP 1 – Calculate the membrane stress for the cylindrical shell. Note that the circumferential membrane stress, $\sigma_{\theta m}$, is determined based on the equations in UG-27(c)(1) and the exact strength of materials solution for the longitudinal membrane stress, σ_{sm}, is used in place of the approximate solution provided in UG-27(c)(2). The shear stress is computed based on the known strength of materials solution.

Note: θ is defined as the angle measured around the circumference from the direction of the applied bending moment to the point under consideration. For this example problem $\theta = 0.0\ deg$ to maximize the bending stress.

$$\sigma_{\theta m} = \frac{1}{E}\left(\frac{PR}{t} + 0.6P\right) = \frac{1}{1.0}\left(\frac{-13.2\left(45.125\right)}{1.0} + 0.6\left(-13.2\right)\right) = -603.57\ \ psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\frac{PD^2}{D_o^2 - D^2} + \frac{4F}{\pi\left(D_o^2 - D^2\right)} \pm \frac{32MD_o \cos[\theta]}{\pi\left(D_o^4 - D^4\right)}\right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(\begin{array}{l}\dfrac{-13.2(90.25)^2}{\left((92.25)^2 - (90.25)^2\right)} + \dfrac{4(-66152.5)}{\pi\left((92.25)^2 - (90.25)^2\right)} \\ \pm\dfrac{32(3048000)(92.25)\cos[0.0]}{\pi\left((92.25)^4 - (90.25)^4\right)}\end{array}\right)$$

$$\sigma_{sm} = \left\{\begin{array}{l}-294.5611 - 230.7616 + 471.1299 = -54.1928 \ psi \\ -294.5611 - 230.7616 - 471.1299 = -996.4526 \ psi\end{array}\right\}$$

$$\tau = \frac{16M_tD_o}{\pi\left(D_o^4 - D^4\right)} = \frac{16(0.0)(92.25)}{\pi\left((92.25)^4 - (90.25)^4\right)} = 0.0 \ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4(\tau)^2}\right)$$

$$\sigma_1 = 0.5\left((-603.57) + (-996.4526) + \sqrt{\left((-603.57) - (-996.4526)\right)^2 + 4(0)^2}\right)$$

$$\sigma_1 = -603.57 \ psi$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2}\right)$$

$$\sigma_2 = 0.5\left((-603.57) + (-996.4526) - \sqrt{\left((-603.57) - (-996.4526)\right)^2 + 4(0)^2}\right)$$

$$\sigma_2 = -996.4526 \ psi$$

$$\sigma_3 = \sigma_r = -0.5P = -0.5(-13.2) = 6.6 \ psi$$

c) STEP 3 – Check the allowable stress acceptance criteria.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]^{0.5}$$

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\begin{array}{l}\left((-603.57) - (-996.4526)\right)^2 + \left((-996.4526) - (6.6)\right)^2 \\ +\left((6.6) - (-603.57)\right)^2\end{array}\right]^{0.5}$$

$$\sigma_e = 875.4367 \ psi$$

$$\left\{\sigma_e = 875.4 \ psi\right\} \le \left\{S = 20000 \ psi\right\} \qquad True$$

Note that VIII-2 uses an acceptance criteria based on von Mises Stress. VIII-1 typically uses the maximum principle stress in the acceptance criteria. Therefore,

$$\max\left[\sigma_1,\, \sigma_2,\, \sigma_3\right] \le S$$

$$\left\{ \max\left[\left|-603.6\right|,\, \left|-996.5\right|,\, 6.6\right] = 996.5\ psi \right\} \le \left\{ S = 20000\ psi \right\} \qquad True$$

d) STEP 4 – For cylindrical and conical shells, if the meridional stress, σ_{sm} is compressive, then check the allowable compressive stress per UG-23(b).

Since σ_{sm} is compressive, $\left\{\sigma_{sm} = -996.5\ psi < 0\right\}$, a compressive stress check is required.

Evaluate per paragraph UG-23(b). The maximum allowable longitudinal compressive stress to be used in the design of cylindrical shells or tubes, either seamless or butt welded, subjected to loadings that produce longitudinal compression in the shell or tube shall be the smaller of the maximum allowable tensile stress value shown in STEP 3 or the value of the factor B determined by the following procedure where the joint efficiency for butt welded joints shall be taken as unity.

1) STEP 4.1 – Using the selected values of t and R, calculate the value of factor A using the following formula:

$$A = \frac{0.125}{\dfrac{R_o}{t}} = \frac{0.125}{\left(\dfrac{46.125}{1.0}\right)} = 0.00271$$

2) STEP 4.2 – Using the value of A calculated in STEP 4.1, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the design temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 4.4.

   Per Section II Part D, Table 1A, a material specification of $SA-516-70N$ is assigned an External Pressure Chart No. CS-2.

3) STEP 4.3 – From the intersection obtained in Step 4.2, move horizontally to the right and read the value of factor B. This is the maximum allowable compressive stress for the values of t and R_o used in STEP 4.1.

$$B = 15800$$

4) STEP 4.4 – For values of A falling to the left of the applicable material/temperature line, the value of B shall be calculated using the following formula:

$$B = \frac{AE}{2} \qquad Not\ required$$

5) STEP 4.5 – Compare the calculated value of B obtained in STEPS 4.3 or 4.4 with the computed longitudinal compressive stress in the cylindrical shell or tube, using the selected values of t and R_o. If the value of B is smaller than the computed compressive stress, a greater value of t must be selected and the design procedure repeated until a value of B is obtained that is greater than the compressive stress computed for the loading on the cylindrical shell or tube.

$$\left\{\sigma_{sm} = \left|-996.5\right| \ psi\right\} \le \left\{B = 15800 \ psi\right\} \qquad True$$

The allowable compressive stress criterion is satisfied.

Section VIII, Division 2 Solution

Evaluate per VIII-2, paragraph 4.4.12.2

The loads transmitted to the cylindrical shell are given in the Table E4.4.6.2. Note that this table is given in terms of the load parameters shown in VIII-2, Table 4.1.1 and Table 4.1.2. (Table E4.4.6.1 of this example). As shown in Table E4.4.6.1, the acceptance criteria is that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition.

In accordance with VIII-2, paragraph 4.4.12.2, the following procedure shall be used to determine the allowable compressive stresses for cylindrical shells that are based on loading conditions. By inspection of the results shown in Table E4.4.6.2 and Table E4.4.6.3, Load Case 5 is determined to be the governing load case. The pressure, net section axial force, bending moment, and radial shear force at the location of interest for Load Case 5 are:

$$0.9P + P_s = -14.7 \ psi \qquad (Conservatively)$$

$$F_5 = -66152.5 \ lbs$$

$$M_5 = 3048000 \ in-lbs$$

$$V_5 = 11257.6 \ lbs$$

Common parameters used in each of the loading conditions are given in VIII-2, paragraph 4.4.12.2.k.

Per VIII-2, paragraph 4.4.12.2.k:

$$A = \frac{\pi\left(D_o^2 - D_i^2\right)}{4} = \frac{\pi\left(92.25^2 - 90.25^2\right)}{4} = 286.6703 \ in^2$$

$$S = \frac{\pi\left(D_o^4 - D_i^4\right)}{32D_o} = \frac{\pi\left(92.25^4 - 90.25^4\right)}{32(92.25)} = 6469.5531 \ in^3$$

$$f_h = \frac{PD_o}{2t} = \frac{14.7(92.25)}{2(1.0)} = 678.0375 \ psi$$

$$f_b = \frac{M}{S} = \frac{3.048E+06}{6469.5531} = 471.1299 \ psi$$

$$f_a = \frac{F}{A} = \frac{66152.5}{286.6703} = 230.7616 \ psi$$

$$f_q = \frac{P\pi D_i^2}{4A} = \frac{14.7(\pi)(90.25)^2}{4(286.6703)} = 328.0341 \ psi$$

$$f_v = \frac{V\sin[\phi]}{A} = \frac{11257.6\sin[90.0]}{286.6703} = 39.2702 \ psi$$

Note: ϕ is defined as the angle measured around the circumference from the direction of the applied shear force to the point under consideration. For this example problem, $\phi = 90^{\circ}$ to maximize the shear force.

$$r_g = 0.25\sqrt{D_o^{\,2} + D_i^2} = 0.25\sqrt{\left(92.25^2 + 90.25^2\right)} = 32.2637\ in$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{(46.125)1.0}} = 93.6459$$

The value of the slenderness factor for column buckling, λ_c is calculated in VIII-2, paragraph 4.4.12.2.b.

Per VIII-2, paragraph 4.4.12.2:

a) External Pressure Acting Alone, (paragraph 4.4.12.2.a) – the allowable hoop compressive membrane stress of a cylinder subject to external pressure acting alone, F_{ha}, is computed using the equations in VIII-2, paragraph 4.4.5.

From Example E4.4.1,

$$F_{ha} = 2257.8645\ psi$$

b) Axial Compressive Stress Acting Alone, (paragraph 4.4.12.2.b) – the allowable axial compressive membrane stress of a cylinder subject to an axial compressive load acting alone, F_{xa}, is computed using the following equations:

The value of the slenderness factor for column buckling, λ_c is dependent on the calculated value of F_{xa}, defined as the allowable compressive membrane stress of a cylinder due to an axial compressive load, with $\lambda_c \le 0.15$. The value of λ_c determines the procedure to be used in obtaining the allowable axial compressive stress, either due to local buckling, $\lambda_c \le 0.15$, or column buckling, $\lambda_c > 0.15$. Therefore, an initial calculation is required to determine the value of F_{xa} with an assumed value of $\lambda_c \le 0.15$. The actual value of λ_c is then calculated and the procedure to obtain the allowable axial compressive stress is determined.

The design factor FS used in VIII-2, paragraph 4.4.12.2.b is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in VIII-2, paragraph 4.4.2. An initial calculation is required to determine the value of F_{xa} by setting $FS = 1.0$, with $F_{ic} = F_{xa}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.b.

1) For $\lambda_c \le 0.15$, (Local Buckling)

$$F_{xa} = \min\left[F_{xa1},\ F_{xa2}\right]$$

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

Since $\dfrac{D_o}{t} \le 135$, calculate F_{xa1} as follows with an initial value of $FS = 1.0$.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.0} = 33600 \ psi$$

The value of F_{xa2} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{xa2} = \frac{F_{xe}}{FS}$$

$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since $\dfrac{D_o}{t} \le 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \dfrac{D_o}{t}\right)},\ 0.9\right]$$

Since $M_x \ge 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389 + \dfrac{92.25}{1.0}},\ 0.9\right] = 0.8499$$

Therefore,

$$F_{xe} = \frac{0.8499(28.3E+06)(1.0)}{92.25} = 260728.1301 \ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{260728.1301}{1.0} = 260728.1301 \ psi$$

$$F_{xa} = \min[33600,\ 260728] = 33600 \ psi$$

With a value of $F_{ic} = F_{xa} = 33600$, in accordance with VIII-2, paragraph 4.4.2, it is determined the value of $FS = 1.667$ since $\{F_{ic} = 33600\} = \{S_y = 33600\}$. Using this computed value of $FS = 1.667$ in paragraph 4.4.12.2.b, F_{xa} is calculated as follows.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$
$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{260728.1301}{1.6670} = 156405.5969 \ psi$$
$$F_{xa} = \min[20155.9688, 156405.5969] = 20155.9688 \ psi$$

With F_{xa} calculated, determine the value of λ_c from paragraph 4.4.12.2.k. For a cylinder with end conditions with one end free and the other end fixed, $K_u = 2.1$.

$$\lambda_c = \frac{K_u L_u}{\pi r_g}\left(\frac{F_{xa} \cdot FS}{E_y}\right)^{0.5} = \frac{2.1(636.0)}{\pi(32.2637)}\left(\frac{20155.9688(1.667)}{28.3E+06}\right)^{0.5} = 0.4540$$

Since $\lambda_c > 0.15$, the allowable axial compressive membrane stress of the cylinder is due to Column Buckling, per VIII-2, paragraph 4.4.12.2.b.2.

2) For $\lambda_c > 0.15$ and $\frac{K_u L_u}{r_g} < 200$ (Column Buckling)

$$\left\{\frac{K_u L_u}{r_g} = \frac{2.1(636.0)}{32.2637} = 41.3964\right\} < \{200\} \qquad \mathit{True}$$

Since $0.15 < \lambda_c < 1.147$, calculate F_{ca} as follows:

$$F_{ca} = F_{xa}\left[1 - 0.74(\lambda_c - 0.15)\right]^{0.3}$$
$$F_{ca} = 20155.9688\left[1 - 0.74(0.4540 - 0.15)\right]^{0.3} = 18672.4331 \ psi$$

c) <u>Compressive Bending Stress, (paragraph 4.4.12.2.c)</u> – the allowable axial compressive membrane stress of a cylindrical shell subject to a bending moment acting across the full circular cross section, F_{ba}, is computed using the following equations.

Similar to the procedure used in paragraph 4.4.12.2.b, the design factor FS used in paragraph 4.4.12.2.c is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in VIII-2, paragraph 4.4.2. An initial calculation is required to determine the value of F_{ba} by setting $FS = 1.0$, with $F_{ic} = F_{ba}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.c.

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

$$\gamma = \frac{S_y D_o}{E_y t} = \frac{33600(92.25)}{(28.3E+06)(1.0)} = 0.1095$$

Since $\dfrac{D_o}{t} \le 100$ and $\gamma < 0.11$, calculate F_{ba} as follows with an initial value of $FS = 1.0$:

$$F_{ba} = \frac{S_y(1.4 - 2.9\gamma)}{FS} = \frac{33600(1.4 - 2.9(0.1095))}{1.0} = 36370.32\ psi$$

With a value of $F_{ic} = F_{ba} = 36370.32$, in accordance with VIII-2, paragraph 4.4.2, it is determined the value of $FS = 1.667$ since $\{F_{ic} = 36370.32\} \ge \{S_y = 33600\}$. Using this computed value of $FS = 1.667$ in paragraph 4.4.12.2.c, F_{ba} is calculated as follows.

$$F_{ba} = \frac{S_y(1.4 - 2.9\gamma)}{FS} = \frac{33600(1.4 - 2.9(0.1095))}{1.667} = 21817.8284\ psi$$

d) Shear Stress, (paragraph 4.4.12.2.d) – the allowable shear stress of a cylindrical shell, F_{va}, is computed using the following equations:

Similar to the procedure used in paragraph 4.4.12.2.b, the design factor FS used in paragraph 4.4.12.2.d is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in VIII-2, paragraph 4.4.2. An initial calculation is required to determine the value of F_{va} by setting $FS = 1.0$, with $F_{ic} = F_{va}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.d.

The value of F_{va} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{va} = \frac{\eta_v F_{ve}}{FS}$$

$$F_{ve} = \alpha_v C_v E_y \left(\frac{t}{D_o}\right)$$

For a value of $M_x = 93.6459$,

$$4.347\left(\frac{D_o}{t}\right) = 4.347\left(\frac{95.25}{1.0}\right) = 401.0108$$

Since $26 < M_x < 4.347\left(\dfrac{D_o}{t}\right)$, calculate C_v as follows:

$$C_v = \frac{1.492}{M_x^{0.5}} = \frac{1.492}{(93.6459)^{0.5}} = 0.1542$$

Since $\left(\dfrac{D_o}{t}\right) \le 500$, calculate α_v as follows:

$$\alpha_v = 0.8$$

It follows then,

$$F_{ve} = 0.8(0.1542)(28.3E+06)\left(\frac{1.0}{92.25}\right) = 37843.7724 \ psi$$

$$\frac{F_{ve}}{S_y} = \frac{37843.7724}{33600} = 1.1263$$

Since $0.48 < \left(\frac{F_{ve}}{S_y}\right) < 1.7$, calculate η_v as follows:

$$\eta_v = 0.43\left(\frac{S_y}{F_{ve}}\right) + 0.1 = 0.43\left(\frac{33600}{37843.7724}\right) + 0.1 = 0.4818$$

Therefore,

$$F_{va} = \frac{0.4818(37843.7724)}{1.0} = 18233.1295 \ psi$$

With a value of $F_{ic} = F_{va} = 18233.1295$, in accordance with VIII-2, paragraph 4.4.2, it is determined the value of $FS = 2.0$ since $\{F_{ic} = 18233.1295\} \le \{0.55S_y = 18480\}$. Using this computed value of $FS = 2.0$ in paragraph 4.4.12.2.d, F_{va} is calculated as follows.

$$F_{va} = \frac{0.4818(37843.7724)}{2.0} = 9116.5648 \, psi$$

e) Axial Compressive Stress and Hoop Compression, (paragraph 4.4.12.2.e) – the allowable compressive stress for the combination of uniform axial compression and hoop compression, F_{xha}, is computed using the following equations:

1) For $\lambda_c \le 0.15$, F_{xha} is computed using the following equation with F_{ha} and F_{xa} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.a and 4.4.12.2.b.1, respectively.

Although, $0.15 < \lambda_c \le 1.2$, the procedure in VIII-2, paragraph 4.4.12.2.e.1 to calculate F_{xha} is required per VIII-2, paragraph 4.4.12.2.e.2 with the modifications noted, (see next step in procedure).

$$F_{xha} = \left[\left(\frac{1}{F_{xa}^2}\right) - \left(\frac{C_1}{C_2 F_{xa} F_{ha}}\right) + \left(\frac{1}{C_2^2 F_{ha}^2}\right)\right]^{-0.5}$$

$$F_{xha} = \left[\begin{array}{l} \left(\frac{1}{(20155.9688)^2}\right) - \\ \left(\frac{0.1344}{0.8241(20155.9688)(2257.8645)}\right) + \\ \left(\frac{1}{(0.8241)^2 (2257.8645)^2}\right) \end{array}\right]^{-0.5} = 1864.3312\ psi$$

Where,

$$C_1 = \frac{(F_{xa} \cdot FS + F_{ha} \cdot FS)}{S_y} - 1.0 = \frac{20155.9688(1.667) + 2257.8645(2.0)}{33600} - 1.0$$

$$C_1 = 0.1344$$

$$C_2 = \frac{f_x}{f_h} = \frac{558.7957}{678.0375} = 0.8241$$

$$f_x = f_a + f_q = 230.7616 + 328.0341 = 558.7957\ psi$$

2) For $0.15 < \lambda_c \leq 1.2$, F_{xha}, is computed from the following equation with $F_{ah1} = F_{xha}$ evaluated using the equations in VIII-2, paragraph 4.4.12.2.e.1, and F_{ah2} using the following procedure. The value of F_{ca} used in the calculation for F_{ah2} is evaluated using the equation in VIII-2, paragraph 4.4.12.2.b.2 with $F_{xa} = F_{xha}$ as determined in VIII-2, paragraph 4.4.12.2.e.1. As noted, the load on the end of a cylinder due to external pressure does not contribute to column buckling and therefore F_{ah1} is compared with f_a rather than f_x. The stress due to the pressure load does, however, lower the effective yield stress and the quantity in $(1 - f_q / S_y)$ accounts for this reduction.

$$F_{xha} = \min[F_{ah1}, F_{ah2}] = \min[1864.3312, 1710.2496] = 1710.2496\ psi$$

$$F_{ah1} = F_{xha} = 1864.3312\ psi$$

$$F_{ah2} = F_{ca}\left(1 - \frac{f_q}{S_y}\right) = 1727.1112\left(1 - \frac{328.0341}{33600}\right) = 1710.2496\ psi$$

Where,

$$F_{ca} = F_{xha}\left[1 - 0.74(\lambda_c - 0.15)\right]^{0.3}$$

$$F_{ca} = 1864.3312\left[1 - 0.74(0.4540 - 0.15)\right]^{0.3} = 1727.1112\ psi$$

3) For $\lambda_c \leq 0.15$, the allowable hoop compressive membrane stress, F_{hxa}, is given by the

following equation.

$$F_{hxa} = \frac{F_{xha}}{C_2}$$

Note: this step is not required since $\lambda_c > 0.15$.

f) <u>Compressive Bending Stress and Hoop Compression, (paragraph 4.4.12.2.f)</u> – the allowable compressive stress for the combination of axial compression due to a bending moment and hoop compression, F_{bha}, is computed using the following equations.

1) An iterative solution procedure is utilized to solve these equations for C_3 with F_{ha} and F_{ba} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.a and 4.4.12.2.c, respectively.

$$F_{bha} = C_3 C_4 F_{ba} = 0.9945(0.0719)(21817.8284) = 1560.0740\ psi$$

Where,

$$C_4 = \left(\frac{f_b}{f_h}\right)\left(\frac{F_{ha}}{F_{ba}}\right) = \left(\frac{471.1299}{678.0375}\right)\left(\frac{2257.8645}{21817.8284}\right) = 0.0719$$

$$C_3^2\left(C_4^2 + 0.6C_4\right) + C_3^{2n} - 1 = 0$$

$$n = 5 - \frac{4F_{ha} \cdot FS}{S_y} = 5 - \frac{4(2257.8645)(2.0)}{33600} = 4.4624$$

1st attempt at solving for C_3, using an interval halving approach, with an initial guess at C_3 as follows:

$$C_3 = \frac{Upper\ Bound + Lower\ Bound}{2} = \frac{1.0 + 0.0}{2} = 0.5$$

The following results are obtained:

$$0.5^2\left((0.0719)^2 + 0.6(0.0719)\right) + 0.5^{2(4.4624)} - 1 = -0.9859$$

2nd attempt at solving for C_3, with an second guess of C_3 as follows:

$$C_3 = \frac{1.0 + 0.5}{2} = 0.75$$

The following results are obtained:

$$0.75^2\left((0.0719)^2 + 0.6(0.0719)\right) + 0.75^{2(4.4624)} - 1 = -0.8961$$

Successive iterations are performed at solving for C_3 until the following value is obtained.

$$C_3 = 0.9945$$

The following results are obtained which satisfy the equation within a tolerance of ± 0.001:

$$0.9945^2\left(\left(0.0719\right)^2+0.6\left(0.0719\right)\right)+0.9945^{2(4.4624)}-1=-0.0003$$

2) The allowable hoop compressive membrane stress, F_{hba}, is given by the following equation.

$$F_{hba}=F_{bha}\left(\frac{f_h}{f_b}\right)=1560.0740\left(\frac{678.0375}{471.1299}\right)=2245.2166\ psi$$

g) Shear Stress and Hoop Compression, (paragraph 4.4.12.2.g) – the allowable compressive stress for the combination of shear, F_{vha}, and hoop compression is computed using the following equations.

Note: This load combination is only applicable for shear stress and hoop compression, in the absence of axial compressive stress and compressive bending stress. It is shown in this example problem for informational purposes only. The effect of shear is accounted for in the interaction equations of paragraphs 4.4.12.2.h and 4.4.12.2.i through the variable K_s.

1) The allowable shear stress is given by the following equation with F_{ha} and F_{va} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.a and 4.4.12.2.d, respectively.

$$F_{vha}=\left[\left(\frac{F_{va}^2}{2C_5F_{ha}}\right)^2+F_{va}^2\right]^{0.5}-\frac{F_{va}^2}{2C_5F_{ha}}$$

$$F_{vha}=\left[\left(\frac{(9116.5648)^2}{2(0.0579)(2257.8645)}\right)^2+(9116.5648)^2\right]^{0.5}-\left[\frac{(9116.5648)^2}{2(0.0579)(2257.8645)}\right]$$

$$F_{vha}=130.7035\ psi$$

Where,

$$C_5=\frac{f_v}{f_h}=\frac{39.2702}{678.0375}=0.0579$$

2) The allowable hoop compressive membrane stress, F_{hva}, is given by the following equation.

$$F_{hva}=\frac{F_{vha}}{C_5}=\frac{130.7035}{0.0579}=2257.4007\ psi$$

h) Axial Compressive Stress, Compressive Bending Stress, Shear Stress, and Hoop Compression, (paragraph 4.4.12.2.h) – the allowable compressive stress for the combination of uniform axial compression, axial compression due to a bending moment, and shear in the presence of hoop compression is computed using the following interaction equations.

1) The shear coefficient is determined using the following equation with F_{va} from VIII-2, paragraph 4.4.12.2.d.

$$K_s=1.0-\left(\frac{f_v}{F_{va}}\right)^2=1.0-\left(\frac{39.2702}{9116.5648}\right)^2=0.9999$$

2) For $\lambda_c \leq 0.15$ the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{xha} and F_{bha} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.e.1 and 4.4.12.2.f.1, respectively.

$$\left(\frac{f_a}{K_s F_{xha}}\right)^{1.7} + \left(\frac{f_b}{K_s F_{bha}}\right) \leq 1.0$$

Note: this step is not required since $\lambda_c > 0.15$.

3) For $0.15 < \lambda_c \leq 1.2$ the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{xha} and F_{bha} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.e.2 and 4.4.12.2.f.1, respectively.

$$\frac{f_a}{K_s F_{xha}} = \frac{230.7616}{0.9999(1710.2496)} = 0.1349$$

Since $\dfrac{f_a}{K_s F_{xha}} \geq 0.2$, the following equation shall be used:

$$\left(\frac{f_a}{K_s F_{xha}}\right) + \left(\frac{8}{9} \cdot \frac{\Delta f_b}{K_s F_{bha}}\right) \leq 1.0$$

$$\left\{\left(\frac{230.7616}{0.9999(1710.2496)}\right) + \left(\frac{8}{9} \cdot \frac{1.0024(471.1299)}{0.9999(1560.0740)}\right) = 0.4041\right\} \leq \{1.0\} \qquad True$$

Where,

$$\Delta = \frac{C_m}{1 - \left(\dfrac{f_a \cdot FS}{F_e}\right)} = \frac{1.0}{1 - \left(\dfrac{230.7616(1.667)}{162990.2785}\right)} = 1.0024$$

$$F_e = \frac{\pi^2 E_y}{\left(\dfrac{K_u L_u}{r_g}\right)^2} = \frac{\pi^2\left(28.3(10)^6\right)}{\left(\dfrac{2.1(636.0)}{32.2637}\right)^2} = 162990.2785 \ psi$$

Note: $C_m = 1.0$ for unbraced skirt supported vessels, see paragraph 4.4.15.

i) <u>Axial Compressive Stress, Compressive Bending Stress, and Shear Stress, (paragraph 4.4.12.2.i)</u> – the allowable compressive stress for the combination of uniform axial compression, axial compression due to a bending moment, and shear in the absence of hoop compression is computed using the following interaction equations.

1) The shear coefficient is determined using the equation in VIII-2, paragraph 4.4.12.2.h.1 with F_{va} from paragraph 4.4.12.2.d.

$$K_s = 1.0 - \left(\frac{f_v}{F_{va}}\right)^2 = 1.0 - \left(\frac{39.2702}{9116.5648}\right)^2 = 0.9999$$

2) For $\lambda_c \leq 0.15$ the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{xa} and F_{ba} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.b.1 and 4.4.12.2.c, respectively.

$$\left(\frac{f_a}{K_s F_{xa}}\right)^{1.7} + \left(\frac{f_b}{K_s F_{ba}}\right) \leq 1.0$$

Note: this step is not required since $\lambda_c > 0.15$.

3) For $0.15 < \lambda_c \leq 1.2$ the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{ca} and F_{ba} evaluated using the equations in VIII-2, paragraphs 4.4.12.2.b.2 and 4.4.12.2.c, respectively. The coefficient Δ is evaluated using the equations in VIII-2, paragraph 4.4.12.2.h.3.

$$\frac{f_a}{K_s F_{ca}} = \frac{230.7616}{0.9999(18672.4331)} = 0.0124$$

Since $\frac{f_a}{K_s F_{ca}} < 0.2$, the following equation shall be used:

$$\left(\frac{f_a}{2K_s F_{ca}}\right) + \left(\frac{\Delta f_b}{K_s F_{ba}}\right) \leq 1.0$$

$$\left\{\left(\frac{230.7616}{2(0.9999)(18672.4331)}\right) + \left(\frac{1.0024(471.1299)}{0.9999(21817.8284)}\right) = 0.0278\right\} \leq \{1.0\} \quad True$$

From VIII-2, paragraph 4.4.12.2.h.3:

$$\Delta = 1.0024$$

$$F_e = 162990.2785 \ psi$$

j) (paragraph 4.4.12.2.j) The maximum deviation , e, may exceed the value e_x given in VIII-2, paragraph 4.4.4.2 if the maximum axial stress is less than F_{xa} for shells designed for axial compression only, or less than F_{xha} for shells designed for combinations of axial compression and external pressure. The change in buckling stress, F'_{xe}, is given by VIII-2, Equation (4.4.114). The reduced allowable buckling stress, $F_{xa(reduced)}$, is determined using VIII-2, Equation (4.4.115) where e is the new maximum deviation, F_{xa} is determined using VIII-2, Equation 4.4.61, and FS_{xa} is the value of the stress reduction factor used to determine F_{xa}.

$$F_{xe}^{'} = \left(0.944 - \left|0.286\log\left[\frac{0.0005e}{e_x}\right]\right|\right)\left(\frac{E_y t}{R}\right)$$

$$F_{xe}^{'} = \left(0.944 - \left|0.286\log\left[\frac{0.0005(0.2501)}{(0.0913)}\right]\right|\right)\left(\frac{(28.3E+06)(1.0)}{46.125}\right) = 76737.5098 \ psi$$

$$F_{xa(reduced)} = \frac{F_{xa} \cdot FS_{xa} - F_{xe}^{'}}{FS_{xa}}$$

$$F_{xa(reduced)} = \frac{20155.9688(1.667) - (76737.5098)}{1.667} = -25877.3304 \ psi$$

From VIII-2, paragraph 4.4.4.1, assuming the measurements are taken using the outside radius:

$$e = \min[e_c,\ 2t] = \min\left[0.2501,\ 2(1.0)\right] = 0.2501 \ in$$

$$e_c = 0.0165t\left(\frac{L_{ec}}{\sqrt{Rt}} + 3.25\right)^{1\ 069} = 0.0165(1.0)\left(\frac{64.3134}{\sqrt{46.125(1.0)}} + 3.25\right)^{1\ 069} = 0.2501 \ in$$

$$L_{ec} = 2R\sin\left[\frac{\pi}{2n}\right] = 2(46.125)\sin\left(\frac{\pi}{2(2.0362)}\right) = 64.3134 \ in$$

$$n = \xi\left(\sqrt{\frac{R}{t}} \cdot \left(\frac{R}{L}\right)\right)^{\psi} = (2.80)\left(\sqrt{\frac{46.125}{1.0}}\left(\frac{46.125}{636.0}\right)\right)^{0\ 4498} = 2.0362$$

$$\xi = \min\left[2.28\left(\frac{R}{t}\right)^{0\ 54}, 2.80\right] = \min\left[2.28\left(\frac{45.625}{1.0}\right)^{0\ 54}, 2.80\right]$$

$$\xi = \min\left[18.05,\ 2.80\right]$$

$$\xi = 2.80$$

$$\psi = \min\left[0.38\left(\frac{R}{t}\right)^{0\ 044}, 0.485\right] = \min\left[0.38\left(\frac{46.125}{1.0}\right)^{0\ 044}, 0.485\right]$$

$$\psi = \min\left[0.4498, 0.485\right]$$

$$\psi = 0.4498$$

From VIII-2, paragraph 4.4.4.2:

$$e_x = 0.002R_m = 0.002(45.625) = 0.0913 \ in$$

$$R_m = \frac{(D_o + D_i)}{4} = \frac{(92.25 + 90.25)}{4} = 45.625 \ in$$

A summary of the allowable compressive stresses are as follows:

Paragraph 4.4.12.2.a, External Pressure Acting Alone

$$F_{ha} = 2257.8645 \ psi$$

Paragraph 4.4.12.2.b, Axial Compressive Stress Acting Alone

$$F_{xa} = 20155.9688 \ psi$$

$$F_{ca} = 18672.4331 \ psi$$

Paragraph 4.4.12.2.c, Compressive Bending Stress

$$F_{ba} = 21817.8284 \ psi$$

Paragraph 4.4.12.2.d, Shear Stress

$$F_{va} = 9116.5648 \ psi$$

Paragraph 4.4.12.2.e, Axial Compressive Stress and Hoop Compression

$$F_{xha} = 1710.2496 \ psi$$

Paragraph 4.4.12.2.f, Compressive Bending Stress and Hoop Compression

$$F_{bha} = 1560.0740 \ psi$$

$$F_{hba} = 2245.2166 \ psi$$

Paragraph 4.4.12.2.g, Shear Stress and Hoop Compression

$$F_{vha} = 130.7035 \ psi$$

$$F_{hva} = 2257.4007 \ psi$$

Table E4.4.6.1: Design Loads and Load Combinations from VIII-2

Table 4.1.1 – Design Loads

Design Load Parameter	Description
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g. catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g. skirts, lugs, saddles, and legs), and appurtenances (e.g. platforms, ladders, etc.) • Weight of vessel contents under operating and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping
L	• Appurtenance Live loading • Effects of fluid flow
E	Earthquake loads (see ASCE 7 for the specific definition of the earthquake load, as applicable)
W	Wind Loads
S	Snow Loads
F	Loads due to Deflagration

Table 4.1.2 – Design Load Combinations

Design Load Combination (1)	General Primary Membrane Allowable Stress (2)
$P+P_s+D$	S
$P+P_s+D+L$	S
$P+P_s+D+S$	S
$0.9P+P_s+D+0.75L+0.75S$	S
$0.9P+P_s+D+(W\ or\ 0.7E)$	S
$0.9P+P_S+D+0.75(W\ or\ 0.7E)+0.75L+0.75S$	S
$0.6D+(W\ or\ 0.7E)$ (3)	S
P_s+D+F	See Annex 4.D

Notes
1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) S is the allowable stress for the load case combination (see paragraph 4.1.5.3.c)
3) This load combination addresses an overturning condition. If anchorage is included in the design, consideration of this load combination is not required.

Table E4.4.6.2: Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	$P = -14.7$
P_s	Static head from liquid or bulk materials (e.g. catalyst)	$P_s = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -66152.5\ lbs$ $D_M = 0.0\ in-lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = 0.0\ lbs$ $L_M = 0.0\ in-lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 0.0\ in-lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 3.048E+06\ in-lbs$ $W_V = 11257.6\ lbs$
S	Snow Loads	$S_F = 0.0\ lbs$ $S_M = 0.0\ in-lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in-lbs$

Based on these loads, the shell is required to be designed for the load case combinations shown in Table E4.4.6.3. Note that this table is given in terms of the load combinations shown in VIII-2, Table 4.1.2 (Table E4.4.6.1 of this example).

Table E4.4.6.3 – Load Case Combination at the Location of Interest

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P+P_s+D$	$P+P_s=-14.7\ psi$ $F_1=-66152.5\ lbs$ $M_1=0.0\ in-lbs$	S
2	$P+P_s+D+L$	$P+P_s=-14.7\ psi$ $F_2=-66152.5\ lbs$ $M_2=0.0\ in-lbs$	S
3	$P+P_s+D+S$	$P+P_s=-14.7\ psi$ $F_3=-66152.5\ lbs$ $M_3=0.0\ in-lbs$	S
4	$0.9P+P_s+D+0.75L+0.75S$	$0.9P+P_s=-13.2\ psi$ $F_4=-66152.5\ lbs$ $M_4=0.0\ in-lbs$	S
5	$0.9P+P_s+D+(W\ or\ 0.7E)$	$0.9P+P_s=-13.2\ psi$ $F_5=-66152.5\ lbs$ $M_5=3048000\ in-lbs$ $V_5=11257.6\ lbs$	S
6	$\left(\begin{array}{l}0.9P+P_S+D+0.75(W\ or\ 0.7E)+\\0.75L+0.75S\end{array}\right)$	$0.9P+P_s=-13.2\ psi$ $F_6=-66152.5\ lbs$ $M_6=2286000\ in-lbs$ $V_6=8443.2\ lbs$	S
7	$0.6D+(W\ or\ 0.7E)$ Anchorage is included in the design. Therefore, consideration of this load combination is not required.	$F_6=-39691.5\ lbs$ $M_6=3048000\ in-lbs$	S
8	P_s+D+F	$P_s=0.0\ psi$ $F_8=-66152.5\ lbs$ $M_8=0.0\ in-lbs$	See Annex 4.D

4.4.7 Example E4.4.7 - Conical Transitions without a Knuckle

Determine if the proposed large and small end cylinder-to-cone transitions are adequately designed considering the following design conditions and applied forces and moments.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $-14.7\ psig @ 300°F$
- Inside Radius (Large End) = $75.0\ in$
- Thickness (Large End) = $1.8125\ in$
- Inside Radius (Small End) = $45.0\ in$
- Thickness (Small End) = $1.125\ in$
- Thickness (Conical Section) = $1.9375\ in$
- Length of Conical Section = $78.0\ in$
- Unsupported Length of Large Cylinder = $732.0\ in$
- Unsupported Length of Small Cylinder = $636.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Yield Strength = $33600\ psi$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle (See E4.3.2) = $21.0375\ deg$
- Axial Force (Large End) = $-99167\ lbs$
- Net Section Bending Moment (Large End) = $5.406E+06\ in-lbs$
- Axial Force (Small End) = $-78104\ lbs$
- Net Section Bending Moment (Small End) = $4.301E+06\ in-lbs$

Adjust variables for corrosion and determine outside dimensions.

$$t_L = 1.8125 - Corrosion\ \ Allowance = 1.8125 - 0.125 = 1.6875\ in$$

$$t_S = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

$$t_C = 1.9375 - Corrosion\ \ Allowance = 1.9375 - 0.125 = 1.8125\ in$$

$$R_L = 75.0 + Uncorroded\ \ Thickness = 75.0 + 1.8125 = 76.8125\ in$$

$$R_S = 45.0 + Uncorroded\ \ Thickness = 45.0 + 1.125 = 46.125\ in$$

$$D_L = 2R_L = 2(76.8125) = 153.625\ in$$

$$D_S = 2R_S = 2(46.125) = 92.25\ in$$

Section VIII, Division 1 Solution

Evaluate per paragraph UG-33(f): Conical heads and Sections. When the cone-to-cylinder junction is not a line of support, the required thickness of a conical head or section under pressure on the convex side, either seamless or of built-up construction with butt joints shall not be less than the required thickness of the adjacent cylindrical shell and, when a knuckle is not provided, the reinforcement requirement of Appendix 1-8 shall be satisfied. When the cone-to-cylinder junction is a line of support the required thickness shall be determined in accordance with the following procedure.

For this example, it is assumed that the cone-to-cylinder junction is a line of support.

Rules for conical reducer sections subject to external pressure are covered in Appendix 1-8. Rules are provided for the design of reinforcement, if needed, and for verification of adequate moment of inertia, when the cone-to-cylinder junction is a line of support, for conical reducer sections and conical heads where all the elements have a common axis and the half-apex angle satisfies $\alpha \le 60\ deg$.

Large End

In accordance with Appendix 1-8(b), reinforcement shall be provided at the large end of the cone when required by (b)(1) or (b)(2). When the large end of the cone is considered a line of support, the moment of inertia for a stiffening ring shall be determined in accordance with (b)(3).

Appendix 1-8(b)(1), for cones attached to a cylinder having a minimum length of $2.0\sqrt{R_L t_s}$, reinforcement shall be provided at the junction of the cone with the large cylinder for conical heads and reducers without knuckles when the value of Δ obtained from Table 1-8.1 using the appropriate ratio $P/S_s E_1$ is less than α.

$$\left\{Cylinder\ Length = 732.0\ in\right\} \ge \left\{2.0\sqrt{R_L t_s} = 2.0\sqrt{(76.8125)(1.6875)} = 22.7703\ in\right\} True$$

$$\frac{P}{S_s E_1} = \frac{14.7}{20000(1.0)} = 0.0007$$

$$\frac{P}{S_s E_1} = 0.0007 \qquad therefore, \left\{\begin{matrix} x_1 = 0.0 \\ \Delta_1 = 0\ deg \end{matrix}\right\} \le x \le \left\{\begin{matrix} x_2 = 0.002 \\ \Delta_2 = 5\ deg \end{matrix}\right\}$$

$$\Delta = \left(\frac{x - x_1}{x_2 - x_1}\right)(\Delta_2 - \Delta_1) + \Delta_1 = \left(\frac{0.0007 - 0.0}{0.002 - 0.0}\right)(5 - 0) + 0 = 1.75\ deg$$

$$\left\{\Delta = 1.75\right\} < \left\{\alpha = 21.0375\right\};\ reinforcement\ is\ required\ at\ the\ large\ end$$

Appendix 1-8(a), since reinforcement is required at the large end, determine the value k. Assuming the reinforcement will be place on the cylinder, if required;

$$k = \frac{y}{S_r E_r} = \frac{20000}{20000(1.0)} = 1.0$$

$$where,$$

$$y = S_s E_s = 20000(1.0) = 20000$$

Appendix 1-8(b)(1), the required area of reinforcement, A_{rL}, shall be at least equal to that indicated by the following equation when Q_L is in compression. At the large end of the cone-to-cylinder juncture, the $PR_L/2$ term is in compression. When f_1 is in tension and the quantity is larger than the $PR_L/2$ term, the design shall be in accordance with U–2(g). The localized stress at the discontinuity shall not exceed the stress values specified in Appendix 1-5(g)(1) and (2).

$$A_{rL} = \frac{kQ_L R_L \tan[\alpha]}{S_s E_1}\left(1-\frac{1}{4}\left(\frac{PR_L - Q_L}{Q_L}\right)\cdot\left(\frac{\Delta}{\alpha}\right)\right)$$

$$A_{rL} = \left\{\begin{array}{l}\dfrac{(1.0)(1061.6955)(76.8125)\cdot\tan[21.0375]}{20000(1.0)}\cdot \\ \left(1-\dfrac{1}{4}\left(\dfrac{(14.7)(76.8125)-1061.6955}{1061.6955}\right)\cdot\left(\dfrac{1.75}{21.0375}\right)\right)\end{array}\right\} = 1.5622\ in^2$$

$$where,$$

$$Q_L = \frac{P(R_L)}{2} + f_1 = \left\{\begin{array}{l}\dfrac{-14.7(76.8125)}{2} + 86.1770 = -478.3949\,\dfrac{lbs}{in\ of\ cir} \\ \dfrac{-14.7(76.8125)}{2} + (-497.1236) = -1061.6955\,\dfrac{lbs}{in\ of\ cir}\end{array}\right\}$$

$$Q_L = |-1061.6955|\,\frac{lbs}{in\ of\ cir} \quad \textit{Use the absolute value of the maximum negative value}$$

$$and,$$

$$f_1 = \frac{F_L}{2\pi R_L} \pm \frac{M_L}{\pi R_L^2} = \left\{\begin{array}{l}\dfrac{-99167}{2\pi(76.8125)} + \dfrac{5.406E+06}{\pi(76.8125)^2} = +86.1770\ \dfrac{lbs}{in\ of\ cir} \\ \dfrac{-99167}{2\pi(76.8125)} - \dfrac{5.406E+06}{\pi(76.8125)^2} = -497.1236\ \dfrac{lbs}{in\ of\ cir}\end{array}\right\}$$

The effective area of reinforcement can be determined in accordance with the following:

$$A_{eL} = 0.55\sqrt{D_L t_L}\left[t_L + \frac{t_c}{\cos[\alpha]}\right]$$

$$A_{eL} = 0.55\sqrt{153.625(1.6875)}\left[1.6875 + \frac{1.8125}{\cos[21.0375]}\right] = 32.1407\ in^2$$

The effective area of available reinforcement due to the excess thickness in the cylindrical shell and conical shell, A_{eL}, exceeds the required reinforcement, A_{rL}.

$$\left\{A_{eL} = 32.1407\ in^2\right\} \geq \left\{A_{rL} = 1.5622\ in^2\right\} \qquad True$$

If this was not true, reinforcement would need to be added to the cylindrical or conical shell using a thick insert plate or reinforcing ring. Any additional area of reinforcement which is required shall be situated within a distance of $\sqrt{R_L t_s}$ from the junction, and the centroid of the added area shall be within a distance of $0.25\sqrt{R_L t_s}$ from the junction.

Appendix 1-8(b)(3), when the cone-to-cylinder or knuckle-to-cylinder juncture is a line of support, the moment of inertia for a stiffening ring at the large end shall be determined by the following procedure.

a) STEP 1 – Assuming that the shell has been designed and D_L, L_L, and t are known, select a member to be used for the stiffening ring and determine the cross-sectional area A_{TL}.

$$A_{TL} = \frac{L_L t_s}{2} + \frac{L_c t_c}{2} + A_s = \frac{732.0(1.6875)}{2} + \frac{83.8196(1.8125)}{2} + 0.0 = 693.5865 \ in^2$$

$$where,$$

$$L_L = 732.0 \ in$$

$$L_c = \sqrt{L^2 + (R_L - R_s)^2} = \sqrt{78.0^2 + (76.8125 - 46.125)^2} = 83.8196 \ in$$

$$A_s = 0.0 \ in^2 \qquad Assume\ no\ stiffening\ ring\ area$$

Calculate factor B using the following formula. If F_L is a negative number, the design shall be in accordance with U-2(g).

$$B = \frac{3}{4}\left(\frac{F_L D_L}{A_{TL}}\right) = \frac{3}{4}\left(\frac{5979.9834(153.625)}{693.5865}\right) = 993.3962 \ psi$$

$$where,$$

$$F_L = PM + f_1 \tan[\alpha] = 14.7(393.7947) + (497.1236)\cdot\tan[21.0375] = 5979.9834 \ \frac{lbs}{in}$$

$$f_1 = |-497.1236| \ \frac{lbs}{in\ of\ circ}$$

$$and,$$

$$M = \frac{-R_L \tan[\alpha]}{2} + \frac{L_L}{2} + \frac{R_L^2 - R_s^2}{3R_L \tan[\alpha]}$$

$$M = \frac{-(76.8125)\cdot\tan[21.0375]}{2} + \frac{732.0}{2} + \frac{(76.8125)^2 - (46.125)^2}{3(76.8125)\cdot\tan[21.0375]} = 393.7947 \ in$$

b) STEP 2 – Enter the right-hand side of the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration at the value of B determined by STEP 1. If different materials are used for the shell and stiffening ring, use the material chart resulting in the larger value of A in STEP 4.

Per Section II Part D, Table 1A, a material specification of $SA-516-70N$ is assigned an External Pressure Chart No. CS-2.

c) STEP 3 – Move horizontally to the left to the material/temperature line for the design metal temperature. For values of B falling below the left end of the material/temperature line, see STEP 5.

d) STEP 4 – Move vertically to the bottom of the chart and read the value of A.

This step is not required as the value of B falls below the left end of the material/temperature line.

e) STEP 5 – For values of B falling below the left end of the material/temperature line for the design temperature, the value of A can be calculated using the following:

$$A = \frac{2B}{E_x} = \frac{2(993.3962)}{28.3E+06} = 0.00007$$

$$where,$$

$$E_x = \min[E_c, E_s, E_r], \ (min\ of\ \ the\ cone,\ shell,\ or\ stiffening\ \ ring)$$

f) STEP 6 – Compute the value of the required moment of inertia from the formulas for I_s or $I_s^{'}$. For the circumferential stiffening ring only,

$$I_s = \frac{AD_L^2 A_{TL}}{14.0} = \frac{0.00007(153.625)^2(693.5865)}{14.0} = 81.8454\ in^4$$

For the shell-cone or ring-shell-cone section,

$$I_s^{'} = \frac{AD_L^2 A_{TL}}{10.9} = \frac{0.00007(153.625)^2(693.5865)}{10.9} = 105.1226\ in^4$$

g) STEP 7 – Determine the available moment of inertia of the ring only, I, or the shell-cone or ring-shell-cone, $I^{'}$.

h) STEP 8 – When the ring only is used,

$$I \geq I_s$$

And when the shell-cone- or ring-shell-cone is used,

$$I^{'} \geq I_s^{'}$$

If the equation is not satisfied, a new section with a larger moment of inertia must be selected, and the calculation shall be done again until the equation is met. The requirements of UG-29(b), (c), (d), (e), and (f) and UG-30 are to be met in attaching stiffening rings to the shell.

VIII-1 does not provide a procedure to calculate the available moment of inertia of the shell-cone or ring-shell-cone junction. The designer must consider the following options.

a) Size a structural member to satisfy the requirement of $I \geq I_s$.

b) Size a structural member to be used in conjunction with the available moment of inertia of the cone and cylinder to satisfy the requirement of $I^{'} \geq I_s^{'}$.

c) The cost of material, fabrication, welding, inspection, and engineering.

<u>Small End</u>

In accordance with Appendix 1-8(c), reinforcement shall be provided at the small end of the cone when required by (c)(1) or (c)(2). When the small end of the cone is considered a line of support, the moment of inertia for a stiffening ring shall be determined in accordance with (c)(3).

Appendix 1-8(c)(1), for cones attached to a cylinder having a minimum length of $1.4\sqrt{R_S t_s}$, reinforcement shall be provided at the junction of the conical shell of a reducer without a flare and the small cylinder.

$$\{Cylinder\ Length = 636.0\ in\} \geq \left\{2.0\sqrt{R_S t_s} = 2.0\sqrt{(46.125)(1.0)} = 9.5081\ in\right\} \qquad True$$

Appendix 1-8(a), since reinforcement is required at the large end, determine the value k. Assuming the reinforcement will be place on the cylinder, if required;

$$k = \frac{y}{S_r E_r} = \frac{20000}{20000(1.0)} = 1.0$$

$$where,$$

$$y = S_s E_s = 20000(1.0) = 20000$$

Appendix 1-8(c)(1), the required area of reinforcement, A_{rs}, shall be at least equal to that indicated by the following equation when Q_s is in compression. At the small end of the cone-to-cylinder juncture, the $PR_s/2$ term is in compression. When f_2 is in tension and the quantity is larger than the $PR_s/2$ term, the design shall be in accordance with U–2(g). The localized stress at the discontinuity shall not exceed the stress values specified in Appendix 1-5(g)(1) and (2).

$$A_{rs} = \frac{kQ_s R_s \tan[\alpha]}{S_s E_1} = \frac{(1.0)(1252.0151)(46.125)\cdot\tan[21.0375]}{20000(1.0)} = 1.1106\ in^2$$

$$where,$$

$$Q_s = \frac{P(R_s)}{2} + f_2 = \begin{Bmatrix} \dfrac{-14.7(46.125)}{2} + 373.9985 = +34.9798\dfrac{lbs}{in\ of\ cir} \\ \dfrac{-14.7(46.125)}{2} + (-912.9963) = -1252.0151\dfrac{lbs}{in\ of\ cir} \end{Bmatrix}$$

$$Q_s = |-1252.0151|\frac{lbs}{in\ of\ cir} \qquad Use\ the\ absolute\ value\ of\ the\ maximum\ negative\ value$$

$$and,$$

$$f_2 = \frac{F_s}{2\pi R_s} \pm \frac{M_s}{\pi R_s^2} = \begin{Bmatrix} \dfrac{-78104}{2\pi(46.125)} + \dfrac{4.301E+06}{\pi(46.125)^2} = +373.9985\ \dfrac{lbs}{in\ of\ cir} \\ \dfrac{-78104}{2\pi(46.125)} - \dfrac{4.301E+06}{\pi(46.125)^2} = -912.9963\ \dfrac{lbs}{in\ of\ cir} \end{Bmatrix}$$

The effective area of reinforcement can be determined in accordance with the following:

$$A_{es} = 0.55\sqrt{D_s t_s}\left[\left(t_s - t\right) + \frac{\left(t_c - t_r\right)}{\cos\left[\alpha\right]}\right]$$

$$A_{es} = 0.55\sqrt{92.25\left(1.0\right)}\left[\left(1.0 - 0.6698\right) + \frac{\left(1.8125 - 0.3339\right)}{\cos\left[21.0375\right]}\right] = 10.1129\ in^2$$

where,

$$t = 0.6698\ in \quad \left(see\ Example\ E4.4.1\right)$$

$$t_r = 0.3339\ in \quad \left(see\ Example\ E4.4.2\right)$$

The effective area of available reinforcement due to the excess thickness in the cylindrical shell and conical shell, A_{es}, exceeds the required reinforcement, A_{rs}.

$$\left\{A_{es} = 10.1129\ in^2\right\} \geq \left\{A_{rs} = 1.1106\ in^2\right\} \qquad True$$

If this was not true, reinforcement would need to be added to the cylindrical or conical shell using a thick insert plate or reinforcing ring. Any additional area of reinforcement which is required shall be situated within a distance of $\sqrt{R_s t_s}$ from the junction, and the centroid of the added area shall be within a distance of $0.25\sqrt{R_s t_s}$ from the junction.

Appendix 1-8(c)(3), when the cone-to-cylinder of knuckle-to-cylinder juncture is a line of support, the moment of inertia for a stiffening ring at the small end shall be determined by the following procedure.

a) STEP 1 – Assuming that the shell has been designed and D_s, L_s, and t are known, select a member to be used for the stiffening ring and determine the cross-sectional area A_{TS}.

$$A_{TS} = \frac{L_s t_s}{2} + \frac{L_c t_c}{2} + A_s = \frac{636.0\left(1.0\right)}{2} + \frac{83.8196\left(1.8125\right)}{2} + 0.0 = 393.9615\ in^2$$

where,

$$L_s = 636.0\ in$$

$$L_c = \sqrt{L^2 + \left(R_L - R_s\right)^2} = \sqrt{78.0^2 + \left(76.8125 - 46.125\right)^2} = 83.8196\ in$$

$$A_s = 0.0\ in^2 \qquad Assume\ no\ stiffening\ ring\ area$$

Calculate factor B using the following formula. If F_s is a negative number, the design shall be in accordance with U-2(g).

$$B=\frac{3}{4}\left(\frac{F_s D_s}{A_{TS}}\right)=\frac{3}{4}\left(\frac{5677.1577(92.25)}{393.9615}\right)=997.0222\ psi$$

where,

$$F_s=PN+f_2\tan[\alpha]=14.7(362.3133)+(912.9963)\cdot\tan[21.0375]=5677.1577\ \frac{lbs}{in}$$

$$f_2=|-912.9963|\ \frac{lbs}{in\ of\ circ}$$

and,

$$N=\frac{R_s\tan[\alpha]}{2}+\frac{L_s}{2}+\frac{R_L^2-R_s^2}{6R_s\tan[\alpha]}$$

$$N=\frac{(46.125)\cdot\tan[21.0375]}{2}+\frac{636.0}{2}+\frac{(76.8125)^2-(46.125)^2}{6(46.125)\cdot\tan[21.0375]}=362.3133\ in$$

b) STEP 2 – Enter the right-hand side of the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration at the value of B determined by STEP 1. If different materials are used for the shell and stiffening ring, use the material chart resulting in the larger value of A in STEP 4.

Per Section II Part D, Table 1A, a material specification of $SA-516-70N$ is assigned an External Pressure Chart No. CS-2.

c) STEP 3 – Move horizontally to the left to the material/temperature line for the design metal temperature. For values of B falling below the left end of the material/temperature line, see STEP 5.

d) STEP 4 – Move vertically to the bottom of the chart and read the value of A.

This step is not required as the value of B falls below the left end of the material/temperature line.

e) STEP 5 – For values of B falling below the left end of the material/temperature line for the design temperature, the value of A can be calculated using the following:

$$A=\frac{2B}{E_x}=\frac{2(997.0222)}{28.3E+06}=0.00007$$

where,

$$E_x=\min[E_c,E_s,E_r],\ \textit{that is the min of the cone, shell, or stiffening ring}$$

f) STEP 6 – Compute the value of the required moment of inertia from the formulas for I_s or I_s'. For the circumferential stiffening ring only,

$$I_s=\frac{AD_s^2A_{TS}}{14.0}=\frac{0.00007(92.25)^2(393.9615)}{14.0}=16.7632\ in^4$$

For the shell-cone or ring-shell-cone section,

$$I_s' = \frac{AD_s^2 A_{TS}}{10.9} = \frac{0.00007(92.25)^2(393.9615)}{10.9} = 21.5307\ in^4$$

g) STEP 7 – Determine the available moment of inertia of the ring only, I, or the shell-cone or ring-shell-cone, I'.

h) STEP 8 – When the ring only is used,

$$I \geq I_s$$

And when the shell-cone- or ring-shell-cone is used,

$$I' \geq I_s'$$

If the equation is not satisfied, a new section with a larger moment of inertia must be selected, and the calculation shall be done again until the equation is met. The requirements of UG-29(b), (c), (d), (e), and (f) and UG-30 are to be met in attaching stiffening rings to the shell.

VIII-1 does not provide a procedure to calculate the available moment of inertia of the shell-cone or ring-shell-cone junction. The designer must consider the following options.

a) Size a structural member to satisfy the requirement of $I \geq I_s$.

b) Size a structural member to be used in conjunction with the available moment of inertia of the cone and cylinder to satisfy the requirement of $I' \geq I_s'$.

c) The cost of material, fabrication, welding, inspection, and engineering.

<u>Section VIII, Division 2 Solution with VIII-1 Allowable Stresses</u>

Evaluate per VIII-2, paragraphs 4.4.13 and 4.3.11.

The design rules in VIII-2, paragraph 4.3.11 shall be satisfied. In these calculations, a negative value of pressure shall be used in all applicable equations.

Per VIII-2, paragraph 4.3.11.3, the length of the conical shell, measured parallel to the surface of the cone, shall be equal to or greater than the following:

$$L_C \geq 2.0\sqrt{\frac{R_L t_C}{\cos[\alpha]}} + 1.4\sqrt{\frac{R_S t_C}{\cos[\alpha]}}$$

$$2.0\sqrt{\frac{75.125(1.8125)}{\cos[21.0375]}} + 1.4\sqrt{\frac{45.125(1.8125)}{\cos[21.0375]}} = 37.2624\ in$$

$$L_C = 78.0 \geq 37.2624 \qquad \qquad \mathit{True}$$

<u>Evaluate the Large End cylinder-to-cone junction per VIII-2, paragraph 4.3.11.4.</u>

a) STEP 1 – Compute the large end cylinder thickness, t_L, using VIII-2, paragraph 4.3.3., (as specified in design conditions)

$$t_L = 1.6875\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the large end using VIII-2, paragraph 4.3.4., (as specified in design conditions)

$$\alpha = 21.0375\ deg$$
$$t_C = 1.8125\ in$$

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all of these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with VIII-2, Part 5. In the calculations, if $0\ deg < \alpha \le 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \le \left\{ \frac{R_L}{t_L} = \frac{75.125}{1.6875} = 44.5185 \right\} \le 500 \qquad True$$

$$1 \le \left\{ \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741 \right\} \le 2 \qquad True$$

$$\left\{ \alpha = 21.0375\ deg \right\} \le 60\ deg \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force, F_L. Calculate the equivalent line load, X_L, using the specified net section axial force, F_L, and bending moment, M_L.

$$X_L = \frac{F_L}{2\pi R_L} \pm \frac{M_L}{\pi R_L^2} = \begin{cases} \dfrac{-99167}{2\pi(75.125)} + \dfrac{5.406E+06}{\pi(75.125)^2} = 94.8111\ \dfrac{lbs}{in} \\ \dfrac{-99167}{2\pi(75.125)} - \dfrac{5.406E+06}{\pi(75.125)^2} = -514.9886\ \dfrac{lbs}{in} \end{cases}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per VIII-2, Table 4.3.3 and VIII-2, Table 4.3.4, respectively. For calculated values of n other than those presented in VIII-2, Table 4.3.3 and Table 4.3.4, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741$$
$$H = \sqrt{\frac{R_L}{t_L}} = \sqrt{\frac{75.125}{1.6875}} = 6.6722$$
$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in VIII-2, Table 4.3.3 and Table 4.3.4 is required. The results of the interpolation are summarized with the following values for C_i (see VIII-2, paragraph 4.3.11.4 and STEP 5 of E4.3.7).

For the applied pressure case both M_{sN} and Q_N are calculated using the following equation

$$M_{sN}, Q_N = -\exp\left[\begin{array}{l} C_1 + C_2\ln[H] + C_3\ln[B] + C_4\left(\ln[H]\right)^2 + C_5\left(\ln[B]\right)^2 + \\ C_6\ln[H]\ln[B] + C_7\left(\ln[H]\right)^3 + C_8\left(\ln[B]\right)^3 + \\ C_9\ln[H]\left(\ln[B]\right)^2 + C_{10}\left(\ln[H]\right)^2\ln[B] \end{array}\right]$$

This results in the following (see VIII-2, paragraph 4.3.11.4 and STEP 5 of E4.3.7):

$$M_{sN} = -10.6148$$
$$Q_N = -4.0925$$

For the Equivalent Line Load case, M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\left[\frac{\left(\begin{array}{l} C_1 + C_3\ln\left[H^2\right] + C_5\ln[\alpha] + C_7\left(\ln\left[H^2\right]\right)^2 + \\ C_9\left(\ln[\alpha]\right)^2 + C_{11}\ln\left[H^2\right]\ln[\alpha] \end{array}\right)}{\left(\begin{array}{l} 1 + C_2\ln\left[H^2\right] + C_4\ln[\alpha] + C_6\left(\ln\left[H^2\right]\right)^2 + \\ C_8\left(\ln[\alpha]\right)^2 + C_{10}\ln\left[H^2\right]\ln[\alpha] \end{array}\right)}\right]$$

This results in the following (see VIII-2, paragraph 4.3.11.4 and STEP 5 of E4.3.7):

$$M_{sN} = -0.4912$$
$$Q_N = -0.1845$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\begin{array}{lll} \textit{Internal Pressure}: & M_{sN} = -10.6148, & Q_N = -4.0925 \\ \textit{Equivalent Line Load}: & M_{sN} = -0.4912, & Q_N = -0.1845 \end{array}$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in VIII-2, Table 4.3.1 for the Large End Junction.

<u>Evaluate the Cylinder at the Large End:</u>

Stress Resultant Calculations:

$$M_{sP} = Pt_L^2 M_{sN} = -14.7(1.6875)^2(-10.6148) = 444.3413\ \frac{in-lbs}{in}$$

$$M_{sX} = X_L t_L M_{sN} = \left\{\begin{array}{l} 94.8111(1.6875)(-0.4912) = -78.5889\ \dfrac{in-lbs}{in} \\ -514.9886(1.6875)(-0.4912) = 426.8741\ \dfrac{in-lbs}{in} \end{array}\right\}$$

$$M_s = M_{sP} + M_{sX} = \begin{Bmatrix} 444.3413 + (-78.5889) = 365.7524 \dfrac{in-lbs}{in} \\ 444.3413 + 426.8741 = 871.2154 \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$Q_P = Pt_L Q_N = -14.7(1.6875)(-4.0925) = 101.5196 \frac{lbs}{in}$$

$$Q_X = X_L Q_N = \begin{Bmatrix} 94.8111(-0.1845) = -17.4926 \dfrac{lbs}{in} \\ -514.9886(-0.1845) = 95.0154 \dfrac{lbs}{in} \end{Bmatrix}$$

$$Q = Q_P + Q_X = \begin{Bmatrix} 101.5196 + (-17.4926) = 84.0270 \dfrac{lbs}{in} \\ 101.5196 + 95.0154 = 196.5350 \dfrac{lbs}{in} \end{Bmatrix}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_L^2 t_L^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(75.125)^2(1.6875)^2}\right]^{0.25} = 0.1142\ in^{-1}$$

$$N_s = \frac{PR_L}{2} + X_L = \begin{Bmatrix} \dfrac{-14.7(75.125)}{2} + 94.8111 = -457.3577 \dfrac{lbs}{in} \\ \dfrac{-14.7(75.125)}{2} + (-514.9886) = -1067.1574 \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_\theta = PR_L + 2\beta_{cy} R_L \left(-M_s \beta_{cy} + Q\right)$$

$$N_\theta = \begin{Bmatrix} -14.7(75.125) + 2(0.1142)(75.125)(-(365.7524)(0.1142) + 84.0270) \\ -14.7(75.125) + 2(0.1142)(75.125)(-(871.2154)(0.1142) + 196.5350) \end{Bmatrix}$$

$$N_\theta = \begin{Bmatrix} -379.2502 \dfrac{lbs}{in} \\ 560.7660 \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations: Determine the axial and hoop membrane and bending stresses.

$$\sigma_{sm} = \frac{N_s}{t_L} = \begin{Bmatrix} \dfrac{-457.3577}{1.6875} = -271.0268\ psi \\ \dfrac{-1067.1574}{1.6875} = -632.3895\ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_s}{t_L^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(365.7524)}{(1.6875)^2(1.0)} = 770.6388 \ psi \\ \dfrac{6(871.2154)}{(1.6875)^2(1.0)} = 1835.6472 \ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_L} = \begin{Bmatrix} \dfrac{-379.2502}{1.6875} = -224.7409 \ psi \\ \dfrac{560.7660}{1.6875} = 332.3058 \ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_L^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(0.3)(365.7524)}{(1.6875)^2(1.0)} = 231.1916 \ psi \\ \dfrac{6(0.3)(871.2154)}{(1.6875)^2(1.0)} = 550.6942 \ psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\begin{Bmatrix} \sigma_{sm} = -271.0268 \ psi \\ \sigma_{sm} = -632.3895 \ psi \end{Bmatrix} \le \{1.5S, \ not \ applicable \ due \ to \ compressive \ stress\}$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = -271.0268 + 770.6388 = 499.6 \ psi \\ \sigma_{sm} - \sigma_{sb} = -271.0268 - 770.6388 = |-1041.7| \ psi \\ \sigma_{sm} + \sigma_{sb} = -632.3895 + 1835.6472 = 1203.3 \ psi \\ \sigma_{sm} - \sigma_{sb} = -632.3895 - 1835.6472 = |-2468.0| \ psi \end{Bmatrix} \le \{S_{PS} = 60000 \ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = -224.7409 \\ \sigma_{\theta m} = 332.3058 \end{Bmatrix} \le \begin{Bmatrix} 1.5S, \ not \ applicable \ due \ to \ compressive \ stress \\ 1.5S = 1.5(20000) = 30000 \ psi \end{Bmatrix} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = -224.7409 + 231.1916 = 6.5 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -224.7409 - 231.1916 = |-455.9| \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 332.3058 + 550.6942 = 883.0 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 332.3058 - 550.6942 = |-218.4| \ psi \end{Bmatrix} \le \{S_{PS} = 60000 \ psi\} \qquad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m} \le F_{ha}$$

$$\sigma_{sm} \le F_{xa}$$

F_{ha} is evaluated using VIII-2, paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he} = \frac{0.4E_y t}{D_o}$$

F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2.b with $\lambda = 0.15$.

In accordance with VIII-2, paragraph 4.4.5.1, the value of F_{ha} is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t = 1.6875\ in$$

$$L \rightarrow Not\ required,\ as\ the\ equation\ for\ F_{he}\ is\ independent\ of\ L$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}

$$F_{he} = \frac{0.4E_y t}{D_o} = \frac{0.4(28.3E+06)(1.6875)}{153.625} = 124344.9959\ psi$$

3) STEP 3 – Calculate the predicted buckling stress, F_{ic}

$$\frac{F_{he}}{S_y} = \frac{124344.9959}{33600} = 3.7007$$

Since $\frac{F_{he}}{S_y} \geq 2.439$, calculate F_{ic} as follows:

$$F_{ic} = S_y = 33600\ psi$$

4) STEP 4 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $F_{ic} = S_y = 33600\ psi$, calculate FS as follows:

$$FS = 1.667$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{33600}{1.667} = 20155.9688\ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ to the allowable hoop compressive membrane stress, F_{ha} per following criteria.

$$\{\sigma_{\theta m} = 224.7\ psi\} \leq \{F_{ha} = 20156.0\ psi\} \qquad True$$

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

In accordance with VIII-2, paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

The design factor FS used in VIII-2, paragraph 4.4.12.2.b is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in VIII-2, paragraph 4.4.2. An initial calculation is required to determine the value of F_{xa} by setting $FS = 1.0$, with

$F_{ic} = F_{xa}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.b.

For $\lambda_c = 0.15$, (Local Buckling)

$$F_{xa} = \min\left[F_{xa1},\, F_{xa2}\right]$$

$$\frac{D_o}{t} = \frac{153.625}{1.6875} = 91.0370$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{732.0}{\sqrt{76.8125(1.6875)}} = 64.2944$$

Since $\dfrac{D_o}{t} \le 135$, calculate F_{xa1} as follows with an initial value of $FS = 1.0$.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.0} = 33600 \;\; psi$$

The value of F_{xa2} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{xa2} = \frac{F_{xe}}{FS}$$

$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since $\dfrac{D_o}{t} \le 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \dfrac{D_o}{t}\right)},\;\; 0.9\right]$$

Since $M_x \ge 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389 + \dfrac{153.625}{1.6875}},\;\; 0.9\right] = 0.8520$$

Therefore,

$$F_{xe} = \frac{0.8520(28.3E+06)(1.6875)}{153.625} = 264854.8413 \;\; psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{264854.8413}{1.0} = 264854.8413 \ psi$$

$$F_{xa} = \min[33600, 264855] = 33600 \ psi$$

With a value of $F_{ic} = F_{xa} = 33600$, in accordance with VIII-2, paragraph 4.4.2, it is determined the value of $FS = 1.667$ since $\{F_{ic} = 33600\} = \{S_y = 33600\}$. Using this computed value of $FS = 1.667$ in paragraph 4.4.12.2.b, F_{xa} is calculated as follows.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{264854.8413}{1.6670} = 158881.1286 \ psi$$

$$F_{xa} = \min[20155.9688, 158881.1286] = 20155.9688 \ psi$$

Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria

$$\{\sigma_{sm} = 632.4 \ psi\} \le \{F_{xa} = 20156.0 \ psi\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

Evaluate the Cone at the Large End:

Stress Resultant Calculations, as determined above.

$$M_{csP} = M_{sP} = 444.3413 \ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \left\{ \begin{array}{l} -78.5889 \ \dfrac{in-lbs}{in} \\ 426.8741 \ \dfrac{in-lbs}{in} \end{array} \right\}$$

$$M_{cs} = M_{csP} + M_{csX} = \left\{ \begin{array}{l} 444.3413 + (-78.5889) = 365.7524 \ \dfrac{in-lbs}{in} \\ 444.3413 + 426.8741 = 871.2154 \ \dfrac{in-lbs}{in} \end{array} \right\}$$

$$Q_c = Q\cos[\alpha] + N_s \sin[\alpha]$$

$$Q_c = \left\{ \begin{array}{l} 84.0270(\cos[21.0375]) + (-457.3577)\sin[21.0375] = -85.7555 \ \dfrac{lbs}{in} \\ 196.5350(\cos[21.0375]) + (-1067.1574)\sin[21.0375] = -199.6519 \ \dfrac{lbs}{in} \end{array} \right\}$$

$$R_C = \frac{R_L}{\cos[\alpha]} = \frac{75.125}{\cos[21.0375]} = 80.4900 \ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_C^2 t_C^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(80.4900)^2(1.8125)^2}\right]^{0.25} = 0.1064 \ in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{Bmatrix} -457.3577(\cos[21.0375]) - 84.0270\sin[21.0375] = -457.0368 \ \dfrac{lbs}{in} \\ -1067.1574(\cos[21.0375]) - 196.5350\sin[21.0375] = -1066.5786 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_{c\theta} = \frac{PR_L}{\cos[\alpha]} + 2\beta_{co}R_C\left(-M_{cs}\beta_{co} - Q_c\right)$$

$$N_{c\theta} = \begin{Bmatrix} \dfrac{-14.7(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(365.7524)(0.1064) - (-85.7555)\right) \\ \dfrac{-14.7(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(871.2154)(0.1064) - (-199.6519)\right) \end{Bmatrix}$$

$$N_{c\theta} = \begin{Bmatrix} -380.9244 \ \dfrac{lbs}{in} \\ 648.7441 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{cpc} = 1.0$$

Stress Calculations: Determine the axial and hoop membrane and bending stresses.

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{Bmatrix} \dfrac{-457.0368}{1.8125} = -252.1582 \ psi \\ \dfrac{-1066.5786}{1.8125} = -588.4572 \ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(365.7524)}{(1.8125)^2(1.0)} = 668.0091 \ psi \\ \dfrac{6(871.2154)}{(1.8125)^2(1.0)} = 1591.1853 \ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{Bmatrix} \dfrac{-380.9244}{1.8125} = -210.1652 \ psi \\ \dfrac{648.7441}{1.8125} = 357.9278 \ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(0.3)(365.7524)}{(1.8125)^2(1.0)} = 200.4027\ psi \\ \dfrac{6(0.3)(871.2154)}{(1.8125)^2(1.0)} = 477.3556\ psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\begin{Bmatrix} \sigma_{sm} = -252.1582\ psi \\ \sigma_{sm} = -588.4572\ psi \end{Bmatrix} \le \{1.5S,\ not\ applicable\ due\ to\ compressive\ stress\}$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = -252.1582 + 668.0091 = 415.6\ psi \\ \sigma_{sm} - \sigma_{sb} = -252.1582 - 668.0091 = |-920.2|\ psi \\ \sigma_{sm} + \sigma_{sb} = -588.4572 + 1591.1853 = 1002.7\ psi \\ \sigma_{sm} - \sigma_{sb} = -588.4572 - 1591.1853 = |-2179.6|\ psi \end{Bmatrix} \le \{S_{PS} = 60000\ psi\} \quad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = -210.1652 \\ \sigma_{\theta m} = 357.9278 \end{Bmatrix} \le \begin{Bmatrix} 1.5S,\ not\ applicable\ due\ to\ compressive\ stress \\ 1.5S = 1.5(20000) = 30000\ psi \end{Bmatrix} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = -210.1652 + 200.4027 = |-9.7|\ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -210.1652 - 200.4027 = -410.6\ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 357.9278 + 477.3556 = 835.3\ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 357.9278 - 477.3556 = |-119.4|\ psi \end{Bmatrix} \le \{S_{PS} = 60000\ psi\} \qquad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

Using the procedure shown above for the cylindrical shell and substituting the cone thickness, t_c for the cylinder thickness, t, the allowable compressive hoop membrane and axial membrane stresses, F_{ha} and F_{xa}, respectively, are calculated as follows.

$$F_{ha} = 20156.0\ psi$$
$$F_{xa} = 20156.0\ psi$$

Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ and axial compressive membrane stress, σ_{sm}, to the allowable hoop compressive membrane stress, F_{ha} and axial compressive membrane stress, F_{xa} per following criteria.

$$\{\sigma_{\theta m} = 210.2\ psi\} \le \{F_{ha} = 20156.0\ psi\} \qquad True$$
$$\{\sigma_{sm} = 588.5\ psi\} \le \{F_{xa} = 20156.0\ psi\} \qquad True$$

Therefore, local buckling due to hoop and axial compressive membrane stress is not a concern.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore the design is complete.

Evaluate the Small End cylinder-to-cone junction per VIII-2, paragraph 4.3.11.5.

a) STEP 1 – Compute the small end cylinder thickness, t_S, using VIII-2, paragraph 4.3.3., (as specified in design conditions)

$$t_S = 1.0\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the small end using VIII-2, paragraph 4.3.4., (as specified in design conditions)

$$\alpha = 21.0375\ deg$$
$$t_C = 1.8125\ in$$

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all of these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with VIII-2, Part 5. In the calculations, if $0\ deg < \alpha \le 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \le \left\{ \frac{R_S}{t_S} = \frac{45.125}{1.0} = 45.125 \right\} \le 500 \qquad True$$

$$1 \le \left(\frac{t_C}{t_S} = \frac{1.8125}{1.0} = 1.8125 \right) \le 2 \qquad True$$

$$\{\alpha = 21.0375\ deg\} \le 60\ deg \qquad True$$

d) STEP 4 – Determine the net section axial force, F_S, and bending moment, M_S, applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force, F_S. Calculate the equivalent line load, X_S, using the specified net section axial force, F_S, and bending moment, M_S.

$$X_S = \frac{F_S}{2\pi R_S} \pm \frac{M_S}{\pi R_S^2} = \begin{Bmatrix} \dfrac{-78104}{2\pi(45.125)} + \dfrac{4.301E+06}{\pi(45.125)^2} = 396.8629\ \dfrac{lbs}{in} \\ \dfrac{-78104}{2\pi(45.125)} - \dfrac{4.301E+06}{\pi(45.125)^2} = -947.8053\ \dfrac{lbs}{in} \end{Bmatrix}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per VIII-2, Table 4.3.5 and VIII-2, Table 4.3.6, respectively. For calculated values of n other than those presented in VIII-2, Table 4.3.5 and Table 4.3.6, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_S} = \frac{1.8125}{1.0} = 1.8125$$

$$H = \sqrt{\frac{R_S}{t_S}} = \sqrt{\frac{45.125}{1.0}} = 6.7175$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in VIII-2, Table 4.3.5 and Table 4.3.6 is required. The results of the interpolation are summarized with the following values for C_i (see VIII-2, paragraph 4.3.11.5 and STEP 5 of E4.3.7)

For the applied pressure case M_{sN} is calculated using the following equation

$$M_{sN} = \exp\left[\begin{array}{l} C_1 + C_2 \ln\left[H^2\right] + C_3 \ln[\alpha] + C_4\left(\ln\left[H^2\right]\right)^2 + C_5\left(\ln[\alpha]\right)^2 + \\ C_6 \ln\left[H^2\right]\ln[\alpha] + C_7\left(\ln\left[H^2\right]\right)^3 + C_8\left(\ln[\alpha]\right)^3 + \\ C_9 \ln\left[H^2\right]\left(\ln[\alpha]\right)^2 + C_{10}\left(\ln\left[H^2\right]\right)^2 \ln[\alpha] \end{array}\right]$$

This results in the following (see VIII-2, paragraph 4.3.11.5 and STEP 5 of E4.3.7)

$$M_{sN} = 9.2135$$

For the applied pressure case Q_N is calculated using the following equation

$$Q_N = \left(\frac{C_1 + C_3 H^2 + C_5 \alpha + C_7 H^4 + C_9 \alpha^2 + C_{11} H^2 \alpha}{1 + C_2 H^2 + C_4 \alpha + C_6 H^4 + C_8 \alpha^2 + C_{10} H^2 \alpha}\right)$$

This results in the following (see VIII-2, paragraph 4.3.11.5 and STEP 5 of E4.3.7)

$$Q_N = -2.7333$$

For the Equivalent Line Load case, M_{sN} is calculated using the following equation

$$M_{sN} = \left(\frac{C_1 + C_3 H + C_5 B + C_7 H^2 + C_9 B^2 + C_{11} HB}{1 + C_2 H + C_4 B + C_6 H^2 + C_8 B^2 + C_{10} HB}\right)$$

This results in the following (see VIII-2, paragraph 4.3.11.5 and STEP 5 of E4.3.7)

$$M_{sN} = 0.4828$$

For the Equivalent Line Load case, Q_N is calculated using the following equation

$$Q_N = \left(\begin{array}{l} C_1 + C_2 \ln[H] + C_3 \ln[B] + C_4\left(\ln[H]\right)^2 + C_5\left(\ln[B]\right)^2 + C_6 \ln[H]\ln[B] + \\ C_7\left(\ln[H]\right)^3 + C_8\left(\ln[B]\right)^3 + C_9 \ln[H]\left(\ln[B]\right)^2 + C_{10}\left(\ln[H]\right)^2 \ln[B] \end{array}\right)$$

This results in the following (see VIII-2, paragraph 4.3.11.5 and STEP 5 of E4.3.7)

$$Q_N = -0.1613$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$Internal\ Pressure: \qquad M_{sN} = 9.2135, \qquad Q_N = -2.7333$$

$$Equivalent\ Line\ Load: \qquad M_{sN} = 0.4828, \qquad Q_N = -0.1613$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in VIII-2, Table 4.3.2. for the Small End Junction

Evaluate the Cylinder at the Small End.

Stress Resultant Calculations.

$$M_{sP} = Pt_S^2 M_{sN} = -14.7(1.0)^2(9.2135) = -135.4385\ \frac{in-lbs}{in}$$

$$M_{sX} = X_S t_S M_{sN} = \begin{Bmatrix} 396.8629(1.0)(0.4828) = 191.6054\ \dfrac{in-lbs}{in} \\ -947.8053(1.0)(0.4828) = -457.6004\ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$M_s = M_{sP} + M_{sX} = \begin{Bmatrix} -135.4385 + (191.6054) = 56.1669\ \dfrac{in-lbs}{in} \\ -135.4385 + (-457.6004) = -593.0389\ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$Q_P = Pt_S Q_N = -14.7(1.0)(-2.7333) = 40.1795\ \frac{lbs}{in}$$

$$Q_X = X_S Q_N = \begin{Bmatrix} 396.8629(-0.1613) = -64.0140\ \dfrac{lbs}{in} \\ -947.8053(-0.1613) = 152.8810\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$Q = Q_P + Q_X = \begin{Bmatrix} 40.1795 + (-64.0140) = -23.8345\ \dfrac{lbs}{in} \\ 40.1795 + 152.8810 = 193.0605\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_S^2 t_S^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(45.1250)^2(1.000)^2}\right]^{0.25} = 0.1914\ in^{-1}$$

$$N_s = \frac{PR_S}{2} + X_S = \begin{Bmatrix} \dfrac{-14.7(45.125)}{2} + 396.8629 = 65.1942\ \dfrac{lbs}{in} \\ \dfrac{-14.7(45.125)}{2} + (-947.8053) = -1279.4741\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_\theta = PR_S + 2\beta_{cy}R_S\left(-M_s\beta_{cy} - Q\right)$$

$$N_\theta = \begin{Bmatrix} -14.7(45.125) + 2(0.1914)(45.125)\left(-(56.1669)(0.1914) - (-23.8345)\right) \\ -14.7(45.125) + 2(0.1914)(45.125)\left(-(-593.0389)(0.1914) - 193.0605\right) \end{Bmatrix}$$

$$N_\theta = \begin{Bmatrix} -437.3238\ \dfrac{lbs}{in} \\ -2037.5216\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations: Determine the axial and hoop membrane and bending stresses:

$$\sigma_{sm} = \frac{N_s}{t_S} = \begin{Bmatrix} \dfrac{65.1942}{1.0} = 65.1942\ psi \\ \dfrac{-1279.4741}{1.0} = -1279.4741\ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(56.1669)}{(1.0)^2(1.0)} = 337.0014\ psi \\ \dfrac{6(-593.0389)}{(1.0)^2(1.0)} = -3558.2334\ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_S} = \begin{Bmatrix} \dfrac{-437.3238}{1.0} = -437.3238\ psi \\ \dfrac{-2037.5216}{1.0} = -2037.5216\ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(0.3)(56.1669)}{(1.0)^2(1.0)} = 101.1004\ psi \\ \dfrac{6(0.3)(-593.0389)}{(1.0)^2(1.0)} = -1067.4700\ psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\left\{\begin{array}{l}\sigma_{sm} = 65.1942\ psi \\ \sigma_{sm} = -1279.4741\ psi\end{array}\right\} \le \left\{\begin{array}{l}1.5S = 1.5(20000) = 30000\ psi \\ 1.5S,\ not\ applicable\ due\ to\ compressive\ stress\end{array}\right\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{sm} + \sigma_{sb} = 65.1942 + 337.0014 = 402.2\ psi \\ \sigma_{sm} - \sigma_{sb} = 65.1942 - 337.0014 = |-271.8|\ psi \\ \sigma_{sm} + \sigma_{sb} = -1279.4741 + (-3558.2334) = |-4837.7|\ psi \\ \sigma_{sm} - \sigma_{sb} = -1279.4741 - (-3558.2334) = 2278.8\ psi\end{array}\right\} \le \{S_{PS} = 60000\ psi\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m} = -437.3238 \\ \sigma_{\theta m} = -2037.5216\end{array}\right\} \le \{1.5S,\ not\ applicable\ due\ to\ compressive\ stress\}$$

$$\left\{\begin{array}{l}\sigma_{\theta m} + \sigma_{\theta b} = -437.3238 + 101.1004 = |-336.2|\ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -437.3238 - 101.1004 = |-538.4|\ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = -2037.5216 + (-1067.4700) = |-3105.0|\ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -2037.5216 - (-1067.4700) = |-970.1|\ psi\end{array}\right\} \le \{S_{PS} = 60000\ psi\} \qquad True$$

Since the hoop membrane stress $\sigma_{\theta m}$ and axial membrane stress σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m} \le F_{ha}$$

$$\sigma_{sm} \le F_{xa}$$

F_{ha} is evaluated using VIII-2, paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he} = \frac{0.4E_y t}{D_o}$$

F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2.b with $\lambda = 0.15$.

In accordance with VIII-2, paragraph 4.4.5.1, the value of F_{ha} is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t = 1.0\ in$$

$$L \rightarrow Not\ required,\ as\ the\ equation\ for\ F_{he}\ is\ independent\ of\ L$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}

$$F_{he} = \frac{0.4E_y t}{D_o} = \frac{0.4(28.3E+06)(1.0)}{92.25} = 122710.0271\ psi$$

3) STEP 3 – Calculate the predicted buckling stress, F_{ic}

$$\frac{F_{he}}{S_y} = \frac{122710.0271}{33600} = 3.6521$$

Since $\dfrac{F_{he}}{S_y} \geq 2.439$, calculate F_{ic} as follows:

$$F_{ic} = S_y = 33600 \ psi$$

4) STEP 4 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $F_{ic} = S_y = 33600 \ psi$, calculate FS as follows:

$$FS = 1.667$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ to the allowable hoop compressive membrane stress, F_{ha} per following criteria.

$$\left\{\sigma_{\theta m} = 2037.5 \ psi\right\} \leq \left\{F_{ha} = 20156.0 \ psi\right\} \qquad True$$

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

In accordance with VIII-2, paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

The design factor FS used in VIII-2, paragraph 4.4.12.2.b is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in VIII-2, paragraph 4.4.2. An initial calculation is required to determine the value of F_{xa} by setting $FS = 1.0$, with $F_{ic} = F_{xa}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.b.

For $\lambda_c = 0.15$, (Local Buckling)

$$F_{xa} = \min\left[F_{xa1}, F_{xa2}\right]$$

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{46.125(1.0)}} = 93.6459$$

Since $\dfrac{D_o}{t} \leq 135$, calculate F_{xa1} as follows with an initial value of $FS = 1.0$.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.0} = 33600 \ psi$$

The value of F_{xa2} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{xa2} = \frac{F_{xe}}{FS}$$

$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since $\frac{D_o}{t} \le 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \frac{D_o}{t}\right)},\ 0.9\right]$$

Since $M_x \ge 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389 + \frac{92.25}{1.0}},\ 0.9\right] = 0.8499$$

Therefore,

$$F_{xe} = \frac{0.8499(28.3E+06)(1.0)}{92.25} = 260728.1301\ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{260728.1301}{1.0} = 260728.1301\ psi$$

$$F_{xa} = \min[33600,\ 260728] = 33600\ psi$$

With a value of $F_{ic} = F_{xa} = 33600$, in accordance with VIII-2, paragraph 4.4.2, it is determined the value of $FS = 1.667$ since $\{F_{ic} = 33600\} = \{S_y = 33600\}$. Using this computed value of $FS = 1.667$ in paragraph 4.4.12.2.b, F_{xa} is calculated as follows.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.667} = 20155.9688\ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{260728.1301}{1.6670} = 156405.5969\ psi$$

$$F_{xa} = \min[20155.9688,\ 156405.5969] = 20155.9688\ psi$$

Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria

$$\{\sigma_{sm} = 1279.5\ psi\} \le \{F_{xa} = 20156.0\ psi\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

Evaluate the Cone at the Small End.

Stress Resultant Calculations as determined above.

$$M_{csP} = M_{sP} = -135.4385\ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \begin{Bmatrix} 191.6054\ \dfrac{in-lbs}{in} \\ -457.6004\ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$M_{cs} = M_{csP} + M_{csX} = \begin{Bmatrix} -135.4385 + 191.6054 = 56.1669\ \dfrac{in-lbs}{in} \\ -135.4385 + (-457.6004) = -593.0389\ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$Q_c = Q\cos[\alpha] + N_s \sin[\alpha]$$

$$Q_c = \begin{Bmatrix} (-23.8345)\cos[21.0375] + 65.1942\sin[21.0375] = 1.1575\ \dfrac{lbs}{in} \\ 193.0605\cos[21.0375] + (-1279.4741)\sin[21.0375] = -279.1120\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$R_c = \frac{R_s}{\cos[\alpha]} = \frac{45.1250}{\cos[21.0375]} = 48.3476\ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_c^{\,2} t_c^{\,2}}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(48.3476)^2(1.8125)^2}\right]^{0.25} = 0.1373\ in^{-1}$$

$$N_{cs} = N_s\cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{Bmatrix} 65.1942\cos[21.0375] - (-23.8345)\sin[21.0375] = 69.4048\ \dfrac{lbs}{in} \\ (-1279.4741)\cos[21.0375] - 193.0605\sin[21.0375] = -1263.4963\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_{c\theta} = \frac{PR_s}{\cos[\alpha]} + 2\beta_{co}R_c\left(-M_{cs}\beta_{co} + Q_c\right)$$

$$N_{c\theta} = \begin{cases} \dfrac{-14.7(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(56.1669)(0.1373) + 1.1575\right) \\ \dfrac{-14.7(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(-593.0389)(0.1373) + (-279.1120)\right) \end{cases}$$

$$N_{c\theta} = \begin{cases} -797.7248 \dfrac{lbs}{in} \\ -3335.2619 \dfrac{lbs}{in} \end{cases}$$

$$K_{cpc} = 1.0$$

Stress Calculations: Determine the axial and hoop membrane and bending stresses:

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{cases} \dfrac{69.4048}{1.8125} = 38.2923 \ psi \\ \dfrac{-1263.4963}{1.8125} = -697.1014 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{cases} \dfrac{6(56.1669)}{(1.8125)^2(1.0)} = 102.5831 \ psi \\ \dfrac{6(-593.0389)}{(1.8125)^2(1.0)} = -1083.1246 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{cases} \dfrac{-797.7248}{1.8125} = -440.1240 \ psi \\ \dfrac{-3335.2619}{1.8125} = -1840.1445 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{cases} \dfrac{6(0.3)(56.1669)}{(1.8125)^2(1.0)} = 30.7749 \ psi \\ \dfrac{6(0.3)(-593.0389)}{(1.8125)^2(1.0)} = -324.9374 \ psi \end{cases}$$

Check Acceptable Criteria:

$$\left\{\begin{array}{l}\sigma_{sm} = 38.2923 \ psi \\ \sigma_{sm} = -697.1014 \ psi\end{array}\right\} \leq \left\{\begin{array}{l}1.5S = 1.5(20000) = 30000 \ psi \\ 1.5S, \ not \ applicable \ due \ to \ compressive \ stress\end{array}\right\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{sm} + \sigma_{sb} = 38.2923 + 102.5831 = 140.9 \ psi \\ \sigma_{sm} - \sigma_{sb} = 38.2923 - 102.5831 = |-64.3| \ psi \\ \sigma_{sm} + \sigma_{sb} = -697.1014 + (-1083.1246) = |-1780.2| \ psi \\ \sigma_{sm} - \sigma_{sb} = -697.1014 - (-1083.1246) = 386.0 \ psi\end{array}\right\} \leq \{S_{PS} = 60000 \ psi\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m} = -440.1240 \\ \sigma_{\theta m} = -1840.1445\end{array}\right\} \leq \{1.5S, \ not \ applicable \ due \ to \ compressive \ stress\}$$

$$\left\{\begin{array}{l}\sigma_{\theta m} + \sigma_{\theta b} = -440.1240 + 30.7749 = |-409.3| \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -440.1240 - 30.7749 = |-470.9| \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = -1840.1445 + (-324.9374) = |-2164.9| \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -1840.1445 - (-324.9374) = |-1515.1| \ psi\end{array}\right\} \leq \{S_{PS} = 60000 \ psi\} \qquad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

Using the procedure shown above for the cylindrical shell and substituting the cone thickness, t_c for the cylinder thickness, t, the allowable compressive hoop membrane and axial membrane stresses, F_{ha} and F_{xa}, respectively, are calculated as follows.

$$F_{ha} = 20156.0 \ psi$$
$$F_{xa} = 20156.0 \ psi$$

Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ and axial compressive membrane stress, σ_{sm}, to the allowable hoop compressive membrane stress, F_{ha} and axial compressive membrane stress, F_{xa} per following criteria.

$$\{\sigma_{\theta m} = 1840.1 \ psi\} \leq \{F_{ha} = 20156.0 \ psi\} \qquad True$$
$$\{\sigma_{sm} = 697.1 \ psi\} \leq \{F_{xa} = 20156.0 \ psi\} \qquad True$$

Therefore, local buckling due to hoop and axial compressive membrane stress is not a concern.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore the design is complete.

4.4.8 Example E4.4.8 - Conical Transitions with a Knuckle

Determine if the proposed design for the large end of a cylinder-to-cone junction with a knuckle is adequately designed considering the following design conditions and applied forces and moments.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $-14.7\ psig@300°F$
- Inside Diameter (Large End) = $120.0\ in$
- Large End Thickness = $1.0\ in$
- Inside Diameter (Small End) = $33.0\ in$
- Small End Thickness = $1.0\ in$
- Knuckle Radius = $10.0\ in$
- Cone Thickness = $1.0\ in$
- Knuckle Thickness = $1.0\ in$
- Length of Conical Section = $73.0\ in$
- Unsupported Length of Large Cylinder = $240.0\ in$
- Unsupported Length of Small Cylinder = $360.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $20000\ psi$
- Yield Strength = $33600\ psi$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle = $30.0\ deg$
- Axial Force (Large End) = $-10000\ lbs$
- Net Section Bending Moment (Large End) = $2.0E+06\ in-lbs$

Section VIII, Division 1 Solution

VIII-1 does not provide design rules for knuckles or flares under pressure on the convex side. However, in accordance with paragraph UG-33(f) for a cone-to-cylinder junction with a knuckle that is a line of support, the moment of inertia calculation of Appendix 1-8 must be performed. The reinforcement calculation however, is not required. The moment of inertia calculation can be performed either by considering the presence of the knuckle or by assuming the knuckle is not present whereby the cone is assumed to intersect the adjacent cylinder.

For this example, it is assumed that the cone-to-cylinder junction with a knuckle is a line of support and the knuckle is not present and the cone is assumed to intersect the adjacent cylinder.

Determine outside dimensions.

$$D_L = 120.0 + 2(Uncorroded\ Thickness) = 122.0\ in$$
$$R_L = 60.0 + Uncorroded\ Thickness = 60.0 + 1.0 = 61.0\ in$$
$$D_s = 33.0 + 2(Uncorroded\ Thickness) = 35.0\ in$$
$$R_s = 16.5 + Uncorroded\ Thickness = 16.5 + 1.0 = 17.5\ in$$

Evaluate per Mandatory Appendix 1-8. The moment of inertia for a stiffening ring at the large end shall be determined by the following procedure.

a) STEP 1 – Assuming that the shell has been designed and D_L, L_L, and t are known, select a member to be used for the stiffening ring and determine the cross-sectional area A_{TL}.

$$A_{TL} = \frac{L_L t_s}{2} + \frac{L_c t_c}{2} + A_s = \frac{240.0(1.0)}{2} + \frac{84.9779(1.0)}{2} + 0.0 = 162.4890\ in^2$$

where,

$$L_L = 240.0\ in$$
$$L_c = \sqrt{L^2 + (R_L - R_s)^2} = \sqrt{73.0^2 + (61.0 - 17.5)^2} = 84.9779\ in$$
$$A_s = 0.0\ in^2 \qquad Assume\ no\ stiffening\ ring\ area$$

Calculate factor B using the following formula. If F_L is a negative number, the design shall be in accordance with U-2(g).

$$B = \frac{3}{4}\left(\frac{F_L D_L}{A_{TL}}\right) = \left\{\begin{array}{l} \dfrac{3}{4}\left(\dfrac{2063.9601(122.0)}{162.4890}\right) = 1162.2470\ psi \\ \dfrac{3}{4}\left(\dfrac{1866.4043(122.0)}{162.4890}\right) = 1051.0003\ psi \end{array}\right\} Use\ maximum\ value$$

Where,

$$F_L = PM + f_1 \tan[\alpha] = \left\{\begin{array}{l} 14.7(134.7106) + (144.9974)\cdot\tan[30] = 2063.9601\ \dfrac{lbs}{in} \\ 14.7(134.7106) + (-197.1793)\cdot\tan[30] = 1866.4043\ \dfrac{lbs}{in} \end{array}\right\}$$

And,

$$f_1 = \frac{F_L}{2\pi R_L} \pm \frac{M_L}{\pi R_L^2} = \left\{\begin{array}{l} \dfrac{-10000}{2\pi(61.0)} + \dfrac{2.0E+06}{\pi(61.0)^2} = 144.9974\ \dfrac{lbs}{in\ of\ cir} \\ \dfrac{-10000}{2\pi(61.0)} - \dfrac{2.0E+06}{\pi(61.0)^2} = -197.1793\ \dfrac{lbs}{in\ of\ cir} \end{array}\right\}$$

And,

$$M = \frac{-R_L \tan[\alpha]}{2} + \frac{L_L}{2} + \frac{R_L^2 - R_s^2}{3R_L \tan[\alpha]}$$

$$M = \frac{-(61.0)\cdot\tan[30]}{2} + \frac{240.0}{2} + \frac{(61.0)^2 - (17.5)^2}{3(61.0)\cdot\tan[30]} = 134.7106\ in$$

b) STEP 2 – Enter the right-hand side of the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration at the value of B determined by STEP 1. If different materials are used for the shell and stiffening ring, use the material chart resulting in the larger value of A in STEP 4.

Per Section II Part D, Table 1A, a material specification of $SA-516,\ Grade\ 70,\ Norm.$ is assigned an External Pressure Chart No. CS-2.

c) STEP 3 – Move horizontally to the left to the material/temperature line for the design metal temperature. For values of B falling below the left end of the material/temperature line, see STEP 5.

d) STEP 4 – Move vertically to the bottom of the chart and read the value of A.

This step is not required as the value of B falls below the left end of the material/temperature line.

e) STEP 5 – For values of B falling below the left end of the material/temperature line for the design temperature, the value of A can be calculated using the following:

$$A = \frac{2B}{E_x} = \frac{2(1162.2470)}{28.3E+06} = 0.00008$$

$$where,$$

$$E_x = \min[E_c, E_s, E_r],\ (min\ of\ the\ cone,\ shell,\ or\ stiffening\ ring)$$

f) STEP 6 – Compute the value of the required moment of inertia from the formulas for I_s or $I_s^{'}$. For the circumferential stiffening ring only,

$$I_s = \frac{AD_L^2 A_{TL}}{14.0} = \frac{0.00008(122.0)^2(162.4890)}{14.0} = 13.8199\ in^4$$

For the shell-cone or ring-shell-cone section,

$$I_s^{'} = \frac{AD_L^2 A_{TL}}{10.9} = \frac{0.00008(122.0)^2(162.4890)}{10.9} = 17.7504\ in^4$$

g) STEP 7 – Determine the available moment of inertia of the ring only, I, or the shell-cone or ring-shell-cone, $I^{'}$.

h) STEP 8 – When the ring only is used,

$$I \geq I_s$$

And when the shell-cone- or ring-shell-cone is used,

$$I^{'} \geq I_s^{'}$$

If the equation is not satisfied, a new section with a larger moment of inertia must be selected, and the calculation shall be done again until the equation is met. The requirements of UG-29(b), (c), (d), (e), and (f) and UG-30 are to be met in attaching stiffening rings to the shell.

VIII-1 does not provide a procedure to calculate the available moment of inertia of the shell-cone or ring-shell-cone junction. The designer must consider the following options.

a) Size a structural member to satisfy the requirement of $I \ge I_s$.

b) Size a structural member to be used in conjunction with the available moment of inertia of the cone and cylinder to satisfy the requirement of $I^{'} \ge I_s^{'}$.

c) The cost of material, fabrication, welding, inspection, and engineering.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraphs 4.4.14 and 4.3.12.

The design rules in VIII-2, paragraph 4.3.12 shall be satisfied. In these calculations, a negative value of pressure shall be used in all applicable equations.

a) STEP 1 – Compute the large end cylinder thickness, t_L, using VIII-2, paragraph 4.4.5, (as specified in design conditions)

$$t_L = 1.0\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the large end using VIII-2, paragraph 4.4.5, (as specified in design conditions).

$$\alpha = 30\ deg$$

$$t_C = 1.0\ in$$

c) STEP 3 – Proportion the transition geometry by assuming a value for the knuckle radius, r_k, and knuckle thickness, t_k, such that the following equations are satisfied. If all of these equations cannot be satisfied, the cylinder-to-cone junction shall be designed in accordance with VIII-2, Part 5.

$$\left\{t_k = 1.0\ in\right\} \ge \left\{t_L = 1.0\ in\right\} \qquad True$$

$$\left\{r_k = 10.0\ in\right\} > \left\{3t_k = 3.0\ in\right\} \qquad True$$

$$\left\{\frac{r_k}{R_L} = \frac{10.0}{60.0} = 0.1667\right\} > \left\{0.03\right\} \qquad True$$

$$\left\{\alpha = 30\ deg\right\} \le \left\{60\ deg\right\} \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition at the location of the knuckle. The thrust load due to pressure shall not be included as part of the axial force, F_L.

$$F_L = -10000\ lbs$$

$$M_L = 2.0E+06\ in-lbs$$

e) STEP 5 – Compute the stresses in the knuckle at the junction using the equations in VIII-2, Table 4.3.7.

Determine if the knuckle is considered to be compact or non-compact.

$$\alpha r_k < 2K_m\left(\left\{R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5}+r_k\right\}t_k\right)^{0.5}$$

$$\{0.5236(10.0)\} < \left\{2(0.7)\left(\left\{50.0\left((0.5236)^{-1}\cdot\tan[0.5236]\right)^{0.5}+10\right\}1\right)^{0.5}\right\}$$

$$\{5.2360\ in\} < \{11.0683\ in\} \qquad\qquad True$$

Where,

$$K_m = 0.7$$

$$\alpha = \frac{30.0}{180}\pi = 0.5236\ rad$$

$$R_k = R_L - r_k = 60.0 - 10.0 = 50.0\ in$$

Therefore, analyze the knuckle junction as a compact knuckle.

Stress Calculations: Determine the hoop and axial membrane stresses at the knuckle:

$$\sigma_{\theta m} = \frac{PK_m\left(R_L\sqrt{R_L t_L}+L_k\sqrt{L_k t_C}\right)+\alpha\left(PL_{1k}r_k-0.5P_e L_{1k}^2\right)}{K_m\left(t_L\sqrt{R_L t_L}+t_C\sqrt{L_k t_C}\right)\alpha t_k r_k}$$

$$\sigma_{sm} = \frac{P_e L_{1k}}{2t_k}$$

Where,

$$L_{1k} = R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5}+r_k = 50.0\left((0.5236)^{-1}\tan[0.5236]\right)^{0.5}+10.0 = 62.5038\ in$$

$$L_k = \frac{R_k}{\cos[\alpha]}+r_k = \frac{50.0}{\cos[0.5236]}+10.0 = 67.7351\ in$$

$$P_e = P + \frac{F_L}{\pi L_{1k}^2 \cos^2\left[\frac{\alpha}{2}\right]} \pm \frac{2M_L}{\pi L_{1k}^3 \cos^3\left[\frac{\alpha}{2}\right]}$$

$$P_e = \begin{Bmatrix} -14.7 + \dfrac{-10000.0}{\pi(62.5038)^2 \cdot \cos^2\left[\dfrac{0.5236}{2}\right]} + \dfrac{2(2.0E+06)}{\pi(62.5038)^3 \cdot \cos^3\left[\dfrac{0.5236}{2}\right]} \\ -14.7 + \dfrac{-10000.0}{\pi(62.5038)^2 \cdot \cos^2\left[\dfrac{0.5236}{2}\right]} - \dfrac{2(2.0E+06)}{\pi(62.5038)^3 \cdot \cos^3\left[\dfrac{0.5236}{2}\right]} \end{Bmatrix}$$

$$P_e = \begin{Bmatrix} -9.7875 \ psi \\ -21.3590 \ psi \end{Bmatrix}$$

Therefore,

$$\sigma_{\theta m} = \begin{Bmatrix} \dfrac{\begin{pmatrix} (-14.7)(0.7)\left(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}\right) + \\ 0.5236\left((-14.7)(62.5038)(10.0) - 0.5(-9.7875)(62.5038)^2\right) \end{pmatrix}}{0.7\left(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}\right) + 0.5236(1.0)(10.0)} = -323.9558 \ psi \\ \dfrac{\begin{pmatrix} (-14.7)(0.7)\left(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}\right) + \\ 0.5236\left((-14.7)(62.5038)(10.0) - 0.5(-21.3590)(62.5038)^2\right) \end{pmatrix}}{0.7\left(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}\right) + 0.5236(1.0)(10.0)} = 396.8501 \ psi \end{Bmatrix}$$

And,

$$\sigma_{sm} = \begin{Bmatrix} \dfrac{P_e L_{1k}}{2t_k} = \dfrac{-9.7875(62.5038)}{2(1.0)} = -305.8780 \ psi \\ \dfrac{P_e L_{1k}}{2t_k} = \dfrac{-21.3590(62.5038)}{2(1.0)} = -667.5093 \ psi \end{Bmatrix}$$

Check Acceptable Criteria:

$$\begin{Bmatrix} \sigma_{\theta m} = -324.0 \ psi \\ \sigma_{\theta m} = 396.9 \ psi \end{Bmatrix} \le \begin{Bmatrix} S, \ not \ applicable \ due \ to \ compressive \ stress \\ 20000 \qquad True \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{sm} = -305.9 \ psi \\ \sigma_{sm} = -667.5 \ psi \end{Bmatrix} \le \{S, \ not \ applicable \ due \ to \ compressive \ stress\}$$

Since the hoop membrane stress $\sigma_{\theta m}$ and axial membrane stress σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m} \le F_{ha}$$

$$\sigma_{sm} \le F_{xa}$$

F_{ha} is evaluated using VIII-2, paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he} = \frac{0.4E_y t}{D_o}$$

F_{xa} is evaluated using VIII-2, paragraph 4.4.12.2.b with $\lambda = 0.15$.

In accordance with VIII-2, paragraph 4.4.5.1, the value of F_{ha} is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t = 1.0\ in$$

$$L \rightarrow Not\ required,\ as\ the\ equation\ for\ F_{he}\ is\ independent\ of\ L$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}

$$F_{he} = \frac{0.4E_y t}{D_o} = \frac{0.4(28.3E+06)(1.0)}{122.0} = 92786.8853\ psi$$

3) STEP 3 – Calculate the predicted buckling stress, F_{ic}

$$\frac{F_{he}}{S_y} = \frac{92786.8853}{33600} = 2.7615$$

Since $\frac{F_{he}}{S_y} \ge 2.439$, calculate F_{ic} as follows:

$$F_{ic} = S_y = 33600\ psi$$

4) STEP 4 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $F_{ic} = S_y = 33600\ psi$, calculate FS as follows:

$$FS = 1.667$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{33600}{1.667} = 20155.9688\ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ to the allowable hoop compressive membrane stress, F_{ha} per following criteria.

$$\{\sigma_{\theta m} = -324.0\ psi\} \le \{F_{ha} = 20156.0\ psi\} \qquad True$$

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

In accordance with VIII-2, paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with

$\lambda = 0.15$.

The design factor FS used in VIII-2, paragraph 4.4.12.2.b is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in VIII-2, paragraph 4.4.2. An initial calculation is required to determine the value of F_{xa} by setting $FS = 1.0$, with $F_{ic} = F_{xa}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.b.

For $\lambda_c = 0.15$, (Local Buckling)

$$F_{xa} = \min\left[F_{xa1}, F_{xa2}\right]$$

$$\frac{D_o}{t} = \frac{122.0}{1.0} = 122.0$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{240.0}{\sqrt{61.0(1.0)}} = 30.7289$$

Since $\dfrac{D_o}{t} \le 135$, calculate F_{xa1} as follows with an initial value of $FS = 1.0$.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.0} = 33600 \; psi$$

The value of F_{xa2} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{xa2} = \frac{F_{xe}}{FS}$$

$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since $\dfrac{D_o}{t} \le 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \dfrac{D_o}{t}\right)}, \; 0.9\right]$$

Since $M_x \ge 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389+\dfrac{122.0}{1.0}},\ 0.9\right] = 0.8004$$

Therefore,

$$F_{xe} = \frac{0.8004(28.3E+06)(1.0)}{122.0} = 185666.5574\ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{185666.5574}{1.0} = 185666.5574\ psi$$

$$F_{xa} = \min[33600,\ 185666.5574] = 33600\ psi$$

With a value of $F_{ic} = F_{xa} = 33600$, in accordance with VIII-2, paragraph 4.4.2, it is determined the value of $FS = 1.667$ since $\{F_{ic} = 33600\} = \{S_y = 33600\}$. Using this computed value of $FS = 1.667$ in paragraph 4.4.12.2.b, F_{xa} is calculated as follows.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.667} = 20155.9688\ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{185666.5574}{1.6670} = 111377.6589\ psi$$

$$F_{xa} = \min[20155.9688,\ 111377.6589] = 20155.9688\ psi$$

Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per the following criteria:

$$\left\{\begin{array}{l}\sigma_{sm} = 305.9\ psi \\ \sigma_{sm} = 667.5\ psi\end{array}\right\} \le \{F_{xa} = 20156.0\ psi\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

f) STEP 6 – The stress acceptance criterion in STEP 6 is satisfied. Therefore the design is complete.

4.5 Shells Openings in Shells and Heads

4.5.1 Example E4.5.1 – Radial Nozzle in Cylindrical Shell

Design an integral nozzle in a cylindrical shell based on the vessel and nozzle data below. The parameters used in this design procedure are shown in Figure E4.5.1.

Vessel and Nozzle Data:

- Design Conditions = $356\ psig @ 300°F$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0
- Shell Material = $SA-516,\ Grade\ 70,\ Norm.$
- Shell Allowable Stress = $20000\ psi$
- Yield Strength = $33600\ psi$
- Nozzle Material = $SA-105$
- Nozzle Allowable Stress = $20000\ psi$
- Shell Inside Diameter = $150.0\ in$
- Shell Thickness = $1.8125\ in$
- Nozzle Outside Diameter = $19.0\ in$
- Nozzle Hub Outside Diameter = $25.5\ in$
- Nozzle Thickness = $1.5\ in$
- Nozzle Hub Thickness = $4.75\ in$
- External Nozzle Projection = $14.1875\ in$
- Internal Nozzle Projection = $0.0 in$

The nozzle is inserted through the shell, i.e. set–in type nozzle, see Fig. UW-16.1(g).

Establish the corroded dimensions.

Shell:

$$D = 150.0 + 2(Corrosion\ Allowance) = 150.0 + 2(0.125) = 150.25\ in$$

$$R = \frac{D}{2} = \frac{150.25}{2} = 75.125\ in$$

$$t = 1.8125 - Corrosion\ Allowance = 1.8125 - 0.125 = 1.6875\ in$$

Nozzle:

$$t_n = 1.5 - Corrosion\ Allowance = 1.5 - 0.125 = 1.375\ in$$

$$t_x = 4.75 - Corrosion\ Allowance = 4.75 - 0.125 = 4.625\ in$$

$$R_n = \frac{D_n - 2(t_n)}{2} = \frac{19.0 - 2(1.375)}{2} = 8.125\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-37.

The required thickness of the shell based on circumferential stress is given by UG-27(c)(1).

$$t_r = \frac{PR}{SE-0.6P} = \frac{356(75.125)}{20000(1.0)-0.6(356)} = 1.3517\ in$$

The required thickness of the nozzle based on circumferential stress is given by UG-27(c)(1).

$$t_{rn} = \frac{PR_n}{SE-0.6P} = \frac{356(8.125)}{20000(1.0)-0.6(356)} = 0.1462\ in$$

a) STEP 1: – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for an integrally reinforced nozzle per Fig. UG-40(e), UG-40(e-1), UG-40(e-2): See Figure E4.5.1 of this example.

$$t_x = 4.625\ in$$

$$L = 7.1875\ in$$

$$\left\{L = 7.1875\ in\right\} < \left\{2.5t_x = 2.5(4.625) = 11.5625\ in\right\}$$

$$\textit{Therefore use UG}-40(e-1)\ \left\{\begin{array}{l} t_n = 1.375\ in \\ t_e = \dfrac{4.625-1.375}{\tan[30]} = 5.6292\ in \\ D_p = 25.5\ in \end{array}\right\}$$

2) Finished opening chord length.

$$d = 2R_n = 2(8.125) = 16.25\ in$$

3) The limits of reinforcement, measured parallel to the vessel wall in the corroded condition.

$$\max\left[d,\ R_n + t_n + t\right] = \max\left[16.25,\ \left\{8.125 + 1.375 + 1.6875\right\}\right] = 16.25\ in$$

4) The limits of reinforcement, measured normal to the vessel wall in the corroded condition.

$$\min\left[2.5t,\ 2.5t_n + t_e\right] = \min\left[2.5(1.6875),\ \left\{2.5(1.375) + 5.6292\right\}\right] = 4.2188\ in$$

b) STEP 2 – Calculate reinforcement strength parameters per UG-37.

1) Strength Reduction Factors:

$$f_{r1} = S_n/S_v = 20000/20000 = 1.0$$

$$f_{r2} = S_n/S_v = 20000/20000 = 1.0$$

$$f_{r3} = \min\left[S_n, S_p\right]/S_v = 20000/20000 = 1.0$$

$$f_{r4} = S_p/S_v = 20000/20000 = 1.0$$

2) Joint Efficiency Parameter: For a nozzle located in a solid plate, $E_1 = 1.0$.

3) Correction Factor for variation of internal pressure stresses on different planes with respect to the axis of the vessel: For a radial nozzle in a cylindrical shell, $F = 1.0$.

c) STEP 3 – Calculate the Areas of Reinforcement, see Fig. UG–37.1 (With Reinforcing Element, per Fig.UG-40(e-1)).

1) Area Required, A:

$$A = dt_r F + 2t_n t_r F(1 - f_{r1})$$
$$A = 16.25(1.3517)(1.0) + 2(1.375)(1.3517)(1.0)(1 - 1.0) = 21.9651\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d\left(E_1 t - F t_r\right) - 2t_n\left(E_1 t - F t_r\right)(1 - f_{r1})$$
$$A_{11} = \left\{\begin{array}{l} 16.25\left(1.0(1.6875) - 1.0(1.3517)\right) - \\ 2(1.375)\left\{\left(1.0(1.6875) - 1.0(1.3517)\right)(1 - 1.0)\right\} \end{array}\right\} = 5.4568\ in^2$$
$$A_{12} = 2\left(t + t_n\right)\left(E_1 t - F t_r\right) - 2t_n\left(E_1 t - F t_r\right)(1 - f_{r1})$$
$$A_{12} = \left\{\begin{array}{l} 2(1.6875 + 1.375)\left(1.0(1.6875) - 1.0(1.3517)\right) - \\ 2(1.375)\left\{\left(1.0(1.6875) - 1.0(1.3517)\right)(1 - 1.0)\right\} \end{array}\right\} = 2.0568\ in^2$$
$$A_1 = \max\left[5.4568,\ 2.0568\right] = 5.4568\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use smaller value:

$$A_{21} = 5(t_n - t_{rn}) f_{r2} t$$
$$A_{21} = 5(1.375 - 0.1462)(1.0)(1.6875) = 10.3680\ in^2$$
$$A_{22} = 2(t_n - t_{rn})(2.5t_n + t_e) f_{r2}$$
$$A_{22} = 2(1.375 - 0.1462)\left(2.5(1.375) + 5.6292\right)(1.0) = 22.2823\ in^2$$
$$A_2 = \min\left[10.3680,\ 22.2823\right] = 10.3680\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3. Use smaller value:

$$A_3 = \min\left[5tt_i f_{r2},\ 5t_i t_i f_{r2},\ 2ht_i f_{r2}\right]$$
$$A_3 = 0.0 \qquad since\ t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.1 of this example:

Outer Nozzle Fillet Weld Leg:	$0.375\ in$
Outer Element Fillet Weld Leg:	$0.0\ in$
Inner Nozzle Fillet Weld Leg:	$0.0\ in$

$$A_{41} = leg^2 f_{r3} = (0.375)^2(1.0) = 0.1406\ in^2$$
$$A_{42} = 0.0\ in^2$$
$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = \left(D_p - d - 2t_n\right)t_e f_{r4} = \left(25.5 - 16.25 - 2\left(1.375\right)\right)\left(4.2188\right)\left(1.0\right) = 27.4222\ in^2$$

Note: The thickness of the reinforcing pad, t_e, exceeds the outside vertical reinforcement zone limit. Therefore, the reinforcement area in the pad is limited to within the zone.

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + \left(A_{41} + A_{42} + A_{43}\right) + A_5$$
$$A_{avail} = 5.4568 + 10.3680 + 0.0 + \left(0.1406 + 0.0 + 0.0\right) + 27.4222 = 43.3876\ in^2$$

d) STEP 4 – Nozzle reinforcement acceptance criterion:

$$\left\{A_{avail} = 43.3876\ in^2\right\} \geq \left\{A = 21.9651\ in^2\right\} \qquad True$$

Therefore, the nozzle is adequately reinforced.

Commentary: The procedure shown above considered the hub thickened section of the nozzle as a separate reinforcement pad, although it is integral with the nozzle. The VIII-1 code does not provide explicit rules for determination of the available area in the nozzle, A_2 for thickened integrally reinforced nozzles. However, the various nozzle sketches in Fig UW-16.1 consistently indicate the nozzle thickness, t_n is that of the upper nozzle neck section. Therefore, it is believed appropriate to remain consistent with the provided nomenclature and evaluate the hub thickened portion of the nozzle as separate reinforcement, paying close attention to the proper use of the strength reduction factor, f_r and the limit of reinforcement measured normal to the vessel.

For comparison purposes, if the value of t_n was re–defined to that of the hub thickened section, where $t_n = 4.625\ in$, the Area of Reinforcement calculations would be as follows.

1) Area Required, A:

$$A = dt_rF + 2t_nt_rF(1 - f_{r1})$$
$$A = 16.25\left(1.3517\right)\left(1.0\right) + 2\left(4.625\right)\left(1.3517\right)\left(1.0\right)\left(1 - 1.0\right) = 21.9651\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d\left(E_1t - Ft_r\right) - 2t_n\left(E_1t - Ft_r\right)\left(1 - f_{r1}\right)$$
$$A_{11} = \left\{\begin{array}{l} 16.25\left(1.0\left(1.6875\right) - 1.0\left(1.3517\right)\right) - \\ 2\left(4.625\right)\left\{\left(1.0\left(1.6875\right) - 1.0\left(1.3517\right)\right)\left(1 - 1.0\right)\right\} \end{array}\right\} = 5.4568\ in^2$$
$$A_{12} = 2\left(t + t_n\right)\left(E_1t - Ft_r\right) - 2t_n\left(E_1t - Ft_r\right)\left(1 - f_{r1}\right)$$
$$A_{12} = \left\{\begin{array}{l} 2\left(1.6875 + 4.625\right)\left(1.0\left(1.6875\right) - 1.0\left(1.3517\right)\right) - \\ 2\left(4.625\right)\left\{\left(1.0\left(1.6875\right) - 1.0\left(1.3517\right)\right)\left(1 - 1.0\right)\right\} \end{array}\right\} = 4.2395\ in^2$$
$$A_1 = \max\left[5.4568,\ 4.2395\right] = 5.4568\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use smaller value:

$$A_{21} = 5(t_n - t_{rn}) f_{r2} t$$
$$A_{21} = 5(4.625 - 0.1462)(1.0)(1.6875) = 37.7899 \; in^2$$
$$A_{22} = 2(t_n - t_{rn})(2.5t_n + t_e) f_{r2}$$
$$A_{22} = 2(4.625 - 0.1462)(2.5(4.625) + 0.0)(1.0) = 103.5723 \; in^2$$
$$A_2 = \min[37.7899, \; 103.5723] = 37.7899 \; in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3. Use smaller value:

$$A_3 = \min[5tt_i f_{r2}, 5t_i t_i f_{r2}, 2ht_i f_{r2}]$$
$$A_3 = 0.0 \qquad since \; t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.1 of this example:

$$Outer\ Nozzle\ Fillet\ Weld\ Leg: \quad 0.375 \; in$$
$$Outer\ Element\ Fillet\ Weld\ Leg: \quad 0.0 \; in$$
$$Inner\ Nozzle\ Fillet\ Weld\ Leg: \quad 0.0 \; in$$

$$A_{41} = leg^2 f_{r3} = (0.375)^2 (1.0) = 0.1406 \; in^2$$
$$A_{42} = 0.0 \; in^2$$
$$A_{43} = 0.0 \; in^2$$

6) Area Available in Element, A_5:

$$A_5 = 0.0 \; in^2$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + (A_{41} + A_{42} + A_{43}) + A_5$$
$$A_{avail} = 5.4568 + 37.7899 + 0.0 + (0.1406 + 0.0 + 0.0) + 0.0 = 43.3873 \; in^2$$

Therefore, it is shown that either procedure will provide equivalent total available area, A_{avail}; both satisfying the required area acceptance criterion.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

The procedure, per VIII-2, paragraph 4.5.5, to design a radial nozzle in a cylindrical shell subject to pressure loading is shown below.

a) STEP 1 – Determine the effective radius of the shell as follows

$$R_{eff} = 0.5D_i = 0.5(150.25) = 75.125 \; in$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall.

For integrally reinforced nozzles:

$$L_R = \min\left[\sqrt{R_{eff}t},\ 2R_n\right]$$

$$L_R = \min\left[\sqrt{(75.125)(1.6875)},\ \ 2(8.125)\right] = \min[11.2594, 16.25] = 11.2594\ in$$

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in VIII-2, paragraph 4.5.13 would need to be checked.

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface.

For set–in nozzles:

$$L_{H1} = \min(1.5t, t_e) + \sqrt{R_n t_n} = \min(1.5 \times 1.6875, 0.0) + \sqrt{8.125(4.625)} = 6.1301\ in$$

$$L_{H2} = L_{pr1} = 14.1875 in$$

$$L_{H3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5\ in$$

$$L_H = \min\left[L_{H1},\ L_{H2},\ L_{H3}\right] + t = \min[6.1301,\ 15.875,\ 13.5] + 1.6875 = 7.8176\ in$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable

$$L_{I1} = \sqrt{R_n t_n} = \sqrt{8.125(4.625)} = 6.1301\ in$$

$$L_{I2} = L_{pr2} = 0.0$$

$$L_{I3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5\ in$$

$$L_I = \min\left[L_{I1},\ L_{I2},\ L_{I3}\right] = \min[6.1301,\ 0.0,\ 13.5] = 0.0$$

e) STEP 5 – Determine the total available area near the nozzle opening (see VIII-2, Figure 4.5.1). Do not include any area that falls outside of the limits defined by L_H, L_R, and L_I.

For set–in nozzles:

$$A_T = A_1 + f_{rn}(A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp}A_5$$

$$A_1 = (tL_R) \cdot \max\left[\left(\frac{\lambda}{5}\right)^{0.85},\ 1.0\right] = 1.6875(11.2594) \cdot \max\left[\left(\frac{1.3037}{5}\right)^{0.85},\ 1.0\right]$$

$$A_1 = 19.0002\ in^2$$

$$\lambda = \min\left[\left\{\frac{(2R_n + t_n)}{\sqrt{(D_i + t_{eff})t_{eff}}}\right\},\ 12.0\right] = \min\left[\frac{2(8.125) + 4.625}{\sqrt{150.25 + 1.6875}(1.6875)},\ 12.0\right] = 1.3037$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 1.6875 + \left(\frac{0.0(0.0)}{11.2594}\right) = 1.6875\ in$$

$$f_{rp} = \min\left[\frac{S_p}{S}, 1\right] = 0.0$$

$$f_{rn} = \min\left[\frac{S_n}{S}, 1\right] = \frac{20000}{20000} = 1.0$$

Since $\{L_H = 7.8176\ in\} \le \{L_{x3} = L_{pr3} + t = 7.1875 + 1.6875 = 8.875\ in\}$, calculate A_2 as follows, see VIII-2, Figure 4.5.13:

$$A_2 = t_n L_H = 4.625(7.8176) = 36.1564\ in^2$$

$$A_3 = t_n L_I = 0.0$$

$$A_{41} = 0.5L_{41}^2 = 0.5(0.375)^2 = 0.0703\ in^2$$

$$A_{42} = 0.5L_{42}^2 = 0.0$$

$$A_{43} = 0.5L_{43}^2 = 0.0$$

$$A_5 = \min\left[A_{5a},\ A_{5b}\right]$$

$$A_{5a} = Wt_e = 0.0$$

$$A_{5b} = L_R t_e = 0.0$$

$$A_5 = 0.0$$

$$A_T = 19.0002 + 1.0(36.1564 + 0.0) + 0.0703 + 0.0 + 0.0 + 0.0 = 55.2269\ in^2$$

f) STEP 6 – Determine the applicable forces

$$f_N = PR_{xn}L_H = 356(10.2644)(7.8176) = 28566.4985\ lbs$$

$$R_{xn} = \frac{t_n}{\ln\left[\dfrac{R_n + t_n}{R_n}\right]} = \frac{4.625}{\ln\left[\dfrac{8.125 + 4.625}{8.125}\right]} = 10.2644\ in$$

$$f_S = PR_{xs}\left(L_R + t_n\right) = 356(75.9656)(11.2594 + 4.625) = 429573.7997\ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\dfrac{R_{eff} + t_{eff}}{R_{eff}}\right]} = \frac{1.6875}{\ln\left[\dfrac{75.125 + 1.6875}{75.125}\right]} = 75.9656\ in$$

$$f_Y = PR_{xs}R_{nc} = 356(75.9656)(8.125) = 219730.4980\ lbs$$

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection.

$$\sigma_{avg} = \frac{\left(f_N + f_S + f_Y\right)}{A_T} = \frac{28566.4985 + 429573.7997 + 219730.4980}{55.2269} = 12274.2866\ psi$$

$$\sigma_{circ} = \frac{PR_{xs}}{t_{eff}} = \frac{356(75.9656)}{1.6875} = 16025.9281 \ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection.

$$P_L = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\}, \sigma_{circ}\right]$$

$$P_L = \max\left[\left\{2(12274.2866) - 16025.9281\right\}, 16025.9281\right] = 16025.9281 \ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy VIII-2, Equation 4.5.56. If the nozzle is subjected to internal pressure, then the allowable stress, S_{allow}, is given by VIII-2, Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by VIII-2, Equation 4.5.58 where F_{ha} is evaluated in VIII-2, paragraph 4.4 for the shell geometry being evaluated (e.g. cylinder, spherical shell, or formed head). The allowable stress shall be the minimum of the shell or nozzle material evaluated at the design temperature.

$$\left\{P_L = 16025.9281 \ psi\right\} \le \left\{S_{allow} = 1.5SE = 1.5(20000)(1.0) = 30000 \ psi\right\}$$

j) STEP 10 – Determine the maximum allowable working pressure at the nozzle intersection.

$$P_{\max 1} = \frac{S_{allow}}{\dfrac{2A_p}{A_T} - \dfrac{R_{xs}}{t_{eff}}} = \frac{1.5(20000)(1.0)}{\left(\dfrac{2(1904.1315)}{55.2269}\right) - \left(\dfrac{75.9656}{1.6875}\right)} = 1253.1320 \ psi$$

$$A_p = \frac{f_N + f_S + f_Y}{P} = \frac{28566.4985 + 429573.7997 + 219730.4980}{356.0} = 1904.1315 \ in^2$$

$$P_{\max 2} = S\left(\frac{t}{R_{xs}}\right) = 20000\left(\frac{1.6875}{75.9656}\right) = 444.28 \ psi$$

$$P_{\max} = \min\left[P_{\max 1}, P_{\max 2}\right] = \min\left[1253.1320, \ 444.28\right] = 444.28 \ psi$$

The nozzle is acceptable because $P_{\max} = 444.28 \ psi$ is greater than the specified design pressure of $356 \ psig$.

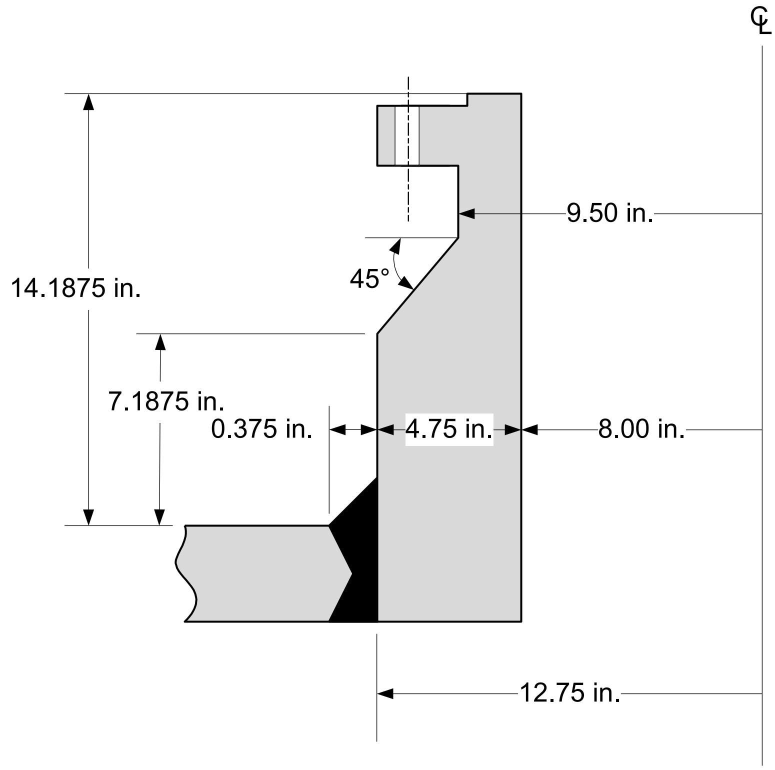


Figure E4.5.1 – Nozzle Detail

4.5.2 Example E4.5.2 – Hillside Nozzle in Cylindrical Shell

Design an integral hillside nozzle in a cylindrical shell based on the vessel and nozzle data below. The parameters used in this design procedure are shown in Figure E4.5.2.

Vessel and Nozzle Data:

- Design Conditions = $356\ psig@300°F$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0
- Shell Material = $SA-516,\ Grade\ 70,\ Norm.$
- Shell Allowable Stress = $20000\ psi$
- Shell Yield Strength = $33600\ psi$
- Nozzle Material = $SA-105$
- Shell Inside Diameter = $150.0\ in$
- Shell Thickness = $1.8125\ in$
- Nozzle Outside Diameter = $11.56\ in$
- Nozzle Thickness = $1.97\ in$
- External Nozzle Projection = $19.0610\ in$
- Internal Nozzle Projection = $0.0\ in$
- Nozzle Offset = $34.875\ in$

The nozzle is inserted through the shell, i.e. set–in type nozzle, see Fig. UW-16.1(d).

Establish the corroded dimensions.

Shell:

$$D = 150.0 + 2\cdot(Corrosion\ Allowance) = 150.0 + 2(0.125) = 150.25\ in$$

$$R = \frac{D}{2} = \frac{150.25}{2} = 75.125\ in$$

$$t = 1.8125 - Corrosion\ Allowance = 1.8125 - 0.125 = 1.6875\ in$$

Nozzle:

$$t_n = 1.97 - Corrosion\ Allowance = 1.97 - 0.125 = 1.845\ in$$

$$R_n = \frac{D_{n,OD} - 2(t_n)}{2} = \frac{11.56 - 2(1.845)}{2} = 3.935\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-37.

The required thickness of the shell based on circumferential stress is given by UG-27(c)(1).

$$t_r = \frac{PR}{SE - 0.6P} = \frac{356(75.125)}{20000(1.0) - 0.6(356)} = 1.3517\ in$$

The required thickness of the nozzle based on circumferential stress is given by UG-27(c)(1).

$$t_{rn} = \frac{PR_n}{SE - 0.6P} = \frac{356(3.935)}{20000(1.0) - 0.6(356)} = 0.0708\ in$$

a) STEP 1 – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for an integrally reinforced nozzle per Fig. UG-40(e), UG-40(e-1), UG-40(e-2): See Figure E4.5.2 of this example:

$$t_x = 1.845\ in$$

$$L \approx 16.0\ in$$

$$\{L = 16.0\ in\} \geq \{2.5t_x = 2.5(1.845) = 4.6125\ in\}$$

$$Therefore\ use\ UG-40(e-2)\quad \begin{Bmatrix} t_n = 1.845\ in \\ t_e = 0.0 \end{Bmatrix}$$

Note: Fig. UG-40 does not provide a sketch for an integral uniform thickness nozzle with full penetration weld inserted through the shell without a reinforcing pad. Therefore, sketch (e-1) was used with $t_e = 0.0$. Additionally, the value of L is approximate and is determined by subtracting the flange thickness from the external nozzle projection, see Figure E4.5.2.

2) Finished opening chord length.

i) Perpendicular to longitudinal axis, see Figure E4.5.2

$$R_m = Mean\ Cylinder\ Radius = R + \frac{t_r}{2} = 75.125 + \frac{1.3517}{2} = 75.8009\ in$$

$$L_{off} = Offset\ Length = 34.875\ in$$

$$x_1 = L_{off} + R_n = 34.875 + 3.935 = 38.81\ in$$

$$x_2 = L_{off} - R_n = 34.875 + 3.935 = 30.94\ in$$

$$y_1 = \sqrt{R_m^2 - x_1^2} = \sqrt{75.8009^2 - 38.81^2} = 65.1119\ in$$

$$y_2 = \sqrt{R_m^2 - x_2^2} = \sqrt{75.8009^2 - 30.94^2} = 69.1989\ in$$

$$d = \sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2} = \sqrt{(38.81 - 30.94)^2 + (69.1989 - 65.1119)^2} = 8.8679\ in$$

ii) Finished opening chord length – parallel to longitudinal axis

$$d = 2R_n = 2(3.935) = 7.870\ in$$

3) The limits of reinforcement, measured parallel to the vessel wall in the corroded condition.

i) Perpendicular to longitudinal axis

$$\max[d,\ R_n + t_n + t] = \max[8.8679,\ \{3.935 + 1.8450 + 1.6875\}] = 8.8679\ in$$

ii) Parallel to longitudinal axis

$$\max[d,\ R_n + t_n + t] = \max[7.870,\ \{3.935 + 1.8450 + 1.6875\}] = 7.870\ in$$

4) The limits of reinforcement, measured normal to the vessel wall in the corroded condition.

$$\min\left[2.5t,\ 2.5t_n+t_e\right]=\min\left[2.5(1.6875),\ \{2.5(1.845)+0.0\}\right]=4.2188\ in$$

b) STEP 2 – Calculate reinforcement strength parameters per UG-37

1) Strength Reduction Factors:

$$f_{r1}=S_n/S_v=20000/20000=1.0$$

$$f_{r2}=S_n/S_v=20000/20000=1.0$$

$$f_{r3}=\min\left[S_n,S_p\right]/S_v=0$$

$$f_{r4}=S_p/S_v=0$$

2) Joint Efficiency Parameter: For a nozzle located in a solid plate, $E_1=1.0$.

3) Correction Factor for variation of internal pressure stresses on different planes with respect to the axis of the vessel. Fig. UG-37 may be used for integrally reinforced openings in cylindrical shells and cones. See UW-16(c)(1).

i) For an opening perpendicular to the longitudinal axis, $d=8.8679\ in\rightarrow F=0.5$.

ii) For an opening parallel to the longitudinal axis, $d=7.870\ in\rightarrow F=1.0$.

c) STEP 3 – Calculate the Areas of Reinforcement perpendicular to the longitudinal axis, $F=0.5$. See Fig. UG-37.1.

1) Area Required, A:

$$A=dt_rF+2t_nt_rF(1-f_{r1})$$

$$A=8.8679(1.3517)(0.5)+2(1.375)(1.3517)(1.0)(1-1.0)=5.9934\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11}=d\left(E_1t-Ft_r\right)-2t_n\left(E_1t-Ft_r\right)(1-f_{r1})$$

$$A_{11}=\left\{\begin{array}{l}8.8679\left(1.0(1.6875)-0.5(1.3517)\right)-\\2(1.375)\left\{\left(1.0(1.6875)-0.5(1.3517)\right)(1-1.0)\right\}\end{array}\right\}=8.9712\ in^2$$

$$A_{12}=2\left(t+t_n\right)\left(E_1t-Ft_r\right)-2t_n\left(E_1t-Ft_r\right)(1-f_{r1})$$

$$A_{12}=\left\{\begin{array}{l}2(1.6875+1.845)\left(1.0(1.6875)-0.5(1.3517)\right)-\\2(1.375)\left\{\left(1.0(1.6875)-0.5(1.3517)\right)(1-1.0)\right\}\end{array}\right\}=7.1473\ in^2$$

$$A_1=\max\left[8.9712,\ 7.1473\right]=8.9712\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use smaller value:

$$A_{21}=5(t_n-t_{rn})f_{r2}t=5(1.845-0.0708)(1.0)(1.6875)=14.9698\ in^2$$

$$A_{22}=5(t_n-t_{rn})f_{r2}t_n=5(1.845-0.0708)(1.0)(1.845)=16.3670\ in^2$$

$$A_2=\min\left[14.9698,\ 16.3670\right]=14.9698\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3. Use smaller value:

$$A_3 = \min\left[5tt_i f_{r2}, 5t_i t_i f_{r2}, 2ht_i f_{r2}\right]$$
$$A_3 = 0.0 \qquad since\ t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.2 of this example:

$$Outer\ Nozzle\ Fillet\ Weld\ Leg: \quad 0.375\ in$$
$$Outer\ Element\ Fillet\ Weld\ Leg: \quad 0.0\ in$$
$$Inner\ Nozzle\ Fillet\ Weld\ Leg: \quad 0.0\ in$$

$$A_{41} = leg^2 f_{r3} = (0.375)^2 (1.0) = 0.1406\ in^2$$
$$A_{42} = 0.0\ in^2$$
$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = \left(D_p - d - 2t_n\right) t_e f_{r4} = 0.0\ in^2$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + \left(A_{41} + A_{42} + A_{43}\right) + A_5$$
$$A_{avail} = 8.9712 + 14.9698 + 0.0 + (0.1406 + 0.0 + 0.0) + 0.0 = 24.0816\ in^2$$

d) STEP 4 – Nozzle reinforcement acceptance criterion:

$$\left\{A_{avail} = 24.0816\ in^2\right\} \geq \left\{A = 5.9934\ in^2\right\} \qquad True$$

Therefore, the nozzle is adequately reinforced in the plane perpendicular to the longitudinal axis.

e) STEP 5 – Calculate the Areas of Reinforcement, parallel to the longitudinal axis, $F = 1.0$. See Fig. UG-37.1.

1) Area Required, A:

$$A = dt_r F + 2t_n t_r F(1 - f_{r1})$$
$$A = 7.870(1.3517)(1.0) + 2(1.375)(1.3517)(1.0)(1 - 1.0) = 10.6379\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d\left(E_1 t - F t_r\right) - 2t_n\left(E_1 t - F t_r\right)\left(1 - f_{r1}\right)$$

$$A_{11} = \left\{\begin{array}{l} 7.870\left(1.0\left(1.6875\right) - 1.0\left(1.3517\right)\right) - \\ 2\left(1.375\right)\left\{\left(1.0\left(1.6875\right) - 1.0\left(1.3517\right)\right)\left(1 - 1.0\right)\right\} \end{array}\right\} = 2.6427\ in^2$$

$$A_{12} = 2\left(t + t_n\right)\left(E_1 t - F t_r\right) - 2t_n\left(E_1 t - F t_r\right)\left(1 - f_{r1}\right)$$

$$A_{12} = \left\{\begin{array}{l} 2\left(1.6875 + 1.845\right)\left(1.0\left(1.6875\right) - 1.0\left(1.3517\right)\right) - \\ 2\left(1.375\right)\left\{\left(1.0\left(1.6875\right) - 1.0\left(1.3517\right)\right)\left(1 - 1.0\right)\right\} \end{array}\right\} = 2.3724\ in^2$$

$$A_1 = \max\left[2.6427,\ 2.3724\right] = 2.6427\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use smaller value:

$$A_{21} = 5(t_n - t_{rn}) f_{r2} t$$

$$A_{21} = 5\left(1.845 - 0.0708\right)\left(1.0\right)\left(1.6875\right) = 14.9698\ in^2$$

$$A_{22} = 5(t_n - t_{rn}) f_{r2} t_n$$

$$A_{22} = 5\left(1.845 - 0.0708\right)\left(1.0\right)\left(1.845\right) = 16.3670\ in^2$$

$$A_2 = \min\left[14.9698,\ 16.3670\right] = 14.9698\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3. Use smaller value:

$$A_3 = \min\left[5 t t_i f_{r2},\ 5 t_i t_i f_{r2},\ 2 h t_i f_{r2}\right]$$

$$A_3 = 0.0 \qquad since\ t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.2 of this example:

Outer Nozzle Fillet Weld Leg :	0.375 *inches*
Outer Element Fillet Weld Leg :	0.0 *inches*
Inner Nozzle Fillet Weld Leg :	0.0 *inches*

$$A_{41} = leg^2 f_{r3} = \left(0.375\right)^2\left(1.0\right) = 0.1406\ in^2$$

$$A_{42} = 0.0\ in^2$$

$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = \left(D_p - d - 2t_n\right) t_e f_{r4} = 0.0\ in^2$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + \left(A_{41} + A_{42} + A_{43}\right) + A_5$$

$$A_{avail} = 2.6427 + 14.9698 + 0.0 + \left(0.1406 + 0.0 + 0.0\right) + 0.0 = 17.7531\ in^2$$

f) STEP 6 – Nozzle reinforcement acceptance criterion:

$$\left\{A_{avail}=17.7531\ in^2\right\}\geq\left\{A=10.6379\ in^2\right\}\qquad True$$

Therefore, the nozzle is adequately reinforced in the plane parallel to the longitudinal axis.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

For a hillside nozzle in a cylindrical shell (see VIII-2, Figure 4.5.4), the design procedure in VIII-2, paragraph 4.5.5 shall be used with the following substitutions from VIII-2, paragraph 4.5.6.

$$R_{nc}=\max\left[\left(\frac{R_{ncl}}{2}\right),\ R_n\right]$$

Where,

$$R_{ncl}=R_{eff}\left(\theta_1-\theta_2\right)$$

$$\theta_1=\cos^{-1}\left[\frac{D_X}{R_{eff}}\right]=\cos^{-1}\left[\frac{34.875}{75.125}\right]=62.3398\ deg=1.0880\ rad$$

$$\theta_2=\cos^{-1}\left[\frac{D_X+R_n}{R_{eff}}\right]=\cos^{-1}\left[\frac{34.875+3.935}{75.125}\right]=58.8952\ deg=1.0279\ rad$$

$$R_{ncl}=75.125\left(1.0880-1.0279\right)=4.5150\ in$$

$$R_{nc}=\max\left[\left(\frac{4.5150}{2}\right),\ 3.935\right]=3.935\ in$$

The procedure in VIII-2, paragraph 4.5.5 is shown below.

a) STEP 1 – Determine the effective radius of the shell as follows:

$$R_{eff}=0.5D_i=0.5\left(150.25\right)=75.125\ in$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall:

For integrally reinforced nozzles:

$$L_R=\min\left[\sqrt{R_{eff}t},\ 2R_n\right]=\min\left[\sqrt{75.125\left(1.6875\right)},\ 2\left(3.935\right)\right]=7.87\ in$$

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in VIII-2, paragraph 4.5.13 would need to be checked.

c) STEP 3 - Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface.

For set–in nozzles:

$$L_{H1}=\min\left[1.5t,t_e\right]+\sqrt{R_nt_n}=\min\left[1.5\times1.6875,0\right]+\sqrt{3.935\left(1.845\right)}=2.6945\ in$$

$$L_{H2} = L_{pr1} = 19.0610 \; in$$

$$L_{H3} = 8\left(t + t_e\right) = 8\left(1.6875 + 0.0\right) = 13.5 \; in$$

$$L_H = \min\left[L_{H1}, L_{H2}, L_{H3}\right] + t = 4.3820 \; in$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable:

$$L_{I1} = \sqrt{R_n t_n} = \sqrt{3.935\left(1.845\right)} = 2.6945$$

$$L_{I2} = L_{pr2} = 0.0$$

$$L_{I3} = 8\left(t + t_e\right) = 8\left(1.6875 + 0.0\right) = 13.5 \; in$$

$$L_I = \min\left[L_{I1}, L_{I2}, L_{I3}\right] = 0.0$$

e) STEP 5 – Determine the total available area near the nozzle opening (see VIII-2, Figures 4.5.1 and 4.5.2). Do not include any area that falls outside of the limits defined by L_H, L_R, and L_I.

For set–in nozzles:

$$A_T = A_1 + f_{rn}(A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp}A_5$$

$$A_1 = \left(tL_R\right)\cdot\max\left[\left(\frac{\lambda}{5}\right)^{0.85}, 1.0\right] = 1.6875\left(7.87\right)\cdot\max\left[\left(\frac{0.6067}{5}\right)^{0.85}, 1.0\right] = 13.2806$$

$$\lambda = \min\left[\left\{\frac{\left(2r_n + t_n\right)}{\sqrt{\left(D_i + t_{eff}\right)t_{eff}}}\right\}, 12.0\right] = \min\left[\left\{\frac{2\left(3.935\right) + 1.845}{\sqrt{\left(150.25 + 1.6875\right)\left(1.6875\right)}}\right\}, 12.0\right]$$

$$\lambda = 0.6067$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 1.6875 + \left(\frac{0.0\left(0.0\right)}{7.87}\right) = 1.6875 \; in$$

$$f_{rn} = \frac{S_n}{S} = \frac{20000}{20000} = 1.0$$

$$f_{rp} = \frac{S_p}{S} = 0.0$$

Since $\left\{t_n = 1.845 \; in\right\} = \left\{t_{n2} = 1.845 \; in\right\}$, calculate A_2 as follows:

$$A_2 = t_n L_H = 1.845\left(4.3820\right) = 8.0848 \; in^2$$

$$A_3 = t_n L_I = 1.845\left(0.0\right) = 0.0$$

$$A_{41} = 0.5L_{41}^2 = 0.5\left(0.375\right) = 0.0703 \; in^2$$

$$A_{42} = 0.5L_{42}^2 = 0.0$$

$$A_{43} = 0.5L_{43}^2 = 0.0$$

$$A_5 = \min\left[A_{5a},\ A_{5b}\right]$$
$$A_{5a} = Wt_e = 0.0$$
$$A_{5b} = L_R t_e = 0.0$$
$$A_5 = 0.0$$

$$A_T = 13.2806 + 1.0(8.0848 + 0.0) + 0.0 + 0.0703 + 0.0 + 0.0 = 21.4357\ in^2$$

f) STEP 6 – Determine the applicable forces:

$$f_N = PR_{xn}L_H = 356(4.7985)(4.3820) = 7485.6216\ lbs$$

$$R_{xn} = \frac{t_n}{\ln\left[\dfrac{R_n + t_n}{R_n}\right]} = \frac{1.845}{\ln\left[\dfrac{3.935 + 1.845}{3.935}\right]} = 4.7985\ in$$

$$f_S = PR_{xs}\left(L_R + t_n\right) = 356(75.9656)(7.87 + 1.845) = 262730.0662\ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\dfrac{R_{eff} + t_{eff}}{R_{eff}}\right]} = \frac{1.6875}{\ln\left[\dfrac{75.125 + 1.6875}{75.125}\right]} = 75.9656\ in$$

$$f_Y = PR_{xs}R_{nc} = 356(75.9656)(3.935) = 106417.1704\ lbs$$

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection:

$$\sigma_{avg} = \frac{\left(f_N + f_S + f_Y\right)}{A_T} = \frac{\left(7485.6216 + 262730.0662 + 106417.1704\right)}{21.4357} = 17570.3550\ psi$$

$$\sigma_{circ} = \frac{PR_{xs}}{t_{eff}} = \frac{356(75.9656)}{1.6875} = 16025.9281\ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection:

$$P_L = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\},\ \sigma_{circ}\right]$$
$$P_L = \max\left[\left\{2(17570.3550) - 16025.9281\right\},\ 16025.9281\right] = 19114.7819\ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy VIII-2, Equation 4.5.56. If the nozzle is subjected to internal pressure, then the allowable stress, S_{allow}, is given by VIII-2, Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by VIII-2, Equation 4.5.58 where F_{ha} is evaluated in VIII-2, paragraph 4.4 for the shell geometry being evaluated (e.g. cylinder, spherical shell, or formed head). The allowable stress shall be the minimum of the shell or nozzle material evaluated at the design temperature.

$$\{P_L = 19114.7819\ psi\} \le \{S_{allow} = 1.5SE = 1.5(20000)(1.0) = 30000\ psi\}$$

j) STEP 10 – Determine the maximum allowable working pressure of the nozzle:

$$P_{\max 1} = \frac{S_{allow}}{\dfrac{2A_p}{A_T} - \dfrac{R_{xs}}{t_{eff}}} = \frac{1.5(20000)(1.0)}{\dfrac{2(1057.9575)}{21.4357} - \dfrac{75.9656}{1.6875}} = 558.7300\,psi$$

$$A_p = \frac{(f_N + f_S + f_Y)}{A_T} = \frac{(7485.6216 + 262730.0662 + 106417.1704)}{356} = 1057.9575\ in^2$$

$$P_{\max 2} = S\left(\frac{t}{R_{xs}}\right) = 20000\left(\frac{1.6875}{75.9656}\right) = 444.28\ psi$$

$$P_{\max} = \min\left[P_{\max 1}, P_{\max 2}\right] = \min\left[558.73,\ 444.28\right] = 444.28\ psi$$

The nozzle is acceptable because $P_{\max} = 444.28\ psi$ is greater than the specified design pressure of $356\ psig$.

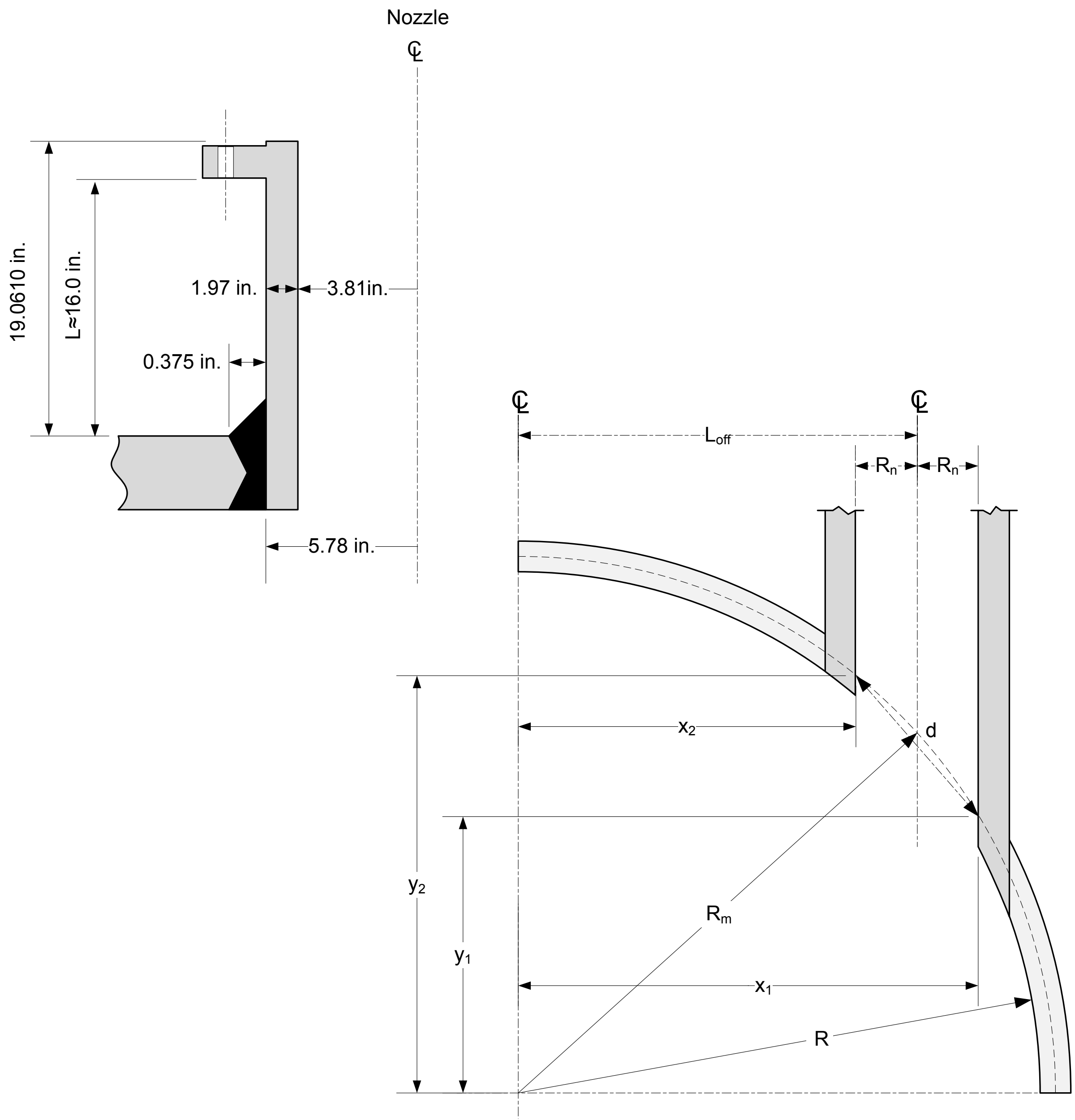


Figure E4.5.2 – Nozzle Detail

4.5.3 Example E4.5.3 – Radial Nozzle in Ellipsoidal Head

Design an integral radial nozzle centrally located in a 2:1 ellipsoidal head based on the vessel and nozzle data below. The parameters used in this design procedure are shown in Figure E4.5.3.

Vessel and Nozzle Data:

- Design Conditions = $356\ psig@300°F$
- Vessel and Nozzle Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0
- Head Material = $SA-516,\ Grade\ 70,\ Norm.$
- Head Allowable Stress = $20000\ psi$
- Head Yield Strength = $33600\ psi$
- Nozzle Material = $SA-105$
- Nozzle Allowable Stress = $20000\ psi$
- Head Inside Diameter = $90.0\ in$
- Height of the Elliptical Head, (2:1) = $22.5\ in$
- Head Thickness = $1.0\ in$
- Nozzle Outside Diameter = $15.94\ in$
- Nozzle Thickness = $2.28\ in$
- External Nozzle Projection = $13.5\ in$
- Nozzle Internal Projection = $0.0\ in$

The nozzle is inserted centrally through the head, i.e. set–in type nozzle, see Fig. UW-16.1(d).

Establish the corroded dimensions.

Ellipsoidal Head:

$$D = 90.0 + 2\left(Corrosion\ \ Allowance\right) = 90.0 + 2\left(0.125\right) = 90.25\ in$$
$$R = \frac{D}{2} = \frac{90.25}{2} = 45.125\ in$$
$$t = 1.0 - Corrosion\ \ Allowance = 1.0 - 0.125 = 0.875\ in$$

Nozzle:

$$t_n = 2.28 - Corrosion\ \ Allowance = 2.28 - 0.125 = 2.155\ in$$
$$R_n = \frac{D_n - 2(t_n)}{2} = \frac{15.94 - 2(2.155)}{2} = 5.815\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-37.

The required thickness of the 2:1 ellipsoidal head based on circumferential stress is given by UG-32(d). However, per UG-37(a), when an opening and its reinforcement are in an ellipsoidal head and located entirely within a circle the center which coincides with the center of the head and the diameter of which is equal to 80% of the shell diameter, t_r is the thickness required for a seamless sphere of radius $K_1 D$, where K_1 is given in Table UG-37.

Per Table UG-37, for a 2:1 ellipsoidal head where, $D/2h = 90.0/2(22.5) = 2 \rightarrow K_1 = 0.9$

The required thickness, t_r, per the UG-37 definition for nozzle reinforcement calculations.

$$t_r = \frac{PDK}{2SE - 0.2P} = \frac{356(90.25)(0.9)}{2(20000)(1.0) - 0.2(356)} = 0.7242\ in$$

The required thickness of the nozzle based on circumferential stress is given by UG-27(c)(1).

$$t_{rn} = \frac{PR_n}{SE - 0.6P} = \frac{356(5.815)}{20000(1.0) - 0.6(356)} = 0.1046\ in$$

a) STEP 1 – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for an integrally reinforced nozzle per Fig. UG-40(e), UG-40(e-1), UG-40(e-2): See Figure E4.5.3 of this example.

$$t_x = 2.155\ in$$
$$L \approx 12\ in$$
$$\{L = 12\ in\} < \{2.5t_x = 2(2.155) = 4.31\ in\}$$
$$Therefore\ use\ UG-40(e-2)\ \begin{Bmatrix} t_n = 2.155\ in \\ t_e = 0.0 \end{Bmatrix}$$

Note: Fig. UG-40 does not provide a sketch for an integral uniform thickness nozzle with full penetration weld inserted through the shell without a reinforcing pad. Therefore, sketch (e-2) was used with $t_e = 0.0$. Additionally, the value of L is approximate and is determined by subtracting the flange thickness from the external nozzle projection, see Figure E4.5.3.

2) Finished opening chord length.

$$d = 2R_n = 2(5.815) = 11.63\ in$$

3) The limits of reinforcement, measured parallel to the vessel wall in the corroded condition.

$$\max[d,\ R_n + t_n + t] = \max[11.63,\ \{5.815 + 2.155 + 1.0\}] = 11.63\ in$$

4) The limits of reinforcement, measured normal to the vessel wall in the corroded condition.

$$\min[2.5t,\ 2.5t_n + t_e] = \min[2.5(0.875),\ \{2.5(2.155) + 0.0\}] = 2.1875\ in$$

b) STEP 2 – Calculate the reinforcement strength parameters per UG-37.

1) Strength Reduction Factors:

$$f_{r1} = S_n/S_v = 20000/20000 = 1.0$$
$$f_{r2} = S_n/S_v = 20000/20000 = 1.0$$
$$f_{r3} = \min[S_n, S_p]/S_v = 0.0$$
$$f_{r4} = S_p/S_v = 0.0$$

2) Joint Efficiency Parameter: For a nozzle located in a solid plate, $E_1 = 1.0$

3) Correction Factor for variation of internal pressure stresses on different planes with respect to the axis of the vessel: For a nozzle in an ellipsoidal head, $F = 1.0$.

c) STEP 3 – Calculate the Areas of Reinforcement, see Fig. UG–37.1

1) Area Required, A:

$$A = dt_r F + 2t_n t_r F(1 - f_{r1})$$
$$A = 11.63(0.7242)(1.0) + 2(2.155)(0.7242)(1.0)(1 - 1.0) = 8.4224\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d\left(E_1 t - Ft_r\right) - 2t_n\left(E_1 t - Ft_r\right)(1 - f_{r1})$$
$$A_{11} = \begin{Bmatrix} 11.63\left(1.0(0.875) - 1.0(0.7242)\right) - \\ 2(2.155)\left(1.0(0.875) - 1.0(0.7242)\right)(1 - 1.0) \end{Bmatrix} = 1.7538\ in^2$$
$$A_{12} = 2\left(t + t_n\right)\left(E_1 t - Ft_r\right) - 2t_n\left(E_1 t - Ft_r\right)(1 - f_{r1})$$
$$A_{12} = \begin{Bmatrix} 2(0.875 + 2.155)\left(1.0(0.875) - 1.0(0.7242)\right) - \\ 2(2.155)\left(1.0(0.875) - 1.0(0.7242)\right)(1 - 1.0) \end{Bmatrix} = 0.9138\ in^2$$
$$A_1 = \max\left[1.7538,\ 0.9138\right] = 1.7538\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use the smaller value:

$$A_{21} = 5(t_n - t_{rn}) f_{r2} t$$
$$A_{21} = 5(2.155 - 0.1046)(1.0)(0.875) = 8.9705\ in^2$$
$$A_{22} = 5(t_n - t_{rn}) f_{r2} t_n$$
$$A_{22} = 5(2.155 - 0.1046)(1.0)(2.155) = 22.0931\ in^2$$
$$A_2 = \min\left[8.9705,\ 22.0931\right] = 8.9705\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3:

$$A_3 = \min\left[5tt_i f_{r2}, 5t_i t_i f_{r2}, 2ht_i f_{r2}\right]$$
$$A_3 = 0.0 \qquad since\ t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.3 of this example:

$$Outer\ Nozzle\ Fillet\ Weld\ Leg: \quad 0.375\ in$$
$$Outer\ Element\ Fillet\ Weld\ Leg: \quad 0.0\ in$$
$$Inner\ Nozzle\ Fillet\ Weld\ Leg: \quad 0.0\ in$$

$$A_{41} = leg^2 f_{r3} = (0.375)^2 (1.0) = 0.1406\ in^2$$
$$A_{42} = 0.0\ in^2$$
$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = (D_p - d - 2t_n) t_e f_{r4} = 0.0\ in^2$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + (A_{41} + A_{42} + A_{43}) + A_5$$
$$A_{avail} = 1.7538 + 8.9705 + 0.0 + (0.1406 + 0.0 + 0.0) + 0.0 = 10.8649\ in^2$$

d) STEP 4 – Nozzle reinforcement acceptance criterion:

$$\{A_{avail} = 10.8649\ in^2\} \geq \{A = 8.4224\ in^2\} \qquad True$$

Therefore, the nozzle is adequately reinforced.

<u>Section VIII, Division 2 Solution with VIII-1 Allowable Stresses</u>

The procedure, per VIII-2, paragraph 4.5.10, to design a radial nozzle in an ellipsoidal head subject to pressure loading is shown below.

a) STEP 1 – Determine the effective radius of the ellipsoidal head as follows.

$$R_{eff} = \frac{0.9D_i}{6}\left[2 + \left(\frac{D_i}{2h}\right)^2\right] = \frac{0.9(90.25)}{6}\left[2 + \left(\frac{90.25}{2(22.625)}\right)^2\right] = 80.9262\ in$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall.

For integrally reinforced set–in nozzles in ellipsoidal heads,

$$L_R = \min\left[\sqrt{R_{eff}t},\ 2R_n\right] = \min\left[\sqrt{80.9262(0.875)},\ 2(5.8150)\right] = 8.4149\ in$$

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in VIII-2, paragraph 4.5.13 would need to be checked.

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface. See VIII-2, Figures 4.5.9 and 4.5.10.

For set–in nozzles in ellipsoidal heads,

$$L_H = \min\left[t + t_e + F_p\sqrt{R_n t_n},\ L_{pr1} + t\right]$$

$$X_o = \min\left[D_R + (R_n + t_n)\cdot\cos[\theta],\ \frac{D_i}{2}\right]$$

$$X_o = \min\left[0.0 + (5.8150 + 2.1550)\cdot\cos[0.0],\ \frac{90.25}{2}\right] = 7.97\ in$$

$$\theta = \arctan\left[\left(\frac{h}{R}\right)\cdot\left(\frac{D_R}{\sqrt{R^2 - D_R^2}}\right)\right] = \arctan\left[\left(\frac{22.625}{45.125}\right)\cdot\left(\frac{0.0}{\sqrt{45.125^2 - 0.0^2}}\right)\right] = 0.0\ rad$$

Since $\{X_o = 7.97\ in\} \le \{0.35D_i = 0.35(90.25) = 31.5875\ in\}$, calculate F_p as follows:

$$F_p = C_n$$

$$C_n = \min\left[\left(\frac{t + t_e}{t_n}\right)^{0.35}, 1.0\right] = \min\left[\left(\frac{0.875 + 0.0}{2.1550}\right)^{0.35}, 1.0\right] = 0.7295$$

$$F_p = 0.7295$$

$$L_H = \min\left[0.875 + 0.0 + (0.7295)\sqrt{5.8150(2.1550)},\ 13.5 + 0.875\right] = 3.4574\ in$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable.

$$L_{pr2} = 0.0$$

$$L_I = \min\left[F_p\sqrt{R_n t_n}, L_{pr2}\right] = 0.0$$

e) STEP 5 – Determine the total available area near the nozzle opening (see VIII-2, Figures 4.5.1 and 4.5.2) where f_{rn} and f_{rp} are given by VIII-2, Equations (4.5.21) and (4.5.22) respectively. Do not include any area that falls outside of the limits defined by L_H, L_R, and L_I.

For set–in nozzles:

$$A_T = A_1 + f_{rn}(A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp}A_5$$

$$A_1 = tL_R = 0.875(8.4149) = 7.3630\ in^2$$

Since $\{t_n = 2.1550\ in\} = \{t_{n2} = 2.1550\ in\}$, calculate A_2 as follows:

$$A_2 = t_n L_H = 2.1550(3.4574) = 7.4507\ in^2$$

$$A_3 = t_n L_I = 0.0$$

$$A_{41} = 0.5L_{41}^2 = 0.5(0.375)^2 = 0.0703\ in^2$$

$$A_{42} = 0.5L_{42}^2 = 0.0$$

$$A_{43} = 0.5L_{43}^2 = 0.0$$

$$t_e = 0.0 \ in$$

$$A_{5a} = Wt_e = 0.0$$

$$A_{5b} = (L_R - t_n)t_e = 0.0$$

$$A_5 = \min\left[A_{5a},\ A_{5b}\right] = 0.0$$

$$f_{rn} = \frac{S_n}{S} = \frac{20000}{20000} = 1.0$$

$$f_{rp} = \frac{S_p}{S} = 0.0$$

$$A_T = 7.363 + 1.0(7.4507 + 0.0) + 0.0703 + 0.0 + 0.0 + 0.0(0.0) = 14.8840 \ in^2$$

f) STEP 6 – Determine the applicable forces.

For set–in nozzles,

$$f_N = PR_{xn}L_H = 356(6.8360)(3.4572) = 8413.4972 \ lbs$$

$$R_{xn} = \frac{t_n}{\ln\left[\dfrac{R_n + t_n}{R_n}\right]} = \frac{2.1550}{\ln\left[\dfrac{5.8150 + 2.1550}{5.8150}\right]} = 6.8360 \ in$$

$$f_S = \frac{PR_{xs}(L_R + t_n)}{2} = \frac{356(81.3629)(8.4149 + 2.1550)}{2} = 153079.5936 \ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\dfrac{R_{eff} + t_{eff}}{R_{eff}}\right]} = \frac{0.875}{\ln\left[\dfrac{80.9262 + 0.875}{80.9262}\right]} = 81.3629 \ in$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 0.875 + \left(\frac{0.0}{8.4149}\right) = 0.875 \ in$$

$$f_Y = \frac{PR_{xs}R_{nc}}{2} = \frac{356(81.3629)(5.8150)}{2} = 84216.2969 \ lbs$$

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection.

$$\sigma_{avg} = \frac{(f_N + f_S + f_Y)}{A_T} = \frac{8413.4972 + 153079.5936 + 84216.2969}{14.884} = 16508.2900 \ psi$$

$$\sigma_{circ} = \frac{PR_{xs}}{2t_{eff}} = \frac{356(81.3629)}{2(0.875)} = 16551.5385 \ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection.

$$P_L = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\}, \sigma_{circ}\right]$$

$$P_L = \max\left[\left\{2(16508.29) - 16551.5385\right\}, 16551.5385\right] = 16551.5385 \ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy VIII-2, Equation 4.5.146. If the nozzle is subjected to internal pressure, then the allowable stress, S_{allow}, is given by VIII-2, Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by VIII-2, Equation 4.5.58.

$$\left\{P_L = 16551.5385\right\} \le \left\{S_{allow} = 1.5SE = 1.5(20000)(1.0) = 30000 \ psi\right\}$$

j) STEP 10 – Determine the maximum allowable working pressure of the nozzle.

$$P_{\max 1} = \frac{S_{allow}}{\left(\dfrac{2A_p}{A_T}\right) - \left(\dfrac{R_{xs}}{2t_{eff}}\right)} = \frac{1.5(20000)(1.0)}{\left(\dfrac{2(690.1949)}{14.884}\right) - \left(\dfrac{81.3629}{2(0.875)}\right)} = 648.6470 \ psi$$

$$A_p = \frac{(f_N + f_S + f_Y)}{P}$$

$$A_p = \frac{8413.4972 + 153079.5936 + 84216.2969}{356} = 690.1949 \ in^2$$

$$P_{\max 2} = 2S\left(\frac{t}{R_{xs}}\right) = 2(20000)\left(\frac{0.875}{81.3629}\right) = 430.1715 \ psi$$

$$P_{\max} = \min\left[P_{\max 1}, P_{\max 2}\right] = \min\left[648.647, \ 430.1715\right] = 430.1715 \ psi$$

The nozzle is acceptable because $P_{\max} = 430.1715 \ psi$ is greater than the specified design pressure of $356 \ psig$.

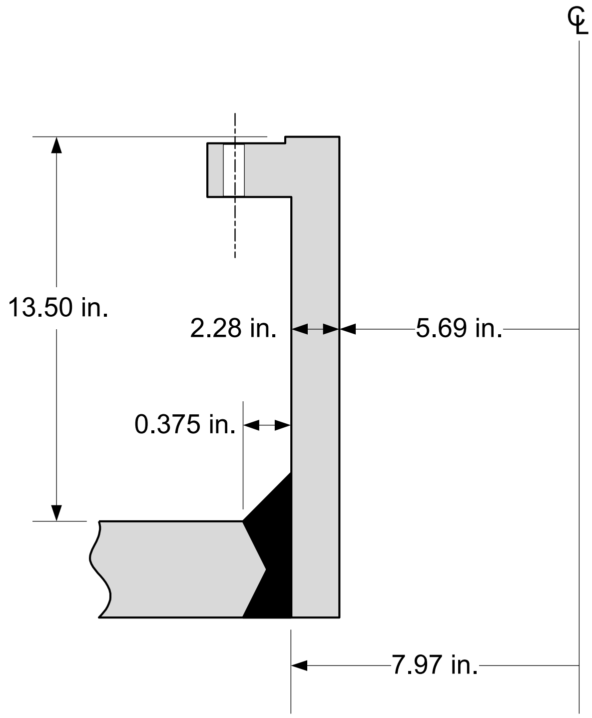


Figure E4.5.3 – Nozzle Details

4.5.4 Example E4.5.4 – Radial Nozzle in Cylindrical Shell

Check the design of an integral radial nozzle in a cylindrical shell based on the vessel and nozzle data below. Verify the adequacy of the attachment welds. The parameters used in this design procedure are shown in Figure E4.5.4.

Vessel and Nozzle Data:

- Design Conditions = $425\ psig@800°F$
- Vessel and Nozzle Corrosion Allowance = $0.0625\ in$
- Weld Joint Efficiency = 1.0
- Shell Allowable Stress = $11400\ psi$
- Nozzle Allowable Stress = $12000\ psi$
- Shell Inside Diameter = $96.0\ in$
- Shell Thickness = $2.0\ in$
- Nozzle Inside Diameter = $16.0\ in$
- Nozzle Thickness (seamless) = $1.75\ in$

The nozzle has a set–on type configuration and the opening does not pass through a vessel Category A joint, see Fig. UW-16.1(n). All category A joints are to be fully radiographed (see UW-3).

Establish the corroded dimensions.

Shell:

$$D = 96.0 + 2(Corrosion\ \ Allowance) = 96.0 + 2(0.0625) = 96.125\ in$$

$$R = \frac{D}{2} = \frac{96.125}{2} = 48.0625\ in$$

$$t = 2.0 - Corrosion\ \ Allowance = 2.0 - 0.0625 = 1.9375\ in$$

Nozzle:

$$t_n = 1.75 - Corrosion\ \ Allowance = 1.75 - 0.0625 = 1.6875\ in$$

$$R_n = \frac{D_n + 2(Corrosion\ \ Allowance)}{2} = \frac{16.0 + 2(0.0625)}{2} = 8.0625\ in$$

<u>Section VIII, Division 1 Solution</u>

Evaluate per UG-37.

The required thickness of the shell based on circumferential stress is given by UG-27(c)(1).

$$t_r = \frac{PR}{SE - 0.6P} = \frac{425(48.0625)}{11400(1.0) - 0.6(425)} = 1.8328\ in$$

The required thickness of the nozzle based on circumferential stress is given by UG-27(c)(1).

$$t_{rn} = \frac{PR_n}{SE - 0.6P} = \frac{425(8.0625)}{12000(1.0) - 0.6(425)} = 0.2917\ in$$

a) STEP 1 – Calculate the required weld sizes per UW-16(d) and Fig. UW-16.1 Sketch (n).

1) Inner perimeter weld:

$$t_{wr} = 0.7t_{\min} = 0.7(0.75) = 0.525\ in$$
$$t_{wact} = 0.875 - 0.0625 = 0.8125\ in$$
$$\{t_{wact} = 0.8125\ in\} > \{t_{wr} = 0.525\ in\} \qquad True$$

2) Outer perimeter weld.

$$Throat_r = 0.5t_{\min} = 0.5(0.75) = 0.375\ in$$
$$Throat_{act} = 0.7(weld\ \ size) = 0.7(0.75) = 0.525\ in$$
$$\{Throat_{act} = 0.525\ in\} > \{Throat_r = 0.375\ in\} \qquad True$$

b) STEP 2 – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for an integrally reinforced nozzle per Fig. UG-40(d). See Figure E4.5.4 of this example.

$$\theta = \arctan\left[\frac{\left(\dfrac{26.0-19.5}{2}\right)}{3.5}\right] = 42.9\ deg$$

Since $\{\theta = 42.9\ deg\} > \{\theta = 30\ deg\}$, Fig. UG-40 sketch (d) applies and $t_e = 3.5\ in$.

2) Finished opening chord length.

$$d = 2R_n = 2(8.0625) = 16.125\ in$$

3) The limits of reinforcement, measured parallel to the vessel wall in the corroded condition.

$$\max[d,\ R_n + t_n + t] = \max\left[16.125,\ \{8.0625 + 1.6875 + 1.9375\}\right] = 16.125\ in$$

4) The limits of reinforcement, measured normal to the vessel wall in the corroded condition.

$$\min[2.5t,\ 2.5t_n + t_e] = \min\left[2.5(1.9375),\ \{2.5(1.6875) + 3.5\}\right] = 4.8438\ in$$

c) STEP 3 – Calculate the reinforcement strength parameters per UG-37.

1) Strength Reduction Factors:

$$f_{r1} = 1.0 \qquad for\ set-on\ type\ nozzle$$
$$f_{r2} = S_n / S_v = 12000/11400 = 1.0526 \rightarrow\ set\ f_{r2} = 1.0$$
$$f_{r3} = \min\left[S_n, S_p\right]/S_v = \min[12000,\ 11400]/11400 = 1.0$$
$$f_{r4} = S_p / S_v = 12000/11400 = 1.0526 \rightarrow\ set\ f_{r4} = 1.0$$

2) Joint Efficiency Parameter: For a nozzle located in a solid plate, $E_1 = 1.0$

3) Correction Factor for variation of internal pressure stresses on different planes with respect to the axis of the vessel: For a radial nozzle in a cylindrical shell, $F = 1.0$.

d) STEP 4 – Calculate the Areas of Reinforcement, see Fig. UG–37.1

1) Area Required, A:

$$A = dt_r F + 2t_n t_r F(1 - f_{r1})$$
$$A = 16.125(1.8328)(1.0) + 2(1.6875)(1.8328)(1.0)(1-1.0) = 29.5539\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d\left(E_1 t - Ft_r\right) - 2t_n\left(E_1 t - Ft_r\right)(1 - f_{r1})$$
$$A_{11} = \begin{Bmatrix} 16.125(1.0(1.9375) - 1.0(1.8328)) - \\ 2(1.6875)(1.0(1.9375) - 1.0(1.8328))(1-1.0) \end{Bmatrix} = 1.6883\ in^2$$
$$A_{12} = 2(t + t_n)\left(E_1 t - Ft_r\right) - 2t_n\left(E_1 t - Ft_r\right)(1 - f_{r1})$$
$$A_{12} = \begin{Bmatrix} 2(1.9375 + 1.6875)(1.0(1.9375) - 1.0(1.8328)) - \\ 2(1.6875)(1.0(1.9375) - 1.0(1.8328))(1-1.0) \end{Bmatrix} = 0.7591\ in^2$$
$$A_1 = \max\left[1.6883,\ 0.7591\right] = 1.6883\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use the smaller value:

$$A_{21} = 5\left(t_n - t_{rn}\right) f_{r2} t$$
$$A_{21} = 5(1.6875 - 0.2917)(1.0)(1.9375) = 13.5218\ in^2$$
$$A_{22} = 2\left(t_n - t_{rn}\right)\left(2.5t_n + t_e\right) f_{r2}$$
$$A_{22} = 2(1.6875 - 0.2917)(2.5(1.6875) + 3.5)(1.0) = 21.5477\ in^2$$
$$A_2 = \min\left[13.5218,\ 21.5477\right] = 13.5218\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3:

$$A_3 = \min\left[5tt_i f_{r2}, 5t_i t_i f_{r2}, 2ht_i f_{r2}\right]$$
$$A_3 = 0.0 \qquad since\ t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.4 of this example:

$Outer\ Nozzle\ Fillet\ Weld\ Leg$: $0.75\ in$

$Outer\ Element\ Fillet\ Weld\ Leg$: $0.0\ in$

$Inner\ Nozzle\ Fillet\ Weld\ Leg$: $0.0\ in$

$$A_{41} = leg^2 f_{r3} = (0.75)^2 (1.0) = 0.5625\ in^2$$
$$A_{42} = 0.0\ in^2$$
$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = \left(D_p - d - 2t_n\right)t_e f_{r4} = \left(26.0 - 16.125 - 2\left(1.6875\right)\right)\left(2.75\right)\left(1.0\right) = 17.875\ in^2$$

Where the value of t_e is calculated as the average thickness of the reinforcing element.

$$t_e = \frac{3.5+2}{2} = 2.75\ in$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + \left(A_{41} + A_{42} + A_{43}\right) + A_5$$

$$A_{avail} = 1.6883 + 13.5218 + 0.0 + \left(0.5625 + 0.0 + 0.0\right) + 17.875 = 33.6476\ in^2$$

e) STEP 5 – Nozzle reinforcement acceptance criterion:

$$\left\{A_{avail} = 33.6476\ in^2\right\} \geq \left\{A = 29.5539\ \ in^2\right\} \qquad\qquad True$$

Therefore, the nozzle is adequately reinforced.

The load to be carried by the welds is calculated in accordance with UG-41.

a) STEP 1 – Per Fig. UG-41.1, sketch (b) Nozzle Attachment Weld Loads and Weld Strength Paths to be Considered; typical nozzle detail with nozzle neck abutting (set–on) the vessel wall.

Per UG-41(b)(1): Weld Load for Strength Path 1-1, W_{1-1}.

$$W_{1-1} = \left(A_2 + A_5 + A_{41} + A_{42}\right)S_v = \left(13.5218 + 17.875 + 0.5625 + 0.0\right)\left(11400\right) = 364336.0\ lbs$$

Per UG-41(b)(2): Total Weld Load, W.

$$W = \left(A - A_1\right)S_v = \left(29.5539 - 1.6883\right)\left(11400\right) = 317667.8\ lbs$$

Since W is smaller than W_{1-1}, W may be used in place of W_{1-1} for comparing weld capacity to weld load.

b) STEP 2 – Determine the allowable stresses of the attachment welds for weld strength path check. The allowable stress of the welds should be considered equal to the lesser of the two allowable stresses joined. Per UW-15(c) and UG-45(c), the allowable stresses for groove/fillet welds in percentages of stress value for the vessel material, used with UG-41 calculations are as follows:

$$Groove\ Weld\ Tension: 74\%$$

$$Groove\ Weld\ Shear: 60\%$$

$$Fillet\ Weld\ Shear: 49\%$$

$$Nozzle\ Neck\ Shear: 70\%$$

1) Groove Weld Shear:

$$S_{gws} = 0.6\left(11400\right) = 6840\ \ psi$$

2) Fillet Weld Shear:

$$S_{fws} = 0.49(11400) = 5586\ psi$$

c) STEP 3 – Determine the Strength of Connection Elements

1) Groove Weld Shear:

$$GWS = \frac{\pi}{2}(Mean\ Diameter\ of\ Weld)(Weld\ Leg)(S_{gws})$$

$$GWS = \frac{\pi}{2}(16.875)(0.8125)(6840) = 147313.7\ lbs$$

2) Fillet Weld Shear:

$$FWS = \frac{\pi}{2}(Nozzle\ OD)(Weld\ Leg)(S_{fws})$$

$$FWS = \frac{\pi}{2}(26.0)(0.75)(5590) = 171224.7\ lbs$$

d) STEP 4 – Check Weld Strength Paths

$$Path_{1-1} = GWS + FWS = 147313.7 + 171224.7 = 318538.4\ lbs$$

e) STEP 5 – Weld Path Acceptance Criteria:

Per UG-41(b)(1):

$$Not\ required,\ see\ STEP\ 1$$

Per UG-42(b)(2):

$$\min\left[Path_{1-1},\ Path_{2-2},\ Path_{3-3}\right] \geq W$$
$$\left\{Path_{1-1} = 318538.4\right\} \geq \left\{W = 317667.8\right\} \qquad True$$

Section VIII, Division 2 Solution

There is no comparable weld detail for this nozzle attachment in VIII-2, Part 4.2. Therefore, no calculation is performed.

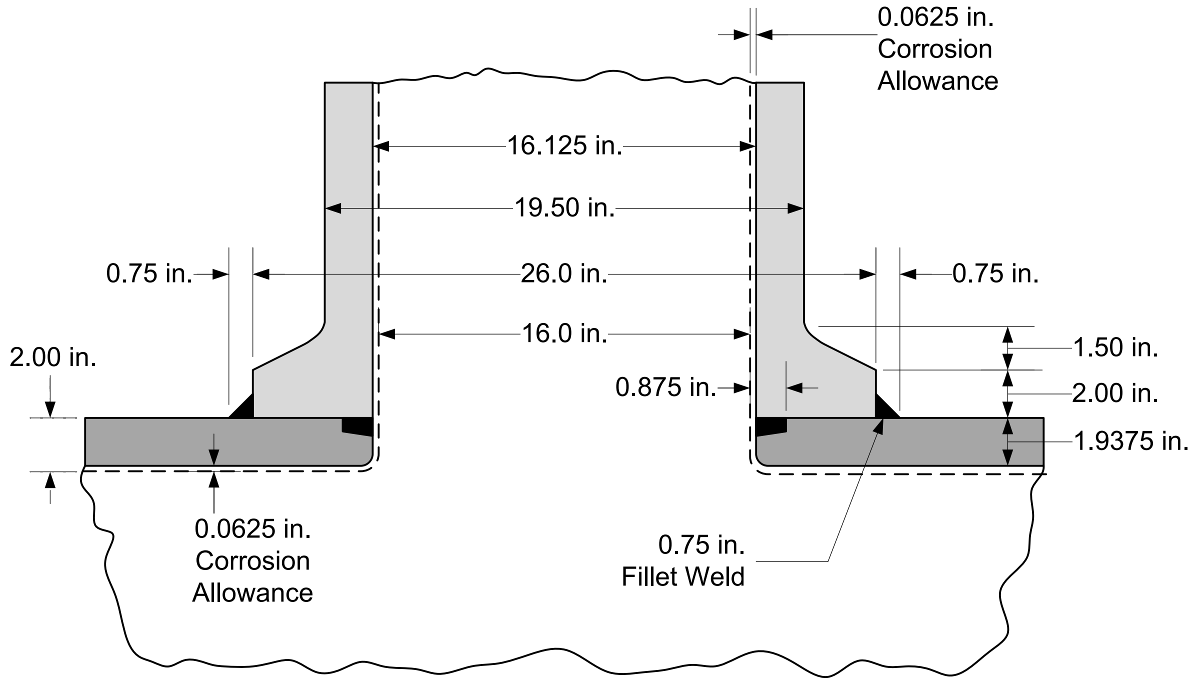


Figure E4.5.4 – Nozzle Details

4.5.5 Example E4.5.5 – Pad Reinforced Radial Nozzle in Cylindrical Shell

Check the design of a radial nozzle in a cylindrical shell based on the vessel and nozzle data below. Verify the adequacy of the attachment welds. Calculate the shear stresses from the applied nozzle loads and compare to the acceptance criteria of UG-45. The parameters used in this design procedure are shown in Figure E4.5.5.

<u>Vessel and Nozzle Data:</u>

- Design Conditions = $500\ psig@400°F$
- Vessel and Nozzle Corrosion Allowance = $0.25\ in$
- Weld Joint Efficiency = 1.0
- Shell Allowable Stress = $13700\ psi$
- Nozzle Allowable Stress = $13700\ psi$
- Reinforcement Pad Allowable Stress = $13700\ psi$
- Shell Inside Diameter = $83.0\ in$
- Shell Thickness = $2.0\ in$
- Nozzle Outside Diameter = $16.0\ in$
- Nozzle Thickness (fabricated from plate) = $0.75\ in$
- Reinforcement Pad Diameter = $28.25\ in$
- Reinforcement Pad Thickness = $1.5\ in$
- Applied Shear Load = $25000\ lbs$
- Applied Torsional Moment = $250000\ in-lbs$

The nozzle has a set–in type configuration and the opening does not pass through a vessel Category A joint, see Fig. UW-16.1(q). All category A joints are to be fully radiographed (see UW-3).

Establish the corroded dimensions.

Shell:

$$D = 83.0 + 2(Corrosion\ \ Allowance) = 83.0 + 2(0.25) = 83.5\ in$$

$$R = \frac{D}{2} = \frac{83.5}{2} = 41.75\ in$$

$$t = 2.0 - Corrosion\ \ Allowance = 2.0 - 0.25 = 1.75\ in$$

Nozzle:

$$t_n = 0.75 - Corrosion\ \ Allowance = 0.75 - 0.25 = 0.5\ in$$

$$R_n = \frac{D_n - 2(Corroded\ \ Nozzle\ Thickness)}{2} = \frac{16.0 - 2(0.5)}{2} = 7.5\ in$$

<u>Section VIII, Division 1 Solution</u>

Evaluate per UG-37.

The required thickness of the shell based on circumferential stress is given by UG-27(c)(1).

$$t_r = \frac{PR}{SE - 0.6P} = \frac{500(41.75)}{13700(1.0) - 0.6(500)} = 1.5578\ in$$

The required thickness of the nozzle based on circumferential stress is given by UG-27(c)(1).

$$t_{rn} = \frac{PR_n}{SE - 0.6P} = \frac{500(7.5)}{13700(1.0) - 0.6(500)} = 0.2799\ in$$

a) STEP 1 – Determine the Minimum Nozzle Thickness per UG-45.

1) For access openings and openings used only for inspection:

$$t_{UG-45} = t_a$$

$$Not\ applicable$$

2) For other nozzles:

$$t_{UG-45} = \max\left[t_a,\ t_b\right]$$

Where,

$$t_b = \min\left[t_{b3},\ \max\left[t_{b1},\ t_{b2}\right]\right]$$

t_a, the minimum neck thickness required for internal or external pressure using UG-27 and UG-28 (plus corrosion allowance), as applicable. The effects of external forces and moments from supplemental loads (see UG-22) shall be considered. Shear stresses caused by UG-22 loadings shall not exceed 70% of the allowable tensile stress for the nozzle material.

$$t_a = t_{rn} + Corrosion\ \ Allowance = 0.2799 + 0.25 = 0.5299\ in$$

t_{b1}, for vessels under internal pressure, the thickness (plus corrosion allowance) required for pressure (assuming $E = 1.0$) for the shell or head at the location where the nozzle neck or other connection attaches to the vessel but in no case less than the minimum thickness specified for the material in UG-16(b).

$$t_{b1} = \max\left[t_{rE=1\ 0} + Corrosion\ \ Allowance,\ t_{UG-16b}\right]$$

$$t_{b1} = \max\left[1.5578 + 0.25,\ 0.0625\right] = 1.8078\ in$$

t_{b2}, for vessels under external pressure, the thickness (plus corrosion allowance) obtained by using the external design pressure as an equivalent internal design pressure (assuming $E = 1.0$) in the formula for the shell or head at the location where the nozzle neck or other connection attaches to the vessel but in no case less than the minimum thickness specified for the material in UG-16(b).

$$t_{b2} = \max\left[t_{rE=1\ 0} + Corrosion\ \ Allowance,\ t_{UG-16b}\right]$$

$$Not\ applicable$$

t_{b3}, the thickness given in Table UG-45 plus the thickness added for corrosion allowance.

$$t_{b3} = t_{TABLE\,UG-45} + Corrosion\ \ Allowance = 0.328 + 0.25 = 0.578\ in$$

Therefore,

$$t_b = \min\left[t_{b3},\ \max\left[t_{b1},\ t_{b2}\right]\right] = \min\left[0.578,\ \max\left[1.8078,\ 0.0\right]\right] = 0.578\ in$$

And,

$$t_{UG-45} = \max\left[t_a,\ t_b\right] = \max\left[0.5299,\ 0.578\right] = 0.578\ in$$

Since $\{t_n = 0.75\ in\} \geq \{t_{UG-45} = 0.578\ in\}$ the nozzle thickness satisfies UG-45 criteria.

b) STEP 2 – Calculate the maximum membrane shear stress due to the superimposed shear and torsion loads and compare to the allowable shear stress.

As specified in the definition of t_a in UG-45:

$$S_s = 0.70S = 0.7(13700) = 9590\ psi$$

Membrane shear stress from shear load:

$$S_{sl} = \frac{Shear\ Load}{\pi r t_n} = \frac{25000}{\pi(7.5)(0.75)} = 1415\ psi$$

Membrane shear stress from torsional moment:

$$S_{tl} = \frac{Torsion\ Load}{2\pi R_n^2 t} = \frac{250000}{2\pi(7.5)^2(0.5)} = 1415\ psi$$

Total membrane shear stress:

$$S_{st} = S_{sl} + S_{tl} = 2122 + 1415 = 3537\ psi$$

Since $\{S_{st} = 3537\ psi\} \leq \{S_s = 9590\ psi\}$ the nozzle is adequately designed for the applied shear loads.

c) STEP 3 – Calculate the required weld sizes per UW-16(d) and Fig. UW-16.1 Sketch (q). See Figure E4.5.5 of this example.

1) Outer nozzle fillet weld, based on throat dimensions:

$$t_c = \min\left[0.25\ in,\ 0.7t_{min}\right]$$

$$t_c = \min\left[0.25\ in,\ 0.7\left(\min\left[0.75\ in,\ thickness\ of\ thinner\ parts\ joined\right]\right)\right]$$

$$t_c = \min\left[0.25,\ 0.7\left(\min\left[0.75,\ 0.5\right]\right)\right] = 0.25\ in$$

$$t_{cact} = 0.7(weld\ leg\ size) = 0.7(0.375) = 0.2625\ in$$

$$\{t_{cact} = 0.2625\ in\} > \{t_c = 0.25\ in\} \qquad True$$

2) Outer reinforcing element fillet weld, based on throat dimensions:

$$Throat_r = 0.5t_{\min} = 0.5\left(\min\left[0.75\ in,\ thickness\ of\ thinner\ parts\ joined\right]\right)$$

$$Throat_r = 0.5\left(\min\left[0.75,\ 1.5\right]\right) = 0.375\ in$$

$$Throat_{act} = 0.7(weld\ leg\ size) = 0.7(0.875) = 0.6125\ in$$

$$\{Throat_{act} = 0.6125\ in\} > \{Throat_r = 0.375\ in\} \qquad True$$

3) Reinforcing element groove weld:

$$t_w = 0.7t_{min} = 0.7\left(\min\left[0.75\ in,\ thickness\ of\ thinner\ parts\ joined\right]\right)$$
$$t_w = 0.7\left(\min\left[0.75,\ 0.5\right]\right) = 0.35\ in$$
$$t_{wact} = 0.375\ in$$
$$\left\{t_{wact} = 0.375\ in\right\} > \left\{t_c = 0.35\ in\right\} \qquad True$$

4) Shell groove weld:

$$t_w = 0.7t_{min} = 0.7\left(\min\left[0.75\ in,\ thickness\ of\ thinner\ parts\ joined\right]\right)$$
$$t_w = 0.7\left(\min\left[0.75,\ 0.5\right]\right) = 0.35\ in$$
$$t_{wact} = 0.375\ in$$
$$\left\{t_{wact} = 0.375\ in\right\} > \left\{t_c = 0.35\ in\right\} \qquad True$$

d) STEP 4 – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for a reinforced nozzle per Fig. UG-40 sketch (b-1). See Figure E4.5.5 of this example.

2) Finished opening chord length.

$$d = 2R_n = 2(7.5) = 15.0\ in$$

3) The limits of reinforcement, measured parallel to the vessel wall in the corroded condition.

$$\max\left[d,\ R_n + t_n + t\right] = \max\left[15.0,\ \left\{7.5 + 0.5 + 1.75\right\}\right] = 15.0\ in$$

4) The limits of reinforcement, measured normal to the vessel wall in the corroded condition.

$$\min\left[2.5t,\ 2.5t_n + t_e\right] = \min\left[2.5(1.75),\ \left\{2.5(0.5) + 1.5\right\}\right] = 2.75\ in$$

e) STEP 5 – Calculate the reinforcement strength parameters per UG-37.

1) Strength Reduction Factors:

$$f_{r1} = S_n / S_v = 13700/13700 = 1.0$$
$$f_{r2} = S_n / S_v = 13700/13700 = 1.0$$
$$f_{r3} = \min\left[S_n, S_p\right] / S_v = \min\left[13700,\ 13700\right]/13700 = 1.0$$
$$f_{r4} = S_p / S_v = 13700/13700 = 1.0$$

2) Joint Efficiency Parameter: For a nozzle located in a solid plate, $E_1 = 1.0$

3) Correction Factor for variation of internal pressure stresses on different planes with respect to the axis of the vessel: For a pad reinforced radial nozzle in a cylindrical shell, $F = 1.0$.

f) STEP 6 – Calculate the Areas of Reinforcement, see Fig. UG–37.1

1) Area Required, A:

$$A = dt_r F + 2t_n t_r F(1 - f_{r1})$$
$$A = 15.0(1.5578)(1.0) + 2(0.5)(1.5578)(1.0)(1 - 1.0) = 23.367\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11} = d\left(E_1 t - F t_r\right) - 2t_n\left(E_1 t - F t_r\right)\left(1 - f_{r1}\right)$$

$$A_{11} = \begin{Bmatrix} 15.0\left(1.0(1.75) - 1.0(1.5578)\right) - \\ 2(0.5)\left(1.0(1.75) - 1.0(1.5578)\right)(1 - 1.0) \end{Bmatrix} = 2.883\ in^2$$

$$A_{12} = 2\left(t + t_n\right)\left(E_1 t - F t_r\right) - 2t_n\left(E_1 t - F t_r\right)\left(1 - f_{r1}\right)$$

$$A_{12} = \begin{Bmatrix} 2(1.75 + 0.5)\left(1.0(1.75) - 1.0(1.5578)\right) - \\ 2(0.5)\left(1.0(1.75) - 1.0(1.5578)\right)(1 - 1.0) \end{Bmatrix} = 0.8649\ in^2$$

$$A_1 = \max\left[2.883,\ 0.8649\right] = 2.883\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use the smaller value:

$$A_{21} = 5\left(t_n - t_{rn}\right) f_{r2} t$$

$$A_{21} = 5(0.5 - 0.2799)(1.0)(1.75) = 1.9259\ in^2$$

$$A_{22} = 2\left(t_n - t_{rn}\right)\left(2.5t_n + t_e\right) f_{r2}$$

$$A_{22} = 2(0.5 - 0.2799)\left(2.5(0.5) + 1.5\right)(1.0) = 1.2106\ in^2$$

$$A_2 = \min\left[1.9259,\ 1.2106\right] = 1.2106\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3:

$$A_3 = \min\left[5tt_i f_{r2}, 5t_i t_i f_{r2}, 2ht_i f_{r2}\right]$$

$$A_3 = 0.0 \qquad since\ t_i = 0.0$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.4 of this example:

Outer Nozzle Fillet Weld Leg :	$0.375\ in$
Outer Element Fillet Weld Leg :	$0.875\ in$
Inner Nozzle Fillet Weld Leg :	$0.0\ in$

$$A_{41} = leg^2 f_{r3} = (0.375)^2(1.0) = 0.1406\ in^2$$

$$A_{42} = leg^2 f_{r4} = (0.875)^2(1.0) = 0.7656\ in^2$$

$$A_{43} = 0.0\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = \left(D_p - d - 2t_n\right) t_e f_{r4} = \left(28.25 - 15.0 - 2(0.5)\right)(1.5)(1.0) = 18.375\ in^2$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + \left(A_{41} + A_{42} + A_{43}\right) + A_5$$

$$A_{avail} = 2.883 + 1.2106 + 0.0 + (0.1406 + 0.7656 + 0.0) + 18.375 = 23.3748\ in^2$$

g) STEP 7 – Nozzle reinforcement acceptance criterion:

$$\left\{A_{avail} = 23.367\ in^2\right\} \geq \left\{A = 23.3748\ \ in^2\right\} \qquad\qquad True$$

Therefore, the nozzle is adequately reinforced.

The load to be carried by the welds is calculated in accordance with UG-41.

a) STEP 1 – Per Fig. UG-41.1, sketch (a) Nozzle Attachment Weld Loads and Weld Strength Paths to be Considered; typical nozzle detail with nozzle neck inserted through (set–in) the vessel wall.

Per UG-41(b)(1):

1) Weld Load for Strength Path 1-1, W_{1-1}.

$$W_{1-1} = \left(A_2 + A_5 + A_{41} + A_{42}\right)S_v = \left(1.2106 + 18.375 + 0.1406 + 0.7656\right)\left(13700\right) = 279504.7\ lbs$$

2) Weld Load for Strength Path 2-2, W_{2-2}.

$$W_{2-2} = \left(A_2 + A_3 + A_{41} + A_{43} + 2t_n t f_{r1}\right)S_v$$
$$W_{2-2} = \left(1.2106 + 0.0 + 0.1406 + 0.0 + 2\left(0.5\right)\left(1.75\right)\left(1.0\right)\right)\left(13700\right) = 41993.2\ lbs$$

3) Weld Load for Strength Path 3-3, W_{3-3}.

$$W_{3-3} = \left(A_2 + A_3 + A_5 + A_{41} + A_{42} + A_{43} + 2t_n t f_{r1}\right)S_v$$
$$W_{3-3} = \left(1.2106 + 0.0 + 18.375 + 0.1406 + 0.7656 + 0 + 2\left(0.5\right)\left(1.75\right)\left(1.0\right)\right)\left(13700\right) = 304712.7\ lbs$$

Per UG-41(b)(2): Total Weld Load, W.

$$W = \left(A - A_1 + 2t_n f_{r1}\left(E_1 t - F t_r\right)\right)S_v$$
$$W = \left(23.367 - 2.883 + 2\left(0.5\right)\left(1.0\right)\left(1.0\left(1.75\right) - 1.0\left(1.5578\right)\right)\right)\left(13700\right) = 283263.9\ lbs$$

Since W is smaller than W_{3-3}, W may be used in place of W_{3-3} for comparing weld capacity to weld load.

b) STEP 2 – Determine the allowable stresses of the attachment welds for weld strength path check. The allowable stress of the welds should be considered equal to the lesser of the two allowable stresses joined. Per UW-15(c) and UG-45(c), the allowable stresses for groove/fillet welds in percentages of stress value for the vessel material, used with UG-41 calculations are as follows:

$$Groove\ Weld\ Tension: 74\%$$
$$Groove\ Weld\ Shear: 60\%$$
$$Fillet\ Weld\ Shear: 49\%$$
$$Nozzle\ Neck\ Shear: 70\%$$

1) Fillet Weld Shear – Outer Nozzle Fillet and Outer Element Fillet:

$$S_{nfws} = S_{efws} = 0.49(13700) = 6713 \ psi$$

2) Groove Weld Tension – Nozzle Groove Weld and Element Groove Weld:

$$S_{ngwt} = S_{egwt} = 0.74(13700) = 10138 \ psi$$

3) Groove Weld Shear:

$$S_{gws} = 0.60(13700) = 8220 \ psi$$

4) Nozzle Wall Shear:

$$S_{nws} = 0.70(13700) = 9590 \ psi$$

c) STEP 3 – Determine the Strength of Connection Elements

1) Outer Nozzle Fillet Weld Shear:

$$ONWS = \frac{\pi}{2}(Nozzle\ OD)(Weld\ Leg)(S_{nfws})$$

$$ONWS = \frac{\pi}{2}(16.0)(0.375)(6713) = 63268.5\ lbs$$

2) Outer Element Fillet Weld Shear:

$$OEWS = \frac{\pi}{2}(Reinforcing\ Element\ OD)(Weld\ Leg)(S_{efws})$$

$$OEWS = \frac{\pi}{2}(28.25)(0.875)(6713) = 260653.2\ lbs$$

3) Nozzle Groove Weld Tension:

$$NGWT = \frac{\pi}{2}(Nozzle\ OD)(Weld\ Leg)(S_{ngwt})$$

$$NGWT = \frac{\pi}{2}(16.0)(0.375)(10138) = 95548.4\ lbs$$

4) Element Groove Weld Tension:

$$EGWT = \frac{\pi}{2}(Nozzle\ OD)(Weld\ Leg)(S_{egwt})$$

$$EGWT = \frac{\pi}{2}(16.0)(0.375)(10138) = 95548.4\ lbs$$

5) Nozzle Wall Shear:

$$NWS = \frac{\pi}{2}(Mean\ Nozzle\ Diameter)(t_n)(S_{nws})$$

$$NWS = \frac{\pi}{2}(15+0.5)(0.5)(9590) = 116745.5\ lbs$$

d) STEP 4 – Check Weld Strength Paths

1) $Path_{1-1} = OEWS + NWS = 260653.2 + 116745.5 = 377398.7\ lbs$

2) $Path_{2-2} = ONWS + EGWT + NGWT = 63268.5 + 95548.4 + 95548.4 = 254365.3\ lbs$

3) $Path_{3-3} = OEWS + NGWT = 260653.2 + 95548.4 = 356201.6\ lbs$

e) STEP 5 – Weld Path Acceptance Criteria:

Per UG-41(b)(1):

$$\{Path_{1-1} = 377398.7\ lbs\} \geq \{W_{1-1} = 279504.7\ lbs\} \qquad True$$
$$\{Path_{2-2} = 254365.3\ lbs\} \geq \{W_{2-2} = 41993.2\ lbs\} \qquad True$$
$$\{Path_{3-3} = 356201.6\ lbs\} \geq \{W_{3-3} = 304712.7\ lbs\} \qquad True$$

Per UG-42(b)(2):

$$\min\left[Path_{1-1},\ Path_{2-2},\ Path_{3-3}\right] \geq W$$
$$\min\left[377398.7,\ 254365.3,\ 356201.6\ lbs\right] \geq \{W = 283263.9\ lbs\} \qquad False$$

$Path_{2-2}$ does not have sufficient strength to resist load W but the weld is acceptable by UG-41(b)(1).

<u>Section VIII, Division 2 Solution</u>

There is no comparable weld detail for this nozzle attachment in VIII-2, Part 4.2. Therefore, no calculation is performed.

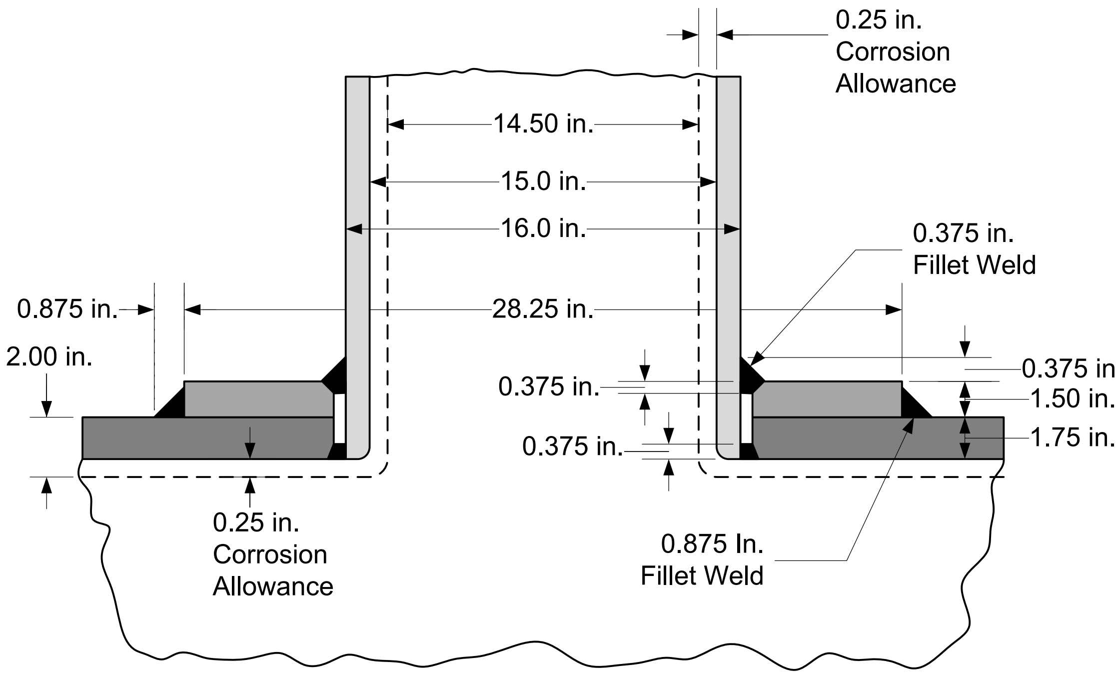


Figure E4.5.5 – Nozzle Details

4.5.6 Example E4.5.6 – Radial Nozzle in an Ellipsoidal Head with Inside Projection

Check the design of a radial nozzle centrally located in a 2:1 ellipsoidal head based on the vessel and nozzle data below. Verify the adequacy of the attachment welds. The parameters used in this design procedure are shown in Figure E4.5.6.

Vessel and Nozzle Data:

- Design Conditions = $150\ psig@400°F$
- Vessel and Nozzle Corrosion Allowance = $0.0\ in$
- Radiography = $Not\ Performed$
- Shell Allowable Stress = $17500\ psi$
- Nozzle Allowable Stress = $12000\ psi$
- Head Inside Diameter = $23.625\ in$
- Head Thickness = $0.1875\ in$
- Nozzle Outside Diameter = $NPS\ 8 \rightarrow 8.625\ in$
- Nozzle Thickness = $SCH\ 20 \rightarrow 0.25\ in$
- Nozzle Internal Projection = $0.500\ in$

The nozzle has a set–in type configuration with an internal projection. The opening does not pass through a vessel Category A joint, see Fig. UW-16.1(i). There is no radiography performed for this vessel.

Establish the dimensions.

Ellipsoidal Head:

$$D = 23.625\ in$$

$$R = \frac{D}{2} = \frac{23.625}{2} = 11.8125\ in$$

$$t = 0.1875\ in$$

Nozzle:

$$t_n = 0.25\ in$$

$$R_n = \frac{D_n - 2(Nozzle\ Thickness)}{2} = \frac{8.625 - 2(0.25)}{2} = 4.0625\ in$$

Section VIII, Division 1 Solution

Evaluate per UG-37.

The required thickness of the 2:1 elliptical head based on circumferential stress is given by UG-32(d). However, per UG-37(a), when an opening and its reinforcement are in an ellipsoidal head and located entirely within a circle the center which coincides with the center of the head and the diameter of which is equal to 80% of the shell diameter, t_r is the thickness required for a seamless sphere of radius $K_1 D$, where K_1 is given in Table UG-37.

Per Table UG-37, for a 2:1 elliptical head where, $D/2h = 2 \rightarrow K_1 = 0.9$

Since no radiography was specified for this vessel, the requirements of UW-11(a)(5)(b) were not satisfied and a joint efficiency of 0.85 is applied to the Category B weld attaching the cylinder to the seamless 2:1 ellipsoidal head. See UW-12(d).

The required thickness, t_r, for the head per UG-37 definition for nozzle reinforcement calculations.

$$t_r = \frac{PDK}{2SE - 0.2P} = \frac{150(23.625)(0.9)}{2(17500)(1.0) - 0.2(150)} = 0.0912\ in$$

The required thickness of the nozzle based on circumferential stress is given by UG-27(c)(1).

$$t_{rn} = \frac{PR_n}{SE - 0.6P} = \frac{150(4.0625)}{12000(1.0) - 0.6(150)} = 0.0512\ in$$

a) STEP 1 – Calculate the required weld sizes per UW-16(d) and Fig. UW-16.1 Sketch (i). See Figure E4.5.6 of this example.

Outer/Inner nozzle fillet weld, based on throat dimensions:

$$t_1\ or\ t_2 \geq \min\left[0.25\ in,\ 0.7t_{min}\right]$$
$$t_1\ or\ t_2 \geq \min\left[0.25\ in,\ 0.7\left(\min\left[0.75\ in,\ thickness\ of\ thinner\ parts\ joined\right]\right)\right]$$
$$t_1\ or\ t_2 \geq \min\left[0.25,\ 0.7\left(\min\left[0.75,\ 0.1875\right]\right)\right] = 0.1313\ in$$
$$t_{1act} = t_{2act} = 0.7\left(weld\ leg\ size\right) = 0.7\left(0.25\right) = 0.175\ in$$
$$\left\{t_{1act} = t_{2act} = 0.175\ in\right\} > \left\{t_1 = t_2 = 0.1313\ in\right\} \qquad True$$

And,

$$t_1 + t_2 \geq 1.25t_{\min}$$
$$t_1 + t_2 \geq 1.25\left(\min\left[0.75\ in,\ thickness\ of\ thinner\ parts\ joined\right]\right)$$
$$\left\{t_1 + t_2 = 0.175 + 0.175 = 0.350\right\} \geq \left\{1.25\left(\min\left[0.75,\ 0.1875\right]\right) = 0.2344\right\} \qquad True$$

b) STEP 2 – Calculate the Limits of Reinforcement per UG-40.

1) Reinforcing dimensions for a reinforced nozzle per Fig. UG-40 sketch (l). See Figure E4.5.6 of this example.

2) Finished opening chord length.

$$d = 2R_n = 2\left(4.0625\right) = 8.125\ in$$

3) The limits of reinforcement, measured parallel to the vessel wall in the corroded condition.

$$\max\left[d,\ R_n + t_n + t\right] = \max\left[8.125,\ \left\{4.0625 + 0.25 + 0.1875\right\}\right] = 8.125\ in$$

4) The limits of reinforcement, measured normal to the vessel wall in the corroded condition.

i) Outside of vessel:

$$\min\left[2.5t,\ 2.5t_n + t_e\right] = \min\left[2.5\left(0.1875\right),\ \left\{2.5\left(0.25\right) + 0.0\right\}\right] = 0.4688\ in$$

ii) Inside of vessel:

$$\min\left[h,\ 2.5t,\ \ 2.5t_i\right]=\min\left[0.5,\ 2.5(0.1875),\ 2.5(0.5)\right]=0.4688\ in$$

c) STEP 3 – Calculate the reinforcement strength parameters per UG-37.

1) Strength Reduction Factors:

$$f_{r1}=S_n/S_v=12000/17500=0.6857$$
$$f_{r2}=S_n/S_v=12000/17500=0.6857$$
$$f_{r3}=\min\left[S_n,S_p\right]/S_v=\min[12000,\ 0.0]/17500=0.0$$
$$f_{r4}=S_p/S_v=0.0/17500=0.0$$

2) Joint Efficiency Parameter: For a nozzle located in a solid plate, $E_1=1.0$

3) Correction Factor for variation of internal pressure stresses on different planes with respect to the axis of the vessel: For a radial nozzle in a ellipsoidal head, $F=1.0$.

d) STEP 4 – Calculate the Areas of Reinforcement, see Fig. UG–37.1

1) Area Required, A:

$$A=dt_rF+2t_nt_rF(1-f_{r1})$$
$$A=8.125(0.0912)(1.0)+2(0.25)(0.0912)(1.0)(1-0.6857)=0.7553\ in^2$$

2) Area Available in the Shell, A_1. Use larger value:

$$A_{11}=d\left(E_1t-Ft_r\right)-2t_n\left(E_1t-Ft_r\right)(1-f_{r1})$$
$$A_{11}=\left\{\begin{array}{l}8.125(1.0(0.1875)-1.0(0.0912))-\\2(0.25)(1.0(0.1875)-1.0(0.0912))(1-0.6857)\end{array}\right\}=0.7673\ in^2$$
$$A_{12}=2\left(t+t_n\right)\left(E_1t-Ft_r\right)-2t_n\left(E_1t-Ft_r\right)(1-f_{r1})$$
$$A_{12}=\left\{\begin{array}{l}2(0.1875+0.25)(1.0(0.1875)-1.0(0.0912))-\\2(0.25)(1.0(0.1875)-1.0(0.0912))(1-0.6857)\end{array}\right\}=0.0691\ in^2$$
$$A_1=\max\left[0.7673,\ 0.0691\right]=0.7673\ in^2$$

3) Area Available in the Nozzle Projecting Outward, A_2. Use the smaller value:

$$A_{21}=5\left(t_n-t_{rn}\right)f_{r2}t$$
$$A_{21}=5(0.25-0.0512)(0.6857)(0.1875)=0.1278\ in^2$$
$$A_{22}=5\left(t_n-t_{rn}\right)f_{r2}t_n$$
$$A_{22}=5(0.25-0.0512)(0.6857)(0.25)=0.1704\ in^2$$
$$A_2=\min\left[0.1278,\ \ 0.1704\right]=0.1278\ in^2$$

4) Area Available in the Nozzle Projecting Inward, A_3:

$$A_3 = \min\left[5tt_i f_{r2}, 5t_i t_i f_{r2}, 2ht_i f_{r2}\right]$$

$$A_3 = \min\left[5(0.1875)(0.25)(0.6857), 5(0.25)^2(0.6857), 2(0.5)(0.25)(0.6857)\right]$$

$$A_3 = \min\left[0.1607, 0.2143, 0.1714\right] = 0.1607\ in^2$$

5) Area Available in Welds, A_{41}, A_{42}, A_{43}, use the following minimum specified weld leg dimensions, see Figure E4.5.4 of this example:

$$Outer\ Nozzle\ Fillet\ Weld\ Leg: \quad 0.25\ in$$

$$Outer\ Element\ Fillet\ Weld\ Leg: \quad 0.0\ in$$

$$Inner\ Nozzle\ Fillet\ Weld\ Leg: \quad 0.25\ in$$

$$A_{41} = leg^2 f_{r2} = (0.25)^2(0.6857) = 0.0429\ in^2$$

$$A_{42} = 0.0\ in^2$$

$$A_{43} = leg^2 f_{r2} = (0.25)^2(0.6857) = 0.0429\ in^2$$

6) Area Available in Element, A_5:

$$A_5 = (D_p - d - 2t_n)t_e f_{r4} = 0.0\ in^2$$

7) Total Available Area, A_{avail}:

$$A_{avail} = A_1 + A_2 + A_3 + (A_{41} + A_{42} + A_{43}) + A_5$$

$$A_{avail} = 0.7673 + 0.1278 + 0.1607 + (0.0429 + 0.0 + 0.0429) + 0.0 = 1.1416\ in^2$$

e) STEP 5 – Nozzle reinforcement acceptance criterion:

$$\left\{A_{avail} = 1.1416\ in^2\right\} \geq \left\{A = 0.7553\ in^2\right\} \qquad True$$

Therefore, the nozzle is adequately reinforced.

The load to be carried by the welds is calculated in accordance with UG-41.

a) STEP 1 – Per Fig. UG-41.1, sketch (a) Nozzle Attachment Weld Loads and Weld Strength Paths to be Considered; typical nozzle detail with nozzle neck inserted through (set–in) the vessel wall.

Per UG-41(b)(1):

1) Weld Load for Strength Path 1-1, W_{1-1}.

$$W_{1-1} = (A_2 + A_5 + A_{41} + A_{42})S_v = (0.1278 + 0.0 + 0.0429 + 0.0)(17500) = 2987.3\ lbs$$

2) Weld Load for Strength Path 2-2, W_{2-2}.

$$W_{2-2} = (A_2 + A_3 + A_{41} + A_{43} + 2t_n t f_{r1})S_v$$

$$W_{2-2} = (0.1278 + 0.1607 + 0.0429 + 0.0429 + 2(0.25)(0.1875)(0.6857))(17500) = 7675.2\ lbs$$

Per UG-41(b)(2): Total Weld Load, W.

$$W = \left(A - A_1 + 2t_n f_{r1}\left(E_1 t - F t_r\right)\right) S_v$$

$$W = \left(0.7553 - 0.7673 + 2\left(0.25\right)\left(0.6857\right)\left(1.0\left(0.1875\right) - 1.0\left(0.0912\right)\right)\right)\left(17500\right) = 367.8 \ lbs$$

Since W is smaller than W_{1-1} and W_{2-2}, W may be used in place of W_{1-1} and W_{2-2} for comparing weld capacity to weld load.

b) STEP 2 – Determine the allowable stresses of the attachment welds for weld strength path check. The allowable stress of the welds should be considered equal to the lesser of the two allowable stresses joined. Per UW-15(c) and UG-45(c), the allowable stresses for groove/fillet welds in percentages of stress value for the vessel material, used with UG-41 calculations are as follows:

$$Groove\ Weld\ Tension : 74\%$$

$$Groove\ Weld\ Shear : 60\%$$

$$Fillet\ Weld\ Shear : 49\%$$

$$Nozzle\ Neck\ Shear : 70\%$$

1) Fillet Weld Shear – Outer Nozzle Fillet and Inner Nozzle Fillet:

$$S_{ofws} = S_{ifws} = 0.49\left(12000\right) = 5880 \ psi$$

2) Nozzle Wall Shear:

$$S_{nws} = 0.70\left(12000\right) = 8400 \ psi$$

c) STEP 3 – Determine the Strength of Connection Elements

1) Outer Nozzle Fillet Weld Shear:

$$ONWS = \frac{\pi}{2}\left(Nozzle\ OD\right)\left(Weld\ Leg\right)\left(S_{ofws}\right)$$

$$ONWS = \frac{\pi}{2}\left(8.625\right)\left(0.25\right)\left(5880\right) = 19915.7 \ lbs$$

2) Inner Nozzle Fillet Weld Shear:

$$INWS = \frac{\pi}{2}\left(Nozzle\ OD\right)\left(Weld\ Leg\right)\left(S_{ifws}\right)$$

$$INWS = \frac{\pi}{2}\left(8.625\right)\left(0.25\right)\left(5880\right) = 19915.7 \ lbs$$

3) Nozzle Wall Shear:

$$NWS = \frac{\pi}{2}\left(Mean\ Nozzle\ Diameter\right)\left(t_n\right)\left(S_{nws}\right)$$

$$NWS = \frac{\pi}{2}\left(8.125 + 0.25\right)\left(0.25\right)\left(8400\right) = 27626.4 \ lbs$$

d) STEP 4 – Check Weld Strength Paths

1) $Path_{1-1} = ONWS + NWS = 19915.7 + 27626.4 = 47542.1\ lbs$

2) $Path_{2-2} = ONWS + INWS = 19915.7 + 19915.7 = 39831.4\ lbs$

e) STEP 5 – Weld Path Acceptance Criteria:

Per UG-41(b)(1):

$$\{Path_{1-1} = 47542.1\ lbs\} \ge \{W_{1-1} = 2987.3\ lbs\} \qquad True$$
$$\{Path_{2-2} = 39831.4\ lbs\} \ge \{W_{2-2} = 7675.2\ lbs\} \qquad True$$

Per UG-42(b)(2):

$$\min\left[Path_{1-1},\ Path_{2-2}\right] \ge W$$
$$\min\left[47542.1,\ 39831.4\ lbs\right] \ge \{W = 367.8\ lbs\} \qquad True$$

Section VIII, Division 2 Solution

There is no comparable weld detail for this nozzle attachment in VIII-2, Part 4.2. Therefore, no calculation is performed.

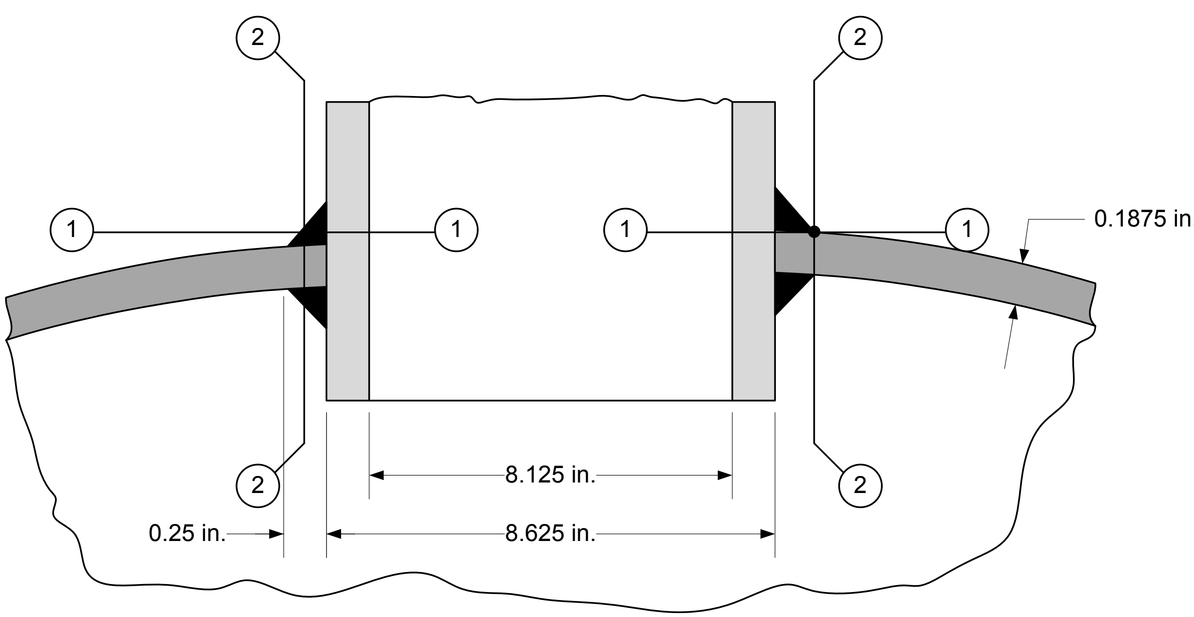


Figure E4.5.6 – Nozzle Details

4.6 Flat Heads

4.6.1 Example E4.6.1 - Flat Unstayed Circular Heads Attached by Bolts

Determine the required thickness for a heat exchanger blind flange.

<u>Blind Flange Data:</u>

- Material = $SA-105$
- Design Conditions = $135\ psig@650°F$
- Flange Bolt-Up Temperature = $100°F$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $17800\ psi$
- Allowable Stress at Flange Bolt-Up Temp. = $20000\ psi$
- Weld Joint Efficiency = 1.0
- Mating flange information and gasket details are provided in Example Problem E4.16.1.

Design rules for unstayed flat heads and covers are provided in UG-34. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.6.

Evaluate the blind flange in accordance with VIII-1, UG-34 and Appendix 2.

The minimum required thickness of a flat unstayed circular head, cover, or blind flange that is attached with bolting that results in an edge moment, see VIII-1, Fig. UG-34, Sketch (j), shall be calculated by the equations shown below. The operating and gasket seating bolt loads, $W = W_{m1}$ and W, and the moment arm of this load, h_G, in these equations shall be computed based on the flange geometry and gasket material as described in VIII-1, paragraph 2-5 and Table 2-6.

a) STEP 1 – Calculate the gasket moment arm, h_G, and the diameter of the gasket load reaction, d in accordance with VIII-1, Table 2-6 and paragraph 2-3, respectively, as demonstrated in Example Problem E4.16.1.

   See Flange Design Procedure, STEP 6: $h_G = 0.875\ in$

   See Gasket Reaction Diameter, STEP 3: $d = G = 29.5\ in$

b) STEP 2 – Calculate the operating and gasket seating bolt loads, $W = W_{m1}$ and W, in accordance with VIII-1, paragraph 2-5, as demonstrated in Example Problem E4.16.1.

   Design Bolt Loads, STEP 1: $W = W_{m1} = 111329.5\ lbs$

   Design Bolt Loads, STEP 4: $W = 237626.3\ lbs$

c) STEP 3 – Identify the appropriate attachment factor, C, from VIII-1, Fig. UG-34 Sketch (j).

   $C = 0.3$

d) STEP 4 - The required thickness of the blind flange is the maximum of the thickness required for the operating and gasket seating conditions.

   $t = \max\left[t_o,\ t_g\right]$

   1) The required thickness in the operating condition is in accordance with VIII-1, UG-34, Equation (2).

$$t_o = d\sqrt{\left(\frac{CP}{SE}\right)+\left(\frac{1.9Wh_G}{SEd^3}\right)}+CA$$

$$t_o = (29.5)\sqrt{\left(\frac{0.3(135)}{17800(1.0)}\right)+\left(\frac{1.9(111329.5)(0.875)}{17800(1.0)(29.5)^3}\right)}+0.125 = 1.6523\ in$$

2) The required thickness in the gasket seating condition is in accordance with VIII-1, UG-34, Equation (2) when $P = 0.0$.

$$t_g = d\sqrt{\frac{1.9Wh_G}{SEd^3}}+CA$$

$$t_g = (29.5)\sqrt{\frac{1.9(237626.3)(0.875)}{20000(1.0)(29.5)^3}}+0.125 = 0.9943\ in$$

$$t = \max[1.6523,\ 0.9943] = 1.6523\ in$$

The required thickness is $1.6523\ in$.

4.6.2 Example E4.6.2 – Flat Un-stayed Non-Circular Heads Attached by Welding

Determine the required thickness for an air-cooled heat exchanger end plate. The end plate is welded to the air-cooled heat exchanger box with a full penetration Category C, Type 7 corner joint.

<u>End Plate Data:</u>

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $400\ psig @ 500°F$
- Short Span Length = $7.125\ in$
- Long Span Length = $9.25\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0

Design rules for unstayed flat heads and covers are provided in UG-34. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.6.

Evaluate the welded end plate in accordance with VIII-1, UG-34 and Appendix 13.

The minimum required thickness of a flat unstayed non-circular head or cover that is not attached with bolting that results in an edge moment shall be calculated by the following equations.

a) STEP 1 – Determine the short and long span dimensions of the non-circular plate, d and D, respectively (in the corroded state) as demonstrated in Example Problem E4.12.1.

$$d = H = 7.375\ in$$

$$D = h = 9.500\ in$$

b) STEP 2 – Calculate the Z factor in accordance with VIII-1, UG-34, Equation (4).

$$Z=\min\left[2.5,\left(3.4-\left(\frac{2.4d}{D}\right)\right)\right]=\min\left[2.5,\left(3.4-\left(\frac{2.4(7.375)}{9.5}\right)\right)\right]=1.5368\ in$$

c) STEP 3 - The appropriate attachment factor, C, is taken from VIII-1, paragraph 13-4(f). For end closures of non-circular vessels constructed of flat plate, the design rules of VIII-1, UG-34 shall be used except that 0.20 shall be used for the value of C in all of the calculations.

$$C=0.20$$

d) STEP 4 - Calculate the required thickness using VIII-1, UG-34, Equation (3).

$$t=d\sqrt{\frac{ZCP}{SE}}+CA=7.375\sqrt{\frac{1.5368(0.20)(400)}{20000(1.0)}}+0.125=0.7032\ in$$

The required thickness is $0.7032\ in$.

4.6.3 Example E4.6.3 – Integral Flat Head with a Centrally Located Opening

Determine if the stresses in the integral flat head with a centrally located opening are within acceptable limits, considering the following design conditions. The head, shell and opening detail is shown in Figure E4.6.3. The vessel is fabricated from Type 304 stainless steel with an allowable stress of 18.8 ksi.

<u>End Plate Data:</u>

- Design Conditions = $100\ psig@100°F$
- Outside diameter of flat head and shell, A = $72\ in$
- Inside diameter of shell, B_s = $70\ in$
- Diameter of central opening, B_n = $40\ in$
- Thickness of the flat head, t = $3.0\ in$
- Thickness of nozzle above the transition, g_{0n} = $0.5625\ in$
- Thickness of nozzle at the flat head, g_{1n} = $1.125\ in$
- Length of nozzle transition, h_n = $2.0\ in$
- Thickness of shell below transition, g_{0s} = $1.0\ in$
- Thickness of shell at head, g_{1s} = $2.0\ in$
- Length of shell transition, h_s = $3.0\ in$
- Allowable stress = $18800\ psi$

Design rules for Integral Flat Head with a Centrally Located Opening are provided in Mandatory Appendix 14. The rules in this appendix are the same as those provided in VIII-2, paragraph 4.6. The design procedure in VIII-2, paragraph 4.6 is used in this example problem with substitute references made to VIII-1, Mandatory Appendix 14 and Appendix 2 paragraphs.

Evaluate the integral flat head with a single, circular, centrally located opening in accordance with VIII-1, Appendix 14.

a) STEP 1 – Determine the design pressure and temperature of the flat head opening.

See the specified data above.

b) STEP 2 – Determine the geometry of the flat head opening.

See Figure E4.6.3 and the specified data above.

c) STEP 3 – Calculate the operating moment, M_o, using the following equation in accordance with VIII-1, paragraph 14-3(a)(1) with reference to paragraphs 2-3, 2-6 and Table 2-6.

$$M_o = M_D + M_T = H_D h_D + M_T h_T$$
$$M_o = 125600(14.44) + 259050(7.5) = 3756225\ in-lbs$$

Where, the flange forces, H_D and H_T, are calculated as follows.

$$H_D = 0.785 B_n^2 P = 0.785(40)^2(100) = 125600\ lbs$$
$$H = 0.785 B_s^2 P = 0.785(70)^2(100) = 384650\ lbs$$
$$H_T = H - H_D = 384650 - 125600 = 259050\ lbs$$

And the moment arms, h_D and h_T, are calculated as follows

$$h_D = R + \frac{g_{1n}}{2} = 13.88 + \frac{1.125}{2} = 14.44\ in$$
$$h_T = \frac{R + g_{1n}}{2} = \frac{13.88 + 1.125}{2} = 7.5\ in$$

Where,

$$R = \frac{B_s - B_n}{2} - g_{1n} = \frac{70 - 40}{2} - 1.125 = 13.88\ in$$

d) STEP 4 – Calculate F, V, and f based on B_n, g_{1n}, g_{0n} and h_n using the equations/direct interpretation from VIII-1 Table 2-7.1 and Fig. 2-7.2, Fig 2-7.3 and Fig. 2-7.6 and designate the resulting values as F_n, V_n, and f_n.

<u>Fig. 2-7.2:</u>

$$g_{rn} = \frac{g_{1n}}{g_{0n}} = \frac{1.125}{0.5625} = 2.0$$
$$h_{0n} = \sqrt{B_n g_{0n}} = \sqrt{(40)(0.5625)} = 4.75\ in$$
$$h_{rn} = \frac{h_n}{h_{0n}} = \frac{2.0}{4.75} = 0.421$$

Interpretation of Fig. 2-7.2, $F_n \approx 0.84$. From the equations of Table 2-7.1, $F_n = 0.843$.

<u>Fig. 2-7.3:</u>

With $g_{rn} = 2.0$ and $h_{rn} = 0.421$:

Interpretation of Fig. 2-7.3, $V_n \approx 0.25$. From the equations of Table 2-7.1, $V_n = 0.252$.

Fig. 2-7.6:

With $g_{rn} = 2.0$ and $h_{rn} = 0.421$:

Interpretation of Fig. 2-7.6, $f_n \approx 1.5$. From the equations of Table 2-7.1, $f_n = 1.518$.

e) STEP 5 – Calculate F, V, and f based on B_s, g_{1s}, g_{0s} and h_s using the equations/direct interpretation from VIII-1 Table 2-7.1 and Fig. 2-7.2, Fig 2-7.3 and Fig. 2-7.6, and designate the resulting values as F_s, V_s, and f_s.

Fig. 2-7.2

$$g_{rs} = \frac{g_{1s}}{g_{0s}} = \frac{2.0}{1} = 2.0$$

$$h_{0s} = \sqrt{B_s g_{0s}} = \sqrt{(70)(1)} = 8.37\ in$$

$$h_{rs} = \frac{h_s}{h_{0s}} = \frac{3.0}{8.37} = 0.359$$

Interpretation of Fig. 2-7.2, $F_s \approx 0.86$. From the equations of Table 2-7.1, $F_s = 0.857$.

Fig. 2-7.3:

With $g_{rs} = 2.0$ and $h_{rs} = 0.359$:

Interpretation of Fig. 2-7.3, $V_s \approx 0.28$. From the equations of Table 2-7.1, $V_s = 0.276$.

Fig. 2-7.6:

With $g_{rs} = 2.0$ and $h_{rs} = 0.359$:

Interpretation of Fig. 2-7.6, $f_s \approx 1.8$. From the equations of Table 2-7.1, $f_s = 1.79$.

f) STEP 6 – Calculate Y, T, U, Z, L, e, and d based on $K = A/B_n$ using the equations/direct interpretation from VIII-1 Fig. 2-7.1.

$$K = \frac{A}{B_n} = \frac{72}{40} = 1.8$$

$$Y = \frac{1}{K-1}\left[0.66845 + 5.71690\left(\frac{K^2 \log_{10} K}{K^2 - 1}\right)\right]$$

$$Y = \frac{1}{(1.8)-1}\left[0.66845 + 5.71690\left(\frac{(1.8)^2 \log_{10}[1.8]}{(1.8)^2 - 1}\right)\right] = 3.47$$

$$T = \frac{K^2\left(1+8.55246\log_{10}K\right)-1}{\left(1.04720+1.9448K^2\right)\left(K-1\right)} = \frac{\left(1.8\right)^2\left(1+8.55246\log_{10}\left[1.8\right]\right)-1}{\left(1.04720+1.9448\left(1.8\right)^2\right)\left(1.8-1\right)} = 1.58$$

$$U = \frac{K^2\left(1+8.55246\log_{10}K\right)-1}{1.36136\left(K^2-1\right)\left(K-1\right)} = \frac{\left(1.8\right)^2\left(1+8.55246\log_{10}\left[1.8\right]\right)-1}{1.36136\left(\left(1.8\right)^2-1\right)\left(\left(1.8\right)-1\right)} = 3.82$$

$$Z = \frac{\left(K^2+1\right)}{\left(K^2-1\right)} = \frac{\left(\left(1.8\right)^2+1\right)}{\left(\left(1.8\right)^2-1\right)} = 1.89$$

$$d = \frac{Ug_{on}^2 h_{on}}{V_n} = \frac{\left(3.82\right)\left(0.5625\right)^2\left(4.75\right)}{0.252} = 23\ in^3$$

$$e = \frac{F_n}{h_{on}} = \frac{0.843}{4.75} = 0.18\ in^{-1}$$

$$L = \frac{te+1}{T} + \frac{t^3}{d} = \frac{\left(3\right)\left(0.18\right)+1}{1.58} + \frac{\left(3\right)^3}{23} = 2.15\ in$$

g) STEP 7 – Calculate the quantity $\left(E\theta\right)^*$ for an opening with an integrally attached nozzle using the following equation, VIII-1, paragraph 14-3(b)(1).

$$\left(E\theta\right)^* = \frac{0.91\left(\frac{g_{1n}}{g_{on}}\right)^2\left(B_{1n}\right)V_n}{f_n h_{0n}} \cdot S_H$$

$$\left(E\theta\right)^* = \frac{0.91\left(\frac{1.125}{0.5625}\right)^2\left(40.5625\right)\left(0.252\right)}{\left(1.518\right)\left(4.75\right)} \cdot \left(52287\right) = 269584\ psi$$

Where, B_{1n} is evaluated from paragraph 2-3 and S_H is evaluated from VIII-1, paragraph 2-7.

$$B_{1n} = B_n + g_{0n} = 40 + 0.5625 = 40.5625\ in$$

$$S_H = \frac{f_n M_o}{Lg_{1n}^2 B_n} = \frac{1.518\left(3756225\right)}{2.15\left(1.125\right)^2\left(40\right)} = 52287\ psi$$

h) STEP 8 – Calculate the quantity M_H using the following equation, VIII-1, paragraph 14-3(c).

$$M_H = \frac{(E\theta)^*}{\dfrac{1.74 h_{0s} V_s}{g_{0s}^3 B_{1s}} + \dfrac{(E\theta)^*}{M_o}\left(1 + \dfrac{F_s t}{h_{0s}}\right)}$$

$$M_H = \frac{269584}{\dfrac{1.74(8.37)(0.276)}{(1)^3(71.0)} + \dfrac{269584}{3756225}\left(1 + \dfrac{(0.857)(3)}{8.37}\right)} = 1792262\ in-lb$$

Where, B_{1s} is evaluated from paragraph 2-3.

$$B_{1s} = B_s + g_{0s} = 70 + 1.0 = 71.0\ in$$

i) STEP 9 – Calculate the quantity X_1 using the following equation, VIII-1, paragraph 14-3(d).

$$X_1 = \frac{M_o - M_H\left(1 + \dfrac{F_s t}{h_{0s}}\right)}{M_o} = \frac{3756225 - 1792262\left(1 + \dfrac{(0.857)(3)}{8.37}\right)}{3756225} = 0.376$$

j) STEP 10 – Calculate the stresses at the shell-to-flat head junction in accordance with VIII-1, paragraph 14-3(e)(1) and the opening-to-flat-head junction in accordance with VIII-1, paragraph 14-3(e)(2).

Longitudinal hub stress in shell:

$$S_{HS} = \frac{1.10 f_s X_1 (E\theta)^* (h_{0s})}{\left(\dfrac{g_{1s}}{g_{0s}}\right)^2 B_s V_s} = \frac{1.10(1.79)(0.376)(269584)(8.37)}{\left(\dfrac{2}{1}\right)^2 (70)(0.276)} = 21621\ psi$$

Radial stress at outside diameter:

$$S_{RS} = \frac{1.91 M_H\left(1 + \dfrac{F_s t}{h_{0s}}\right)}{B_s t^2} + \frac{0.64 F_s M_H}{B_s h_{0s} t} = \left(\begin{array}{l}\dfrac{1.91(1792262)\left(1 + \dfrac{(0.857)(3)}{8.37}\right)}{(70)(3)^2} + \\ \dfrac{0.64(0.857)(1792262)}{(70)(8.37)(3)}\end{array}\right) = 7663\ psi$$

Tangential stress at outside diameter:

$$S_{TS} = \frac{X_1 (E\theta)^* t}{B_s} - \frac{0.57M_H \left(1 + \frac{F_s t}{h_{0s}}\right)}{B_s t^2} + \frac{0.64ZF_s M_H}{B_s h_{0s} t}$$

$$S_{TS} = \left(\begin{array}{l} \frac{(0.376)(269584)(3)}{70} - \frac{0.57(1792262)\left(1 + \frac{(0.857)(3)}{8.37}\right)}{(70)(3)^2} + \\ \frac{0.64(1.89)(0.857)(1792262)}{(70)(8.37)(3)} \end{array} \right)$$

$$S_{TS} = 3286 \ psi$$

Longitudinal hub stress in central opening:

$$S_{HO} = X_1 S_H = (0.376)(52287) = 19672 \ psi$$

Radial stress at central opening:

$$S_{RO} = X_1 S_R = (0.376)(8277) = 3114 \ psi$$

Where, S_R is evaluated from VIII-1, paragraph 2-7.

$$S_R = \frac{(1.33te+1)M_o}{Lt^2 B_n} = \frac{(1.33(3)(0.18)+1)(3756225)}{(2.15)(3)^2(40)} = 8277 \ psi$$

Tangential stress at diameter of central opening:

$$S_{TO} = X_1 S_T + \frac{0.64Z_1 F_s M_H}{B_s h_{os} t} = \left(\begin{array}{l} (0.376)(20582) + \\ \frac{0.64(2.89)(0.857)(1792262)}{(70)(8.37)(3)} \end{array} \right) = 9362 \ psi$$

Where, Z_1 is calculated as follows and S_T is evaluated from VIII-1, paragraph 2-7.

$$Z_1 = \frac{2K^2}{K^2 - 1} = \frac{2(1.8)^2}{\left((1.8)^2 - 1\right)} = 2.89$$

$$S_T = \frac{YM_o}{t^2 B_n} - ZS_R = \frac{(3.47)(3756225)}{(3)^2(40)} - (1.89)(8277) = 20582 \ psi$$

k) STEP 11 – Check the flange stress acceptance criteria in VIII-1, paragraph 14-3(f) with reference to paragraph 2-8. If the stress criteria are satisfied, then the design is complete. If the stress criteria are not satisfied, then re-proportion the flat head and/or opening dimensions and go to STEP 3.

Shell-to-flat-head junction:

$$\left\{S_{HS} = 21621\ psi\right\} \le \left\{1.5S_f = 1.5(18800) = 28200\ psi\right\} \qquad True$$

$$\left\{S_{RS} = 7663\ psi\right\} \le \left\{S_f = 18800\ psi\right\} \qquad True$$

$$\left\{S_{TS} = 3286\ psi\right\} \le \left\{S_f = 18800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_{HS} + S_{RS})}{2} = \frac{(21621 + 7663)}{2} = 14642\ psi \le S_f = 18800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_{HS} + S_{TS})}{2} = \frac{(21621 + 3286)}{2} = 12454\ psi \le S_f = 18800\ psi\right\} \qquad True$$

Opening-to-flat-head junction

$$\left\{S_{HO} = 19672\ psi \le 1.5S_f = 1.5(18800) = 28200\ psi\right\} \qquad True$$

$$\left\{S_{RO} = 3114\ psi\right\} \le \left\{S_f = 18800\ psi\right\} \qquad True$$

$$\left\{S_{TO} = 9362\ psi\right\} \le \left\{S_f = 18800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_{HO} + S_{RO})}{2} = \frac{(19672 + 3114)}{2} = 11393\ psi \le S_f = 18800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_{HO} + S_{TO})}{2} = \frac{(19672 + 9362)}{2} = 14517\ psi \le S_f = 18800\ psi\right\} \qquad True$$

Stress acceptance criteria are satisfied, the design is complete.

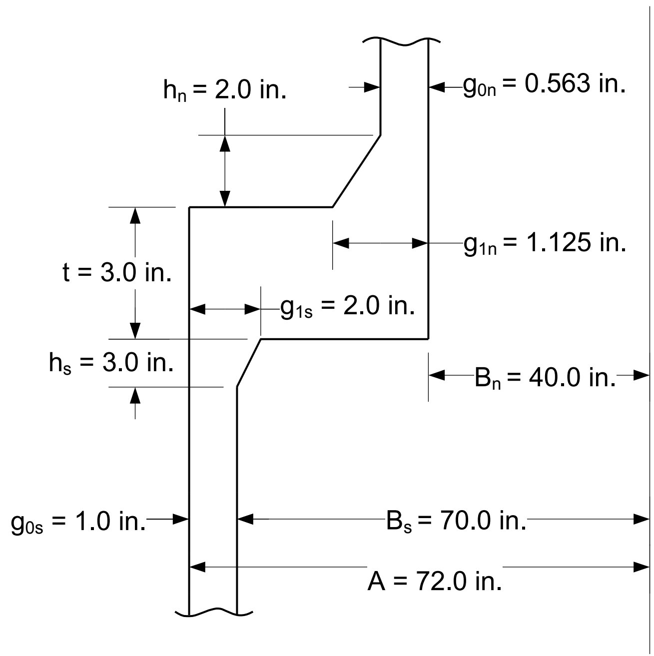


Figure E4.6.3 – Head, Shell and Nozzle Geometry

4.7 Spherically Dished Bolted Covers

4.7.1 Example E4.7.1 – Thickness Calculation for a Type D Head

Determine if the proposed Type D spherically dished bolted cover, used in a heat exchanger application, is adequately designed considering the following design conditions. The spherically dished head is seamless. See Figure E4.7.1 for details.

Tubeside Data:

- Design Conditions = $213\ psig@400°F$
- Corrosion Allowance (CAT) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Shellside Data:

- Design Conditions = $305\ psig@250°F$
- Corrosion Allowance (CAS) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Flange Data:

- Material = $SA-105$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Allowable Stress at Tubeside Design Temperature = $20000\ psi$
- Allowable Stress at Shellside Design Temperature = $20000\ psi$

Head Data:

- Material = $SA-515,\ Grade\ 60$
- Allowable Stress At Ambient Temperature = $17100\ psi$
- Allowable Stress at Tubeside Design Temperature = $17100\ psi$
- Allowable Stress at Shellside Design Temperature = $17100\ psi$
- Yield Stress at Shellside Design Temperature = $28800\ psi$
- Modulus of Elasticity at Shellside Design Temp. = $28.55E+06\ psi$

Bolt Data

- Material = $SA-193,\ Grade\ B7$
- Diameter = $0.75\ in$
- Cross-Sectional Root Area = $0.302\ in^2$
- Number of Bolts = 20
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Allowable Stress at Tubeside Design Temperature = $25000\ psi$
- Allowable Stress at Shellside Design Temperature = $25000\ psi$

Gasket Data

- Material = Solid Flat Metal (Iron/Soft Steel)
- Gasket Factor = 5.5
- Gasket Seating Factor = $18000\ psi$
- Inside Diameter = $16.1875\ in$
- Outside Diameter = $17.0625\ in$

Design rules for spherically dished bolted covers with ring type gaskets are provided in Mandatory Appendix 1-6 with reference to Mandatory Appendix 2. The rules in the paragraphs of Appendix 1-6 are the same as those provided in VIII-2, paragraph 4.7. The rules in the paragraphs of Appendix 2 are the same as those provided in VIII-2, paragraph 4.16 with noted differences as outlined in Example Problems E4.16.1 and E4.16.2.

The calculations are performed using dimensions in the corroded condition and the uncorroded condition, and the more severe case shall control. This example only evaluates the spherically dished bolted cover in the corroded condition.

Per VIII-1 Appendix 1-6(g), the thickness of the head for a Type D Head Configuration Fig. 1-6 Sketch (d) shall be determined by the following equations.

a) Internal pressure (pressure on the concave side) – the head thickness shall be determined using Appendix 1-6, Equation (9).

$$L = L + CAS = 16.0 + 0.125 = 16.125$$

$$t = \left(\frac{5PL}{6S}\right) = \frac{5(213)(16.125)}{6(17100)} = 0.1674\ in$$

This calculated thickness is increased for corrosion allowance on both the shell and tube side.

$$t = t + CAS + CAT$$

$$t = 0.1674 + 0.125 + 0.125 = 0.4174\ in$$

b) External pressure (pressure on the convex side) – the head thickness shall be determined in accordance with the rules of paragraph UG-33(c). As noted in paragraph UG-33(c), the required thickness of a hemispherical head having pressure on the convex side shall be determined in the same manner as outlined in paragraph UG-28(d) for determining the thickness for a spherical shell.

1) STEP 1 - Assume an initial thickness, t, for the spherical shell and calculate the value of factor A using the following formula:.
The specified head thickness shall consider corrosion from the tubeside and shellside, however, when calculating the outside radius of the head, R_o, the uncorroded head thickness is conservatively used in the calculations.

$$A = \frac{0.125}{\left(\dfrac{R_o}{t}\right)} = \frac{0.125}{\left(\dfrac{16.875}{0.625}\right)} = 0.00463$$

Where,

$$R_o = L + Uncorroded\ Thickness = 16.0 + 0.875 = 16.875$$
$$t = t - CAS - CAT$$
$$t = 0.875 - 0.125 - 0.125 = 0.625\ in$$

2) STEP 2 - Using the value of A calculated in STEP 1, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically to an intersection with the material/temperature line for the tubeside temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 5.

Per Section II Part D, Table 1A, a material specification of $SA-515-60$ is assigned an External Pressure Chart No. CS-2.

3) STEP 3 - From the intersection obtained in Step 2, move horizontally to the right and read the value of factor B.

$$B = 14700$$

4) STEP 4 - Using the value of B obtained in STEP 3, calculate the value of the maximum allowable external working pressure P_a using the following formula..

$$P_a = \frac{B}{\left(\frac{R_o}{t}\right)} = \frac{17000}{\left(\frac{16.875}{0.625}\right)} = 544.4\ psi$$

5) STEP 5 - For values of A falling to the left of the applicable material/temperature line, the value of P_a can be calculated using the following formula.

$$P_a = \frac{0.0625E}{\left(\frac{R_o}{t}\right)^2} \qquad Not\ required$$

6) STEP 6 - If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2.

Since $\{P_a = 544.4\ psi\} > \{P = 305\ psi\}$, the specified head thickness is acceptable for external pressure.

The flange thickness of the head for a Type D Head Configuration is determined per Appendix 1-6, Equation (10). To compute the required flange thickness, the flange operating and gasket seating moments are determined using VIII-1, Appendix 2.

Establish the design conditions and gasket reaction diameter.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$Tubeside\ Conditions: P = 213\ psig\ at\ 400°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 2-5.1.

$$m = 5.5$$
$$y = 18000 \ psi$$

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5(GOD - GID) = 0.5(17.0625 - 16.1875) = 0.4375 \ in$$

From Table 2-5.2, Facing Sketch Detail 2, Column I,

$$b_o = \frac{w+N}{4} = \frac{(0.125+0.4375)}{4} = 0.1406 \ in$$

Where,

$$w = raised \ nubbin \ width = 0.125 \ in$$

For $b_o \leq 0.25 \ in$,

$$b = b_o = 0.1406 \ in$$

Therefore, from paragraph 2-3, the location of the gasket reaction is calculated as follows.

$$G = mean \ diameter \ of \ the \ gasket \ contact \ face$$
$$G = 0.5(17.0625 + 16.1875) = 16.625 \ in$$

Paragraph 2-5 – Calculate the design bolts load for the operating and gasket seating conditions.

a) STEP 1 – Paragraph 2-5(c)(1), determine the design bolt load for the operating condition.

$$W_{m1} = H + H_p = \frac{\pi}{4}G^2P + 2b\pi GmP \quad for \ non-self-energized \ gaskets$$

$$W_{m1} = \frac{\pi}{4}(16.625)^2(213) + 2(0.1406)\pi(16.625)(5.5)(213) = 63442.9 \ lbs$$

b) STEP 2 – Paragraph 2-5(c)(2), determine the design bolt load for the gasket seating condition.

$$W_{m2} = \pi bGy$$
$$W_{m2} = \pi(0.1406)(16.625)(18000) = 132181.1 \ lbs$$

c) STEP 3 – Paragraph 2-5(d), determine the total required and actual bolt areas.

The total cross-sectional area of bolts A_m required for both the operating conditions and gasket seating is determined as follows.

$$A_m = \max[A_{m1}, A_{m2}] = \max[4.4532, 5.7221] = 5.7221 \ in^2$$

Where,

$$A_{m1} = \frac{W_{m1}}{S_b} = \frac{63442.9}{25000} = 2.5377\ in^2$$

$$A_{m2} = \frac{W_{m2}}{S_a} = \frac{132181.1}{25000} = 5.2872\ in^2$$

The actual bolt area A_b is calculated as follows.

$$A_b = (Number\ of\ bolts)(Root\ area\ of\ one\ bolt) = 20(0.302) = 6.04\ in^2$$

Verify that the actual bolt area is equal to or greater than the total required area.

$$\left\{A_b = 6.04\ in^2\right\} \geq \left\{A_m = 5.2872\ in^2\right\} \qquad True$$

d) STEP 4 – Paragraph 2-5(e), determine the flange design bolt load.

For operating conditions,

$$W = W_{m1} = 111329.5\ lbs$$

For gasket seating,

$$W = \frac{(A_m + A_b)S_a}{2} = \frac{(5.2872 + 6.04)25000}{2} = 141590.0\ lbs$$

Commentary: VIII-1, Appendix 2 does not include an overall step-by-step procedure to design a flange. However, an organized procedure of the steps taken when designing a flange is presented in VIII-2, paragraph 4.16.7. The procedure is applicable to VIII-1, Appendix 2 and is presented in this example problem in an effort to assist the designer.

The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$Tubeside\ Conditions: P = 213\ psig\ at\ 400°F$$

$$Shellside\ Conditions: P = 305\ psig\ at\ 250°F$$

b) STEP 2 – Determine the design bolt loads for operating condition W, and the gasket seating condition W, and the corresponding actual bolt load area A_b, paragraph 2-5.

$$W = 63442.9\ lbs \qquad Operating\ Condition$$

$$W = 141590.0\,lbs \qquad Gasket\ Seating$$

$$A_b = 6.04\ in^2$$

c) STEP 3 – Determine an initial flange geometry (see Figure E4.7.1), in addition to the information required to determine the bolt load, the following geometric parameters are required.

1) Flange bore

$$B = \left[16.25 + 2(CAT)\right] = \left[16.25 + 2(0.125)\right] = 16.50\ in$$

2) Bolt circle diameter

$$C = 18.125\ in$$

3) Outside diameter of the flange

$$A = \left[19.625 - 2(CAS)\right] = \left[19.625 - 2(0.125)\right] = 19.375\ in$$

4) Flange thickness, (see Figure E4.7.1)

$$T = t - Flange\ \ Extension = 2.3125 - 0.1875 = 2.125\ in$$

5) Thickness of the hub at the large end

$$Not\ \ Applicable$$

6) Thickness of the hub at the small end

$$Not\ \ Applicable$$

7) Hub length

$$Not\ \ Applicable$$

d) STEP 4 – Determine the flange stress factors using the equations/direct interpretation from Table 2-7.1 and Fig. 2-7.1 – Fig. 2-7.6.

$$Not\ \ Applicable$$

e) STEP 5 – Determine the flange forces, paragraph 2-3.

Tubeside Conditions:

$$H_D = \frac{\pi}{4}B^2P = \frac{\pi}{4}(16.5)^2(213) = 45544.7\ lbs$$

$$H = \frac{\pi}{4}G^2P = \frac{\pi}{4}(16.625)^2(213) = 46237.3\ lbs$$

$$H_T = H - H_D = 46237.3 - 45544.7 = 692.6\ lbs$$

$$H_G = W - H = 63442.9 - 46237.3 = 17205.6\ lbs \qquad Operating$$

Shellside Conditions:

$$H_D = \frac{\pi}{4}B^2P = \frac{\pi}{4}(16.5)^2(305) = 65216.5\ lbs$$

$$H = \frac{\pi}{4}G^2P = \frac{\pi}{4}(16.625)^2(305) = 66208.4\ lbs$$

$$H_T = H - H_D = 66208.4 - 65216.5 = 991.9\ lbs$$

$$H_G = Not\ \ Applicable$$

f) STEP 6 – Determine the flange moment for the operating condition using paragraph 2-6 for internal pressure and paragraph 2-11 for external pressure. In these equations, h_D is determined from paragraph 1-6(b), and h_T and h_G are determined from Table 2-6.

For internal pressure (Tubeside Conditions):

$$M_o = H_D h_D + H_T h_T + H_G h_G$$
$$M_o = 45544.7(0.8125) + 692.6(0.7813) + 17205.6(0.75)$$
$$M_o = 50450.4\ in - lbs$$

For external pressure (Shellside Conditions):

$$M_o = H_D (h_D - h_G) + H_T (h_T - h_G)$$
$$M_o = 65216.5(0.8125 - 0.75) + 991.9(0.7813 - 0.75)$$
$$M_o = 4107.1\ in - lbs$$

From paragraph 1-6(b),

$$h_D = \frac{C - B}{2} = \frac{18.125 - 16.50}{2} = 0.8125\ in$$

From Table 2-6 for loose type flanges.

$$h_G = \frac{C - G}{2} = \frac{18.125 - 16.625}{2} = 0.75\ in$$

$$h_T = \frac{h_D + h_T}{2} = \frac{0.8125 + 0.75}{2} = 0.7813\ in$$

g) STEP 7 – Determine the flange moment for the gasket seating condition using paragraph 2-6.

For internal pressure (Tubeside Conditions):

$$M_o = W\frac{(C - G)}{2} = 141590.0\left(\frac{(18.125 - 16.625)}{2}\right) = 106192.5\ in - lbs$$

For external pressure (Shellside Conditions):

$$M_o = W h_G = (141590.0)(0.75) = 106192.5\ in - lbs$$

Paragraph 1-6(g)(2) – the flange thickness of the head for a Type D Head Configuration shall be determined by the following equations. When determining the flange design moment for the design condition, M_o, using paragraph 2-6, the following modifications must be made. An additional moment term, M_r, computed using paragraph (1-6(b) shall be added to M_o as defined in paragraph 2-6. Note that this term may be positive or negative depending on the location of the head–to–flange ring intersection with relation to the flange ring centroid. Since the head–to–flange ring intersection is above the flange centroid, the sign of the M_r value is negative.

$$T = \max\left[T_g,\ T_o\right] = \max\left[T_g,\ \max\left[T_{o(tubeside)},\ T_{o(shellside)}\right]\right]$$

Where,

$$T_g = F + \sqrt{F^2 + J} + CAS + CAT$$

$$F = \frac{PB\sqrt{4L^2 - B^2}}{8S(A-B)}$$

$$J = \left(\frac{M_o}{SB}\right)\left(\frac{A+B}{A-B}\right)$$

And,

$$T_o = F + \sqrt{F^2 + J} + CAS + CAT$$

$$F = \frac{PB\sqrt{4L^2 - B^2}}{8S(A-B)}$$

$$J = \left(\frac{M_o}{SB}\right)\left(\frac{A+B}{A-B}\right)$$

a) STEP 1 – Calculate the additional moment, M_r, using paragraph (1-6(b) as follows.

$$M_r = H_r h_r$$

Where,

$$H_r = \left(0.785B^2 P\cot\left[\beta_1\right]\right)$$

$$\beta_1 = \arcsin\left[\frac{B}{2L+t}\right] = \arcsin\left[\frac{(16.5)}{2(16.125)+(0.625)}\right] = \begin{Bmatrix} 0.5258\ rad \\ 30.1259\ deg \end{Bmatrix}$$

And,

$$L = 16.0 + CAT = 16.0 + 0.125 = 16.125\ in$$

$$t = t - CAT - CAS = 0.875 - 0.125 - 0.125 = 0.625\ in$$

Referencing Figure E4.7.1,

$$h_r = \frac{T}{2} - X = \frac{2.125}{2} - 0.8125 = 0.25\ in$$

For internal pressure (Tubeside Conditions),

$$H_r = \left(0.785B^2 P\cot\left[\beta_1\right]\right)$$

$$H_r = \left[(0.785)(16.5)^2(213)\cot\left[30.1259\right]\right] = 78447.1\ lbs$$

$$M_r = H_r h_r = 78447.1(0.25) = 19611.8\ in-lbs$$

For external pressure (Shellside Conditions),

$$H_r = \left(0.785 B^2 P \cot[\beta_1]\right)$$

$$H_r = \left[(0.785)(16.5)^2(305)\cot[30.1259]\right] = 112330.3\ lbs$$

$$M_r = H_r h_r = 112330.3(0.25) = 28082.6\ in-lbs$$

b) STEP 2 – Calculate the modified flange moment for the design condition, M_o, using paragraph 2-6 including the additional moment, M_r.

For internal pressure (Tubeside Conditions),

$$M_{o(tubeside)} = M_o - M_r = 50450.4 - 19611.8 = 30838.6\ in-lbs$$

For external pressure (Shellside Conditions),

$$M_{o(shellside)} = M_o - M_r = 4107.1 - 28082.6 = -23975.5\ in-lbs$$

c) STEP 3 – Calculate the flange thickness for the gasket seating condition, T_g.

$$T_g = F + \sqrt{F^2 + J} + CAS + CAS$$

$$T_g = 0.0 + \sqrt{(0.0)^2 + 4.0154} + 0.125 + 0.125 = 2.2539\ in$$

Where,

$$F = \frac{PB\sqrt{4L^2 - B^2}}{8S(A-B)} = \frac{0.0(16.5)\sqrt{4(16.125)^2 - (16.5)^2}}{8(20000)(19.375-16.5)} = 0.0$$

$$J = \left(\frac{M_o}{SB}\right)\left(\frac{A+B}{A-B}\right) = \left(\frac{106192.5}{(20000)(16.5)}\right)\cdot\left(\frac{(19.375+16.5)}{(19.375-16.5)}\right) = 4.0154$$

d) STEP 4 – Calculate the flange thickness for the operating conditions, $T_{o(tubeside)}$ and $T_{o(shellside)}$.

For internal pressure (Tubeside Conditions),

$$T_o = F + \sqrt{F^2 + J} + CAS + CAS$$

$$T_{o(tubeside)} = 0.2117 + \sqrt{(0.2117)^2 + 1.1661} + 0.125 + 0.125 = 1.5621\ in$$

Where,

$$F = \frac{PB\sqrt{4L^2 - B^2}}{8S(A-B)} = \frac{213(16.5)\sqrt{4(16.125)^2 - (16.5)^2}}{8(20000)(19.375-16.5)} = 0.2117$$

$$J = \left(\frac{M_o}{SB}\right)\left(\frac{A+B}{A-B}\right) = \left(\frac{30838.6}{(20000)(16.5)}\right)\cdot\left(\frac{(19.375+16.5)}{(19.375-16.5)}\right) = 1.1661$$

For external pressure (Shellside Conditions),

$$T_o = F + \sqrt{F^2 + J} + CAS + CAS$$

$$T_{o(shellside)} = 0.3031 + \sqrt{(0.3031)^2 + 0.9066} + 0.125 + 0.125 = 1.5517\ in$$

Where,

$$F = \frac{PB\sqrt{4L^2 - B^2}}{8S(A-B)} = \frac{|305|(16.5)\sqrt{4(16.125)^2 - (16.5)^2}}{8(20000)(19.375-16.5)} = 0.3031$$

$$J = \left(\frac{M_o}{SB}\right)\left(\frac{A+B}{A-B}\right) = \left(\frac{|-23975.5|}{(20000)(16.5)}\right)\cdot\left(\frac{(19.375+16.5)}{(19.375-16.5)}\right) = 0.9066$$

e) STEP 5 – Determine the required flange thickness using the thicknesses determined in STEP 3 and STEP 4.

$$T = \max\left[T_g,\ T_o\right] = \max\left[T_g,\ \max\left[T_{o(tubeside)},\ T_{o(shellside)}\right]\right]$$

$$T = \max\left[2.2539,\ \max\left[1.5621,\ 1.5517\right]\right] = 2.2539\ in$$

Since the specified head thickness, $\{t = 0.875\ in\} > \{t_{req} = 0.4174\ in\}$ and the specified flange thickness, $\{T = 2.125\ in\} > \{T_{req} = 2.2539\ in\}$ for both internal pressure (tubeside conditions) and external pressure (shellside conditions), the proposed Type D spherically dished bolted cover is adequately designed.

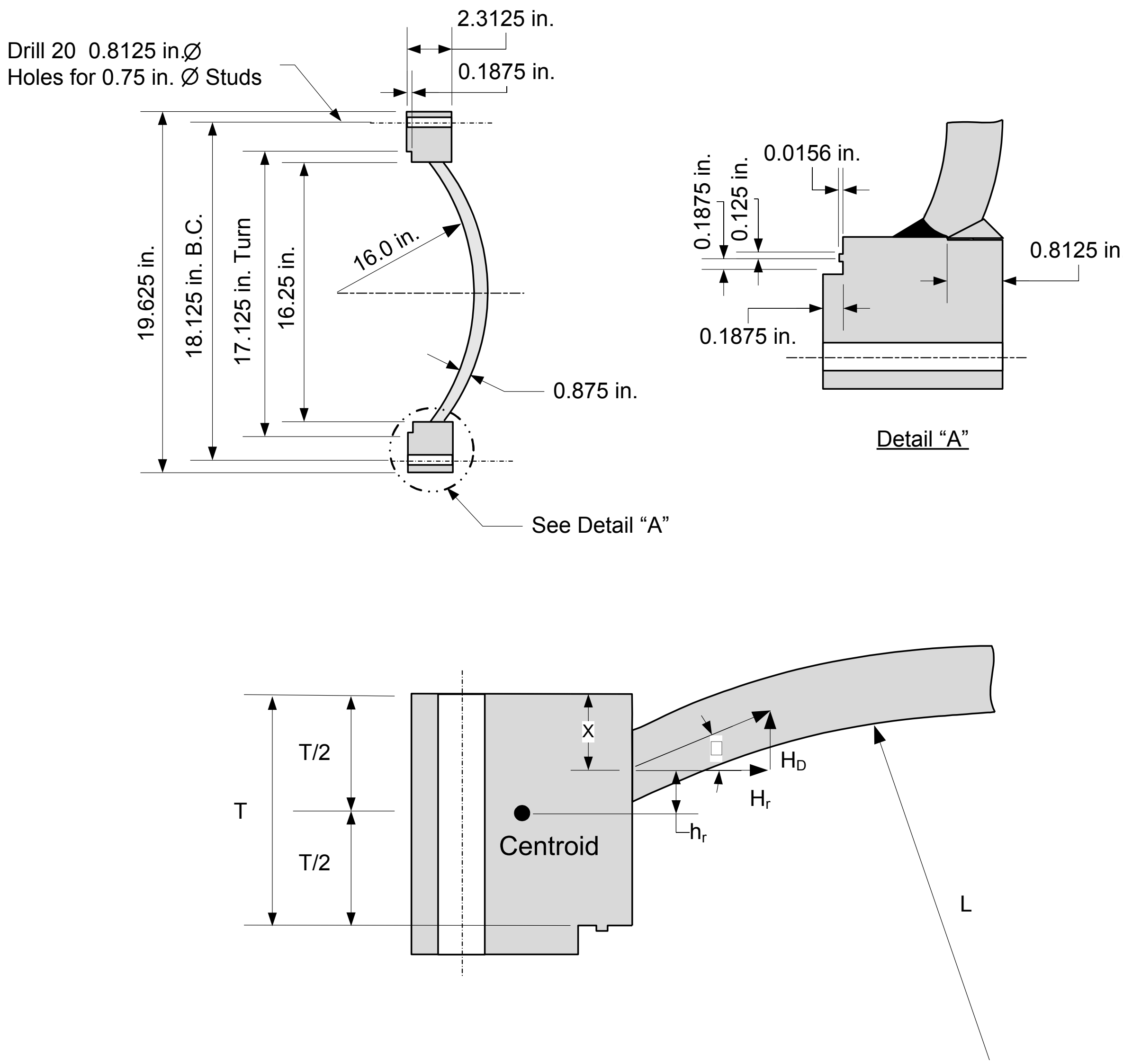


Figure E4.7.1 – Floating Head Geometry

4.7.2 Example E4.7.2 – Thickness Calculation for a Type D Head Using the Alternative Rule in VIII-2, Paragraph 4.7.5.3

Mandatory Appendix 1-6(h) indicates that the equations for the bolted heads with a dished cover are approximate in that they do not take into account continuity between the flange ring and the dished head. A more exact method of analysis which takes the continuity of the flange and head into account may be used if it meets the requirements of U-2(g). The alternate design method provided in VIII-2, paragraph 4.7.5.3 satisfies this requirement.

Determine if the proposed Type D spherically dished bolted cover is adequately designed, considering the following design conditions. The spherically dished head is seamless. Evaluate using the alternative procedure in VIII-2, paragraph 4.7.5.3.

Tubeside Data:

- Design Conditions = $213\ psig@400°F$
- Corrosion Allowance (CAT) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Shellside Data:

- Design Conditions = $305\ psig@250°F$
- Corrosion Allowance (CAS) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Flange Data:

- Material = $SA-105$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Allowable Stress at Tubeside Design Temperature = $20000\ psi$
- Allowable Stress at Shellside Design Temperature = $20000\ psi$

Head Data:

- Material = $SA-515,\ Grade\ 60$
- Allowable Stress At Ambient Temperature = $17100\ psi$
- Allowable Stress at Tubeside Design Temperature = $17100\ psi$
- Allowable Stress at Shellside Design Temperature = $17100\ psi$
- Yield Stress at Shellside Design Temperature = $28800\ psi$
- Modulus of Elasticity at Shellside Design Temp. = $28.55E+06\ psi$

Bolt Data

- Material = $SA-193,\ Grade\ B7$
- Diameter = $0.75\ in$
- Cross-Sectional Root Area = $0.302\ in^2$
- Number of Bolts = 20
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Allowable Stress at Tubeside Design Temperature = $25000\ psi$

- Allowable Stress at Shellside Design Temperature = $25000\ psi$

Gasket Data

- Material = Solid Flat Metal (Iron/Soft Steel)
- Gasket Factor = 5.5
- Gasket Seating Factor = $18000\ psi$
- Inside Diameter = $16.1875\ in$
- Outside Diameter = $17.0625\ in$

Per VIII-2, paragraph 4.7.5.3, the following procedure can be used to determine the required head and flange thickness of a Type D head. This procedure accounts for the continuity between the flange ring and the head, and represents a more accurate method of analysis.

a) STEP 1 – Determine the design pressure and temperature of the flange joint. When evaluating external pressure, a negative value of the pressure is used in all equations of this procedure.

$$Tubeside\ Conditions: P = 213\ psig\ at\ 400°F$$

$$Shellside\ Conditions: P = 305\ psig\ at\ 250°F$$

b) STEP 2 – Determine an initial Type D head configuration geometry (see Figure E4.7.1). The following geometry parameters are required.

1) Flange bore

$$B = \left[16.25 + 2\left(CAT\right)\right] = \left[16.25 + 2\left(0.125\right)\right] = 16.50\ in$$

2) Bolt circle diameter

$$C = 18.125\ in$$

3) Outside diameter of the flange

$$A = \left[19.625 - 2\left(CAS\right)\right] = \left[19.625 - 2\left(0.125\right)\right] = 19.375\ in$$

4) Flange thickness, (see VIII-2, Figure E4.7.1)

$$T = T - Flange\ Extension - CAT - CAS$$

$$T = 2.3125 - 0.1875 - 0.125 - 0.125 = 1.875\ in$$

5) Mean head radius, (see VIII-2, Figure 4.7.5)

$$R = \frac{\left(L + t_{uncorroded} - CAS\right) + \left(L + CAT\right)}{2}$$

$$R = \frac{\left(16.0 + 0.875 - 0.125\right) + \left(16.0 + 0.125\right)}{2} = 16.4375\ in$$

6) Head thickness

$$t = t - CAT - CAS = 0.875 - 0.125 - 0.125 = 0.625\ in$$

7) Inside depth of flange to the base of the head, (see Figure 4.7.5)

$$q = q - CAS = 0.9512 - 0.125 = 0.8262\ in$$

c) STEP 3 – Select a gasket configuration and determine the location of the gasket reaction, G, and the design bolt loads for the gasket seating, W_g, and operating conditions, W_o, using the rules of VIII-2, paragraph 4.16. Computations for the following parameters are shown in Example Problem 4.7.1.

$$G = 16.625\ in$$

$$W_g = 141590.0\ lbs$$

$$W_o = 63442.9\ lbs$$

d) STEP 4 – Determine the geometry parameters

$$h_1 = \frac{(C-G)}{2} = \frac{(18.125-16.625)}{2} = 0.75\ in$$

$$h_2 = \frac{(G-B)}{2} = \frac{(16.625-16.5)}{2} = 0.0625\ in$$

$$d = \frac{(A-B)}{2} = \frac{(19.375-16.5)}{2} = 1.4375\ in$$

$$n = \frac{T}{t} = \frac{1.875}{0.625} = 3.0$$

$$K = \frac{A}{B} = \frac{19.375}{16.5} = 1.1742$$

$$\phi = \arcsin\left[\frac{B}{2R}\right] = \arcsin\left[\frac{16.5}{2(16.4375)}\right] = 30.1259\ deg$$

$$e = q - \frac{1}{2}\left[T - \frac{t}{\cos[\phi]}\right] = 0.8262 - \frac{1}{2}\left[1.875 - \frac{0.625}{\cos[30.1259]}\right] = 0.25\ in$$

$$k_1 = 1 - \left(\frac{1-2v}{2\lambda}\right)\cot[\phi] = 1 - \left[\frac{1-2(0.3)}{2(6.5920)}\right]\cot[30.1259] = 0.9477$$

$$k_2 = 1 - \left(\frac{1+2v}{2\lambda}\right)\cot[\phi] = 1 - \left[\frac{1+2(0.3)}{2(6.5920)}\right]\cot[30.1259] = 0.7907$$

Where,

$$\lambda = \left[3\left(1-v^2\right)\left(\frac{R}{t}\right)^2\right]^{0.25} = \left\{3\left(1-0.3^2\right)\left(\frac{16.4375}{0.625}\right)^2\right\}^{0.25} = 6.5920$$

e) STEP 5 – Determine the shell discontinuity geometry factors

$$C_1 = \frac{0.275n^3t \cdot \ln[K]}{k_1} - e = \left(\frac{0.275(3.0)^3(0.625)\cdot\ln[1.1742]}{0.9477}\right) - (0.25) = 0.5364$$

$$C_2 = \frac{1.1\lambda n^3 t \ln[K]}{Bk_1} + 1 = \left(\frac{1.1(6.5920)(3.0)^3(0.625)\cdot\ln[1.1742]}{(16.5)(0.9477)}\right) + 1 = 2.2566$$

$$C_4 = \frac{\lambda\sin[\phi]}{2}\left(k_2 + \frac{1}{k_1}\right) + \frac{B}{4nd} + \frac{1.65e}{tk_1}$$

$$C_4 = \left[\begin{array}{l}\dfrac{(6.5920)\sin[30.1259]}{2}\left(0.7907 + \dfrac{1}{0.9477}\right) + \\ \dfrac{16.5}{4(3.0)(1.4375)} + \dfrac{1.65(0.25)}{(0.625)(0.9477)}\end{array}\right] = 4.7065$$

$$C_5 = \frac{1.65}{tk_1}\left(1 + \frac{4\lambda e}{B}\right) = \left(\frac{1.65}{(0.625)(0.9477)}\right)\left(1 + \frac{4(6.5920)(0.25)}{(16.5)}\right) = 3.8986$$

f) STEP 6 – Determine the shell discontinuity load factors for the operating and gasket seating conditions.

<u>Operating Condition – Tubeside:</u>

$$C_{3o} = \frac{\pi B^2 P}{4}\left[e\cot[\phi] + \frac{2q(T-q)}{B} - h_2\right] - W_o h_1$$

$$C_{3o} = \left(\frac{\pi(16.5)^2(213)}{4}\right)\left[\begin{array}{l}(0.25)\cot[30.1259] + \\ \dfrac{2(0.8262)(1.875-0.8262)}{(16.5)} - 0.0625\end{array}\right] - 63442.9(0.75)$$

$$C_{3o} = -26023.3317\ in-lbs$$

$$C_{6o} = \frac{\pi B^2 P}{4}\left(\frac{4q - B\cot[\phi]}{4nd} - \frac{0.35}{\sin[\phi]}\right)$$

$$C_{6o} = \frac{\pi(16.5)^2(213)}{4}\left(\frac{4(0.8262)-(16.5)\cot[30.1259]}{4(3.0)(1.4375)} - \frac{0.35}{\sin[30.1259]}\right)$$

$$C_{6o} = -98109.2705\ lbs$$

<u>Operating Condition – Shellside:</u>

$$C_{3o} = \frac{\pi B^2 P}{4}\left[e\cot[\phi] + \frac{2q(T-q)}{B} - h_2\right] - W_o h_1$$

$$C_{3o} = \left(\frac{\pi(16.5)^2(-305)}{4}\right)\left[\begin{array}{l}(0.25)\cot[30.1259]+ \\ \dfrac{2(0.8262)(1.875-0.8262)}{(16.5)}-0.0625\end{array}\right]-63442.9(0.75)$$

$$C_{3o} = -78452.8191\ in-lbs$$

$$C_{6o} = \frac{\pi B^2 P}{4}\left(\frac{4q - B\cot[\phi]}{4nd} - \frac{0.35}{\sin[\phi]}\right)$$

$$C_{6o} = \frac{\pi(16.5)^2(-305)}{4}\left(\frac{4(0.8262)-(16.5)\cot[30.1259]}{4(3.0)(1.4375)} - \frac{0.35}{\sin[30.1259]}\right)$$

$$C_{6o} = 140485.1056\ lbs$$

Gasket Seating Condition:

$$C_{3g} = -W_g h_1 = -(141590.0)(0.75) = -106192.5\ in-lbs$$

$$C_{6g} = 0.0$$

g) STEP 7 – Determine the shell discontinuity force and moment for the operating and gasket condition.

Operating Condition – Tubeside:

$$V_{do} = \frac{C_2 C_{6o} - C_{3o} C_5}{C_2 C_4 - C_1 C_5}$$

$$V_{do} = \frac{(2.2566(-98109.2705)) - (-26023.3317(3.8986))}{(2.2566(4.7065)) - (0.5364(3.8986))} = -14061.7\ lbs$$

$$M_{do} = \frac{C_1 C_{6o} - C_{3o} C_4}{C_2 C_4 - C_1 C_5}$$

$$M_{do} = \frac{(0.5364(-98109.2705)) - (-26023.3317(4.7065))}{(2.2566(4.7065)) - (0.5364(3.8986))} = 8189.6\ in-lbs$$

Operating Condition – Shellside:

$$V_{do} = \frac{C_2 C_{6o} - C_{3o} C_5}{C_2 C_4 - C_1 C_5}$$

$$V_{do} = \frac{(2.2566(-140485.1056)) - (-78452.8191(3.8986))}{(2.2566(4.7065)) - (0.5364(3.8986))} = 73026.1\ lbs$$

$$M_{do} = \frac{C_1 C_{6o} - C_{3o} C_4}{C_2 C_4 - C_1 C_5}$$

$$M_{do} = \frac{\left(0.5364\left(140485.1056\right)\right)-\left(-78452.8191\left(4.7065\right)\right)}{\left(2.2566\left(4.7065\right)\right)-\left(0.5364\left(3.8986\right)\right)} = 52124.5\ in-lbs$$

Gasket Seating Condition:

$$V_{dg} = \frac{C_2C_{6g}-C_{3g}C_5}{C_2C_4-C_1C_5} = \frac{\left(2.2566\left(0.0\right)\right)-\left(-106192.5\left(3.8986\right)\right)}{\left(2.2566\left(4.7065\right)\right)-\left(0.5364\left(3.8986\right)\right)} = 48537.8\ lbs$$

$$M_{dg} = \frac{C_1C_{6g}-C_{3g}C_4}{C_2C_4-C_1C_5} = \frac{\left(0.5364\left(0.0\right)\right)-\left(-106192.5\left(4.7065\right)\right)}{\left(2.2566\left(4.7065\right)\right)-\left(0.5364\left(3.8986\right)\right)} = 58596.2\ in-lbs$$

h) STEP 8 – Calculate the stresses in the head and at the head to flange junction using VIII-2, Table 4.7.1 and check the stress criteria for both the operating and gasket conditions.

Calculated Stresses – Operating Conditions – Tubeside:

$$S_{hm} = \frac{PR}{2t}+P_e = \frac{213\left(16.4375\right)}{2\left(0.625\right)}+0.0 = 2801.0\ psi$$

$$S_{hl} = \frac{PR}{2t}+\frac{V_{do}\cos\left[\phi\right]}{\pi Bt}+P_e$$

$$S_{hl} = \frac{213\left(16.4375\right)}{2\left(0.625\right)}+\frac{\left(-14061.7\right)\cos\left[30.1259\right]}{\pi\left(16.5\right)\left(0.625\right)}+0.0 = 2425.5\ psi$$

$$S_{hb} = \frac{6M_{do}}{\pi Bt^2} = \frac{6\left(8189.6\right)}{\pi\left(16.5\right)\left(0.625\right)^2} = 2426.7\ psi$$

$$S_{hlbi} = S_{hl}-S_{hb} = 2425.5-2426.7 = -1.2\ psi$$

$$S_{hlbo} = S_{hl}+S_{hb} = 2425.5+2426.7 = 4852.2\ psi$$

$$S_{fm} = \frac{1}{\pi BT}\left(\frac{\pi B^2P}{4}\left(\frac{4q}{B}-\cot\left[\phi\right]\right)-V_{do}\right)\left(\frac{K^2+1}{K^2-1}\right)+P_e$$

$$S_{fm} = \left(\frac{1}{\pi\left(16.5\right)\left(1.875\right)}\right)\left(\begin{array}{l}\left(\dfrac{\pi\left(16.5\right)^2\left(213\right)}{4}\right)\\ \left(\dfrac{4\left(0.8262\right)}{\left(16.5\right)}-\cot\left[30.1259\right]\right)-\\ \left(-14061.7\right)\end{array}\right)\left(\frac{\left(1.1742\right)^2+1}{\left(1.1742\right)^2-1}\right)+0.0$$

$$S_{fm} = -3573.7\ psi$$

$$S_{fb} = \frac{0.525n}{Btk_1}\left(V_{do} - \frac{4M_{do}\lambda}{B}\right)$$

$$S_{fb} = \frac{0.525(3.0)}{(16.5)(0.625)(0.9477)}\left((-14061.7) - \frac{4(8189.6)(6.5920)}{(16.5)}\right) = -4375.2 \ psi$$

$$S_{fmbo} = S_{fm} - S_{fb} = -3573.7 - (-4375.2) = 801.5 \ psi$$

$$S_{fmbi} = S_{fm} + S_{fb} = -3573.7 + (-4375.2) = -7948.9 \ psi$$

Acceptance Criteria – Operating Conditions – Tubeside:

$$\{S_{hm} = 2801.0 \ psi\} \le \{S_{ho} = 17100 \ psi\} \qquad True$$

$$\{S_{hl} = 2425.5 \ psi\} \le \{1.5S_{ho} = 1.5(17100) = 25650 \ psi\} \qquad True$$

$$\{S_{hlbi} = -1.2 \ psi\} \le \{1.5S_{ho} = 1.5(17100) = 25650 \ psi\} \qquad True$$

$$\{S_{hlbo} = 4852.2 \ psi\} \le \{1.5S_{ho} = 1.5(17100) = 25650 \ psi\} \qquad True$$

$$\{S_{fm} = -3573.7 \ psi\} \le \{S_{fo} = 20000 \ psi\} \qquad True$$

$$\{S_{fmbo} = 801.5 \ psi\} \le \{1.5S_{fo} = 1.5(20000) = 30000 \ psi\} \qquad True$$

$$\{S_{fmbi} = -7948.9 \ psi\} \le \{1.5S_{fo} = 1.5(20000) = 30000 \ psi\} \qquad True$$

Calculated Stresses – Operating Conditions – Shellside:

$$S_{hm} = \frac{PR}{2t} + P_e = \frac{(-305)(16.4375)}{2(0.625)} + (-305) = -4315.8 \ psi$$

$$S_{hl} = \frac{PR}{2t} + \frac{V_{do}\cos[\phi]}{\pi Bt} + P_e$$

$$S_{hl} = \frac{(-305)(16.4375)}{2(0.625)} + \frac{(73026.1)\cos[30.1259]}{\pi(16.5)(0.625)} + (-305) = -2366.2 \ psi$$

$$S_{hb} = \frac{6M_{do}}{\pi Bt^2} = \frac{6(52124.5)}{\pi(16.5)(0.625)^2} = 15445.4 \ psi$$

$$S_{hlbi} = S_{hl} - S_{hb} = -2366.2 - 15445.4 = -17811.6 \ psi$$

$$S_{hlbi} = S_{hl} + S_{hb} = -2366.2 + 15445.4 = 13079.2 \ psi$$

$$S_{fm} = \frac{1}{\pi BT}\left(\frac{\pi B^2 P}{4}\left(\frac{4q}{B} - \cot[\phi]\right) - V_{do}\right)\left(\frac{K^2+1}{K^2-1}\right) + P_e$$

$$S_{fm} = \left(\frac{1}{\pi(16.5)(1.875)}\right)\left(\begin{array}{l}\left(\dfrac{\pi(16.5)^2(-305)}{4}\right) \\ \left(\dfrac{4(0.8262)}{(16.5)} - \cot[30.1259]\right) - \\ (73026.1)\end{array}\right)\left(\frac{(1.1742)^2+1}{(1.1742)^2-1}\right) + (-305)$$

$$S_{fm} = 1394.4 \ psi$$

$$S_{fb} = \frac{0.525n}{Btk_1}\left(V_{do} - \frac{4M_{do}\lambda}{B}\right)$$

$$S_{fb} = \frac{0.525(3.0)}{(16.5)(0.625)(0.9477)}\left(73026.1 - \frac{4(52124.5)(6.5920)}{(16.5)}\right) = -1655.4 \ psi$$

$$S_{fmbo} = S_{fm} - S_{fb} = 1394.4 - (-1655.4) = 3049.8 \ psi$$

$$S_{fmbo} = S_{fm} + S_{fb} = 1394.4 + (-1655.4) = -261.0 \ psi$$

Acceptance Criteria – Operating Conditions – Shellside:

$\{S_{hm} = -4315.8 \ psi\} \le \{S_{ho} = 17100 \ psi\}$ *True*

$\{S_{hl} = -2366.2 \ psi\} \le \{1.5S_{ho} = 1.5(17100) = 25650 \ psi\}$ *True*

$\{S_{hlbi} = -17811.6 \ psi\} \le \{1.5S_{ho} = 1.5(17100) = 25650 \ psi\}$ *True*

$\{S_{hlbo} = 13079.2 \ psi\} \le \{1.5S_{ho} = 1.5(17100) = 25650 \ psi\}$ *True*

$\{S_{fm} = 1394.4 \ psi\} \le \{S_{fo} = 20000 \ psi\}$ *True*

$\{S_{fmbo} = 3049.8 \ psi\} \le \{1.5S_{fo} = 1.5(20000) = 30000 \ psi\}$ *True*

$\{S_{fmbi} = -261.0 \ psi\} \le \{1.5S_{fo} = 1.5(20000) = 30000 \ psi\}$ *True*

Calculated Stresses – Gasket Seating Conditions:

$$S_{hm} = 0.0$$

$$S_{hl} = \frac{V_{dg}\cos[\phi]}{\pi Bt} = \frac{(48537.8)\cos[30.1259]}{\pi(16.5)(0.625)} = 1295.8 \ psi$$

$$S_{hb}=\frac{6M_{dg}}{\pi Bt^2}=\frac{6(58596.2)}{\pi(16.5)(0.625)^2}=17363.1\ psi$$

$$S_{hlbi}=S_{hl}-S_{hb}=1295.8-17363.1=-16067.3\ psi$$

$$S_{hlbo}=S_{hl}+S_{hb}=1295.8+17363.1=18658.9\ psi$$

$$S_{fm}=\frac{1}{\pi BT}\left(-V_{dg}\right)\left(\frac{K^2+1}{K^2-1}\right)$$

$$S_{fm}=\left(\frac{1}{\pi(16.5)(1.875)}\right)\left(-(48537.8)\right)\left(\frac{(1.1742)^2+1}{(1.1742)^2-1}\right)=-3136.5\ psi$$

$$S_{fb}=\frac{0.525n}{Btk_1}\left(V_{dg}-\frac{4M_{dg}\lambda}{B}\right)$$

$$S_{fb}=\frac{0.525(3.0)}{(16.5)(0.625)(0.9477)}\left(48537.8-\frac{4(58596.2)(6.5920)}{(16.5)}\right)=-7268.5\ psi$$

$$S_{fmbo}=S_{fm}-S_{fb}=-3136.5-(-7268.5)=4132.0\ psi$$

$$S_{fmbi}=S_{fm}+S_{fb}=-3136.5+(-7268.5)=-10405.0\ psi$$

<u>Acceptance Criteria – Gasket Seating Conditions:</u>

$$\left\{S_{hm}=0.0\ psi\right\}\le\left\{S_{hg}=17100\ psi\right\}\qquad True$$

$$\left\{S_{hl}=1295.8\ psi\right\}\le\left\{1.5S_{hg}=1.5(17100)=25650\ psi\right\}\qquad True$$

$$\left\{S_{hlbi}=-16067.3\ psi\right\}\le\left\{1.5S_{hg}=1.5(17100)=25650\ psi\right\}\qquad True$$

$$\left\{S_{hlbo}=18658.9\ psi\right\}\le\left\{1.5S_{hg}=1.5(17100)=25650\ psi\right\}\qquad True$$

$$\left\{S_{fm}=-3136.5\ psi\right\}\le\left\{S_{fg}=20000\ psi\right\}\qquad True$$

$$\left\{S_{fmbo}=4132.0\ psi\right\}\le\left\{1.5S_{fg}=1.5(20000)=30000\ psi\right\}\qquad True$$

$$\left\{S_{fmbi}=-10405.0\ psi\right\}\le\left\{1.5S_{fg}=1.5(20000)=30000\ psi\right\}\qquad True$$

Since the calculated stresses in both the head and flange ring are shown to be within the acceptance criteria, for both internal pressure (tubeside conditions) and external pressure (shellside conditions), the proposed Type D spherically dished bolted cover is adequately designed.

4.8 Quick-Actuating (Quick Opening) Closures

4.8.1 Example E4.8.1 – Review of Requirements for Quick-Actuating Closures

An engineer is tasked with developing a design specification for an air filter vessel to be equipped with a quick-actuating closure that is to be constructed in accordance with VIII-1, paragraph UG-35.2.

Design rules for quick–actuating (quick opening) closures are provided in UG-35.2. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.8.

As part of developing the design specification, the following items need to be considered.

a) Scope

Specific calculation methods are not given in paragraph UG-35.2. However, both general and specific design requirements are provided.

b) General Design Requirements
Quick-actuating closures shall be designed such that:

1) The locking elements will be engaged prior to or upon application of the pressure and will not disengage when operated as intended until the pressure is released.
2) The failure of a single locking component while the vessel is pressurized will not:
   i) Cause or allow the closure to be opened or leaked; or
   ii) Result in the failure of any other locking component or holding element; or
   iii) Increase the stress in any other locking or holding element by more than 50% above the allowable stress of the component.
3) All locking components can be verified to be fully engaged by visual observation or other means prior to application if pressure to the vessel.
4) When installed:
   i) It may be determined by visual external observation that the holding elements are in satisfactory condition.
   ii) All vessels shall be provided with a pressure-indicating device visible from the operating area and suitable to detect pressure at the closure.

c) Specific Design Requirements

Quick-actuating closures that are held in position by positive locking devices and that are fully released by partial rotation or limited movement of the closure itself or the locking mechanism and any closure that is other than manually operates shall be so designed that when the vessel is installed the following conditions are met:

1) The closure and its holding elements are fully engaged in their intended operating position before pressure can be applied in the vessel.
2) Pressure tending to force the closure open or discharge the contents clear of the vessel shall be released before the closure can be fully opened for access.

The designer shall consider the effects of cyclic loading, other loadings, and mechanical wear on the holding and locking components.

d) Alternative Designs for Manually Operated Closures

Quick-actuating closures that are held in position by a locking mechanism designed for manual operation shall be designed such that if an attempt is made to open the closure when the vessel is under pressure, the closure will leak prior to full disengagement of the locking components and release of the closure. Any leakage shall be directed away from the normal position of the operator.

e) Supplementary Requirements

Additional design information for the Manufacturer and guidance on installation is provided in VIII-1, Nonmandatory Appendix FF.

4.9 Braced and Stayed Surfaces

4.9.1 Example E4.9.1 - Braced and Stayed Surfaces

Determine the required thickness for a flat plate with welded staybolts considering the following design condition. Verify that the welded staybolts are adequately designed. See Figure E4.9.1

Vessel Data:

- Plate Material = $SA-516,\ Grade\ 70$
- Design Conditions = $100\ psig@300°F$
- Staybolt Material = $SA-675,\ Grade\ 70$
- Staybolt Diameter = $1.5\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress Plate Material = $20000\ psi@300°F$
- Allowable Stress Staybolt Material = $20000\ psi@300°F$
- Staybolt Pattern = $Equilateral\ Triangle$
- Staybolt Pitch = $p_s = p_{horizontal} = p_{diagonal} = 15.0\ in$

Design rules for braced and stayed surfaces are provided in UG-47, UG-48, UG-49, and UG-50. The rules in these paragraphs are the same as those provided in VIII-2, paragraph 4.9 with the exception that VIII-2 only includes rules for welded stays. UW-19 also provides requirements for welded-in stays.

a) STEP 1 – Evaluate per UG-47. Calculate the required thickness of the flat plate, the load carried by each staybolt, and the required diameter of the staybolt.

The minimum required thickness for braced and stayed flat plates and those parts that, by these rules, require staying as flat plates with braces or staybolts of uniform diameter symmetrically spaced, shall be calculated by the following equation.

Assume, $C = 2.2$ from UG-47 with the Welded Staybolt Construction per Figure UW-19.1 Sketch (e).

$$t = p_s\sqrt{\frac{P}{SC}} = 15.0\sqrt{\frac{100.0}{20000(2.2)}} = 0.7151\ in$$

b) STEP 2 – Evaluate per UG-50. UG-50(a) – The required area of a staybolt or stay at its minimum cross section, usually located at the root of the thread, exclusive of any corrosion allowance, is obtained by dividing the load on the staybolt computed in accordance with UG-50(b) by the allowable stress value for the staybolt material, and multiplying the result by 1.10.

UG-50(b) – The area supported by a staybolt or stay shall be computed on the basis of the full pitch dimensions, with a deduction for the area occupied by the stay. The load carried by a stay is the product of the area supported by the stay and the maximum allowable working pressure.

UG-50(c) – Stays made of parts joined by welding shall be checked for strength using a joint efficiency of 60% for the weld.

1) The area of the flat plate supported by the staybolt, A_p, is calculated as follows.

$$A_p = \left(p_{horizontal}\cdot p_{diagonal}\cdot\cos[\theta]\right) - A_{sb} = 15.0\left(15.0\cdot\cos[30]\right) - 1.7671 = 193.0886\ in^2$$

Where,

$$\theta = 30\ deg, \qquad See\ Figure\ E4.9.1$$

$$A_{sb} = \frac{\pi}{4}(1.5)^2 = 1.7671\ in^2$$

2) The load carried by the staybolt, L_{sb}, is calculated as follows.

$$L_{sb} = A_p \cdot P = 193.0886(100.0) = 19308.9\ lbs$$

3) The required area of the staybolt, A_{rsb}, is calculated as follows.

$$A_{rsb} = 1.10\left(\frac{L_{sb}}{S_{sb}}\right) = 1.10\left(\frac{19308.9}{20000}\right) = 1.0620\ in^2$$

Since $\left\{A_{sb} = 1.7671\ in^2\right\} > \left\{A_{rsb} = 1.0620\ in^2\right\}$, the staybolt is adequately designed.

4) If the stays are made of parts by welding, the allowable load on the welds shall not exceed the product of the weld area (based on the weld dimension parallel to the staybolt), the allowable stress of the material being welded, and a weld joint factor of 60%.

$$L_{sb} \le L_a$$

Where,

$$L_a = E\left(t \cdot \pi d_{sb}\right)S_{sb} = 0.6\left(0.7151(\pi)(1.5)\right)20000 = 40438.0\ lbs$$

Since $\left\{L_{sb} = 19308.9\ lbs\right\} \le \left\{L_a = 40438.0\ lbs\right\}$, the staybolt is adequately designed.

c) STEP 3 – Evaluate per UW-19(a)(1). Welded-in staybolts shall meet the following requirements:

1) The configuration is in accordance with the typical arrangements shown in Figure UW-19.1.

$$Construction\ per\ Figure\ UW-19.1(e) \qquad Satisfied$$

2) The required thickness of the plate shall not exceed 1.5 in (38 mm), but if greater than 0.75 in (19 mm), the staybolt pitch shall not exceed 20 in (500 mm).

$$\left\{t = 0.7151\ in\right\} \le \left\{1.5\ in\right\} \qquad Satisfied$$

3) The provisions of UG-47 and UG-49 shall be followed.

$$Satisfied$$

4) The required area of the staybolt shall be determined in accordance with the requirements in UG-50.

$$Satisfied$$

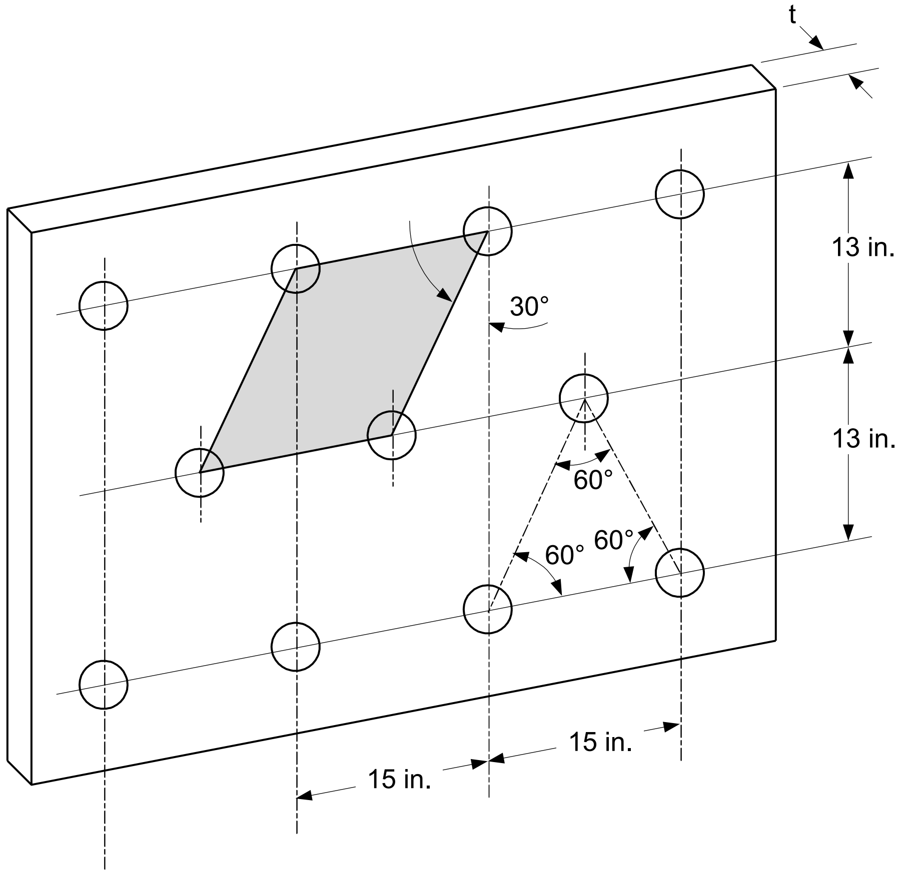


Figure E4.9.1 – Stayed Plate Detail

4.10 Ligaments

4.10.1 Example E4.10.1 - Ligaments

Determine the ligament efficiency and corresponding efficiency to be used in the design equations of UG-27 for a group of tube holes in a cylindrical shell as shown in Figure E4.10.1.

Design rules for ligaments are provided in UG-53. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.10.

Evaluate the tube hole installation pattern in accordance with UG-53.

Commentary: As shown in Figure E4.10.1, three ligaments are produced; longitudinal, circumferential, and diagonal. UG-53(c) states that in addition to the longitudinal ligament, diagonal and circumferential ligaments shall also be examined with the least equivalent longitudinal ligament efficiency used to determine the minimum required wall thickness and the maximum allowable working pressure. Considering only pressure loading, the circumferential ligament can be half as strong as the longitudinal ligament. This is because the circumferential ligament is subject to longitudinal stress which is essentially half of circumferential stress. By inspection, the circumferential ligament is greater than the longitudinal ligament and thus will the not govern the design. Therefore, the circumferential ligament efficiency is not explicitly calculated.

Paragraph UG-53(d) – when a cylindrical shell is drilled for holes so as to form diagonal ligaments, as shown in Fig. UG-53.4, the efficiency of these ligaments shall be determined by VIII-1, Figs. UG-53.5 and UG-53.6. Fig. UG-53.5 is used when either or both longitudinal and circumferential ligaments exist with diagonal ligaments. The procedure to determine the equivalent longitudinal ligament efficiency is described in UG-53(e).

a) STEP 1 – Compute the value of p'/p_1.

$$Diagonal\ Pitch,\ p' = 3.75\ in$$

$$Unit\ Length\ of\ Ligament,\ p_1 = 4.5\ in$$

$$\frac{p'}{p_1} = \frac{3.75}{4.5} = 0.8333$$

b) STEP 2 – Compute the efficiency of the longitudinal ligament in accordance with Fig. UG-53.5, Note 4.

$$E_{long} = 100\left(\frac{p_1 - d}{p_1}\right) = 100\left(\frac{4.5 - 2.25}{4.5}\right) = 50\%$$

Where,

$$Diameter\ of\ Tube\ Holes,\ d = 2.25\ in$$

c) STEP 3 – Compute the diagonal efficiency in accordance with Fig. UG-53.5, Note 2.

$$E_{diag} = \frac{J + 0.25 - \left(1 - 0.01 \cdot E_{long}\right)\sqrt{0.75 + J}}{0.00375 + 0.005J}$$

$$E_{diag} = \frac{0.6944 + 0.25 - \left(1 - 0.01(50)\right)\sqrt{(0.75 + 0.6944)}}{0.00375 + 0.005(0.6944)} = 47.56\%$$

Where,

$$J = \left(\frac{p'}{p_1}\right)^2 = \left(\frac{3.75}{4.5}\right)^2 = 0.6944$$

Alternatively, STEP 3 can be replaced with the following procedure.

STEP 3 (Alternate) – Enter Fig. UG-53.5 at the vertical line corresponding to the value of the longitudinal efficiency, E_{long}, and follow this line vertically to the point where it intersects the diagonal line representing the ratio of the value of p'/p_1. Then project this point horizontally to the left, and read the diagonal efficiency of the ligament on the scale at the edge of the diagram.

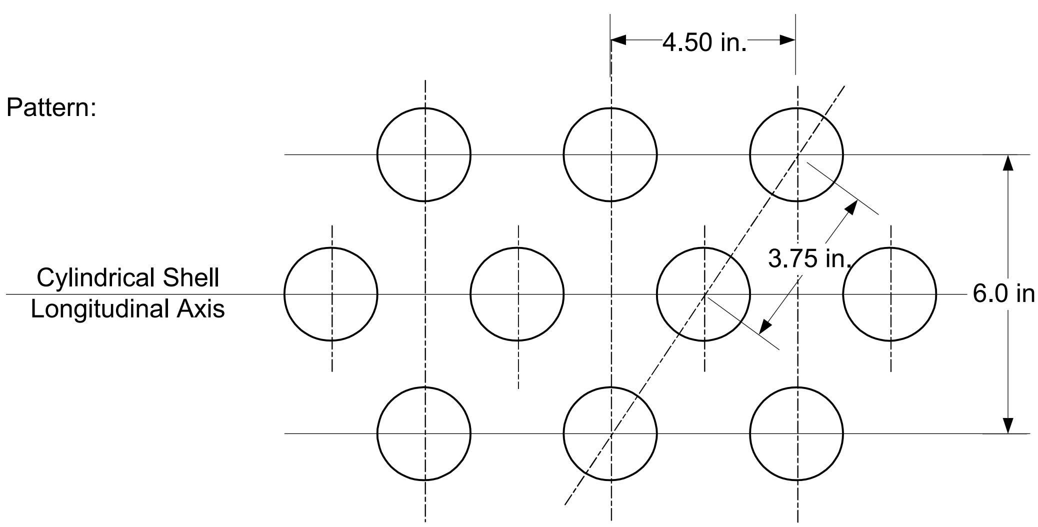
$$E_{diag} \approx 47.5\%$$

d) STEP 4 – The minimum shell thickness and the maximum allowable working pressure shall be based on the ligament that has the lower efficiency.

$$E = \min\left[E_{long}, E_{diag}\right] = \min\left[50\%, 47.5\%\right] = 47.5\%$$

In accordance with UG-53(i) when ligaments occur in cylindrical shells made from welded pipe or tubes and their calculated efficiency is less than 85% (longitudinal) or 50% (circumferential), the efficiency to be used in UG-27 to determine the minimum required thickness is the calculated ligament efficiency. In this case, the appropriate stress value in tension may be multiplied by the factor 1.18.

- Installation Pattern:



- All Finished Hole Diameters are 2.25 in.

Figure E4.10.1 – Installation Pattern

4.11 Jacketed Vessels

4.11.1 Example E4.11.1 - Partial Jacket

Design a partial jacket to be installed on the outside diameter of a section of a tower.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $350\ psig@300°F$
- Vessel ID = $90.0\ in$
- Nominal Thickness = $1.125\ in$
- Allowable Stress = $22400\ psi$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0

Jacket Data:

- Jacket Type per Figure 9-2 = $Type\ 2$
- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $150\ psig@400°F$
- Jacket ID = $96.0\ in$
- Allowable Stress = $22400\ psi$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0

Notes:

1) Jacket closure will be made using closure bars similar to details in Fig. 9-5 of Mandatory Appendix 9.
2) Full penetration welds will be used in the closure.

Establish the corroded dimensions.

$$R_j = R_j + Corrosion\ \ Allowance = 48.0 + 0.125 = 48.125\ in$$

$$ID\ of\ Jacket = 2(48.125) = 96.25\ in$$

$$OD\ of\ Inner\ \ Shell = 90 + 2(1.125 - 0.125) = 92.0\ in$$

$$t_s = t_s - 2(Corrosion\ \ Allowance) = 1.125 - 2(0.125) = 0.8750\ in$$

$$R_s = \frac{OD\ of\ Inner\ \ Shell}{2} = 46.0\ in$$

Design rules for jacketed vessels are provided in Mandatory Appendix 9. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.11.

Evaluate the partial jacket per Mandatory Appendix 9:

a) STEP 1– Determine required thickness of partial jacket per UG-27(c)(1).

$$t_{rj} = \frac{R_j P_j}{S_j E - 0.6P_j} = \frac{48.125(150)}{20000(1.0) - 0.6(150)} = 0.3626\ in$$

$$t_{rj} = t_{rj} + Corrosion\ \ Allowance = 0.3626 + 0.125 = 0.4876\ in$$

$$Select\ the\ next\ available\ plate\ thickness > 0.4876,\ \ use\ t_j = 0.5\ in$$

b) STEP 2– Determine maximum jacket space, j, to ensure that proposed jacket is acceptable.

$$j_{specified} = \frac{(ID\ \ of\ \ Jacket) - (OD\ \ of\ \ Inner\ \ Shell)}{2} = \frac{(96.25 - 92.0)}{2} = 2.125\ in$$

The maximum of j is determined from paragraph 9-5(c)(5).

$$j = \left(\frac{2S_c t_s^2}{P_j R_j}\right) - \left(\frac{(t_s + t_j)}{2}\right) = \left(\frac{2(20000)(0.875)^2}{150(48.125)}\right) - \left(\frac{0.875 + 0.5}{2}\right) = 3.5549\ in$$

$$\{j = 3.5549\ in\} \geq \{j_{specified} = 2.125\ in\}$$

The design is acceptable.

c) STEP 3 – Determine thickness of jacket closures. Use closure detail in Fig. 9-5, Sketch (f-2).

$$t_{rc} = 1.414\sqrt{\frac{P_j R_s j}{S_c}} = 1.414\sqrt{\frac{150(48.125)(2.125)}{20000}} = 1.2384\ in$$

$$t_{rc} = t_{rc} + Corrosion\ \ Allowance = 1.2384 + 0.125 = 1.3634\ in$$

Use end closure plates with a wall thickness of $1.375\ in$.

d) STEP 4 – Determine weld sizes for the closure to shell weld per Fig. 9-5, Sketch (f-2).

Jacket to closure weld:

- To be full penetration with backing strip.
- Fillet weld to be equal to t_j as a minimum.

Closure to shell weld (a full penetration weld is to be used)

$$t_c = t_{rc} - Corrosion\ \ Allowance = 1.375 - 0.125 = 1.25\ in$$

$$t_s = 0.875$$

$$Y = a + b \geq \min\left[1.5t_c,\ 1.5t_s\right] = \min\left[1.5(1.25),\ 1.5(0.875)\right] = 1.3125$$

$$Y = a + b \geq 1.3125\ in$$

$$Z = Y - \frac{t_s}{2} = 1.3125 - \frac{0.875}{2} = 0.8750\ in$$

4.11.2 Example E4.11.2 - Half-Pipe Jacket

Design a half-pipe jacket for a section of a tower using the information shown below.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $350\ psig@300°F$
- Vessel ID = $90.0\ in$
- Nominal Thickness = $1.125\ in$
- Allowable Stress = $20000\ psi$
- Corrosion Allowance = $0.125\ in$
- Applied Net Section Bending Moment = $4.301E+06\ in-lbs$
- Applied Axial Force = $-78104.2\ lbs$

Half-Pipe Jacket Data:

- Material = $SA-106,\ Grade\ B$
- Design Conditions = $150\ psig@400°F$
- Jacket ID = $NPS\ 4\ \left(STD\ WT\right)\rightarrow 0.237\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0
- Corrosion Allowance = $0.0\ in$

Establish the corroded dimensions.

Vessel:

$$D_0 = 90.0 + 2t_s = 90.0 + 2\left(1.125\right) = 90.25\ in$$

$$t_s = t_s - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

$$D = D + 2\left(Corrosion\ \ Allowance\right) = 90.0 + 2\left(0.125\right) = 90.25\ in$$

Half-Pipe Jacket:

$$d_J = 4.5 - 2t_j = 4.5 - 2\left(0.237\right) = 4.026\ in$$

$$r = \frac{dj}{2} = \frac{4.026}{2} = 2.013\ in$$

Design rules for half-pipe jackets are provided in Nonmandatory Appendix EE. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.11.6. However, the design rules from VIII-2 also provide specific requirements for the use of partial penetration and fillet welds. For informational purposes, this check is shown prior to the first step of the example problem procedure.

Supplementary check for the acceptability of half-pipe jackets per VIII-2, paragraph 4.11.6. Verify the acceptability of a half pipe jacket in accordance with the requirements VIII-2, paragraphs 4.11.6.1 and 4.11.6.2.

Specified nominal size of 4 NPS is acceptable per VIII-2, paragraph 4.11.6.1.

Material of construction is SA-106, Grade B per ASME Section II Part D, Table

$$S_{yT} = 31\ ksi\ \left(@\ 300°F\right)$$

$$S_u = 60\ ksi$$

$$\left\{\frac{S_{yT}}{S_u} = \frac{31}{60} = 0.52\right\} \le 0.625 \qquad\qquad True$$

Therefore, partial penetration welds can be used. In addition, the vessel is not in cyclic service; therefore requirements of paragraph 4.11.6.2 are satisfied.

Note: This VIII-2, paragraph 4.11.3.3 requirement is not in VIII-1 Nonmandatory Appendix EE. This check is not required in VIII-1 because the above criteria will always be satisfied because of the allowable stress basis used in VIII-1.

a) STEP 1 – Calculate the minimum required thickness for the $NPS\ 4$ STD WT half-pipe jacket.

$$T = \frac{P_1 r}{0.85S_1 - 0.6P_1} = \frac{150\left(2.0130\right)}{0.85\left(20000\right) - 0.6\left(150\right)} = 0.0179\ in$$

Since $\left\{t_j = 0.237\ in\right\} \ge \left\{T = 0.0179\ in\right\}$, the thickness of STD. WT pipe is acceptable for the half-pipe jacket.

b) STEP 2 – Calculate maximum permissible pressure in the half-pipe, P', to verify that $P' \ge P_1$.

$$P' = \frac{F}{K}$$

Where,

$F = \max\left[\left(1.5S - S'\right),\ 1.5S\right]$. The value of K is determined from VIII-1, Fig. EE-1, EE-2, or EE-3. For VIII-2 designs, the value of K or (K_p for VIII-2), is also provided below for informational purposes.

In order to compute P', the parameter S', defined as the actual longitudinal stress in the shell, must be computed. This stress may be computed using the following thin-wall equations for a cylindrical shell.

Note: Per VIII-1, paragraph EE-2, when the combination of axial forces and pressure stress is such that S' would be a negative number, then S' shall be taken as zero.

$$S' = \ Pressure\ Stress + Axial\ Stress \pm Bending\ Stress$$

$$S' = \frac{PD}{4t_s} + \frac{F}{A} \pm \frac{Mc}{I}$$

$$S' = \begin{cases} \dfrac{350(90.25)}{4(1.0)} + \dfrac{-78104.2}{286.6703} + \dfrac{4.301E+06\left(\dfrac{92.25}{2}\right)}{298408.1359} = 8269.2283\ psi \\ \dfrac{350(90.25)}{4(1.0)} + \dfrac{-78104.2}{286.6703} - \dfrac{4.301E+06\left(\dfrac{92.25}{2}\right)}{298408.1359} = 6959.6156\ psi \end{cases}$$

Where,

$$I = \frac{\pi}{64}\left(D_o^4 - D^4\right) = \frac{\pi}{64}\left((92.25)^4 - (90.25)^4\right) = 298408.1359\ in^4$$

$$A = \frac{\pi}{4}\left(D_o^2 - D^2\right) = \frac{\pi}{4}\left((92.25)^2 - (90.25)^2\right) = 286.6703\ in^2$$

Therefore,

$$F = \min\left[\left(1.5S - S'\right),\ 1.5S\right]$$
$$F = \min\left[\left(1.5(20000) - 8269.2283\right), 1.5(20000)\right] = 21730.7717\ psi$$

The value of K is interpreted from Fig. EE-3, with $D = 90.25\ in$ and $t_s = 1.0\ in$.

$$K \approx 11$$

For VIII-2 designs, the coefficients for the equation K_p are obtained from VIII-2, Table 4.11.3 for NPS 4 and shell nominal thickness of $1.0\ in$.

$$C_1 = -2.5016604E+02, \qquad C_2 = 1.7178270E+02, \qquad C_3 = -4.6844914E+01$$
$$C_4 = 6.6874346E+00, \qquad C_5 = -5.2507555E-01, \qquad C_6 = 2.1526948E-02$$
$$C_7 = -3.6091550E-04, \qquad C_8 = C_9 = C_{10} = 0.0$$

With the a vessel diameter, $D = 90.25\ in$, the value of K_p is calculated as,

$$K_p = C_1 + C_2 D^{0.5} + C_3 D + C_4 D^{1.5} + C_5 D^2 + C_6 D^{2.5} + C_7 D^3 + C_8 D^{3.5} + C_9 D^4 + C_{10} D^{4.5}$$
$$K_p = 11.2903$$

Therefore, the maximum permissible pressure in the half-pipe is calculated as,

$$P' = \frac{F}{K} = \begin{cases} \dfrac{21730.7717}{11} = 1975.5\ psi & VIII-1 \\ \dfrac{21730.7717}{11.2903} = 1924.7\ psi & VIII-2 \end{cases} \geq \left\{P_1 = 150\ psi\right\} \qquad True$$

Since $P' \geq P_1$, the half-pipe design is acceptable.

4.12 NonCircular Vessels

4.12.1 Example E4.12.1 - Unreinforced Vessel of Rectangular Cross Section

Using the data shown below, design a rectangular vessel per Appendix 13, Fig. 13-2(a) Sketch (1)).

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $400\ psig@500°F$
- Inside Length (Short Side) = $7.125\ in$
- Inside Length (Long Side) = $9.25\ in$
- Overall Vessel Length = $40.0\ in$
- Thickness (Short Side) = $1.0\ in$
- Thickness (Long Side) = $1.0\ in$
- Thickness (End Plate) = $0.75\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency (Corner Joint) = 1.0
- Tube Outside Diameter = $1.0000\ in$
- Tube Pitch = $2.3910\ in$

Adjust variables for corrosion.

$$h = 9.25 + 2\left(Corrosion\ \ Allowance\right) = 9.25 + 2\left(0.125\right) = 9.50\ in$$

$$H = 7.125 + 2(Corrosion\ \ Allowance) = 7.125 + 2\left(0.125\right) = 7.375\ in$$

$$t_1 = 1.0 - Corrosion\ \ Allowance = 1.0 - 0.125 = 0.875\ in$$

$$t_2 = 1.0 - Corrosion\ \ Allowance = 1.0 - 0.125 = 0.875\ in$$

$$t_5 = 0.75 - Corrosion\ \ Allowance = 0.75 - 0.125 = 0.625\ in$$

Design rules for vessels of noncircular cross section are provided in Mandatory Appendix 13. The rules in this paragraph produce the same results as those provided in VIII-2, paragraph 4.12. However, the nomenclature and formatting of the equations in VIII-2 are significantly different. Therefore, the example problem will be shown twice, the first time using VIII-1 nomenclature and equations and secondly using the VIII-2 design procedure, nomenclature, and equations.

Section VIII, Division 1 Solution

Evaluate per Mandatory Appendix 13.Paragraph 13-4(h) – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. These equations are conservatively applicable to vessels of aspect ratio less than four. Vessels with aspect ratios less than four may be designed in accordance with the provisions of U-2(g).

$$Aspect\ \ Ratio = \frac{L_v}{h} = \frac{40.0}{9.5} = 4.21 \qquad\qquad Satisfied$$

Paragraph 13-4(g) – The ligament efficiencies e_m and e_b shall only be applied to the calculated stresses for the plates containing the ligaments. When e_m and e_b are less than the joint efficiency E, the membrane and bending stresses calculated on the gross area of the section shall be divided by e_m and e_b, respectively, to obtain the stresses based on the net area for the section. The allowable design stresses for membrane and membrane plus bending shall be calculated as described in paragraph 13-4(b) using $E = 1.0$. When e_m and e_b are greater than the joint efficiency E, the membrane and bending stresses shall be calculated as if there were no ligaments in the plate. The allowable design stresses for membrane and membrane plus bending shall be calculated as described in paragraph 13-4(b) using the appropriate E factor required by paragraph UW-12.

Paragraph 13-6 – It is assumed that the holes drilled in the long side plates (tube sheet and plug sheet) are of uniform diameter. Therefore, e_m and e_b shall be the same value and calculated in accordance with paragraph UG-53.

$$e_m = e_b = \frac{p-d}{p} = \frac{2.3910-1.0}{2.3910} = 0.5818$$

Paragraph 13-5 – Calculate the equation constants.

$$b = 1.0\ (unit\ width)$$

$$c_i = c_o = \frac{t_1}{2} = \frac{t_2}{2} = \frac{0.875}{2} = 0.4375\ in \rightarrow Note: \begin{Bmatrix} The\ sign\ of\ c_i\ is\ positive\ (+) \\ The\ sign\ of\ c_o\ is\ negative\ (-) \end{Bmatrix}$$

$$I_1 = \frac{bt_1^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558\ in^4$$

$$I_2 = \frac{bt_2^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558\ in^4$$

$$\alpha = \frac{H}{h} = \frac{7.375}{9.5} = 0.7763$$

$$K = \frac{I_2}{I_1}\alpha = \left(\frac{0.0558}{0.0558}\right)0.7763 = 0.7763$$

Paragraph 13-7(a) – Calculate the membrane and membrane plus bending stresses.

<u>The membrane stress on the short side plate, Equation (1):</u>

$$S_m = \frac{Ph}{2(t_1)} = \frac{400(9.5)}{2(0.875)} = 2171.4\ psi$$

<u>The bending stress at Location N, short side plate, Equation (3):</u>

$$S_{bN} = \frac{Pc}{12I_1}\left[-1.5H^2 + h^2\left(\frac{1+\alpha^2 K}{1+K}\right)\right]$$

$$S_{bN} = \frac{400(\pm 0.4375)}{12(0.0558)}\left[-1.5(7.375)^2 + (9.5)^2\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$

$$S_{bN} = \begin{Bmatrix} -1831.7\ psi & Inside\ Surface \\ 1831.7\ psi & Outside\ Surface \end{Bmatrix}$$

The bending stress at Location Q, short side plate, Equation (4):

$$S_{bQ} = \frac{Ph^2c}{12I_1}\left[\frac{1+\alpha^2 K}{1+K}\right]$$

$$S_{bQ} = \frac{400(9.5)^2(\pm 0.4375)}{12(0.0558)}\left[\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right]$$

$$S_{bQ} = \begin{Bmatrix} 19490.8\ psi & Inside\ Surface \\ -19490.8\ psi & Outside\ Surface \end{Bmatrix}$$

The membrane stress on the long side plate, Equation (2):

$$S_m = \frac{PH}{2t_2 e_m} = \frac{400(7.375)}{2(0.875)(0.5818)} = 2897.4\ psi$$

The bending stress at Location M, long side plate, Equation (5):

$$S_{bM} = \frac{Ph^2c}{12I_2 e_b}\left[-1.5 + \left(\frac{1+\alpha^2 K}{1+K}\right)\right]$$

$$S_{bM} = \frac{400(9.5)^2(\pm 0.4375)}{12(0.0558)(0.5818)}\left[-1.5 + \left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$

$$S_{bM} = \begin{Bmatrix} -27310.9\ psi & Inside\ Surface \\ 27310.9\ psi & Ouside\ Surface \end{Bmatrix}$$

The bending stress at Location Q, long side plate, Equation (6):

$$S_{bQ} = \frac{Ph^2c}{12I_2}\left[\frac{1+\alpha^2 K}{1+K}\right]$$

$$S_{bQ} = \frac{400(9.5)^2(\pm 0.4375)}{12(0.0558)}\left[\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right]$$

$$S_{bQ} = \begin{Bmatrix} 19490.8\ psi & Inside\ Surface \\ -19490.8\ psi & Outside\ Surface \end{Bmatrix}$$

Paragraphs 13-4(b), 13-4(c), and 13-7, Equations (7) through (10) – Acceptance Criteria:

Short side plate, Membrane Stress:

$$\{S_m = 2171.4\ psi\} \le \{SE = 20000(1.0) = 20000\ psi\} \qquad True$$

Short side plate at Location N, Membrane + Bending Stress:

$$\begin{Bmatrix} S_m + S_{bN} = 2171.4 + (-1831.7) = 339.7\ psi \\ S_m + S_{bN} = 2171.4 + (1831.7) = 4003.1\ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m + S_{bN} = 339.7\ psi \\ S_m + S_{bN} = 4003.1\ psi \end{Bmatrix} \le \{1.5SE = 1.5(20000)(1.0) = 30000\ psi\} \qquad True$$

Short side plate at Location Q, Membrane + Bending Stress:

$$\begin{Bmatrix} S_m + S_{bQ} = 2171.4 + 19490.8 = 21662.2\ psi \\ S_m + S_{bQ} = 2171.4 + (-19490.8) = -17319.4\ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m + S_{bQ} = 21662.2\ psi \\ S_m + S_{bQ} = -17319.4\ psi \end{Bmatrix} \le \{1.5SE = 1.5(20000)(1.0) = 30000\ psi\} \qquad True$$

Long side plate, Membrane Stress:

$$\{S_m = 2897.4\ psi\} \le \{S = 20000\ psi\} \qquad True$$

Long side plate at Location M, Membrane + Bending Stress:

$$\begin{Bmatrix} S_m + S_{bM} = 2897.4 + (-27310.9) = -24413.5\ psi \\ S_m + S_{bM} = 2897.4 + 27310.9 = 30208.3\ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m + S_{bM} = -24413.5\ psi \\ S_m + S_{bM} = 30208.3\ psi \end{Bmatrix} \le \{1.5SE = 1.5(20000)(1.0) = 30000\ psi\} True,\ False$$

Long side plate at Location Q, Membrane + Bending Stress:

$$\begin{Bmatrix} S_m + S_{bQ} = 2897.4 + 19490.8 = 22388.2\ psi \\ S_m + S_{bQ} = 2897.4 + (19490.8) = -16593.4\ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m + S_{bQ} = 22388.2\ psi \\ S_m + S_{bQ} = -16593.4\ psi \end{Bmatrix} \le \{1.5SE = 1.5(20000)(1.0) = 30000\ psi\} \qquad True$$

The acceptance criteria for membrane and membrane plus bending stresses are satisfied at all locations with the exception of the membrane plus bending stress at Location M on the long side plate, $\{S_m + S_{bM}\}$. However, the overstress is less than 1%.

<u>Section VIII, Division 2 Solution with VIII-1 Allowable Stresses</u>

<u>VIII-2, paragraph 4.12.2, General Design Requirements</u>.

Paragraph 4.12.2.7 – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. These equations are conservatively applicable to vessels of aspect ratio less than four. Vessels with aspect ratios less than four may be designed in accordance with VIII-2, paragraph 4.12.5 and may be designed in accordance with the provisions of Part 5.

$$Aspect\ Ratio = \frac{L_v}{h} = \frac{40.0}{9.5} = 4.21 \qquad\qquad Satisfied$$

Paragraph 4.12.2.9 – The openings in this noncircular vessel meet the requirements of VIII-2, paragraph 4.5.2.

<u>VIII-2, paragraphs 4.12.3, 4.12.4 and 4.12.5.</u>

These paragraphs are not applicable to this design.

<u>VIII-2, paragraph 4.12.6, Weld Joint Factors and Ligament Efficiency</u>.

Paragraph 4.12.6.1– The non-circular vessel is constructed with corner joints typical of VIII-2, paragraph 4.2. Therefore, the weld joint efficiencies E_m and E_b are set to 1.0 at stress calculation locations in the corners of the vessel. Since there are no welds or hole pattern in the short side plates of the vessel, the weld joint efficiencies E_m and E_b are set to 1.0 for these stress calculation locations. For the stress calculation locations on the long side plates that do not contain welded joints, but do contain a hole pattern, the weld joint efficiencies E_m and E_b are set equal to the ligament efficiencies e_m and e_b, respectively.

Paragraph 4.12.6.3 – It is assumed that the holes drilled in the long side plates (tube sheet and plug sheet) are of uniform diameter. Therefore, e_m and e_b shall be the same value and calculated in accordance with paragraph 4.10 .

$$e_m = e_b = \frac{p-d}{p} = \frac{2.3910-1.0}{2.3910} = 0.5818$$

<u>VIII-2, paragraph 4.12.7, Design Procedure</u>.

a) STEP 1 – The design pressure and temperature are listed in the information given above.
b) STEP 2 – The vessel to be designed is a Type 1 vessel (VIII-1, Fig. 13-2(a), Sketch (1)).
c) STEP 3 – The vessel configuration and wall thicknesses of the pressure containing plates are listed in the information given above.
d) STEP 4 – Determine the location of the neutral axis from the inside and outside surfaces. Since the section under evaluation does not have stiffeners, but has uniform diameter holes, then $c_i = c_o = t/2$ where t is the thickness of the plate .

$$c_i = c_o = \frac{t_1}{2} = \frac{t_2}{2} = \frac{0.875}{2} = 0.4375\ in$$

e) STEP 5 – Determine the weld joint factor and ligaments efficiencies as applicable, see VIII-2, paragraph 4.12.6, and determine the factors E_m and E_b .

f) STEP 6 – Complete the stress calculation for the selected noncircular vessel Type, see VIII-2, Table 4.12.1, and check the acceptance criteria.

For non-circular vessel Type 1, the applicable table for stress calculations is VIII-2, Table 4.12.2 and the corresponding details are shown in VIII-2, Figure 4.12.1

.Equation Constants.

$$b = 1.0 \; (unit \;\; width)$$

$$J_{2s} = 1.0$$

$$J_{3s} = 1.0$$

$$J_{2l} = 1.0$$

$$J_{3l} = 1.0$$

$$I_1 = \frac{bt_1^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558 \; in^4$$

$$I_2 = \frac{bt_2^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558 \; in^4$$

$$\alpha = \frac{H}{h} = \frac{7.375}{9.5} = 0.7763$$

$$K = \frac{I_2}{I_1}\alpha = \left(\frac{0.0558}{0.0558}\right)0.7763 = 0.7763$$

Nomenclature for Stress Results

S_m^s membrane stress, short side.

S_{bi}^{sC}, S_{bo}^{sC} bending stress, short side at point C on the inside and outside surfaces, respectively.

S_{bi}^{sB}, S_{bo}^{sB} bending stress, short side at point B on the inside and outside surfaces, respectively.

S_m^l membrane stress, long side.

S_{bi}^{lA}, S_{bo}^{lA} bending stress, long side at point A on the inside and outside surfaces, respectively.

S_{bi}^{lB}, S_{bo}^{lB} bending stress, long side at point B on the inside and outside surfaces, respectively.

Membrane and Bending Stresses – Critical Locations of Maximum Stress

$$S_m^s = \frac{Ph}{2(t_1)E_m} = \frac{400(9.5)}{2(0.875)(1.0)} = 2171.4 \; psi$$

$$S_{bi}^{sC} = -S_{bo}^{sC}\left(\frac{c_i}{c_o}\right) = \frac{PbJ_{2s}c_i}{12I_1E_b}\left[-1.5H^2 + h^2\left(\frac{1+\alpha^2K}{1+K}\right)\right]$$

$$S_{bi}^{sC} = \frac{400(1.0)(1.0)(0.4375)}{12(0.0558)(1.0)}\left[-1.5(7.375)^2 + (9.5)^2\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$

$$S_{bi}^{sC} = -1831.7 \ psi$$

$$S_{bo}^{sC} = -S_{bi}^{sC}\left(\frac{c_o}{c_i}\right)1831.7 = -(-1831.7)\left(\frac{0.4375}{0.4375}\right) = 1831.7 \ psi$$

$$S_{bi}^{sB} = -S_{bo}^{sB}\left(\frac{c_i}{c_o}\right) = \frac{Pbh^2J_{3s}c_i}{12I_1E_b}\left[\frac{1+\alpha^2K}{1+K}\right]$$

$$S_{bi}^{sB} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(0.0558)(1.0)}\left[\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$

$$S_{bi}^{sB} = 19490.8 \ psi$$

$$S_{bo}^{sB} = -S_{bi}^{sB}\left(\frac{c_o}{c_i}\right) = -19490.8\left(\frac{0.4375}{0.4375}\right) = -19490.8 \ psi$$

$$S_m^l = \frac{PH}{2t_2E_m} = \frac{400(7.375)}{2(0.875)(0.5818)} = 2897.4 \ psi$$

$$S_{bi}^{lA} = -S_{bo}^{lA}\left(\frac{c_i}{c_o}\right) = \frac{Pbh^2J_{2l}c_i}{12I_2E_b}\left[-1.5 + \left(\frac{1+\alpha^2K}{1+K}\right)\right]$$

$$S_{bi}^{lA} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(1.0)(0.0558)(0.5818)}\left[-1.5 + \left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$

$$S_{bi}^{lA} = -27310.9 \ psi$$

$$S_{bo}^{lA} = -S_{bi}^{lA}\left(\frac{c_o}{c_i}\right) = -(-27310.9)\left(\frac{0.4375}{0.4375}\right) = 27310.9 \ psi$$

$$S_{bi}^{lB} = -S_{bo}^{lB}\left(\frac{c_i}{c_o}\right) = \frac{Pbh^2J_{3l}c_i}{12I_2E_b}\left[\frac{1+\alpha^2K}{1+K}\right]$$

$$S_{bi}^{lB} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(0.0558)(1.0)}\left[\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$

$$S_{bi}^{lB} = 19490.8 \ psi$$

$$S_{bo}^{lB} = -S_{bi}^{lB}\left(\frac{c_o}{c_i}\right) = -19490.8\left(\frac{0.4375}{0.4375}\right) = -19490.8 \ psi$$

Acceptance Criteria – Critical Locations of Maximum Stress

$$\left\{S_m^s = 2171.4 \ psi\right\} \le \left\{S = 20000 \ psi\right\} \qquad True$$

$$\begin{Bmatrix} S_m^s + S_{bi}^{sC} = 2171.4 + (-1831.7) = 339.7 \ psi \\ S_m^s + S_{bo}^{sC} = 2171.4 + 1831.7 = 4003.1 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m^s + S_{bi}^{sC} = 339.7 \ psi \\ S_m^s + S_{bo}^{sC} = 4003.1 \ psi \end{Bmatrix} \le \left\{1.5S = 1.5(20000) = 30000 \ psi\right\} \qquad True$$

$$\begin{Bmatrix} S_m^s + S_{bi}^{sB} = 2171.4 + 19490.8 = 21662.2 \ psi \\ S_m^s + S_{bo}^{sB} = 2171.4 + (-19490.8) = -17319.4 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m^s + S_{bi}^{sB} = 21662.2 \ psi \\ S_m^s + S_{bo}^{sB} = -17319.4 \ psi \end{Bmatrix} \le \left\{1.5S = 1.5(20000) = 30000 \ psi\right\} \qquad True$$

$$\left\{S_m^l = 2897.4 \ psi\right\} \le \left\{S = 20000 \ psi\right\} \qquad True$$

$$\begin{Bmatrix} S_m^l + S_{bi}^{lA} = 2897.4 + (-27310.9) = -24413.5 \ psi \\ S_m^l + S_{bo}^{lA} = 2897.4 + 27310.9 = 30208.3 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m^l + S_{bi}^{lA} = -24413.5 \ psi \\ S_m^l + S_{bo}^{lA} = 30208.3 \ psi \end{Bmatrix} \le \left\{1.5S = 1.5(20000) = 30000 \ psi\right\} \qquad True, \ False$$

$$\begin{Bmatrix} S_m^l + S_{bi}^{lB} = 2897.4 + 19490.8 = 22388.2 \ psi \\ S_m^l + S_{bo}^{lB} = 2897.4 + (-19490.8) = -16593.4 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m^l + S_{bi}^{lB} = 22388.2 \ psi \\ S_m^l + S_{bo}^{lB} = -16593.4 \ psi \end{Bmatrix} \le \left\{1.5S = 1.5(20000) = 30000 \ psi\right\} \qquad True$$

The acceptance criteria for membrane and membrane plus bending stresses are satisfied at all locations with the exception of the membrane plus bending stress at Location A on the long side plate, $\left\{S_m^l + S_{bo}^{lA}\right\}$. However, the overstress is less than 1%.

4.12.2 Example E4.12.2 - Reinforced Vessel of Rectangular Cross Section

Using the data shown below, design a rectangular vessel with reinforcement per Appendix 13 Fig. 13-2(a) Sketch (4). The stiffeners are attached with continuous fillets welds on both sides of the member see Fig UG-30.

Vessel Data:

- Material = $SA-516, \ Grade \ 70$
- Design Conditions = $50 \, psig \, @ \, 200°F$
- Inside Length (Short Side) = $30.0 \, in$
- Inside Length (Long Side) = $60.0 \ in$
- Overall Vessel Length = $240.0 \ in$

- Unstiffened Span Length (pitch) = $12.0\ in$
- Thickness (Short Side) = $0.4375\ in$
- Thickness (Long Side) = $0.4375\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0
- Yield Stress at Design Temperature = $34800\ psi$
- Modulus of Elasticity at Design Temperature = $28.8E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$

Stiffener Data:

- Material = $SA-36$
- Allowable Stress = $16600\ psi$
- Stiffener Yield Stress at Design Temperature = $33000\ psi$
- Modulus of Elasticity at Design Temperature = $28.8E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$
- Stiffener Cross Sectional Area = $3.83\ in^2$
- Stiffener Moment of Inertia = $11.3\ in^4$
- Stiffener Height = $4.125\ in$
- Stiffener Centerline Distance (Short Side) = $34.125\ in$
- Stiffener Centerline Distance (Long Side) = $64.125\ in$

Required variables.

$$h = 60.0\ in$$
$$H = 30.0\ in$$
$$t_1 = 0.4375\ in$$
$$t_2 = 0.4375\ in$$

Design rules for vessels of noncircular cross section are provided in Mandatory Appendix 13. The rules in this paragraph produce the same results as those provided in VIII-2, paragraph 4.12. However, the nomenclature and formatting of the equations in VIII-2 are significantly different. Therefore, the example problem will be shown twice, the first time using VIII-1 nomenclature and equations and secondly using the VIII-2 design procedure, nomenclature, and equations.

Section VIII, Division 1 Solution

Evaluate per Appendix 13.

Paragraph 13-4(c) – For a vessel with reinforcement, when the reinforcing members and the shell plate does not have the same S and S_y values at the design temperature, the total stress shall be determined at the innermost and outermost fibers for each material. The appropriate c values (with proper signs, 13-5) for the composite section properties shall be used in the bending equations. The total stresses at the innermost and outermost fibers for each material shall be compared to the allowable design stress 13-4(b) for each material.

Paragraph 13-4(h) – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. These equations are conservatively applicable to vessels of aspect ratio less than four. Vessels with aspect ratios less than four may be designed in accordance with the provisions of U-2(g).

$$Aspect\ Ratio = \frac{L_v}{h} = \frac{240.0}{60.0} = 4.0$$

Paragraph 13-4(g) – In this example problem, there are no ligaments in either the short side or long side plates. Therefore the allowable design stresses for membrane and membrane plus bending shall be calculated as described in paragraph 13-4(b) using the appropriate E factor required by paragraph UW-12.

Paragraph 13-8(b) – The rules of this paragraph cover only the types of reinforced rectangular cross section vessels shown in Fig. 13-2(a) sketches (4), (5), and (6) where welded-on reinforcement members are in a plane perpendicular to the long axis of the vessel; however, the spacing between reinforcing members need not be uniform. All reinforcement members attached to two opposite plates shall have the same moment of inertia. Reinforcing members shall be placed on the outside of the vessel and shall be attached to the plates of the vessel by welding on each side of the reinforcing member. For continuous reinforcement, the welding may be continuous or intermittent.

Paragraph 13-8(d)(1) – The basic maximum distance between reinforcing member centerlines shall be determined by Eqn (1) of UG-47. This distance is then used to calculate a value of β for the short side H and for the long side h. A value of J is then obtained for each value from Table 13-8(d). The values thus obtained are used in the applicable equations in paragraph 13-8(d)(5) to determine the values of p_1 and p_2. The maximum distance between any reinforcing member centerlines shall not be greater than the least of the values of p_1 and p_2.

For the short side plate, $\{H = 30.0\ in\} \geq \{p = 12.0\ in\}$, Equation (1a) and Table 13-8(d)(5).

Calculate the basic maximum distance between reinforcing members, per paragraph UG-47.

$$p = t_1\sqrt{\frac{SC}{P}} = 0.4375\sqrt{\frac{(20000)2.1}{50}} = 12.6800\ in$$

Determine β per paragraph 13-5.

$$\beta = \frac{H}{p} = \frac{30.0}{12.6800} = 2.3659$$

Calculate Stress Parameter J from Table 13-8(d).

$$\beta = \min\left[\max\left[\beta, \frac{1}{\beta}\right], 4.0\right]$$

$$\beta = \min\left[\max\left[2.3659, \frac{1}{2.3659}\right], 4.0\right] = 2.3659$$

$$From\ interpolation,\ J = 2.2206$$

Calculate p_1 from Equation 1(1a).

$$p_1 = t_1\sqrt{\frac{SJ}{P}} = 0.4375\sqrt{\frac{(20000)2.2206}{50}} = 13.0390\ in$$

For the long side plate, $\{h = 60.0\ in\} \geq \{p = 12.0\ in\}$, Equation (1c) and Table 13-8(d)(5).

Calculate the basic maximum distance between reinforcing members, per paragraph UG-47.

$$p = t_1\sqrt{\frac{SC}{P}} = 0.4375\sqrt{\frac{(20000)2.1}{50}} = 12.6800\ in$$

Determine β per paragraph 13-5.

$$\beta = \frac{h}{p} = \frac{60.0}{12.6800} = 4.7391$$

Calculate Stress Parameter J from Table 13-8(d).

$$\beta = \min\left[\max\left[\beta, \frac{1}{\beta}\right], 4.0\right]$$

$$\beta = \min\left[\max\left[4.7391, \frac{1}{4.7391}\right], 4.0\right] = 4.0$$

$$\textit{From direct observation, } J = 2.0$$

Calculate p_2 from Equation (1c).

$$p_2 = t_2\sqrt{\frac{SJ}{P}} = 0.4375\sqrt{\frac{(20000)2.0}{50}} = 12.3744\ in$$

Therefore,

$$p = \min[p_1,\ p_2] = \min[13.0390,\ 12.3744] = 12.3744\ in$$

Since $\{p_{design} = 12.0\} < \{p_{allow} = 12.3744\}$, the design is acceptable.

Paragraph 13-8(d)(3) – The allowable effective width of shell plate w shall not be greater than the least value of p computed in paragraph 13-8(d)(5) nor greater than the actual value of p if the actual value of p is less than that permitted in paragraph 13-8(d)(5). One half of w shall be considered to be effective on each side of the reinforcing member centerline, but the effective widths shall not overlap. The effective width shall not be greater than the actual width available.

For the short side plate, calculate w per paragraph 13-8(d)(5), Equation (2) and Table 13-8(e).

$$w = \frac{t_1\Delta}{\sqrt{S_y}}\left(\frac{E_y}{E_{ya}}\right) = \frac{0.4375(6000)}{\sqrt{33000}}\left(\frac{28.8E+06}{29.4E+06}\right) = 14.1552\ in$$

Where,

$$\Delta = 6000\ \sqrt{psi}$$

Therefore,

$$w = \min\left[p,\ \min\left[w,\ p_1\right]\right] = \min\left[12.0, \min\left[14.1552,\ 13.0390\right]\right] = 12.0\ in$$

For the long side plate, calculate w per paragraph 13-8(d)(5), Equation (2) and Table 13-8(e).

$$w = \frac{t_2\Delta}{\sqrt{S_y}}\left(\frac{E_y}{E_{ya}}\right) = \frac{0.4375(6000)}{\sqrt{33000}}\left(\frac{28.8E+06}{29.4E+06}\right) = 14.1552\ in$$

Therefore,

$$w = \min\left[p,\ \min\left[w,\ p_2\right]\right] = \min\left[12.0,\ \min\left[14.1552,\ 12.3744\right]\right] = 12.0\ in$$

Paragraph 13-8(d)(2) – Equation (2) of paragraph 13-8(d)(5) is used to compute the maximum effective width of the shell plate which can be used in computing the effective moment of inertia I_{11} and I_{21} of the composite section (reinforcement and shell plate acting together) at locations where the shell plate is in compression.

VIII-1 does not provide rules for computing the effective moment of inertia I_{11} and I_{21}. This example provides one possible method of satisfying U-2(g); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

A composite structure may include the use of two or more different materials, each carrying a part of the load. Unless all the various materials used have the same Modulus of Elasticity, the evaluation of the composite section will need to consider the ratio of the moduli. Although the material specifications for the shell plate and stiffeners are different, their Moduli of Elasticity are the same; therefore, no adjustment to the procedure to calculate the composite section moment of inertia is required.

Calculate the short side stiffener/plate composite section neutral axis as follows, see Figure E4.12.2.

$$\overline{y} = \frac{A_{stif}\left(t_1 + \dfrac{h_s}{2}\right) + A_{plate}\left(\dfrac{t_1}{2}\right)}{\left(A_{stif} + A_{plate}\right)}$$

$$\overline{y} = \frac{3.83\left(0.4375 + \dfrac{4.125}{2}\right) + 0.4375(12.0)\left(\dfrac{0.4375}{2}\right)}{\left(3.83 + 0.4375(12.0)\right)} = 1.1810\ in$$

Calculate the short side composite section moment of inertia, I_{11}, using parallel axis theorem.

$$I_{11} = I_{stif} + A_{stif}\left(t_1 + \frac{h_s}{2} - \bar{y}\right)^2 + \frac{w_1\left(t_1\right)^3}{12} + w_1\left(t_1\right)\left(\bar{y} - \frac{t_1}{2}\right)^2$$

$$I_{11} = \begin{Bmatrix} 11.3 + 3.83\left(0.4375 + \dfrac{4.125}{2} - 1.1810\right)^2 + \\ \dfrac{12.0\left(0.4375\right)^3}{12} + 12.0\left(0.4375\right)\left(1.1810 - \dfrac{0.4375}{2}\right)^2 \end{Bmatrix} = 22.9081\ in^4$$

Since the stiffener is continuous around the vessel with a consistent net section, the plate thicknesses of the short side and long side are equal, $t_1 = t_2$, the pitch of stiffeners are equal, $w_1 = w_2$, it follows that $\bar{y}$ for the short side and long side plates are equal and $I_{11} = I_{21}$.

Determine the location of the neutral axis from the inside and outside surfaces. If the section under evaluation has stiffeners, then c_i and c_o are determined from the cross section of the combined plate and stiffener section using strength of materials concepts.

For the short side plate,

$$c_i = \bar{y} = 1.1810\ in \rightarrow The\ sign\ of\ c_i\ is\ positive\ (+)$$
$$c_o = t_1 + h_s - \bar{y} = 0.4375 + 4.125 - 1.1810 = 3.3815\ in \rightarrow The\ sign\ of\ c_o\ is\ negative\ (-)$$

For the long side plate,

$$c_i = \bar{y} = 1.1810\ in \rightarrow The\ sign\ of\ c_i\ is\ positive\ (+)$$
$$c_o = t_1 + h_s - \bar{y} = 0.4375 + 4.125 - 1.1810 = 3.3815\ in \rightarrow The\ sign\ of\ c_o\ is\ negative\ (-)$$

The reinforcing member does not have the same allowable stress as the vessel; therefore, the stress at the interface of the components of the composite section shall be determined. Since the interface between components is oriented below the composite section neutral axis, the bending equation used to compute the stress shall be that considered acting on the inside surface. The distance between the composite section neutral axis and the interface of the components is calculated as follows.

For the short side and long side plates, respectively,

$$c_{i(interface)} = \bar{y} - t_1 = 1.1810 - 0.4375 = 0.7435\ in$$
$$c_{i(interface)} = \bar{y} - t_2 = 1.1810 - 0.4375 = 0.7435\ in$$

Paragraph 13-5 – Calculate the equation constants.

$$\alpha_1 = \frac{H_1}{h_1} = \frac{H + 2\left(t_1\right) + h_s}{h + 2\left(t_2\right) + h_s} = \frac{30 + 2\left(0.4375\right) + 4.125}{60 + 2\left(0.4375\right) + 4.125} = 0.5385$$

$$k = \frac{I_{21}}{I_{11}}\alpha_1 = \left(\frac{22.9081}{22.9081}\right)0.5385 = 0.5385$$

Paragraph 13-8(e) – Calculate the membrane and membrane plus bending stresses.

The membrane stress on the short side plate, Equation (3):

$$S_m = \frac{Php}{2(A_1 + pt_1)} = \frac{50(60.0)12.0}{2(3.83 + 12.0(0.4375))} = 1982.4 \ psi$$

The bending stress at Location N, short side plate, Equation (5):

$$S_{bN} = \frac{Ppc}{24I_{11}}\left[-3H^2 + 2h^2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bN} = \begin{cases} \dfrac{50(12.0)(1.1810)}{24(22.9081)}\left[-3(30.0)^2 + 2(60.0)^2\left(\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right] \\ \dfrac{50(12.0)(-3.3815)}{24(22.9081)}\left[-3(30.0)^2 + 2(60.0)^2\left(\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right] \end{cases}$$

$$S_{bN} = \begin{cases} 3493.6 \ psi & Inside \ Surface \\ -10003.1 \ psi & Outside \ Surface \end{cases}$$

The bending stress at Location Q, short side plate, Equation (6):

$$S_{bQ} = \frac{Ph^2 pc}{12I_{11}}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bQ} = \begin{cases} \dfrac{50(60.0)^2(12.0)(1.1810)}{12(22.9081)}\left[\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] \\ \dfrac{50(60.0)^2(12.0)(-3.3815)}{12(22.9081)}\left[\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] \end{cases}$$

$$S_{bQ} = \begin{cases} 6973.5 \ psi & Inside \ Surface \\ -19966.9 \ psi & Outside \ Surface \end{cases}$$

The membrane stress on the long side plate, Equation (4):

$$S_m = \frac{PHp}{2(A_2 + pt_2)} = \frac{50(30.0)(12.0)}{2(3.83 + 12.0(0.4375))} = 991.2 \ psi$$

The bending stress at Location M, long side plate, Equation (7):

$$S_{bM} = \frac{Ph^2 pc}{24I_{21}}\left[-3+2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bM} = \begin{Bmatrix} \dfrac{50(60.0)^2(12.0)(+1.1810)}{24(22.9081)}\left[-3+2\left(\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right] \\ \dfrac{50(60.0)^2(12.0)(-3.3815)}{24(22.9081)}\left[-3+2\left(\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right] \end{Bmatrix}$$

$$S_{bM} = \begin{Bmatrix} -6946.0 \ psi & Inside \ Surface \\ 19888.2 \ psi & Outside \ Surface \end{Bmatrix}$$

The bending stress at Location Q, long side plate, Equation (8):

$$S_{bQ} = \frac{Ph^2 pc}{12I_{21}}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bQ} = \begin{Bmatrix} \dfrac{50(60.0)^2(12.0)(1.1810)}{12(22.9081)}\left[\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] \\ \dfrac{50(60.0)^2(12.0)(-3.3815)}{12(22.9081)}\left[\dfrac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] \end{Bmatrix}$$

$$S_{bQ} = \begin{Bmatrix} 6973.5 \ psi & Inside \ Surface \\ -19966.9 \ psi & Outside \ Surface \end{Bmatrix}$$

Paragraph 13-8(e) – (re–visited) Calculate the membrane plus bending stresses at the interface.

The bending stress at Location N, short side plate, Equation (5):

$$S_{bN} = \frac{Ppc}{24I_{11}}\left[-3H^2+2h^2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bN} = \frac{50(12.0)(0.7435)}{24(22.9081)}\left[-3(30.0)^2+2(60.0)^2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bN} = 2199.4 \ psi$$

The bending stress at Location Q, short side plate, Equation (6):

$$S_{bQ} = \frac{Ph^2 pc}{12I_{11}}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bQ} = \frac{50(60.0)^2(12.0)(0.7435)}{12(22.9081)}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right]$$

$$S_{bQ} = 4390.2 \ psi$$

The bending stress at Location M, long side plate, Equation (7):

$$S_{bM} = \frac{Ph^2 pc}{24I_{21}}\left[-3+2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bM} = \frac{50(60.0)^2(12.0)(0.7435)}{24(22.9081)}\left[-3+2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bM} = -4372.9 \ psi$$

The bending stress at Location Q, long side plate, Equation (8):

$$S_{bQ} = \frac{Ph^2 pc}{12I_{21}}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bQ} = \frac{50(60.0)^2(12.0)(0.7435)}{12(22.9081)}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right]$$

$$S_{bQ} = 4390.2 \ psi$$

Paragraphs 13-4(b), 13-4(c), and 13-8, Equations (9) through (12) – Acceptance Criteria:

Paragraph 13-4(b) – Membrane stresses due to pressure and mechanical loads shall not exceed the design stress, S. NOTE – see paragraph 13-4(g) for the inclusion of the joint efficiency factor, E, as applicable.

Shell Material, $SA-516, \ Grade \ 70$: $S_{ms} = SE = 20000(1.0) = 20000 \ psi$

Reinforcement Material, $SA-36$: $S_{mr} = SE = 16600(1.0) = 16600 \ psi$

Note – membrane stress for a composite reinforced bar or shapes and plate sections, etc., shall not exceed the following limits;

$$S = \min\left[S_{ms}, S_{mr}\right] = \min\left[20000, 16600\right] = 16600 \ psi$$

Paragraph 13-4(b)(2) – Any combination of membrane plus bending tension or compression stress for other cross sections (such as composite reinforced bar or shapes and plate sections, etc.), shall not exceed the following limits;

$$\min\left[1.5SE, 2/3S_y\right]$$

Paragraph 13-4(c)(2) – When the reinforcing members and the shell plate do not have the same S and S_y values at the design temperature, the total stress shall be determined at the innermost and outermost fibers for each material. The total stresses at the innermost and outermost fibers for each material shall be compared to the allowable design stress for each material.

Shell Material, $SA-516, \ Grade \ 70$:

$$S = \min\left[1.5(20000)(1.0) = 30000 \ psi, 2/3(34800) = 23200\right] = 23200 \ psi$$

Reinforcement Material, $SA-36$

$$S = \min\left[1.5(16600)(1.0) = 24900\ psi,\ 2/3(33000) = 22000\right] = 22000\ psi$$

Short side plate, Membrane Stress:

$$\{S_m = 1982.4\ psi\} \le \{S = 16000\ psi\} \qquad True$$

Short side plate at Location N, Membrane + Bending Stress:

$$\left\{\begin{array}{l} S_m + S_{bN} = 1982.4 + 3493.6 = 5476.0\ psi \\ S_m + S_{bN} = 1982.4 + (-10003.1) = -8020.7\ psi \end{array}\right\} \le \left\{\begin{array}{l} S = 23200\ psi \\ S = 22000\ psi \end{array}\right\} True,\ True$$

Short side plate at Location Q, Membrane + Bending Stress:

$$\left\{\begin{array}{l} S_m + S_{bQ} = 1982.4 + 6973.5 = 8955.9\ psi \\ S_m + S_{bQ} = 1982.4 + (-19966.9) = -17984.5\ psi \end{array}\right\} \le \left\{\begin{array}{l} S = 23200\ psi \\ S = 22000\ psi \end{array}\right\} True,\ True$$

Long side plate, Membrane Stress:

$$\{S_m = 991.2\ psi\} \le \{16600\ psi\} \qquad True$$

Long side plate at Location M, Membrane + Bending Stress:

$$\left\{\begin{array}{l} S_m + S_{bM} = 991.2 + (-6946.0) = -5954.8\ psi \\ S_m + S_{bM} = 991.2 + 19888.2 = 20879.4\ psi \end{array}\right\} \le \left\{\begin{array}{l} S = 23200\ psi \\ S = 22000\ psi \end{array}\right\} \quad True,\ True$$

Long side plate at Location Q, Membrane + Bending Stress:

$$\left\{\begin{array}{l} S_m + S_{bQ} = 991.2 + 6973.5 = 7964.7\ psi \\ S_m + S_{bQ} = 991.2 + (-19966.9) = -18975.7\ psi \end{array}\right\} \le \left\{\begin{array}{l} S = 23200\ psi \\ S = 22000\ psi \end{array}\right\} True,\ True$$

Paragraph 13-8(e) – Acceptance Criteria for the membrane plus bending stresses at the interface.

Short side plate at Location N, Membrane + Bending Stress:

$$\{S_m + S_{bN} = 1982.4 + 2199.4 = 4181.8\ psi\} \le \{S = 22000\ psi\} \qquad True$$

Short side plate at Location Q, Membrane + Bending Stress:

$$\{S_m + S_{bQ} = 1982.4 + 4390.2 = 6372.6\ psi\} \le \{S = 22000\ psi\} \qquad True$$

Long side plate at Location M, Membrane + Bending Stress:

$$\{S_m + S_{bM} = 991.2 + (-4372.9) = -3381.7\ psi\} \le \{S = 22000\ psi\} \qquad True$$

Long side plate at Location Q, Membrane + Bending Stress:

$$\{S_m + S_{bQ} = 991.2 + 4390.2 = 5381.4\ psi\} \le \{S = 22000\ psi\} \qquad True$$

The acceptance criteria for membrane and membrane plus bending stresses are satisfied at all locations; therefore the design is complete.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.12.

VIII-2, paragraph 4.12.2, General Design Requirements.

Paragraph 4.12.2.3.c – For a vessel with reinforcement, when the reinforcing member does not have the same allowable stress as the vessel, the total stress shall be determined at the inside and outside surfaces of each component of the composite section. The total stresses at the inside and outside surfaces shall be compared to the allowable stress.

i) For locations of stress below the neutral axis, the bending equation used to compute the stress shall be that considered acting on the inside surface.

ii) For locations of stress above the neutral axis, the bending equation used to compute the stress shall be that considered acting on the outside surface.

Paragraph 4.12.2.7 – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. These equations are conservatively applicable to vessels of aspect ratio less than four. Vessels with aspect ratios less than four may be designed in accordance with VIII-2, paragraph 4.12.5 and may be designed in accordance with the provisions of Part 5.

$$Aspect\ Ratio = \frac{L_v}{h} = \frac{240.0}{60.0} = 4.0$$

Paragraph 4.12.2.9

There are no specified openings for this example problem.

VIII-2, paragraph 4.12.3, Requirements for Vessels with Reinforcement.

Paragraph 4.12.3.1 – Design rules are provided for Type 4 configurations where the welded-on reinforcement members are in a plane perpendicular to the long axis of the vessel. All reinforcement members attached to two opposite plates shall have the same moment of inertia.

Paragraph 4.12.3.5 – Reinforcing members shall be placed on the outside of the vessel and shall be attached to the plates of the vessel by welding on each side of the reinforcing member. For continuous reinforcement, the welding may be continuous or intermittent.

Paragraph 4.12.3.6 – The maximum distance between reinforcing members is computed in VIII-2, paragraph 4.12.3 and are covered in STEP 3 of the Design Procedure in VIII-2, paragraph 4.12.7.

VIII-2, paragraphs 4.12.4 and 4.12.5.

These paragraphs are not applicable to this design.

VIII-2, paragraph 4.12.6, Weld Joint Factors and Ligament Efficiency.

Paragraph 4.12.6.1 – The non-circular vessel is constructed with corner joints typical of VIII-2, paragraph 4.2. Therefore, the weld joint efficiencies E_m and E_b are set to 1.0 at stress calculation locations in the corners of the vessel. Since there are no welds or hole pattern in either the short side plates or long side plates of the vessel, the weld joint efficiencies E_m and E_b are set to 1.0 for these stress calculation locations.

<u>VIII-2, paragraph 4.12.7, Design Procedure</u>.

a) STEP 1 – The design pressure and temperature are listed in the information given above.

b) STEP 2 – The vessel to be designed is a Type 4 vessel.

c) STEP 3 – The vessel configuration and wall thicknesses of the pressure containing plates are listed in the information given above. The vessel has stiffeners; therefore, calculate the maximum spacing and size of the stiffeners per VIII-2, paragraph 4.12.3.

VIII-2, paragraph 4.12.3.6.a – The maximum distance between reinforcing member centerlines is given by VIII-2, Equation (4.12.1). In the equations for calculating stresses for reinforced noncircular vessels, the value of p shall be the sum of one-half the distances to the next reinforcing member on each side.

For the short side plate, where $\{H = 30.0\ in\} \geq \{p = 12.0\ in\}$

$$p_1 = t_1\sqrt{\frac{SJ_1}{P}} = 0.4375\sqrt{\frac{(20000)2.2206}{50}} = 13.0390\ in$$

Where,

$$J_1 = -0.26667 + \frac{24.222}{(\beta_{1\max})} - \frac{99.478}{(\beta_{1\max})^2} + \frac{194.59}{(\beta_{1\max})^3} - \frac{169.99}{(\beta_{1\max})^4} + \frac{55.822}{(\beta_{1\max})^5}$$

$$J_1 = -0.26667 + \frac{24.222}{(2.3659)} - \frac{99.478}{(2.3659)^2} + \frac{194.59}{(2.3659)^3} - \frac{169.99}{(2.3659)^4} + \frac{55.822}{(2.3659)^5}$$

$$J_1 = 2.2206$$

$$\beta_{1\max} = \min\left[\max\left[\beta_1, \frac{1}{\beta_1}\right], 4.0\right]$$

$$\beta_{1\max} = \min\left[\max\left[2.3659, \frac{1}{2.3659}\right], 4.0\right] = 2.3659$$

$$\beta_1 = \frac{H}{p_{b1}} = \frac{30.0000}{12.680} = 2.3659 \qquad (\textit{for rectangular vessels})$$

$$p_{b1} = t_1\sqrt{\frac{2.1S}{P}} = t_1\sqrt{\frac{2.1S}{P}} = 0.4375\sqrt{\frac{2.1(20000)}{50}} = 12.680\ in$$

For the long side plate, where $\{h = 60.0\ in\} \geq \{p = 12.0\ in\}$

$$p_2 = t_2\sqrt{\frac{SJ_2}{P}} = 0.4375\sqrt{\frac{(20000)2.0000}{50}} = 12.3794\ in$$

Where,

$$J_2 = -0.26667 + \frac{24.222}{(\beta_{1\max})} - \frac{99.478}{(\beta_{1\max})^2} + \frac{194.59}{(\beta_{1\max})^3} - \frac{169.99}{(\beta_{1\max})^4} + \frac{55.822}{(\beta_{1\max})^5}$$

$$J_2 = -0.26667 + \frac{24.222}{(4.0)} - \frac{99.478}{(4.0)^2} + \frac{194.59}{(4.0)^3} - \frac{169.99}{(4.0)^4} + \frac{55.822}{(4.0)^5}$$

$$J_2 = 2.0000$$

$$\beta_{2\max} = \min\left[\max\left[\beta_2, \frac{1}{\beta_2}\right], 4.0\right]$$

$$\beta_{2\max} = \min\left[\max\left[4.7319, \frac{1}{4.7319}\right], 4.0\right] = 4.0$$

$$\beta_2 = \frac{h}{p_{b2}} = \frac{60.0000}{12.680} = 4.7319 \qquad (\mathit{for\ rectangular\ vessels})$$

$$p_{b2} = t_2\sqrt{\frac{2.1S}{P}} = t_2\sqrt{\frac{2.1S}{P}} = 0.4375\sqrt{\frac{2.1(20000)}{50}} = 12.680\ in$$

Therefore,

$$p = \min[p_1, p_2] = \min[13.0390, 12.3744] = 12.3744\ in$$

Since $\{p_{design} = 12.0\} < \{p_{allow} = 12.3744\}$, the design is acceptable.

VIII-2, paragraph 4.12.3.6.b – The allowable effective widths of shell plate, w_1 and w_2 shall not be greater than the value given by VIII-2, Equation (4.12.16) or VIII-2, Equation (4.12.17), nor greater than the actual value of p if this value is less than that computed in VIII-2, paragraph 4.12.3.6.a. One half of w shall be considered to be effective on each side of the reinforcing member centerline, but the effective widths shall not overlap. The effective width shall not be greater than the actual width available.

$$w_1 = \min[p, \min[w_{\max}, p_1]] = \min[12.0, \min[14.1552, 13.0390]] = 12.0\ in$$

$$w_2 = \min[p, \min[w_{\max}, p_2]] = \min[12.0, \min[14.1552, 12.3744]] = 12.0\ in$$

Where,

$$w_{\max} = \frac{t_1\Delta}{\sqrt{S_y}}\left(\frac{E_y}{E_{ya}}\right) = \frac{t_2\Delta}{\sqrt{S_y}}\left(\frac{E_y}{E_{ya}}\right) = \frac{0.4375(6000)}{\sqrt{33000}}\left(\frac{28.8E+06}{29.4E+06}\right) = 14.1552\ in$$

$$\Delta = 6000\ \sqrt{psi} \qquad \mathit{From\ Table}\ 4.12.14$$

VIII-2, paragraph 4.12.3.6.c – At locations, other than in the corner regions where the shell plate is in tension, the effective moments of inertia, I_{11} and I_{21}, of the composite section (reinforcement and shell plate acting together) shall be computed based on the values of w_1 and w_2 computed in VIII-2, paragraph 4.12.3.6.b.

NOTE – A composite structure may include the use of two or more different materials, each carrying a part of the load. Unless all the various materials used have the same Modulus of Elasticity, the evaluation of the composite section will need to consider the ratio of the moduli. Although the material specifications for the shell plate and stiffeners are different, their Moduli of Elasticity are the same; therefore, no adjustment to the procedure to calculate the composite section moment of inertia is required.

Calculate the short side stiffener/plate composite section neutral axis as follows, see Figure E4.12.2.

$$\bar{y} = \frac{A_{stif}\left(t_1 + \frac{h_s}{2}\right) + A_{plate}\left(\frac{t_1}{2}\right)}{\left(A_{stif} + A_{plate}\right)}$$

$$\bar{y} = \frac{3.83\left(0.4375 + \frac{4.125}{2}\right) + 0.4375(12.0)\left(\frac{0.4375}{2}\right)}{\left(3.83 + 0.4375(12.0)\right)} = 1.1810\ in$$

Calculate the short side composite section moment of inertia, I_{11}, using parallel axis theorem.

$$I_{11} = I_{stif} + A_{stif}\left(t_1 + \frac{h_s}{2} - \bar{y}\right)^2 + \frac{w_1\left(t_1\right)^3}{12} + w_1\left(t_1\right)\left(\bar{y} - \frac{t_1}{2}\right)^2$$

$$I_{11} = \left\{\begin{array}{l} 11.3 + 3.83\left(0.4375 + \frac{4.125}{2} - 1.1810\right)^2 + \\ \frac{12.0\left(0.4375\right)^3}{12} + 12.0\left(0.4375\right)\left(1.1810 - \frac{0.4375}{2}\right)^2 \end{array}\right\} = 22.9081\ in^4$$

Since the stiffener is continuous around the vessel with a consistent net section, the plate thicknesses of the short side and long side are equal, $t_1 = t_2$, the pitch of stiffeners are equal, $w_1 = w_2$, it follows that $\bar{y}$ for the short side and long side plates are equal and $I_{11} = I_{21}$.

d) STEP 4 – Determine the location of the neutral axis from the inside and outside surfaces. If the section under evaluation has stiffeners, then c_i and c_o are determined from the cross section of the combined plate and stiffener section using strength of materials concepts.

For the short side plate,

$$c_i = \bar{y} = 1.1810\ in$$

$$c_o = t_1 + h_s - \bar{y} = 0.4375 + 4.125 - 1.1810 = 3.3815\ in$$

For the long side plate,

$$c_i = \bar{y} = 1.1810\ in$$

$$c_o = t_2 + h_s - \bar{y} = 0.4375 + 4.125 - 1.1810 = 3.3815\ in$$

The reinforcing member does not have the same allowable stress as the vessel; therefore, the stress at the interface of the components of the composite section shall be determined. Since the interface between components is oriented below the composite section neutral axis, the bending equation used to compute the stress shall be that considered acting on the inside surface. The distance between the composite section neutral axis and the interface of the components is calculated as follows.

For the short side and long side plates, respectively,

$$c_{i(interface)} = \bar{y} - t_1 = 1.1810 - 0.4375 = 0.7435\ in$$

$$c_{i(interface)} = \bar{y} - t_2 = 1.1810 - 0.4375 = 0.7435\ in$$

e) STEP 5 – Determine the weld joint factor and ligaments efficiencies, as applicable, see VIII-2, paragraph 4.12.6, and determine the factors E_m and E_b.

$$E_m = E_b = 1.0$$

f) STEP 6 – Complete the stress calculation for the selected noncircular vessel Type, see VIII-2, Table 4.12.1, and check the acceptance criteria.

For non-circular vessel Type 4, the applicable table for stress calculations is VIII-2, Table 4.12.5 and the corresponding details are shown in VIII-2, Figure 4.12.4.

Equation Constants.

$$\alpha_1 = \frac{H_1}{h_1} = \frac{H + 2(t_1) + h_s}{h + 2(t_2) + h_s} = \frac{30 + 2(0.4375) + 4.125}{60 + 2(0.4375) + 4.125} = 0.5385$$

$$k = \frac{I_{21}}{I_{11}}\alpha_1 = \left(\frac{22.9081}{22.9081}\right)0.5385 = 0.5385$$

Nomenclature for Stress Results

S_m^s membrane stress, short side.

S_{bi}^{sC}, S_{bo}^{sC} bending stress, short side at point C on the inside and outside surfaces, respectively.

S_{bi}^{sB}, S_{bo}^{sB} bending stress, short side at point B on the inside and outside surfaces, respectively.

S_m^l membrane stress, long side.

S_{bi}^{lA}, S_{bo}^{lA} bending stress, long side at point A on the inside and outside surfaces, respectively.

S_{bi}^{lB}, S_{bo}^{lB} bending stress, long side at point B on the inside and outside surfaces, respectively.

Membrane and Bending Stresses – Critical Locations of Maximum Stress

$$S_m^s = \frac{Php}{2(A_1 + t_1 p)E_m} = \frac{50(60.0)12.0}{2(3.83 + 0.4375(12.0))1.0} = 1982.4\ psi$$

$$S_{bi}^{sC} = -S_{bo}^{sC}\left(\frac{c_i}{c_o}\right) = \frac{Ppc_i}{24I_{11}E_b}\left[-3H^2 + 2h^2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{sC} = \frac{50(12.0)1.1810}{24(22.9081)1.0}\left[-3(30.0)^2 + 2(60.0)^2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{sC} = 3493.6\ psi$$

$$S_{bo}^{sC} = -S_{bi}^{sC}\left(\frac{c_o}{c_i}\right) = -3493.6\left(\frac{3.3815}{1.1810}\right) = -10003.1\ psi$$

$$S_{bi}^{sB} = -S_{bo}^{sB}\left(\frac{c_i}{c_o}\right) = \frac{Ph^2 pc_i}{12I_{11}E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{sB} = \frac{50(60.0)^2(12.0)(1.1810)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right]$$

$$S_{bi}^{sB} = 6973.5\ psi$$

$$S_{bo}^{sB} = -S_{bi}^{sB}\left(\frac{c_o}{c_i}\right) = -6973.5\left(\frac{3.3815}{1.1810}\right) = -19966.9\ psi$$

$$S_m^l = \frac{PHp}{2(A_2 + t_2 p)E_m} = \frac{50(30.0)(12.0)}{2(3.83+0.4375(12.0))1.0} = 991.2\ psi$$

$$S_{bi}^{lA} = -S_{bo}^{lA}\left(\frac{c_i}{c_o}\right) = \frac{Ph^2 pc_i}{24I_{21}E_b}\left[-3 + 2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{lA} = \frac{50(60.0)^2(12.0)(1.1810)}{24(22.9081)1.0}\left[-3 + 2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{lA} = -6946.0\ psi$$

$$S_{bo}^{lA} = -S_{bi}^{lA}\left(\frac{c_o}{c_i}\right) = -(-6946.0)\left(\frac{3.3815}{1.1810}\right) = 19888.2\ psi$$

$$S_{bi}^{lB} = -S_{bo}^{lB}\left(\frac{c_i}{c_o}\right) = \frac{Ph^2 pc_i}{12I_{21}E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{lB} = \frac{50(60.0)^2(12.0)(1.1810)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right]$$

$$S_{bi}^{lB} = 6973.5\ psi$$

$$S_{bo}^{lB} = -S_{bi}^{lB}\left(\frac{c_o}{c_i}\right) = -6973.5\left(\frac{3.3815}{1.1810}\right) = -19966.9\ psi$$

Calculate the bending stresses at the interface of the shell plate and stiffener at the Critical Locations of Maximum Stress.

$$S_{bi}^{sC} = \frac{Ppc_i}{24I_{11}E_b}\left[-3H^2 + 2h^2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{sC} = \frac{50(12.0)(0.7435)}{24(22.9081)1.0}\left[-3(30.0)^2 + 2(60.0)^2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{sC} = 2199.4 \ psi$$

$$S_{bi}^{sB} = \frac{Ph^2 pc_i}{12I_{11}E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{sB} = \frac{50(60.0)^2(12.0)(0.7435)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] = 4390.2 \ psi$$

$$S_{bi}^{lA} = \frac{Ph^2 pc_i}{24I_{21}E_b}\left[-3 + 2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{lA} = \frac{50(60.0)^2(12.0)(0.7435)}{24(22.9081)1.0}\left[-3 + 2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right] = -4372.9 \ psi$$

$$S_{bi}^{lB} = \frac{Ph^2 pc_i}{12I_{21}E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{lB} = \frac{50(60.0)^2(12.0)(0.7435)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] = 4390.2 \ psi$$

Acceptance Criteria – Critical Locations of Maximum Stress: The stiffener allowable stress, S_{stif}, is used for the membrane stress and membrane plus bending stress for the outside fiber stress acceptance criteria, while the plate allowable stress, S, is used for the membrane plus bending stress for inside fiber allowable stress criteria.

$$\left\{S_m^s = 1982.4 \ psi\right\} \le \left\{S = 20000 \ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_m^s + S_{bi}^{sC} = 1982.4 + 3493.5 = 5476.0 \ psi \\ S_m^s + S_{bo}^{sC} = 1982.4 + (-10003.1) = -8020.7 \ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(20000) = 30000 \ psi \\ 1.5S = 1.5(16600) = 24900 \ psi \end{array}\right\} True$$

$$\left\{\begin{array}{l} S_m^s + S_{bi}^{sB} = 1982.4 + 6973.5 = 8955.9 \ psi \\ S_m^s + S_{bo}^{sB} = 1982.4 + (-19966.9) = -17984.5 \ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(20000) = 30000 \ psi \\ 1.5S = 1.5(16600) = 24900 \ psi \end{array}\right\} True$$

$$\left\{S_m^l = 991.2 \ psi\right\} \le \left\{S = 16600 \ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lA} = 991.2 + (-6946.0) = -5954.8 \ psi \\ S_m^l + S_{bo}^{lA} = 991.2 + 19888.2 = 20879.4 \ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(20000) = 30000 \ psi \\ 1.5S = 1.5(16600) = 24900 \ psi \end{array}\right\} True$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lB} = 991.2 + 6973.5 = 7964.7 \ psi \\ S_m^l + S_{bo}^{lB} = 991.2 + (-19966.9) = -18975.7 \ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(20000) = 30000 \ psi \\ 1.5S = 1.5(16600) = 24900 \ psi \end{array}\right\} True$$

The allowable stress of the shell plate and stiffener is limited by the stiffener. Therefore, at the interface of the shell plate and stiffener, the allowable stress used in the acceptance criteria is that of the stiffener.

$$\left\{S_m^s + S_{bi}^{sC} = 1982.4 + 2199.4 = 4181.8 \ psi\right\} \le \left\{1.5S = 1.5(16600) = 24900 \ psi\right\} \quad True$$

$$\left\{S_m^s + S_{bi}^{sB} = 1982.4 + 4390.2 = 6372.6 \ psi\right\} \le \left\{1.5S = 1.5(16600) = 24900 \ psi\right\} \quad True$$

$$\left\{S_m^l + S_{bi}^{lA} = 991.2 + (-4372.9) = -3381.7 \ psi\right\} \le \left\{1.5S = 1.5(16600) = 24900 \ psi\right\} \quad True$$

$$\left\{S_m^l + S_{bi}^{lB} = 991.2 + 4390.2 = 5381.4 \ psi\right\} \le \left\{1.5S = 1.5(16600) = 24900 \ psi\right\} \quad True$$

The acceptance criteria for membrane and membrane plus bending stresses are satisfied at all locations; therefore the design is complete.

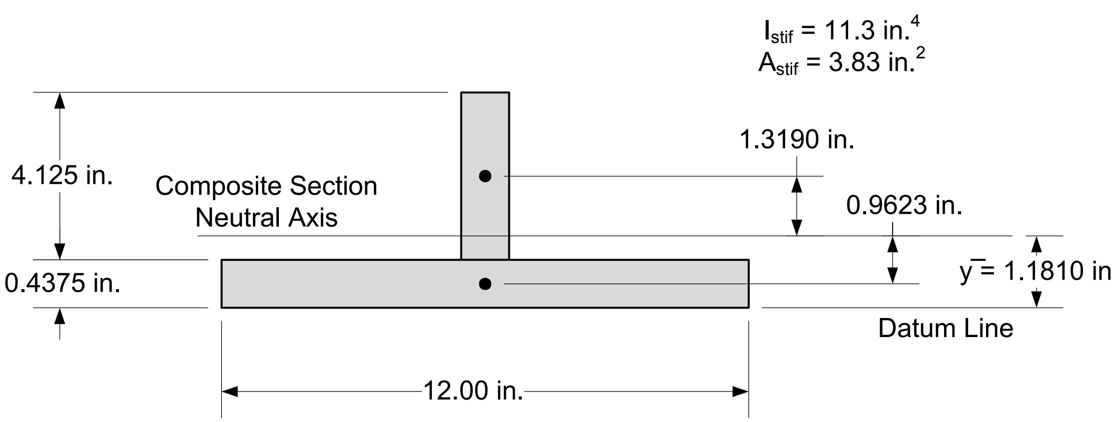


Figure E4.12.2 – Composite Section Details

4.13 Layered Vessels

4.13.1 Example E4.13.1 – Layered Cylindrical Shell

Determine the required total thickness of the layered cylindrical shell for the following design conditions. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with ULW-50 through ULW-57.

Vessel Data:

- Material = $SA-724,\ Grade\ B$
- Design Conditions = $5400\ psig\ @\ 300°F$
- Inside Diameter = $84.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $26800\ psi$
- Weld Joint Efficiency = 1.0
- Thickness of each layer = $0.3125\ in$

Section VIII, Division 1 Solution

Evaluate per ULW-16 and UG-27(c)(1).

$$t = \frac{PR}{SE-0.6P}$$

$$Note:\{P = 5400\ \ psi\} \le \{0.385SE = 0.385(26800)(1.0) = 10318\ \ psi\}$$

$$R = \frac{84.0}{2} = 42\ in$$

$$t = \frac{5400(42)}{26800(1.0)-0.6(5400)} = 9.6265\ in$$

The required thickness for all layers is $9.6265\ in$

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

In accordance with Part 4, paragraph 4.13.4.1, determine the total thickness of the layered cylindrical shell using Part 4, paragraph 4.3.1

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{84}{2}\left(\exp\left[\frac{5400}{26800(1.0)}\right]-1\right) = 9.3755\ in$$

The required thickness for all layers is $9.3755\ in$

4.13.2 Example E4.13.2 – Layered Hemispherical Head

Determine the required total thickness of the layered hemispherical head for the following design conditions. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with ULW-50 through ULW-57.

Vessel Data:

- Material = $SA-724,\ Grade\ B$
- Design Conditions = $5400\ psig @ 300°F$
- Inside Diameter = $84.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $26800\ psi$
- Weld Joint Efficiency = 1.0
- Thickness of each layer = $0.3125\ in$

Section VIII, Division 1 Solution

Evaluate per ULW-16 and UG-32(f).

$$t = \frac{PL}{2SE - 0.2P}$$

$$Note: \{P = 5400\ psi\} \le \{0.665SE = 0.665(26800)(1.0) = 17822\ psi\}$$

$$L = \frac{D}{2} = \frac{84}{2} = 42\ in$$

$$t = \frac{PL}{2SE - 0.2P} = \frac{5400(42)}{2(26800)(1.0) - 0.2(5400)} = 4.3184\ in$$

The required thickness is $4.3184\ in$

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

In accordance with Part 4, paragraph 4.13.4.1, determine the total thickness of the layered hemispherical head using Part 4, paragraph 4.3.3.

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right] - 1\right) = \frac{84}{2}\left(\exp\left[\frac{0.5(5400)}{26800(1.0)}\right] - 1\right) = 4.4518\ in$$

The required thickness for all layers is $4.4518\ in$

4.13.3 Example E4.13.3 – Maximum Permissible Gap in a Layered Cylindrical Shell

Determine the maximum permissible gap between any two layers in accordance with ULW-77 for the cylindrical shell in Example Problem E4.13.1. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with ULW-50 through ULW-57.

Vessel Data:

- Material = $SA-302,\ Grade\ B$
- Design Conditions = $4800\ psig\ @\ 300°F$
- Inside Diameter = $84.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $26800\ psi$
- Weld Joint Efficiency = 1.0
- Thickness of each layer = $0.3125\ in$
- Number of layers = 20
- Specified design cycles in the UDS = 1000
- Stress amplitude at 1000 cycles = $80600\ psi$
- Elastic modulus = $28.3E+6\ psi$

Section VIII, Division 1 Solution

Per ULW-77, the maximum height of any gap shall not exceed 0.1875 in.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

In accordance with VIII-2, paragraph 4.13.12.3, determine the maximum permissible gap between any two layers, consider the outermost layer.

$$K_c = \sqrt{\frac{4S_a}{3S_m} + 0.25} - 0.5 = \sqrt{\frac{4(80600)}{3(26800)} + 0.25} - 0.5 = 1.5640$$

$$N = \frac{2}{K_c}\left(\frac{S_a}{S_m}\right) = \frac{2}{1.5640}\left(\frac{80600}{26800}\right) = 3.8459$$

$$R_g = \frac{84 + 20(0.3125)}{2} = 45.125\ in$$

$$h = 0.55\left(N - 0.5 - \frac{P}{S_m}\right)\frac{R_g S_m}{E_y}$$

$$h = 0.55\left(3.8459 - 0.5 - \frac{5400}{26800}\right)\frac{(45.125)(5400)}{28.3E6} = 0.0149\ in$$

The maximum permissible gap is $0.0149\ in$.

4.14 Evaluation of Vessels Outside of Tolerance

4.14.1 Example E4.14.1 – Shell Tolerances

A pressure vessel is constructed from NPS 30 long seam welded pipe. During construction, examination of the vessel shell indicates peaking at the long seam weld. Peaking is known to lead to in-service damage. Determine if the design is acceptable.

Vessel Data

- Material = $SA\text{-}53,\ Grade\ B$
- Design Conditions = $325\ psig@600\ ^\circ F$
- Pipe Outside Diameter = $30\ in$
- Wall Thickness = $0.5\ in$
- Joint Efficiency = $100\ \%$
- Corrosion Allowance = $0.063\ in$
- Allowable Stress = $17100\ psi@600\ ^\circ F$
- Material Yield Strength = $26800\ psi@600\ ^\circ F$

Examination Data

- Peaking distortion δ = $0.33\ in$

Section VIII, Division 1 Solution

Evaluate per UG-80.

The maximum inside diameter is:

$$ID_{\max} = 30 - 2(0.5) + 0.33 = 29.33\ in$$

The minimum inside diameter is:

$$ID_{min} = 30 - 2(0.5) = 29\ in$$

In accordance with UG-80

$$\frac{ID_{\max} - ID_{min}}{ID_{nom}} = \frac{0.33\ in}{29.0} \cdot 100.0 = 1.14\% \geq 1\%$$

The out of roundness is not acceptable in accordance with UG-80.

4.14.2 Example E4.14.2 - Local Thin Area

For the vessel in Example Problem 1, an arch strike was removed during fabrication by blend grinding that has resulted in a region of local metal loss. Determine whether the local thin area is acceptable using Appendix 32.

Vessel Data

- Material = $SA\text{-}53,\ Grade\ B$
- Pipe Outside Diameter = $30\ in$

- Wall Thickness = $0.5\ in$
- Design Conditions = $325\ psig @ 600\ °F$
- Joint Efficiency = $100\,\%$
- Future Corrosion Allowance = $0.063\ in$
- Allowable Stress = $17100\ psi @ 600\ °F$
- Supplemental Loads = $Negligible$
- The vessel is not in cyclic service (subject to less than 150 cycles)

Examination Data

Based on inspection data, the thickness profile in the longitudinal direction has a length of 8.0 in. and a maximum measured thickness of 0.36 in. The critical thickness profile in the circumferential direction has a length of 10.0 in. The region of local metal loss is located 45 in away from the nearest structural discontinuity and is the only region of local metal loss found in the vessel during inspection.

Section VIII, Division 1 Solution

Evaluate per Mandatory Appendix 32, paragraph 32-4.

$$t_{nom} = 0.5\ in$$

$$t = t_{nom} - CA = 0.5 - 0.063 = 0.437\ in$$

$$R = \frac{30}{2} - 0.437 = 14.563\ in$$

$$L = 8.0\ in$$

$$C = 10.0\ in$$

$$t_L = 0.36 - CA = 0.36 - 0.63 = 0.2970$$

$$\left\{\frac{t_L}{t} = \frac{0.2970}{0.4370} = 0.6796\right\} \geq 0.9 \qquad False$$

$$\{L = 8.0\} \leq \left\{\sqrt{Rt} = \sqrt{14.563(0.437)} = 2.5227\right\} \qquad False$$

$$\{C = 10.0\} \leq \{2L = 16\} \qquad False$$

$$\{t - t_L = 0.437 - 0.2970 = 0.140\} \leq 0.1875 \qquad True$$

The LTA has to be repaired in accordance with Appendix 32.

4.15 Supports and Attachments

4.15.1 Example E4.15.1 - Horizontal Vessel with Zick's Analysis

Determine if the stresses in the horizontal vessel induced by the proposed saddle supports are with acceptable limits. The vessel is supported by two symmetric equally spaced saddles welded to the vessel, without reinforcing plates or stiffening rings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined. See Figure E4.15.1

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $2074\ psig@175°F$
- Outside Diameter = $66.0\ in$
- Thickness = $3.0\ in$
- Corrosion Allowance = $0.125\ in$
- Formed Head Type = $2:1\ Elliptical$
- Head Height (Based on OD) = $16.5\ in$
- Allowable Stress = $20000\ psi$
- Weld Joint Efficiency = 1.0
- Shell Tangent to Tangent Length = $292.0\ in$

Saddle Data:

- Material = $SA-516,\ Grade\ 70$
- Saddle Center Line to Head Tangent Line = $41.0\ in$
- Saddle Contact Angle = $123.0\ deg$
- Width of Saddles = $8.0\ in$
- Vessel Load per Saddle = $50459.0\ lbs$

Adjust the vessel inside diameter and thickness by the corrosion allowance.

$$ID = ID + 2(Corrosion\ \ Allowance) = 60.0 + 2(0.125) = 60.25\ in$$

$$t = t - Corrosion\ \ Allowance = 3.0 - 0.125 = 2.875\ in$$

$$R_m = \frac{OD + ID}{4} = \frac{66.0 + 60.25}{4} = 31.5625\ in$$

Section VIII, Division 1 Solution

VIII-1 does not provide rules for saddle supported vessels. However, UG-22 requires consideration of such loadings and the provisions of U-2(g) apply. This example provides one possible method of satisfying U-2(g); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

Zick's analysis is one of the accepted analysis procedures for determining the stresses in the shell of a horizontal drum support on two saddle supports. The Zick's analysis procedure is provided in VIII-2, paragraph 4.15.3, and this procedure will be used in this example problem.

VIII-2, paragraph 4.15.3.1, Application of Rules

a) The stress calculation method is based on linear elastic mechanics and covers modes of failure by excessive deformation and elastic instability.
b) Saddle supports for horizontal vessels shall be configured to provide continuous support for at least one-third of the shell circumference, or $\theta = 120.0\ deg$.

Since $\left\{\theta = 123.0\ deg\right\} \geq \left\{\theta_{req} = 120.0\ deg\right\}$ the geometry is acceptable.

VIII-2, paragraph 4.15.3.2, Moment and Shear Force

The vessel is composed of a cylindrical shell with formed heads at each end that is supported by two equally spaced saddle supports. The moment at the saddle, M_1, the moment at the center of the vessel, M_2, and the shear force at the saddle, T, may be computed if the distance between the saddle centerline and head tangent line satisfies the following limit.

$$\left\{a = 41.0\ in\right\} \leq \left\{0.25L = 0.25(292.0) = 73.0\ in\right\} \qquad \textit{Satisfied}$$

Bending Moment at the Saddle

$$M_1 = -Qa\left(1 - \frac{1 - \dfrac{a}{L} + \dfrac{R_m^2 - h_2^2}{2aL}}{1 + \dfrac{4h_2}{3L}}\right)$$

$$M_1 = -(50459.0)(41.0)\left(1 - \frac{1 - \left(\dfrac{41.0}{292.0}\right) + \dfrac{(31.5625)^2 - (16.5)^2}{2(41.0)(292.0)}}{1 + \dfrac{4(16.5)}{3(292.0)}}\right) = -356913.7\ in-lbs$$

Bending Moment at the Center of the Vessel

$$M_2 = \frac{QL}{4}\left(\frac{1 + \dfrac{2\left(R_m^2 - h_2^2\right)}{L^2}}{1 + \dfrac{4h_2}{3L}} - \frac{4a}{L}\right)$$

$$M_2 = \frac{50459.0(292.0)}{4}\left(\frac{1 + \dfrac{2\left[(31.5625)^2 - (16.5)^2\right]}{(292.0)^2}}{1 + \dfrac{4(16.5)}{3(292.0)}} - \frac{4(41.0)}{292.0}\right) = 1414775.7\ in-lbs$$

Shear Force at the Saddle

$$T=\frac{Q\left(L-2a\right)}{L+\dfrac{4h_2}{3}}=\frac{50459.0\left[292.0-2\left(41.0\right)\right]}{292.0+\dfrac{4\left(16.5\right)}{3}}=33746.5\ lbs$$

VIII-2, paragraph 4.15.3.3, Longitudinal Stress

a) The longitudinal membrane plus bending stresses in the cylindrical shell between the supports are given by the following equations.

At the top of shell:

$$\sigma_1=\frac{PR_m}{2t}-\frac{M_2}{\pi R_m^2 t}=\frac{2074\left(31.5625\right)}{2\left(2.875\right)}-\frac{1414775.7}{\pi\left(31.5625\right)^2\left(2.875\right)}=11227.2\ psi$$

Note: A load combination that includes zero internal pressure and the vessel full of contents would provide the largest compressive stress at the top of the shell, and should be checked as part of the design.

At the bottom of the shell:

$$\sigma_2=\frac{PR_m}{2t}+\frac{M_2}{\pi R_m^2 t}=\frac{2074\left(31.5625\right)}{2\left(2.875\right)}+\frac{1414775.7}{\pi\left(31.5625\right)^2\left(2.875\right)}=11541.7\ psi$$

b) The longitudinal stresses in the cylindrical shell at the support location are given by the following equations. The values of these stresses depend on the rigidity of the shell at the saddle support. The cylindrical shell may be considered as suitably stiffened if it incorporates stiffening rings at, or on both sides of the saddle support, or if the support is sufficiently close defined as $a\le 0.5R_m$ to the elliptical head.

Since $\left\{a=41.0\ in\right\}>\left\{0.5R_m=0.5\left(31.5625\right)=15.7813\ in\right\}$, the criterion is not satisfied.

Therefore, for an unstiffened shell, calculate the maximum values of longitudinal membrane plus bending stresses at the saddle support as follows.

At points A and B in VIII-2, Figure 4.15.5:

$$\sigma_3^*=\frac{PR_m}{2t}-\frac{M_1}{K_1\pi R_m^2 t}=\frac{2074\left(31.5625\right)}{2\left(2.875\right)}-\frac{-356913.7}{0.1114\left(\pi\right)\left(31.5625\right)^2\left(2.875\right)}$$

$$\sigma_3^*=11740.5\ psi$$

Where the coefficient K_1 is found in VIII-2, Table 4.15.1,

$$K_1=\frac{\Delta+\sin\Delta\cdot\cos\Delta-\dfrac{2\sin^2\Delta}{\Delta}}{\pi\left(\dfrac{\sin\Delta}{\Delta}-\cos\Delta\right)}=\frac{1.4181+\sin\left[1.4181\right]\cdot\cos\left[1.4181\right]-\dfrac{2\sin^2\left[1.4181\right]}{1.4181}}{\pi\left(\dfrac{\sin\left[1.4181\right]}{1.4181}-\cos\left[1.4181\right]\right)}$$

$$K_1=0.1114$$

$$\Delta = \frac{\pi}{6} + \frac{5\theta}{12} = \frac{\pi}{6} + \frac{5\left[(123.0)\left(\frac{\pi}{180}\right)\right]}{12} = 1.4181\ rad$$

At the bottom of the shell:

$$\sigma_4^* = \frac{PR_m}{2t} + \frac{M_1}{K_1^* \pi R_m^2 t} = \frac{2074(31.5625)}{2(2.875)} + \frac{-356913.7}{0.2003(\pi)(31.5625)^2(2.875)}$$

$$\sigma_4^* = 11186.4\ psi$$

Where the coefficient K_1^* is found in VIII-2, Table 4.15.1,

$$K_1^* = \frac{\Delta + \sin\Delta \cdot \cos\Delta - \frac{2\sin^2\Delta}{\Delta}}{\pi\left(1 - \frac{\sin\Delta}{\Delta}\right)} = \frac{1.4181 + \sin[1.4181]\cdot\cos[1.4181] - \frac{2\sin^2[1.4181]}{1.4181}}{\pi\left(1 - \frac{\sin[1.4181]}{1.4181}\right)}$$

$$K_1^* = 0.2003$$

c) Acceptance Criteria:

$$\left\{\left|\sigma_1\right| = \left|11227.2\right|\ psi\right\} \le \left\{SE = 20000(1.0) = 20000\ psi\right\} \qquad True$$

$$\left\{\left|\sigma_2\right| = \left|11541.7\right|\ psi\right\} \le \left\{SE = 20000(1.0) = 20000\ psi\right\} \qquad True$$

$$\left\{\left|\sigma_3^*\right| = \left|11740.5\right|\ psi\right\} \le \left\{SE = 20000(1.0) = 20000\ psi\right\} \qquad True$$

$$\left\{\left|\sigma_4^*\right| = \left|11186.4\right|\ psi\right\} \le \left\{SE = 20000(1.0) = 20000\ psi\right\} \qquad True$$

Since all calculated stresses are positive (tensile), the compressive stress check per VIII-2, paragraph 4.15.3.3.c.2 is not required.

VIII-2, paragraph 4.15.3.4, Shear Stresses

The shear stress in the cylindrical shell without stiffening ring(s) that is not stiffened by a formed head, $\{a = 41.0\ in\} > \{0.5R_m = 0.5(31.5625) = 15.7813\ in\}$, is calculated as follows.

$$\tau_2 = \frac{K_2 T}{R_m t} = \frac{1.1229(33746.5)}{31.5625(2.875)} = 417.6\ psi$$

Where the coefficient K_2 is found in VIII-2, Table 4.15.1,

$$K_2 = \frac{\sin\alpha}{\pi - \alpha + \sin\alpha\cdot\cos\alpha} = \frac{\sin[1.9648]}{\pi - (1.9648) + \sin[1.9648]\cdot\cos[1.9648]} = 1.1229$$

$$\alpha = 0.95\left(\pi - \frac{\theta}{2}\right) = 0.95\left(\pi - \frac{123.0\left(\frac{\pi}{180}\right)}{2}\right) = 1.9648\ rad$$

Acceptance Criteria, where $C = 0.8$ for ferritic materials:

$$\left\{\left|\tau_2\right| = \left|417.6\right|\ psi\right\} \le \left\{CS = 0.8(20000) = 16000\ psi\right\}\ True$$

Per VIII-2, paragraph 4.15.3.5, Circumferential Stress

a) Maximum circumferential bending moment - the distribution of the circumferential bending moment at the saddle support is dependent on the use of stiffeners at the saddle location. For a cylindrical shell without a stiffening ring, the maximum circumferential bending moment is shown in VIII-2, Figure 4.15.6 Sketch (a) and is calculated as follows.

$$M_\beta = K_7 Q R_m = (0.0504)(50459.0)(31.5625) = 80267.7\ in-lbs$$

Where the coefficient K_7 is found in VIII-2, Table 4.15.1,

When $\dfrac{a}{R_m} \ge 1.0$, $K_7 = K_6$

$$\left\{\frac{a}{R_m} = \frac{41.0}{31.5625} = 1.2990\right\} \ge 1.0 \rightarrow K_7 = K_6 = 0.0504$$

$$K_6 = \left(\frac{\dfrac{3\cos\beta}{4}\left(\dfrac{\sin\beta}{\beta}\right)^2 - \dfrac{5\sin\beta\cos^2\beta}{4\beta} + \dfrac{\cos^3\beta}{2} - \dfrac{\sin\beta}{4\beta} + \dfrac{\cos\beta}{4} - \beta\sin\beta\left[\left(\dfrac{\sin\beta}{\beta}\right)^2 - \dfrac{1}{2} - \dfrac{\sin 2\beta}{4\beta}\right]}{2\pi\left[\left(\dfrac{\sin\beta}{\beta}\right)^2 - \dfrac{1}{2} - \dfrac{\sin 2\beta}{4\beta}\right]}\right)$$

$$K_6 = \left(\frac{\begin{array}{l} \dfrac{3\cos[2.0682]}{4}\left(\dfrac{\sin[2.0682]}{2.0682}\right)^2 - \dfrac{5\sin[2.0682]\cos^2[2.0682]}{4(2.0682)} + \\ \dfrac{\cos^3[2.0682]}{2} - \dfrac{\sin[2.0682]}{4(2.0682)} + \dfrac{\cos[2.0682]}{4} - \\ (2.0682)\sin[2.0682]\left[\left(\dfrac{\sin[2.0682]}{2.0682}\right)^2 - \dfrac{1}{2} - \left(\dfrac{\sin[2(2.0682)]}{4(2.0682)}\right)\right] \end{array}}{2\pi\left[\left(\dfrac{\sin[2.0682]}{2.0682}\right)^2 - \dfrac{1}{2} - \left(\dfrac{\sin[2(2.0682)]}{4(2.0682)}\right)\right]} \right) = 0.0504$$

$$\beta = \pi - \frac{\theta}{2} = \pi - \frac{123.0\left(\dfrac{\pi}{180}\right)}{2} = 2.0682 \ rad$$

b) Width of cylindrical shell - the width of the cylindrical shell that contributes to the strength of the cylindrical shell at the saddle location shall be determined as follows.

$$\{x_1, x_2\} \le \left\{0.78\sqrt{R_m t} = 0.78\sqrt{31.5625(2.875)} = 7.4302 \ in\right\}$$

If the width $(0.5b + x_1)$ extends beyond the limit of a, as shown in VIII-2, Figure 4.15.2, then the width x_1 shall be reduced such as not to exceed a.

$$\{(0.5b + x_1) = 0.5(8.0) + 7.4302 = 11.4302 \ in\} \le \{a = 41.0 \ in\} \qquad \textit{Satisfied}$$

c) Circumferential stresses in the cylindrical shell without stiffening ring(s).

The maximum compressive circumferential membrane stress in the cylindrical shell at the base of the saddle support shall be calculated as follows.

$$\sigma_6 = \frac{-K_5 Q k}{t(b + x_1 + x_2)} = \frac{-0.7492(50459.0)(0.1)}{2.875(8.0 + 7.4302 + 7.4302)} = -57.5 \ psi$$

Where the coefficient K_5 is found in Table 4.15.1,

$$K_5 = \frac{1 + \cos\alpha}{\pi - \alpha + \sin\alpha \cdot \cos\alpha} = \frac{1 + \cos[1.9648]}{\pi - (1.9648) + \sin[1.9648] \cdot \cos[1.9648]} = 0.7492$$

$$k = 0.1 \qquad \textit{when the vessel is welded to the saddle support}$$

The circumferential compressive membrane plus bending stress at Points G and H of VIII-2,

Figure 4.15.6 Sketch (a) is determined as follows.

If $L \geq 8R_m$, then the circumferential compressive membrane plus bending stress shall be computed using VIII-2, Equation (4.15.24).

Since $\{L = 292.0\ in\} \geq \{8R_m = 8(31.5625) = 252.5\ in\}$, the criterion is satisfied.

$$\sigma_7 = \frac{-Q}{4t(b + x_1 + x_2)} - \frac{3K_7 Q}{2t^2}$$

$$\sigma_7 = \frac{-(50459.0)}{4(2.875)(8 + 7.4302 + 7.4302)} - \frac{3(0.0504)(50459.0)}{2(2.875)^2} = -653.4\ psi$$

The stresses at σ_6 and σ_7 may be reduced by adding a reinforcement or wear plate at the saddle location that is welded to the cylindrical shell.

A wear plate was not specified in this problem.

Acceptance Criteria:

$$\{|\sigma_6| = |57.5|\ psi\} \leq \{S = 20000\ psi\} \qquad True$$

$$\{|\sigma_7| = |653.4|\ psi\} \leq \{1.25S = 1.25(20000) = 25000\ psi\} \qquad True$$

VIII-2, paragraph 4.15.3.6, Horizontal Splitting Force

The horizontal force at the minimum section at the low point of the saddle is given by VIII-2, Equation (4.15.42). The saddle shall be designed to resist this force.

$$F_h = Q\left(\frac{1 + \cos\beta - 0.5\sin^2\beta}{\pi - \beta + \sin\beta \cdot \cos\beta}\right)$$

$$F_h = (50459.0)\left(\frac{1 + \cos[2.0682] - 0.5\sin^2[2.0682]}{\pi - (2.0682) + \sin[2.0682] \cdot \cos[2.0682]}\right) = 10545.1\ lbs$$

Note: The horizontal splitting force is equal to the sum of all of the horizontal reactions at the saddle due to the weight loading of the vessel. The splitting force is used to calculate tension stress and bending stress in the web of the saddle. The following provides one possible method of calculating the tension and bending stress in the web and its acceptance criteria. However, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

The membrane stress is given by,

$$\left\{\sigma_t = \frac{F_h}{A_s}\right\} \leq \{0.6S_y\}$$

where A_s is the cross-sectional area of the web at the low point of the saddle with units of in^2, and S_y is the yield stress of the saddle material with units of psi.

The bending stress is given by,

$$\left\{\sigma_b = \frac{F_h \cdot d \cdot c}{I}\right\} \le \left\{0.66S_y\right\}$$

where d is the moment arm of the horizontal splitting force, measured from the center of gravity of the saddle arc to the bottom of the saddle baseplate with units of in, c is the distance from the centroid of the saddle composite section to the extreme fiber with units of in, I is the moment of inertia of the composite section of the saddle with units of in^4, and S_y is the yield stress of the saddle material with units of psi.

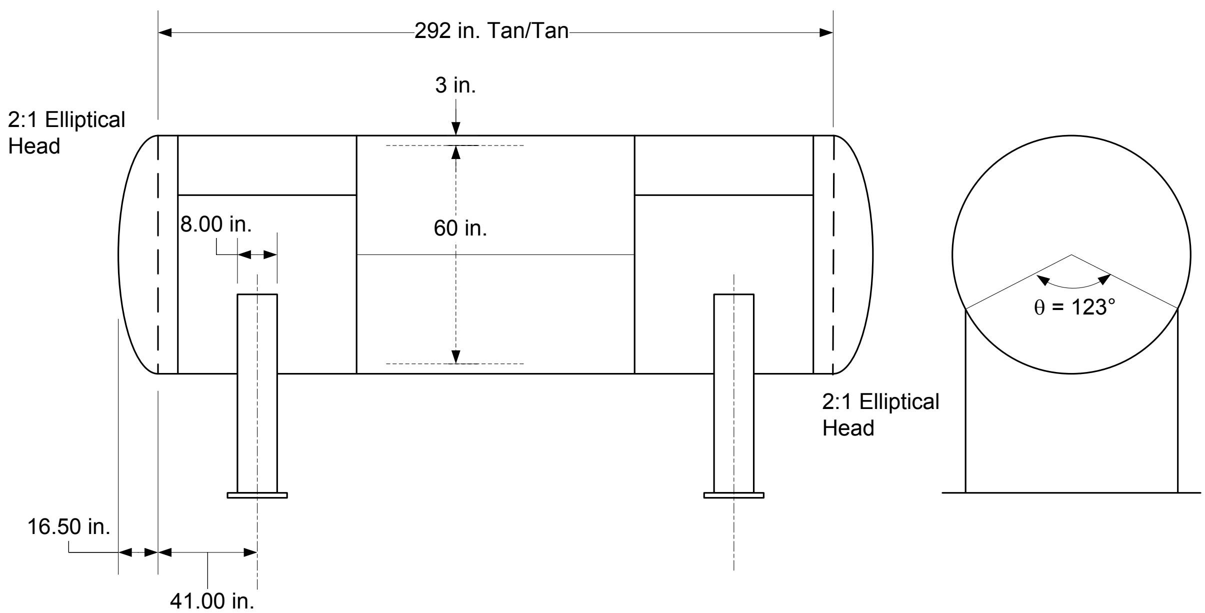


Figure E4.15.1 – Saddle Details

4.15.2 Example E4.15.2 – Vertical Vessel, Skirt Design

Determine if the proposed cylindrical vessel skirt is adequately designed considering the following loading conditions.

Skirt Data:

- Material = $SA-516,\ Grade\ 70$
- Design Temperature = $300°F$
- Skirt Inside Diameter = $150.0\ in$
- Thickness = $0.625\ in$
- Corrosion Allowance = $0.0\ in$
- Length of Skirt = $147.0\ in$
- Allowable Stress at Design Temperature = $20000\ psi$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength at Design Temperature = $33600\ psi$
- Applied Axial Force = $-427775\ lbs$
- Applied Net Section Bending Moment = $21900435\ in-lbs$

Adjust variable for corrosion and determine outside dimensions.

$$D = 150.0 + 2(Corrosion\ Allowance) = 150.0 + 2(0.0) = 150.0\ in$$

$$R = 0.5D = 0.5(150.0) = 75.0\ in$$

$$t = 0.625 - Corosion\ Allowance = 0.625 - 0.0 = 0.625\ in$$

$$D_o = 150.0 + 2(Uncorroded\ Thickness) = 150.0 + 2(0.625) = 151.25\ in$$

$$R_o = 0.5D_o = 0.5(151.25) = 75.625\ in$$

Section VIII, Division 1 Solution

VIII-1 does not provide rules on the loadings to be considered in the design of a vessel. However, UG-22 requires consideration of such loadings and the provisions of U-2(g) apply. This example provides one possible method of satisfying U-2(g); however, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

This example uses VIII-2, paragraph 4.1 which provides specific requirements to account for both loads and load case combinations used in the design of a vessel. These loads and load case combinations (Table 4.1.1 and Table 4.1.2 of VIII-2, respectively) are shown in this example problem in Table E4.15.2.1 for reference.

Additionally, VIII-1 does not provide a procedure for the calculation of combined stresses. Paragraph 4.3.10.2, in VIII-2, does provide a procedure and this procedure is used in this example problem with modifications to address specific requirements of VIII-1.

In accordance with VIII-2, paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.15.2.2 and Table E4.15.2.3, Load Case 6 is determined to be a potential governing load case. The pressure, net section axial force, and bending moment at the location of interest for Load Case 6 are:

$P = P_s = 0.0\ psi$

$F_6 = -427775\ lbs$

$M_6 = 21900435\ in-lbs$

a) STEP 1 – Calculate the membrane stress for the cylindrical shell. Note that the circumferential membrane stress, $\sigma_{\theta m}$, is determined based on the equations in UG-27(c)(1) and the exact strength of materials solution for the longitudinal membrane stress, σ_{sm}, is used in place of the approximate solution provided in UG-27(c)(2). The shear stress is computed based on the known strength of materials solution. For the skirt, weld joint efficiency is set as $E = 1.0$.

Note: θ is defined as the angle measured around the circumference from the direction of the applied bending moment to the point under consideration. For this example problem $\theta = 0.0\ deg$ to maximize the bending stress.

$$\sigma_{\theta m} = \frac{1}{E}\left(\frac{PR}{t} + 0.6P\right) = \frac{1}{1.0}\left(\frac{0.0(75.0)}{0.625} + 0.6(0.0)\right) = 0.0\ psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\frac{PD^2}{D_o^2 - D^2} + \frac{4F}{\pi\left(D_o^2 - D^2\right)} \pm \frac{32MD_o\cos[\theta]}{\pi\left(D_o^4 - D^4\right)}\right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(0.0 + \frac{4(-427775)}{\pi\left((151.25)^2 - (150.0)^2\right)} \pm \frac{32(21900435)(151.25)\cos[0.0]}{\pi\left((151.25)^4 - (150.0)^4\right)}\right)$$

$$\sigma_{sm} = \begin{Bmatrix} -1446.4001 + 1974.6020 = 528.2019\ psi \\ -1446.4001 - 1974.6020 = -3421.0021\ psi \end{Bmatrix}$$

$$\tau = \frac{16M_tD_o}{\pi\left(D_o^4 - D^4\right)} = \frac{16(0.0)(151.25)}{\pi\left((151.25)^4 - (150.0)^4\right)} = 0.0\ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{(\sigma_{\theta m} - \sigma_{sm})^2 + 4(\tau)^2}\right)$$

$$\sigma_1 = \begin{Bmatrix} 0.5\left(0 + (528.2019) + \sqrt{(0 - (528.2019))^2 + 4(0)^2}\right) = 528.2019\ psi \\ 0.5\left(0 + (-3421.0021) + \sqrt{(0 - (-3421.0021))^2 + 4(0)^2}\right) = 0.0\ psi \end{Bmatrix}$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2}\right)$$

$$\sigma_2 = \left\{ \begin{array}{l} 0.5\left(0 + (528.2019) - \sqrt{\left(0 - (528.2019)\right)^2 + 4(0)^2}\right) = 0.0 \ \ psi \\ 0.5\left(0 + (-3421.0021) - \sqrt{\left(0 - (-3421.0021)\right)^2 + 4(0)^2}\right) = -3421.0021 \ \ psi \end{array} \right\}$$

$$\sigma_3 = \sigma_r = -0.5P = 0.0 \ \ psi$$

c) STEP 3 – Check the allowable stress acceptance criteria.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]^{0.5}$$

$$\sigma_e = \left\{ \begin{array}{l} \dfrac{1}{\sqrt{2}}\left[\left(0 - (528.2019)\right)^2 + \left((528.2019) - 0\right)^2 + \left((0) - (0)\right)^2\right]^{0.5} = 528.2 \ \ psi \\ \dfrac{1}{\sqrt{2}}\left[\left(0 - (-3421.0021)\right)^2 + \left((-3421.0021) - 0\right)^2 + \left((0) - (0)\right)^2\right]^{0.5} = 3421.0 \ \ psi \end{array} \right\}$$

$$\left\{ \begin{array}{l} \sigma_e = 528.2 \\ \sigma_e = 3421.0 \end{array} \right\} \le \left\{ S = 20000 \ \ psi \right\} \qquad \textit{True}$$

Note that VIII-2 uses an acceptance criteria based on von Mises Stress. VIII-1 typically uses the maximum principle stress in the acceptance criteria. Therefore,

$$\max\left[\sigma_1, \ \sigma_2, \ \sigma_3\right] \le S$$

$$\left\{ \max\left[528.2, \ |-3421.0|, \ 0.0\right] = 3421.0 \ \ psi \right\} \le \left\{ S = 20000 \ \ psi \right\} \qquad \textit{True}$$

Since the maximum tensile principal stress is less than the acceptance criteria, the shell section is adequately designed.

d) STEP 4 – For cylindrical and conical shells, if the meridional stress, σ_{sm} is compressive, then check the allowable compressive stress per UG-23(b).

Since σ_{sm} is compressive, $\left\{\sigma_{sm} = -3421.0 \ \ psi < 0\right\}$, a compressive stress check is required.

Evaluate per paragraph UG-23(b). The maximum allowable longitudinal compressive stress to be used in the design of cylindrical shells or tubes, either seamless or butt welded, subjected to loadings that produce longitudinal compression in the shell or tube shall be the smaller of the maximum allowable tensile stress value shown in STEP 3 or the value of the factor B determined by the following procedure where the joint efficiency for butt welded joints shall be taken as unity.

1) STEP 4.1 – Using the selected values of t and R, calculate the value of factor A using the following formula:

$$A = \frac{0.125}{\left(\dfrac{R_o}{t}\right)} = \frac{0.125}{\left(\dfrac{75.625}{0.625}\right)} = 0.0010$$

2) STEP 4.2 – Using the value of A calculated in STEP 4.1, enter the applicable material chart in Subpart 3 of Section II, Part D for the material under consideration. Move vertically

to an intersection with the material/temperature line for the design temperature. Interpolation may be made between lines for intermediate temperatures. In cases where the value of A falls to the right of the material/temperature line, assume and intersection with the horizontal projection of the upper end of the material/temperature line. For values of A falling to the left of the material/temperature line, see STEP 4.4.

Per Section II Part D, Table 1A, a material specification of $SA-516,\ Grade\ 70,\ Normalized$ is assigned an External Pressure Chart No. CS-2.

3) STEP 4.3 – From the intersection obtained in Step 4.2, move horizontally to the right and read the value of factor B. This is the maximum allowable compressive stress for the values of t and R_o used in STEP 4.1.

$$B = 12300\ psi$$

4) STEP 4.4 – For values of A falling to the left of the applicable material/temperature line, the value of B shall be calculated using the following formula:

$$B = \frac{AE}{2} \qquad Not\ required$$

5) STEP 4.5 – Compare the calculated value of B obtained in STEPS 4.3 or 4.4 with the computed longitudinal compressive stress in the cylindrical shell or tube, using the selected values of t and R_o. If the value of B is smaller than the computed compressive stress, a greater value of t must be selected and the design procedure repeated until a value of B is obtained that is greater than the compressive stress computed for the loading on the cylindrical shell or tube.

$$\left\{\sigma_{sm} = \left|-3421.0\right|\ psi\right\} \le \left\{B = 12300\ psi\right\} \qquad True$$

The allowable compressive stress criterion is satisfied.

Section VIII, Division 2 Solution with VIII-1 Allowable Stresses

Evaluate per VIII-2, paragraph 4.3.10.

The loads transmitted to the base of the skirt are given in the Table E4.15.2.2. Note that this table is given in terms of the load parameters shown in VIII-2, Table 4.1.1 and Table 4.1.2. (Table E4.15.2.1 of this example). As shown in Table E4.15.2.1, the acceptance criteria is that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition.

In accordance with VIII-2, paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.15.2.3, Load Case 6 is determined to be a potential governing load case. The pressure, net section axial force, and bending moment at the location of interest for Load Case 6 are:

$$P = P_s = 0.0\ psi$$
$$F_6 = -427775\ lbs$$
$$M_6 = 21900435\ in-lbs$$

Determine applicability of the rules of VIII-2, paragraph 4.3.10 based on satisfaction of the following requirements.

The section of interest is at least $2.5\sqrt{Rt}$ away from any major structural discontinuity.

$$2.5\sqrt{Rt} = 2.5\sqrt{(75.0)(0.625)} = 17.1163\ in$$

Shear force is not applicable.

The shell R/t ratio is greater than 3.0, or:

$$\left\{R/t = \frac{75.0}{0.625} = 120.0\right\} > 3.0 \qquad\qquad True$$

a) STEP 1 – Calculate the membrane stress for the skirt, with a weld joint efficiency of $E = 1.0$. Note that the maximum bending stress occurs at $\theta = 0.0\ deg$.

$$\sigma_{\theta m} = \frac{P}{E(D_o - D)} = \frac{P}{E(151.25 - 150.0)} = 0.0\ psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\frac{PD^2}{D_o^2 - D^2} + \frac{4F}{\pi\left(D_o^2 - D^2\right)} \pm \frac{32MD_o\cos[\theta]}{\pi\left(D_o^4 - D^4\right)}\right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(0.0 + \frac{4(-427775)}{\pi\left((151.25)^2 - (150.0)^2\right)} \pm \frac{32(21900435)(151.25)\cos[0.0]}{\pi\left((151.25)^4 - (150.0)^4\right)}\right)$$

$$\sigma_{sm} = \begin{Bmatrix} -1446.4001 + 1974.6020 = 528.2019\ psi \\ -1446.4001 - 1974.6020 = -3421.0021\ psi \end{Bmatrix}$$

$$\tau = \frac{16M_tD_o}{\pi\left(D_o^4 - D^4\right)} = \frac{16(0.0)(151.25)}{\pi\left((151.25)^4 - (150.0)^4\right)} = 0.0\ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{(\sigma_{\theta m} - \sigma_{sm})^2 + 4(\tau)^2}\right)$$

$$\sigma_1 = \begin{Bmatrix} 0.5\left(0 + (528.2019) + \sqrt{(0 - (528.2019))^2 + 4(0)^2}\right) = 528.2019\ psi \\ 0.5\left(0 + (-3421.0021) + \sqrt{(0 - (-3421.0021))^2 + 4(0)^2}\right) = 0.0\ psi \end{Bmatrix}$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{(\sigma_{\theta m} - \sigma_{sm})^2 + 4\tau^2}\right)$$

$$\sigma_2 = \begin{Bmatrix} 0.5\left(0 + 528.2019 - \sqrt{(0 - (528.2019))^2 + 4(0)^2}\right) = 0.0\ psi \\ 0.5\left(0 + (-3421.0021) - \sqrt{(0 - (-3421.0021))^2 + 4(0)^2}\right) = -3421.0021\ psi \end{Bmatrix}$$

$$\sigma_3 = \sigma_r = -0.5P = 0.0\ psi$$

c) STEP 3 – Check the allowable stress acceptance criteria.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]^{0.5}$$

$$\sigma_e = \begin{Bmatrix} \frac{1}{\sqrt{2}}\left[(0-(528.2019))^2+((528.2019)-0)^2+((0)-(0))^2\right]^{0.5} = 528.2\ psi \\ \frac{1}{\sqrt{2}}\left[(0-(-3421.0021))^2+((-3421.0021)-0)^2+((0)-(0))^2\right]^{0.5} = 3421.0\ psi \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_e = 528.2 \\ \sigma_e = 3421.0 \end{Bmatrix} \leq \{S = 20000\ psi\} \qquad True$$

Since the equivalent stress is less than the acceptance criteria, the shell section is adequately designed.

d) STEP 4 – For cylindrical and conical shells, if the axial membrane stress, σ_{sm} is compressive, then VIII-2, Equation (4.3.45) shall be satisfied where F_{xa} is evaluated using paragraph 4.4.12.2 with $\lambda = 0.15$.

$$\sigma_{sm} \leq F_{xa}$$

Since σ_{sm} is compressive, $\{\sigma_{sm} = -3421.0021\ psi < 0\}$, a buckling check is required.

VIII-2, paragraph 4.4.12.2.b – Axial Compressive Stress Acting Alone.

In accordance with VIII-2, paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

The design factor FS used in VIII-2, paragraph 4.4.12.2.b is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in VIII-2, paragraph 4.4.2. An initial calculation is required to determine the value of F_{xa} by setting $FS = 1.0$, with $F_{ic} = F_{xa}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.b.

For $\lambda_c = 0.15$, (Local Buckling)

$$F_{xa} = \min\left[F_{xa1},\ F_{xa2}\right]$$

$$\frac{D_o}{t} = \frac{151.25}{0.625} = 242.0$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{147.0}{\sqrt{75.625(0.625)}} = 21.3818$$

Since $135 < \frac{D_o}{t} \leq 600$, calculate F_{xa1} as follows with an initial value of $FS = 1.0$.

$$F_{xa1} = \frac{466 S_y}{FS\left(331 + \frac{D_o}{t}\right)} = \frac{466(33600)}{1.0\left(331 + \frac{151.25}{0.625}\right)} = 27325.6545 \ psi$$

The value of F_{xa2} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{xa2} = \frac{F_{xe}}{FS}$$

$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since $\frac{D_o}{t} \leq 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \frac{D_o}{t}\right)}, \ 0.9\right]$$

Since $M_x \geq 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389 + \frac{151.25}{0.625}}, \ 0.9\right] = 0.6482$$

Therefore,

$$F_{xe} = \frac{0.6482(28.3E+06)(0.625)}{151.25} = 75801.9008 \ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{75801.9008}{1.0} = 75801.9008 \ psi$$

$$F_{xa} = \min[27325.6545, \ 75801.9008] = 27325.6545 \ psi$$

With a value of $F_{ic} = F_{xa} = 27325.6545$, in accordance with VIII-2, paragraph 4.4.2, the value of FS is determined as follows.

Since $\{0.55S_y = 0.55(33600) = 18480\} \leq \{F_{ic} = 27325.6545\} \leq \{S_y = 33600\}$,

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{27325.6545}{33600}\right) = 1.8044$$

Using this computed value of $FS = 1.8044$ in paragraph 4.4.12.2.b, F_{xa} is calculated as follows.

$$F_{xa1} = \frac{466S_y}{FS\left(331 + \frac{D_o}{t}\right)} = \frac{466(33600)}{1.8044\left(331 + \frac{151.25}{0.625}\right)} = 15143.9007 \ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{75801.9008}{1.8044} = 42009.4773 \ psi$$

$$F_{xa} = \min[15143.9007,\ 42009.4773] = 15143.9007 \ psi$$

Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria

$$\{\sigma_{sm} = 3421.0 \ psi\} \le \{F_{xa} = 15143.9 \ psi\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

Table E4.15.2.1: Design Loads and Load Combinations from VIII-2

Table 4.1.1 – Design Loads

Design Load Parameter	Description
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g. catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g. skirts, lugs, saddles, and legs), and appurtenances (e.g. platforms, ladders, etc.) • Weight of vessel contents under operating and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping
L	• Appurtenance Live loading • Effects of fluid flow
E	Earthquake loads (see ASCE 7 for the specific definition of the earthquake load, as applicable)
W	Wind Loads
S	Snow Loads
F	Loads due to Deflagration

Table 4.1.2 – Design Load Combinations

Design Load Combination (1)	General Primary Membrane Allowable Stress (2)
$P+P_s+D$	S
$P+P_s+D+L$	S
$P+P_s+D+S$	S
$0.9P+P_s+D+0.75L+0.75S$	S
$0.9P+P_s+D+(W\,or\,0.7E)$	S
$0.9P+P_S+D+0.75(W\,or\,0.7E)+0.75L+0.75S$	S
$0.6D+(W\,or\,0.7E)$ (3)	S
P_s+D+F	See Annex 4.D

Notes
1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) S is the allowable stress for the load case combination (see paragraph 4.1.5.3.c)
3) This load combination addresses an overturning condition. If anchorage is included in the design, consideration of this load combination is not required.

Table E4.15.2.2: Design Loads (Net-Section Axial Force and Bending Moment) at the Base of The Skirt

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a); The skirt is not pressurized.	$P = 0.0$
P_s	Static head from liquid or bulk materials (e.g. catalyst); The skirt does not contain liquid head.	$P_s = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest.	$D_F = -363500\ lbs$ $D_M = 0.0\ in-lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = -85700\ lbs$ $L_M = 90580\ in-lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 18550000\ in-lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 29110000\ in-lbs$
S	Snow Loads	$S_F = 0.0\ lbs$ $S_M = 0.0\ in-lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in-lbs$

Based on these loads, the skirt is required to be designed for the load case combinations shown in Table E4.15.2.3. Note that this table is given in terms of the load combinations shown in VIII-2, Table 4.1.2 (Table E4.15.2.1 of this example).

Table E4.15.2.3: Load Case Combination at the Base of The Skirt

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P+P_s+D$	$P=P_s=0.0\ psi$ $F_1=-363500\ lbs$ $M_1=0.0\ in-lbs$	S
2	$P+P_s+D+L$	$P=P_s=0.0\ psi$ $F_2=-449200\ lbs$ $M_2=90580\ in-lbs$	S
3	$P+P_s+D+S$	$P=P_s=0.0\ psi$ $F_3=-363500\ lbs$ $M_3=0.0\ in-lbs$	S
4	$0.9P+P_s+D+0.75L+0.75S$	$P=P_s=0.0\ psi$ $F_4=-427775\ lbs$ $M_4=67935\ in-lbs$	S
5	$0.9P+P_s+D+(W\ or\ 0.7E)$	$P=P_s=0.0\ psi$ $F_5=-363500\ lbs$ $M_5=29110000\ in-lbs$	S
6	$\begin{pmatrix}0.9P+P_S+D+0.75(W\ or\ 0.7E)+\\ 0.75L+0.75S\end{pmatrix}$	$P=P_s=0.0\ psi$ $F_6=-427775\ lbs$ $M_6=21900435\ in-lbs$	S
7	$0.6D+(W\ or\ 0.7E)$ Anchorage is included in the design; therefore, consideration of this load combination is not required.	$F_7=-218100\ lbs$ $M_7=29110000\ in-lbs$	S
8	P_s+D+F	$P=P_s=0.0\ psi$ $F_8=-363500\ lbs$ $M_8=0.0\ in-lbs$	See Annex 4.D

4.16 Flanged Joints

4.16.1 Example E4.16.1 - Integral Type

Determine if the stresses in the heat exchanger girth flange are with acceptable limits, considering the following design conditions. The flange is of an integral type and is attached to a cylindrical shell with a Category C, Type 1 butt weld and has been 100% radiographically examined. See Figure E4.16.1.

General Data:

- Cylinder Material = $SA-516,\ Grade\ 70$
- Design Conditions = $135\ psig@650°F$
- Allowable Stress at Design Temperature = $18800\ psi$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Corrosion Allowance = $0.125\ in$

Flange Data

- Material = $SA-105$
- Allowable Stress at Design Temperature = $17800\ psi$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Modulus of Elasticity at Design Temperature = $26.0E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$

Bolt Data

- Material = $SA-193,\ Grade\ B7$
- Allowable Stress at Design Temperature = $25000\ psi$
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Diameter = $0.75\ in$
- Number of Bolts = 44
- Root area = $0.302\ in^2$

Gasket Data

- Material = Flat Metal Jacketed (Iron/Soft Steel)
- Gasket Factor = 3.75
- Seating Stress = $7600\ psi$
- Inside Diameter = $29.0\ in$
- Outside Diameter = $30.0\ in$

Design rules for bolted flange connections with ring type gaskets are provided in VIII-1 Mandatory Appendix 2. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.16. However, there are differences to be noted, including; a step-by-step design procedure, nomenclature with regard to operating and gasket seating bolt loads, the inclusion of a flange moment due to externally applied axial forces and bending moment, and minor

differences in bolt spacing criteria. Therefore, while the example problem will be presented for use with VIII-1, Appendix 2, references to VIII-2 paragraphs will be provided, as applicable.

Evaluate the girth flange in accordance with VIII-1, Appendix 2.

Establish the design conditions and gasket reaction diameter, (VIII-2, paragraph 4.16.6).

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

$$P = 135\ psig\ at\ 650°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 2-5.1 (VIII-2, Table 4.16.1).

$$m = 3.75$$
$$y = 7600\ psi$$

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5(GOD - GID) = 0.5(30.0 - 29.0) = 0.500\ in$$

From Table 2-5.2 (VIII-2, Table 4.16.3), Facing Sketch Detail 2, Column II,

$$b_o = \frac{w + 3N}{8} = \frac{0.125 + 3(0.500)}{8} = 0.2031\ in$$

Where,

$$w = raised\ nubbin\ width = 0.125\ in$$

For $b_o \le 0.25\ in$,

$$b = b_o = 0.2031\ in$$

Therefore, from paragraph 2-3 the location of the gasket reaction is calculated as follows .

$$G = mean\ diameter\ of\ the\ gasket\ contact\ face$$
$$G = 0.5(30.0 + 29.0) = 29.5\ in$$

Paragraph 2-5 – Calculate the design bolt load for the operating and gasket seating conditions, (VIII-2, paragraph 4.16.6).

a) STEP 1 – Paragraph 2-5(c)(1), determine the design bolt load for the operating condition.

$$W_{m1} = H + H_p = \frac{\pi}{4}G^2P + 2b\pi GmP \qquad for\ non-self-energized\ gaskets$$

$$W_{m1} = \frac{\pi}{4}(29.5)^2(135) + 2(0.2031)\pi(29.5)(3.75)(135) = 111329.5\ lbs$$

b) STEP 2 – Paragraph 2-5(c)(2), determine the design bolt load for the gasket seating condition.

$$W_{m2} = \pi bGy \qquad for\ non-self-energized\ gaskets$$
$$W_{m2} = \pi(0.2031)(29.5)(7600) = 143052.5\ lbs$$

c) STEP 3 – Paragraph 2-5(d), determine the total required and actual bolt areas.

The total cross-sectional area of bolts A_m required for both the operating conditions and gasket seating is determined as follows.

$$A_m = \max\left[A_{m1},\, A_{m2}\right] = \max\left[4.4532,\, 5.7221\right] = 5.7221\ in^2$$

Where,

$$A_{m1} = \frac{W_{m1}}{S_b} = \frac{111329.5}{25000} = 4.4532\ in^2$$

$$A_{m2} = \frac{W_{m2}}{S_a} = \frac{143052.5}{25000} = 5.7221\ in^2$$

The actual bolt area A_b is calculated as follows.

$$A_b = \left(Number\ of\ bolts\right)\left(Root\ area\ of\ one\ bolt\right) = 44\left(0.302\right) = 13.2880\ in^2$$

Verify that the actual bolt area is equal to or greater than the total required area.

$$\left\{A_b = 13.2880\ in^2\right\} \geq \left\{A_m = 5.7221\ in^2\right\} \qquad True$$

For vessels in lethal service or when specified by the user or his designated agent, the maximum bolt spacing shall not exceed the following:

$$B_{s\max} = 2a + \frac{6t}{m+0.5} = 2\left(0.75\right) + \frac{6\left(1.4375\right)}{\left(3.75+0.5\right)} = 3.5294\ in$$

Where,

$$a = nominal\ bolt\ diameter$$

$$t = flange\ thickness$$

$$m = gasket\ \ factor$$

d) STEP 4 – Paragraph 2-5(e), determine the flange design bolt load.

For operating conditions,

$$W = W_{m1} = 111329.5\ lbs$$

For gasket seating,

$$W = \frac{\left(A_m + A_b\right)S_a}{2} = \frac{\left(5.7221+13.2880\right)25000}{2} = 237626.3\ lbs$$

Commentary: VIII-2 design procedure to determine the design bolt load, paragraph 4.16.6:

1) The nomenclature differences include:

W_o, bolt load for operating conditions and flange design load bolt load (VIII-1 W_{m1}, W).

W_{gs}, design bolt load for the gasket seating condition (VIII-1, W_{m2}).

W_g, flange design bolt load for gasket seating (VIII-1, W).

2) When calculating the required total cross-sectional area of bolts using the VIII-2 procedure,

the designer has the ability to add an externally applied net-section axial force, F_A and bending moment, M_E to the bolt load W_o for the operating condition. This is shown in the following equation.

$$A_m = \max\left[\left(\frac{W_o + F_A + \dfrac{4M_E}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right]$$

3) The bolt spacing criteria of VIII-1 and VIII-2 are similar, but there are some minor differences regarding the application of the two. This will be further discussed later in the example problem.

Commentary: VIII-1, Appendix 2 does not include an overall step-by-step procedure to design a flange. However, an organized procedure of the steps taken when designing a flange is presented in VIII-2, paragraph 4.16.7. The procedure is applicable to VIII-1, Appendix 2 and is presented in this example problem in an effort to assist the designer.

VIII-2, paragraph 4.16.7, Flange Design Procedure. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings. The procedure incorporates both a strength check and a rigidity check for flange rotation.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$P = 135\ psig\ at\ 650°F$$

b) STEP 2 – Determine the design bolt loads for operating condition W, and the gasket seating condition W, and the corresponding actual bolt load area A_b, (VIII-1, paragraph 2-5).

$$W = 111329.5\ lbs \qquad Operating\ Condition$$

$$W = 237626.3\ lbs \qquad Gasket\ Seating$$

$$A_b = 13.2880\ in^2$$

c) STEP 3 – Determine an initial flange geometry (see Figure E4.16.1) in addition to the information required to determine the bolt load, the following geometric parameters are required, (VIII-1, paragraph 2-3).

1) Flange bore

$$B = \left[26.0 + 2\left(Corrosion\ Allowance\right)\right] = \left[26.0 + 2\left(0.125\right)\right] = 26.25\ in$$

2) Bolt circle diameter

$$C = 31.25\ in$$

3) Outside diameter of the flange

$$A = 32.875\ in$$

4) Flange thickness

$$t = 1.625 - 0.1875 = 1.4375\ in$$

5) Thickness of the hub at the large end

$$g_1 = \left[0.5\left(Hub\ OD\ at\ Back\ of\ Flange - Uncorroded\ Bore\right) - Corrosion\ Allowance\right]$$

$$g_1 = \left[0.5\left(27.625 - 26.0\right) - 0.125\right] = 0.6875\ in$$

6) Thickness of the hub at the small end

$$g_0 = \left(Hub\ Thickness\ at\ Cylinder\ Attachment - Corrosion\ Allowance\right)$$

$$g_0 = \left(0.4375 - 0.125\right) = 0.3125\ in$$

7) Hub length

$$h = 2.125\ in$$

d) STEP 4 – Determine the flange stress factors using the equations/direct interpretation from Table 2-7.1 and Fig. 2-7.1 – Fig. 2-7.6 and paragraph 2-3.

Fig. 2-7.1:

$$K = \frac{A}{B} = \frac{32.875}{26.25} = 1.2524$$

$$Y = \frac{1}{K-1}\left[0.66845 + 5.71690\left(\frac{K^2 \log_{10} K}{K^2 - 1}\right)\right]$$

$$Y = \frac{1}{1.2524-1}\left[0.66845 + 5.71690\frac{(1.2524)^2 \log_{10}[1.2524]}{(1.2524)^2 - 1}\right] = 8.7565$$

$$T = \frac{K^2\left(1 + 8.55246\log_{10} K\right) - 1}{\left(1.04720 + 1.9448K^2\right)\left(K-1\right)} = \frac{(1.2524)^2\left(1 + 8.55246\log_{10}[1.2524]\right) - 1}{\left(1.04720 + 1.9448(1.2524)^2\right)\left(1.2524 - 1\right)} = 1.8175$$

$$U = \frac{K^2\left(1 + 8.55246\log_{10} K\right) - 1}{1.36136\left(K^2 - 1\right)\left(K-1\right)} = \frac{(1.2524)^2\left(1 + 8.55246\log_{10}[1.2524]\right) - 1}{1.36136\left((1.2524)^2 - 1\right)\left(1.2524 - 1\right)} = 9.6225$$

$$Z = \frac{\left(K^2 + 1\right)}{\left(K^2 - 1\right)} = \frac{\left((1.2524)^2 + 1\right)}{\left((1.2524)^2 - 1\right)} = 4.5180$$

Fig. 2-7.2:

$$\frac{h}{h_o} = \frac{h}{\sqrt{Bg_0}} = \frac{2.125}{\sqrt{(26.25)(0.3125)}} = \frac{2.125}{2.8641} = 0.7419$$

$$\frac{g_1}{g_0} = \frac{0.6875}{0.3125} = 2.2000$$

Interpretation of Fig. 2-7.2, $F \approx 0.77$. From the equations of Table 2-7.1, $F = 0.7677$.

Fig. 2-7.3:

With $\frac{h}{h_o} = 0.7419$ and $\frac{g_1}{g_0} = 2.2000$:

Interpretation of Fig. 2-7.3, $V \approx 0.16$. From the equations of Table 2-7.1, $V = 0.1577$.

Fig. 2-7.6:

With $\frac{h}{h_o} = 0.7419$ and $\frac{g_1}{g_0} = 2.2000$:

Interpretation of Fig. 2-7.6, $f = 1.0$. From the equations of Table 2-7.1, $f = 1.0$.

Paragraph 2-3:

$$d = \frac{Ug_0^2 h_o}{V} = \frac{(9.6225)(0.3125)^2(2.8641)}{0.1577} = 17.0665\ in$$

$$e = \frac{F}{h_o} = \frac{0.7677}{2.8641} = 0.2680$$

$$L = \frac{te+1}{T} + \frac{t^3}{d} = \frac{1.4375(0.2680)+1}{1.8175} + \frac{(1.4375)^3}{17.0665} = 0.9362$$

Commentary: VIII-2, Table 4.16.4 provides the equations to determine the flange stress factors Y, T, U, Z as a function of K as provided for in VIII-1, Fig. 2-7.1, and the flange factors d, e, L located in VIII-1, paragraph 2-3. Similarly, VIII-2, Table 4.16.5 provides regressed curve-fit equations of the flange stress factors F, V, f as provided in VIII-1, Fig. 2-7.2, Fig. 2-7.3 and Fig. 2-7.6, respectively. The curve-fit equations are shown for informational purposes with the variable substitution of $X_h = h/h_o$ and $X_g = g_1/g_o$.

$$F = \begin{pmatrix} 0.897697 - 0.297012\ln\left[X_g\right] + 9.5257\left(10^{-3}\right)\ln\left[X_h\right] + \\ 0.123586\left(\ln\left[X_g\right]\right)^2 + 0.0358580\left(\ln\left[X_h\right]\right)^2 - \\ 0.194422\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right) - 0.0181259\left(\ln\left[X_g\right]\right)^3 + \\ 0.0129360\left(\ln\left[X_h\right]\right)^3 - 0.0377693\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right)^2 + \\ 0.0273791\left(\ln\left[X_g\right]\right)^2\left(\ln\left[X_h\right]\right) \end{pmatrix}$$

$$F = \begin{pmatrix} 0.897697 - 0.297012\ln[2.20] + 9.5257\left(10^{-3}\right)\ln[0.7419] + \\ 0.123586\left(\ln[2.20]\right)^2 + 0.0358580\left(\ln[0.7419]\right)^2 - \\ 0.194422\left(\ln[2.20]\right)\left(\ln[0.7419]\right) - 0.0181259\left(\ln[2.20]\right)^3 + \\ 0.0129360\left(\ln[0.7419]\right)^3 - 0.0377693\left(\ln[2.20]\right)\left(\ln[0.7419]\right)^2 + \\ 0.0273791\left(\ln[2.20]\right)^2\left(\ln[0.7419]\right) \end{pmatrix}$$

$$F = 0.7695$$

$$For\ 0.5 \le X_h \le 2.0,$$

$$V = \left(\begin{array}{l} 0.0144868 - \dfrac{0.135977}{X_g} - \dfrac{0.0461919}{X_h} + \dfrac{0.560718}{X_g^2} + \dfrac{0.0529829}{X_h^2} + \\ \dfrac{0.244313}{X_g X_h} + \dfrac{0.113929}{X_g^3} - \dfrac{0.00929265}{X_h^3} - \dfrac{0.0266293}{X_g X_h^2} - \dfrac{0.217008}{X_g^2 X_h} \end{array} \right)$$

$$V = \left(\begin{array}{l} 0.0144868 - \dfrac{0.135977}{2.20} - \dfrac{0.0461919}{0.7419} + \dfrac{0.560718}{(2.20)^2} + \dfrac{0.0529829}{(0.7419)^2} + \\ \dfrac{0.244313}{(2.20)(0.7419)} + \dfrac{0.113929}{(2.20)^3} - \dfrac{0.00929265}{(0.7419)^3} - \dfrac{0.0266293}{(2.20)(0.7419)^2} - \dfrac{0.217008}{(2.20)^2(0.7419)} \end{array} \right)$$

$$V = 0.1577$$

$$f = \max\left[1.0, \left(\dfrac{\begin{array}{l} 0.0927779 - 0.0336633 X_g + 0.964176 X_g^2 + \\ 0.0566286 X_h + 0.347074 X_h^2 - 4.18699 X_h^3 \end{array}}{\begin{array}{l} 1 - 5.96093\left(10^{-3}\right) X_g + 1.62904 X_h + \\ 3.49329 X_h^2 + 1.39052 X_h^3 \end{array}} \right) \right]$$

$$f = \max\left[1.0, \left(\dfrac{\begin{array}{l} 0.0927779 - 0.0336633(2.20) + 0.964176(2.20)^2 + \\ 0.0566286(0.7419) + 0.347074(0.7419)^2 - 4.18699(0.7419)^3 \end{array}}{\begin{array}{l} 1 - 5.96093\left(10^{-3}\right)(2.20) + 1.62904(0.7419) + \\ 3.49329(2.20)^2 + 1.39052(0.7419)^3 \end{array}} \right) \right]$$

$$f = 1.0$$

e) STEP 5 – Determine the flange forces, (VIII-1, paragraph 2-3).

$$H_D = \frac{\pi}{4} B^2 P = \frac{\pi}{4}(26.25)^2(135) = 73060.4\ lbs$$

$$H = \frac{\pi}{4} G^2 P = \frac{\pi}{4}(29.5)^2(135) = 92271.5\ lbs$$

$$H_T = H - H_D = 92271.5 - 73060.4 = 19211.1\ lbs$$

$$H_G = W - H = 111329.5 - 92227.5 = 19058.0\ lbs \qquad Operating$$

f) STEP 6 – Determine the flange moment for the operating condition using paragraph 2-6. In these equations, h_D, h_T, and h_G are determined from Table 2-6.

For internal pressure,

$$h_D = \frac{C-B-g_1}{2} = \frac{31.25-26.25-0.6875}{2} = 2.1563\ in$$

$$h_G = \frac{C-G}{2} = \frac{31.25-29.5}{2} = 0.875\ in$$

$$h_T = \frac{1}{2}\left[\frac{C-B}{2}+h_G\right] = \frac{1}{2}\left[\frac{31.25-26.25}{2}+0.875\right] = 1.6875\ in$$

Paragraph 2-6:

$$M_o = H_D h_D + H_T h_T + H_G h_G$$
$$M_o = 73060.4(2.1563)+19211.1(1.6875)+19058.0(0.875)$$
$$M_o = 206634.6\ in-lbs$$

For vessels in lethal service or when specified by the user or his designated agent, the bolt spacing correction factor B_{SC} shall be applied in calculating the flange stress in paragraph 2-7. The flange moment M_o without correction for bolt spacing is used for the calculation of the rigidity index in paragraph 2-14. When the bolt spacing exceeds $2a+t$, multiply M_o by the bolt spacing correction factor B_{SC} for calculating flange stress, where:

$$B_{SC} = \sqrt{\frac{B_s}{2a+t}}$$

Commentary:

1) The equations for maximum bolt spacing B_{smax} and bolt spacing correction factor B_{SC} used in the VIII-2 are the same as in VIII-1 and are located in Table 4.16.11. However, VIII-2 does not have the supplemental check on bolt spacing of $2a+t$.

2) As is the case with VIII-1 design, VIII-2 also notes that the flange moment M_o without correction for bolt spacing is used for the calculation of the rigidity index located in Table 4.16.10.

3) The bolt spacing correction factor B_{SC} and the split loose flange factor of VIII-1, paragraph 2-9, F_s, are directly incorporated into the calculations for determining the flange moments for both operating and gasket seating.

$$M_o = abs\left[\left(\left(H_D h_D + H_T h_T + H_G h_G\right)B_{SC} + M_{oe}\right)F_s\right] \quad Internal\ Pressure$$

$$M_g = \frac{W_g\left(C-G\right)B_{SC}F_s}{2} \qquad Gasket\ Seating$$

4) As previously noted, the VIII-2 procedure provides the designer the ability to add an externally applied net-section axial force and bending moment to the bolt load for the operating condition. These externally applied loads induce a bending moment, referenced as M_{oe}, which is calculated from Equation 4.16.16,

$$M_{oe} = 4M_E\left[\frac{I}{0.3846I_p + I}\right]\cdot\left[\frac{h_D}{(C-2h_D)}\right] + F_A h_D$$

g) STEP 7 – Determine the flange moment for gasket seating condition using paragraph 2-6.

For internal pressure,

$$M_o = W\frac{(C-G)}{2} = 237626.3\left(\frac{(31.25-29.5)}{2}\right) = 207923.0\ in-lbs$$

h) STEP 8 – Determine the flange stresses for the operating and gasket seating conditions using the equations in paragraph 2-7.

Note: Paragraph 2-3 – If $B < 20g_1$, the designer may substitute the value of B_1 for B in the equation for S_H, where,

For integral flanges when $f \geq 1.0$,

$$B_1 = B + g_o$$

For integral flanges when $f < 1.0$ and for loose type flanges,

$$B_1 = B + g_1$$

Operating Condition

$$S_H = \frac{fM_o}{Lg_1^2B} = \frac{(1.0)(206634.6)}{(0.9362)(0.6875)^2(26.25)} = 17789.3\ psi$$

$$S_R = \frac{(1.33te+1)M_o}{Lt^2B} = \frac{[(1.33)(1.4375)(0.2680)+1](206634.6)}{(0.9362)(1.4375)^2(26.25)} = 6153.9\ psi$$

$$S_T = \frac{YM_o}{t^2B} - ZS_R = \frac{(8.7565)(206634.6)}{(1.4375)^2(26.25)} - 4.5180(6153.9) = 5553.8\ psi$$

Gasket Seating Condition

$$S_H = \frac{fM_o}{Lg_1^2B} = \frac{(1.0)(207923.0)}{(0.9362)(0.6875)^2(26.25)} = 17900.3\ psi$$

$$S_R = \frac{(1.33te+1)M_o}{Lt^2B} = \frac{[(1.33)(1.4375)(0.2680)+1](207923.0)}{(0.9362)(1.4375)^2(26.25)} = 6192.3\ psi$$

$$S_T = \frac{YM_o}{t^2B} - ZS_R = \frac{(8.7565)(207923.0)}{(1.4375)^2(26.25)} - 4.5180(6192.3) = 5588.3\ psi$$

i) STEP 9 – Check the flange stress acceptance criteria. The criteria below shall be evaluated. If the stress criteria are satisfied, go to STEP 10. If the stress criteria are not satisfied, re-proportion the flange dimensions and go to STEP 4.

Allowable normal stress – The criteria to evaluate the normal stresses for the operating and gasket seating conditions are shown in paragraph 2-8, for integral type flanges with hub welded to the neck, pipe or vessel wall.

Operating Condition

$$S_H \le \min\left[1.5S_f,\ 2.5S_n\right]$$

$$\left\{S_H = 17789.3\ psi\right\} \le \left\{\min\left[1.5(17800),\ 2.5(18800)\right] = 26700\ psi\right\} \qquad True$$

$$\left\{S_R = 6153.9\ psi\right\} \le \left\{S_f = 17800\ psi\right\} \qquad True$$

$$\left\{S_T = 5553.8\ psi\right\} \le \left\{S_f = 17800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_H + S_R)}{2} = \frac{(17789.3 + 6153.9)}{2} = 11971.6\ psi\right\} \le \left\{S_{fo} = 17800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_H + S_T)}{2} = \frac{(17789.3 + 5553.9)}{2} = 11671.6\ psi\right\} \le \left\{S_{fo} = 17800\ psi\right\} \qquad True$$

Gasket Seating Condition

$$S_H \le \min\left[1.5S_f,\ 2.5S_n\right]$$

$$\left\{S_H = 17900.3\ psi\right\} \le \left\{\min\left[1.5(20000),\ 2.5(20000)\right] = 30000\ psi\right\} \qquad True$$

$$\left\{S_R = 6192.3\ psi\right\} \le \left\{S_f = 20000\ psi\right\} \qquad True$$

$$\left\{S_T = 5588.3\ psi\right\} \le \left\{S_f = 17800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_H + S_R)}{2} = \frac{(17900.3 + 6192.3)}{2} = 12046.3\ psi\right\} \le \left\{S_f = 20000\ psi\right\} \qquad True$$

$$\left\{\frac{(S_H + S_T)}{2} = \frac{(17900.3 + 5588.3)}{2} = 11744.3\ psi\right\} \le \left\{S_f = 20000\ psi\right\} \qquad True$$

j) STEP 10 – Check the flange rigidity criterion in Table 2-14. If the flange rigidity criterion is satisfied, then the design is complete. If the flange rigidity criterion is not satisfied, then re-proportion the flange dimensions and go to STEP 3.

Operating Condition

$$J = \frac{52.14 V M_o}{L E_y g_0^2 K_I h_o} \le 1.0$$

$$\left\{J = \frac{52.14(0.1577)(206634.6)}{(0.9362)(26.0E+06)(0.3125)^2(0.3)(2.8641)} = 0.8319\right\} \le 1.0 \qquad True$$

Where,

$$K_I = 0.3 \ \textit{for integral flanges}$$

Gasket Seating Condition

$$J = \frac{52.14VM_o}{LE_y g_o^2 K_I h_o} \le 1.0$$

$$\left\{ J = \frac{52.14(0.1577)(207922.9)}{(0.9362)(29.4E+06)(0.3125)^2(0.3)(2.8641)} = 0.7403 \right\} \le 1.0 \qquad \textit{True}$$

Where,

$$K_I = 0.3 \ \textit{for integral flanges}$$

Since the acceptance criteria are satisfied, the design is complete.

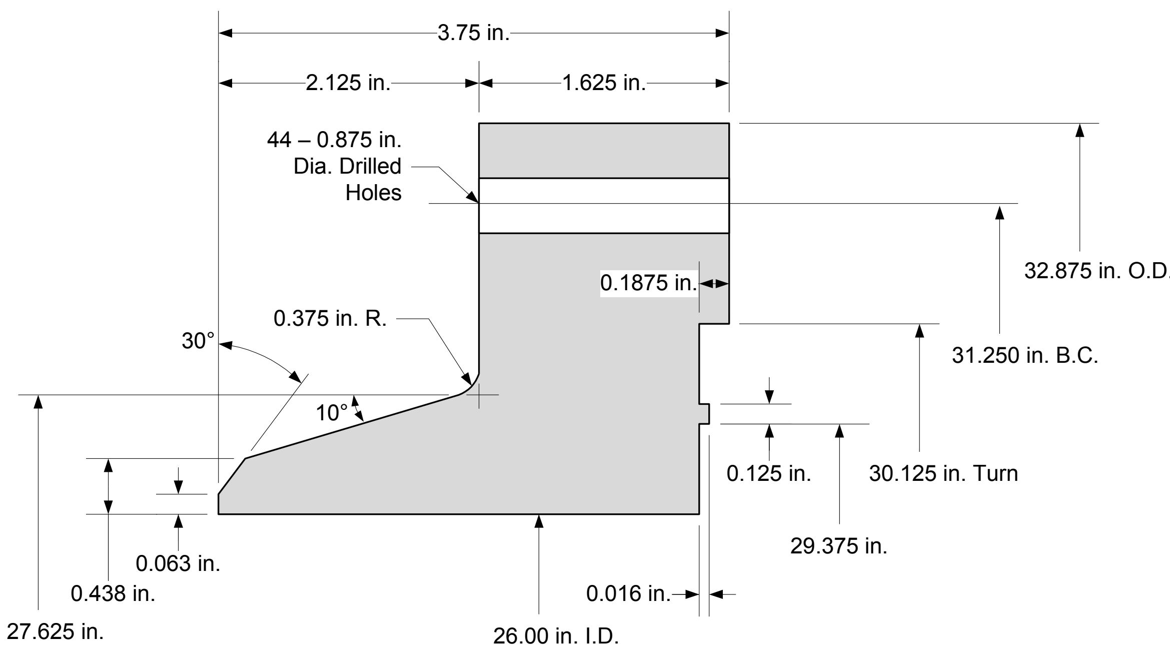


Figure E4.16.1 – Flanged Joints

4.16.2 Example E4.16.2 - Loose Type

Determine if the stresses in the ASME B16.5, Class 300, NPS 20 Slip–on Flange are with acceptable limits, considering the following design conditions. The flange is of a loose type with hub and is attached to a cylindrical shell with Category C fillet welds, see VIII-1 Appendix 2, Figure 2-4 Sketch 3.

General Data:

- Cylinder Material = $SA-516,\ Grade\ 70$
- Design Conditions = $450\ psig\ @650°F$
- Allowable Stress at Design Temperature = $18800\ psi$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Corrosion Allowance = $0.0\ in$

Flange Data

- Material = $SA-105$
- Allowable Stress at Design Temperature = $17800\ psi$
- Allowable Stress at Ambient Temperature = $20000\ psi$
- Modulus of Elasticity at Design Temperature = $26.0E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$

Bolt Data

- Material = $SA-193,\ Grade\ B7$
- Allowable Stress at Design Temperature = $25000\ psi$
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Diameter = $1.25\ in$
- Number of Bolts = 24
- Root area = $0.929\ in^2$

Gasket Data

- Material = Kammprofile
- Gasket Factor = 2.0
- Seating Stress = $2500\ psi$
- Inside Diameter = $20.875\ in$
- Outside Diameter = $22.875\ in$

Design rules for bolted flange connections with ring type gaskets are provided in VIII-1 Mandatory Appendix 2. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.16. However, there are differences to be noted, including; a step-by-step design procedure, nomenclature with regard to operating and gasket seating bolt loads, the inclusion of a flange moment due to externally applied axial forces and bending moment, and minor differences in bolt spacing criteria. Therefore, while the example problem will be presented for use with VIII-1, Appendix 2, references to VIII-2 paragraphs will be provided, as applicable.

Evaluate the flange in accordance with VIII-1, Appendix 2.

Establish the design conditions and gasket seating reaction diameter, (VIII-2, paragraph 4.16.6).

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

$$P = 450\ psig\ at\ 650°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 2-5.1 (VIII-2, Table 4.16.1).

$$m = 2.0$$

$$y = 2500\ psi$$

Note: Table 2-5.1 (VIII-2, Table 4.16.1) gives a list of many commonly used gasket materials and contact facings with suggested design values of m and y that have generally proved satisfactory in actual service when using effective seating width b given in Table 2-5.2 (VIII-2, Table 4.16.3). The design values and other details given in this table re suggested only and are not mandatory.

For this example, gasket manufacturer's suggested m and y values were used.

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5(GOD - GID) = 0.5(22.875 - 20.875) = 1.0\ in$$

From Table 2-5.2 (VIII-2, Table 4.16.3), Facing Sketch Detail 1a, Column II,

$$b_o = \frac{N}{2} = \frac{1.0}{2} = 0.500\ in$$

For $b_o > 0.25\ in$,

$$b = C_b\sqrt{b_o} = (0.5)\sqrt{0.500} = 0.3536\ in$$

Where,

$$C_b = 0.5,\ for\ US\ Customary\ Units$$

Therefore, from paragraph 2-3 the location of the gasket reaction is calculated as follows.

$$G = outside\ diameter\ of\ gasket\ contact\ face\ less\ 2b$$

$$G = G_C - 2b = 22.875 - 2(0.3536) = 22.1678\ in$$

Where,

$$G_C = \min[Gasket\ OD,\ Flange\ Face\ OD] = \min[22.875,\ 23.0] = 22.875\ in$$

Paragraph 2-5 – Calculate the design bolt load for the operating and gasket seating conditions, (VIII-2, paragraph 4.16.6).

a) STEP 1 – Paragraph 2-5(c)(1), determine the design bolt load for the operating condition.

$$W_{m1} = H + H_p = \frac{\pi}{4}G^2 P + 2b\pi GmP \qquad for\ non-self-energized\ gaskets$$

$$W_{m1} = \frac{\pi}{4}(22.1678)^2(450) + 2(0.3536)\pi(22.1678)(2.0)(450) = 218005.0\ lbs$$

b) STEP 2 – Paragraph 2-5(c)(2), determine the design bolt load for the gasket seating condition.

$$W_{m2} = \pi bGy \qquad for\ non-self-energized\ gaskets$$

$$W_{m2} = \pi(0.3536)(22.1678)(2500) = 61563.7\ lbs$$

c) STEP 3 – Paragraph 2-5(d), determine the total required and actual bolt areas.

The total cross-sectional area of bolts A_m required for both the operating conditions and gasket seating is determined as follows.

$$A_m = \max\left[A_{m1},\ A_{m2}\right] = \max\left[8.7202,\ 2.4625\right] = 8.7202\ in^2$$

Where,

$$A_{m1} = \frac{W_{m1}}{S_b} = \frac{218005.0}{25000} = 8.7202\ in^2$$

$$A_{m2} = \frac{W_{m2}}{S_a} = \frac{61563.7}{25000} = 2.4625\ in^2$$

The actual bolt area A_b is calculated as follows.

$$A_b = (Number\ of\ bolts)(Root\ area\ of\ one\ bolt) = 24(0.929) = 22.2960\ in^2$$

Verify that the actual bolt area is equal to or greater than the total required area.

$$\left\{A_b = 22.2960\ in^2\right\} \ge \left\{A_m = 8.7202\ in^2\right\} \qquad True$$

For vessels in lethal service or when specified by the user or his designated agent, the maximum bolt spacing shall not exceed the following:

$$B_{s\max} = 2a + \frac{6t}{m+0.5} = 2(1.25) + \frac{6(2.44)}{(2.0+0.5)} = 8.3560\ in$$

Where,

$$a = nominal\ bolt\ diameter$$

$$t = flange\ thickness$$

$$m = gasket\ factor$$

d) STEP 4 – Paragraph 2-5(e), determine the flange design bolt load.

For operating conditions,

$$W = W_{m1} = 218005.0\ lbs$$

For gasket seating,

$$W = \frac{(A_m + A_b)S_a}{2} = \frac{(8.7202 + 22.2960)25000}{2} = 387702.5\ lbs$$

Commentary: VIII-2 design procedure to determine the design bolt load, paragraph 4.16.6:

1) The nomenclature differences include:

   W_o, bolt load for operating conditions and flange design load bolt load.(VIII-1 W_{m1}, W).

   W_{gs}, design bolt load for the gasket seating condition (VIII-1, W_{m2}).

   W_g, flange design bolt load for gasket seating (VIII-1, W).

2) When calculating the required total cross-sectional area of bolts using the VIII-2 procedure, the designer has the ability to add an externally applied net-section axial force, F_A and bending moment, M_E to the bolt load W_o for the operating condition. This is shown in the following equation.

$$A_m = \max\left[\left(\frac{W_o + F_A + \frac{4M_E}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right]$$

3) The bolt spacing criteria of VIII-1 and VIII-2 are similar, but there are some minor differences regarding the application of the two. This will be further discussed later in the example problem.

Commentary: VIII-1, Appendix 2 does not include an overall step-by-step procedure to design a flange. However, an organized procedure of the steps taken when designing a flange is presented in VIII-2, paragraph 4.16.7. The procedure is applicable to VIII-1, Appendix 2 and is presented in this example problem in an effort to assist the designer.

VIII-2, paragraph 4.16.7, Flange Design Procedure. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings. The procedure incorporates both a strength check and a rigidity check for flange rotation.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$P = 450\ psig\ at\ 650°F$$

b) STEP 2 – Determine the design bolt loads for operating condition W, and the gasket seating condition W, and the corresponding actual bolt load area A_b, (VIII-1, paragraph 2-5).

$$W = 218005.0\ lbs \qquad Operating\ Condition$$

$$W = 387702.5\ lbs \qquad Gasket\ Seating$$

$$A_b = 22.2960\ in^2$$

c) STEP 3 – Determine an initial flange geometry, in addition to the information required to determine the bolt load, the following geometric parameters are required, (VIII-1, paragraph 2-3). The flange is an ASME B16.5, Class 300, NPS 20 Slip–on Flange.

   1) Flange bore

$$B = 20.20\ in$$

2) Bolt circle diameter

$$C = 27.0 \; in$$

3) Outside diameter of the flange

$$A = 30.5 \; in$$

4) Flange thickness

$$t = 2.44 \; in$$

5) Thickness of the hub at the large end

$$g_1 = 1.460 \; in$$

6) Thickness of the hub at the small end

$$g_0 = 1.460 \; in$$

7) Hub length

$$h = 1.25 \; in$$

d) STEP 4 – Determine the flange stress factors using the equations/direct interpretation from Table 2-7.1 and Fig. 2-7.1 – Fig. 2-7.6 and paragraph 2-3.

<u>Fig. 2-7.1:</u>

$$K = \frac{A}{B} = \frac{30.5}{20.20} = 1.5099$$

$$Y = \frac{1}{K-1}\left[0.66845 + 5.71690\left(\frac{K^2 \log_{10} K}{K^2 - 1}\right)\right]$$

$$Y = \frac{1}{1.5099-1}\left[0.66845 + 5.71690\frac{(1.5099)^2 \log_{10}[1.5099]}{(1.5099)^2 - 1}\right] = 4.8850$$

$$T = \frac{K^2(1 + 8.55246\log_{10} K) - 1}{(1.04720 + 1.9448K^2)(K-1)} = \frac{(1.5099)^2(1 + 8.55246\log_{10}[1.5099]) - 1}{\left(1.04720 + 1.9448(1.5099)^2\right)(1.5099 - 1)} = 1.7064$$

$$U = \frac{K^2(1 + 8.55246\log_{10} K) - 1}{1.36136(K^2 - 1)(K - 1)} = \frac{(1.5099)^2(1 + 8.55246\log_{10}[1.5099]) - 1}{1.36136\left((1.5099)^2 - 1\right)(1.5099 - 1)} = 5.3681$$

$$Z = \frac{(K^2 + 1)}{(K^2 - 1)} = \frac{\left((1.5099)^2 + 1\right)}{\left((1.5099)^2 - 1\right)} = 2.5627$$

<u>Fig. 2-7.4:</u>

$$\frac{h}{h_o} = \frac{h}{\sqrt{Bg_0}} = \frac{1.25}{\sqrt{(20.20)(1.46)}} = \frac{1.25}{5.4307} = 0.2302$$

$$\frac{g_1}{g_0} = \frac{1.460}{1.460} = 1.0$$

Interpretation of Fig. 2-7.4, $F_L \approx 3.3$. From the equations of Table 2-7.1, $F_L = 3.2609$.

Fig. 2-7.5:

With $\dfrac{h}{h_o} = 0.2302$ and $\dfrac{g_1}{g_0} = 1.0$:

Interpretation of Fig. 2-7.5, $V_L \approx 11.4$. From the equations of Table 2-7.1, $V_L = 11.3725$.

Fig. 2-7.6:

With $\dfrac{h}{h_o} = 0.2302$ and $\dfrac{g_1}{g_0} = 1.0$:

Interpretation of Fig. 2-7.6, $f = 1.0$. From the equations of Table 2-7.1, $f = 1.0$.

Paragraph 2-3:

$$d = \frac{Ug_0^2 h_o}{V_L} = \frac{(5.3681)(1.460)^2(5.4307)}{11.3725} = 5.4642\ in$$

$$e = \frac{F_L}{h_o} = \frac{3.2609}{5.4307} = 0.6005$$

$$L = \frac{te+1}{T} + \frac{t^3}{d} = \frac{2.44(0.6005)+1}{1.7064} + \frac{(2.44)^3}{5.4642} = 4.1032$$

Commentary: VIII-2, Table 4.16.4 provides the equations to determine the flange stress factors Y, T, U, Z as a function of K as provided for in VIII-1, Fig. 2-7.1, and the flange factors d, e, L located in VIII-1, paragraph 2-3. Similarly, VIII-2, Table 4.16.5 provides regressed curve-fit equations of the flange stress factors F_L, V_L, f as provided in VIII-1, Fig. 2-7.2, Fig. 2-7.3 and Fig. 2-7.6, respectively. The curve-fit equations are shown for informational purposes with the variable substitution of $X_h = h/h_o$ and $X_g = g_1/g_o$.

$$F_L = \left(\frac{\left\{ \begin{array}{l} 0.941074 + 0.176139\left(\ln\left[X_g\right]\right) - 0.188556\left(\ln\left[X_h\right]\right) + \\ 0.0689847\left(\ln\left[X_g\right]\right)^2 + 0.523798\left(\ln\left[X_h\right]\right)^2 - \\ 0.513894\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right) \end{array} \right\}}{\left\{ \begin{array}{l} 1 + 0.379392\left(\ln\left[X_g\right]\right) + 0.184520\left(\ln\left[X_h\right]\right) - \\ 0.00605208\left(\ln\left[X_g\right]\right)^2 - 0.00358934\left(\ln\left[X_h\right]\right)^2 + \\ 0.110179\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right) \end{array} \right\}} \right)$$

$$F_L = \left(\frac{\left\{ \begin{array}{l} 0.941074 + 0.176139\left(\ln\left[1.0\right]\right) - 0.188556\left(\ln\left[0.2302\right]\right) + \\ 0.0689847\left(\ln\left[1.0\right]\right)^2 + 0.523798\left(\ln\left[0.2302\right]\right)^2 - \\ 0.513894\left(\ln\left[1.0\right]\right)\left(\ln\left[0.2302\right]\right) \end{array} \right\}}{\left\{ \begin{array}{l} 1 + 0.379392\left(\ln\left[1.0\right]\right) + 0.184520\left(\ln\left[0.2302\right]\right) - \\ 0.00605208\left(\ln\left[1.0\right]\right)^2 - 0.00358934\left(\ln\left[0.2302\right]\right)^2 + \\ 0.110179\left(\ln\left[1.0\right]\right)\left(\ln\left[0.2302\right]\right) \end{array} \right\}} \right)$$

$$F_L = 3.2556$$

$For\ 0.1 \le X_h \le 0.25,$

$$\ln\left[V_L\right] = \left(\begin{array}{l} 6.57683 - 0.115516 X_g + 1.39499\sqrt{X_g}\left(\ln\left[X_g\right]\right) + \\ 0.307340\left(\ln\left[X_g\right]\right)^2 - 8.30849\sqrt{X_g} + 2.62307\left(\ln\left[X_g\right]\right) + \\ 0.239498 X_h\left(\ln\left[X_h\right]\right) - 2.96125\left(\ln\left[X_h\right]\right) + \dfrac{7.035052\left(10^{-4}\right)}{X_h} \end{array} \right)$$

$$\ln\left[V_L\right] = \left(\begin{array}{l} 6.57683 - 0.115516(1.0) + 1.39499\sqrt{1.0}\left(\ln 1.0\right) + \\ 0.307340\left(\ln\left[1.0\right]\right)^2 - 8.30849\sqrt{1.0} + 2.62307\left(\ln\left[1.0\right]\right) + \\ 0.239498(0.2302)\left(\ln\left[0.2302\right]\right) - 2.96125\left(\ln\left[0.2302\right]\right) + \dfrac{7.035052\left(10^{-4}\right)}{0.2302} \end{array} \right)$$

$$\ln\left[V_L\right] = 2.4244$$

$$V_L = \exp\left[2.4244\right] = 11.2955$$

$$f = 1.0$$

e) STEP 5 – Determine the flange forces, paragraph 2-3.

$$H_D = \frac{\pi}{4}B^2P = \frac{\pi}{4}(20.20)^2(450) = 144213.2\ lbs$$

$$H = \frac{\pi}{4}G^2P = \frac{\pi}{4}(22.1678)^2(450) = 173679.1\ lbs$$

$$H_T = H - H_D = 173679.1 - 144213.2 = 29465.9\ lbs$$

$$H_G = W - H = 218005.0 - 173679.1 = 44325.9\ lbs \quad Operating$$

f) STEP 6 – Determine the flange moment for the operating condition using paragraph 2-6. In these equations, h_D, h_T, and h_G are determined from Table 2-6.

For internal pressure,

$$h_D = \frac{C-B}{2} = \frac{27.0-20.20}{2} = 3.40\ in$$

$$h_G = \frac{C-G}{2} = \frac{27.0-22.1678}{2} = 2.4161\ in$$

$$h_T = \frac{h_D + h_G}{2} = \frac{3.40+2.4161}{2} = 2.9081\ in$$

Paragraph 2-6:

$$M_o = H_D h_D + H_T h_T + H_G h_G$$
$$M_o = 144213.2(3.40) + 29465.9(2.9081) + 44325.9(2.4161)$$
$$M_o = 683110.5\ in-lbs$$

For vessels in lethal service or when specified by the user or his designated agent, the bolt spacing correction factor B_{SC} shall be applied in calculating the flange stress in paragraph 2-7. The flange moment M_o without correction for bolt spacing is used for the calculation of the rigidity index in paragraph 2-14. When the bolt spacing exceeds $2a+t$, multiply M_o by the bolt spacing correction factor B_{SC} for calculating flange stress, where:

$$B_{SC} = \sqrt{\frac{B_s}{2a+t}}$$

Commentary:

1) The equations for maximum bolt spacing B_{smax} and bolt spacing correction factor B_{SC} used in the VIII-2 are the same as in VIII-1 and are located in Table 4.16.11. However, VIII-2 does not have the supplemental check on bolt spacing of $2a+t$.

2) As is the case with VIII-1 design, VIII-2 also notes that the flange moment M_o without correction for bolt spacing is used for the calculation of the rigidity index located in Table 4.16.10.

3) The bolt spacing correction factor B_{SC} and the split loose flange factor of VIII-1, paragraph 2-9, F_s, are directly incorporated into the calculations for determining the flange moments for both operating and gasket seating.

$$M_o = abs\left[\left(\left(H_D h_D + H_T h_T + H_G h_G\right)B_{SC} + M_{oe}\right)F_s\right] \quad Internal\ Pressure$$

$$M_g = \frac{W_g\left(C-G\right)B_{SC}F_s}{2} \qquad Gasket\ Seating$$

4) As previously noted, the VIII-2 procedure provides the designer the ability to add an externally applied net-section axial force and bending moment to the bolt load for the operating condition. These externally applied loads induce a bending moment, referenced as M_{oe}, which is calculated from Equation 4.16.16,

$$M_{oe} = 4M_E\left[\frac{I}{0.3846I_p + I}\right]\cdot\left[\frac{h_D}{\left(C-2h_D\right)}\right] + F_A h_D$$

g) STEP 7 – Determine the flange moment for gasket seating condition using paragraph 2-6.

For internal pressure,

$$M_o = W\frac{\left(C-G\right)}{2} = 387702.5\frac{\left(27.0-22.1678\right)}{2} = 936728.0\ in-lbs$$

h) STEP 8 – Determine the flange stresses for the operating and gasket seating conditions using the equations in paragraph 2-7.

Note: Paragraph 2-3 – If $B < 20g_1$, the designer may substitute the value of B_1 for B in the equation for S_H, where,

For integral flanges when $f \geq 1.0$,

$$B_1 = B + g_o$$

For integral flanges when $f < 1.0$ and for loose type flanges,

$$B_1 = B + g_1$$

Operating Condition

$$S_H = \frac{fM_o}{Lg_1^2B} = \frac{\left(1.0\right)\left(683110.5\right)}{\left(4.1032\right)\left(1.460\right)^2\left(20.20\right)} = 3866.4\ psi$$

$$S_R = \frac{\left(1.33te+1\right)M_o}{Lt^2B} = \frac{\left[\left(1.33\right)\left(2.44\right)\left(0.6005\right)+1\right]\left(683110.5\right)}{\left(4.1032\right)\left(2.44\right)^2\left(20.20\right)} = 4082.0\ psi$$

$$S_T = \frac{YM_o}{t^2B} - ZS_R = \frac{\left(4.8850\right)\left(683110.5\right)}{\left(2.44\right)^2\left(20.20\right)} - 2.5627\left(4082.0\right) = 17286.6\ psi$$

Gasket Seating Condition

$$S_H = \frac{fM_o}{Lg_1^2 B} = \frac{(1.0)(936728.0)}{(4.1032)(1.460)^2(20.20)} = 5301.9 \ psi$$

$$S_R = \frac{(1.33te+1)M_o}{Lt^2 B} = \frac{\left[(1.33)(2.44)(0.6005)+1\right](936728.0)}{(4.1032)(2.44)^2(20.20)} = 5597.5 \ psi$$

$$S_T = \frac{YM_o}{t^2 B} - ZS_R = \frac{(4.8850)(936728.0)}{(2.44)^2(20.20)} - 2.5627(5597.5) = 23704.6 \ psi$$

i) STEP 9 – Check the flange stress acceptance criteria. The criteria below shall be evaluated. If the stress criteria are satisfied, go to STEP 10. If the stress criteria are not satisfied, re-proportion the flange dimensions and go to STEP 4.

Allowable normal stress – The criteria to evaluate the normal stresses for the operating and gasket seating conditions are shown in VIII-1, paragraph 2-8, for loose type flanges with a hub.

Operating Condition

$$S_H \le \min\left[1.5S_f, \ 2.5S_n\right]$$

$$\left\{S_H = 3866.4 \ psi\right\} \le \left\{\min\left[1.5(17800), \ 2.5(18800)\right] = 26700 \ psi\right\} \qquad True$$

$$\left\{S_R = 4082.0 \ psi\right\} \le \left\{S_f = 17800 \ psi\right\} \qquad True$$

$$\left\{S_T = 17286.6 \ psi\right\} \le \left\{S_f = 17800 \ psi\right\} \qquad True$$

$$\left\{\frac{(S_H + S_R)}{2} = \frac{(3866.4 + 4082.0)}{2} = 3974.2 \ psi\right\} \le \left\{S_f = 17800 \ psi\right\} \qquad True$$

$$\left\{\frac{(S_H + S_T)}{2} = \frac{(3866.4 + 17286.6)}{2} = 10576.5 \ psi\right\} \le \left\{S_f = 17800 \ psi\right\} \qquad True$$

Gasket Seating Condition

$$S_H \le \min\left[1.5S_f, \ 2.5S_n\right]$$

$$\left\{S_H = 5301.9 \ psi\right\} \le \left\{\min\left[1.5(20000), \ 2.5(20000)\right] = 30000 \ psi\right\} \qquad True$$

$$\left\{S_R = 5597.5 \ psi\right\} \le \left\{S_f = 17800 \ psi\right\} \qquad True$$

$$\left\{S_T = 23704.6 \ psi\right\} \le \left\{S_f = 17800 \ psi\right\} \qquad False$$

$$\left\{\frac{(S_H + S_R)}{2} = \frac{(5301.9 + 5597.5)}{2} = 5449.7 \ psi\right\} \le \left\{S_f = 20000 \ psi\right\} \qquad True$$

$$\left\{\frac{(S_H + S_T)}{2} = \frac{(5301.9 + 23704.6)}{2} = 14503.3 \ psi\right\} \le \left\{S_f = 20000 \ psi\right\} \qquad True$$

Since the acceptance criteria is not satisfied for the Tangential Flange Stress, S_T re-proportion the flange dimensions and go to STEP 4 to re-evaluate the design.

j) STEP 10 – Check the flange rigidity criterion in Table 2-14. If the flange rigidity criterion is satisfied, then the design is complete. If the flange rigidity criterion is not satisfied, then re-proportion the flange dimensions and go to STEP 3.

Operating Condition

$$J = \frac{52.14 V_L M_o}{L E_y g_0^2 K_L h_o} \le 1.0$$

$$\left\{ J = \frac{52.14(11.3725)(683110.5)}{(4.1032)(26.0E+06)(1.460)^2(0.2)(5.4307)} = 1.6399 \right\} \le 1.0 \quad \textit{Not Satisfied}$$

Where,

$$K_L = 0.2 \;\; \textit{for loose type flanges}$$

Gasket Seating Condition

$$J = \frac{52.14 V_L M_o}{L E_y g_o^2 K_L h_o} \le 1.0$$

$$\left\{ J = \frac{52.14(11.3725)(936728.0)}{(4.1032)(29.4E+06)(1.460)^2(0.2)(5.4307)} = 1.9887 \right\} \le 1.0 \quad \textit{Not Satisfied}$$

Where,

$$K_L = 0.2 \;\; \textit{for loose type flanges}$$

Since the flange rigidity criterion is not satisfied for either the operating condition or the gasket seating condition, the flange dimensions should be re-proportioned and the design procedure shall be performed beginning with STEP 3.

NOTE: Although the proposed ASME B16.5 slip-on flange is shown not to satisfy the Tangential Flange Stress, S_T, in the gasket seating condition and the flange rigidity acceptance criteria of VIII-1 Appendix 2 Rules for Bolted Flange Connections, ASME B16.5–2009, Table II–2–1.1, Pressure–Temperature Ratings for Group 1.1 Materials, states an ASME Class 300 flange is permitted to operate at a pressure of 550 psi for a coincident temperature of 650°F.

4.17 Clamped Connections

4.17.1 Example E4.17.1 - Flange and Clamp Design Procedure

Using the data shown below, determine if the clamp design meets the design requirements of Section VIII, Division 1.

Data (Refer to Figure E4.17.1)

- Design Conditions = $3000\ psi@200°F$
- Corrosion Allowance = $0.0\ in$

Clamp

- Material = $SA-216,\ Grade\ WCB$
- Inside Diameter = $43.75\ in$
- Thickness = $7.625\ in$
- Width = $28.0\ in$
- Gap = $14.0\ in$
- Lug height = $15.0\ in$
- Lug Width = $28.0\ in$
- Lip Length = $2.75\ in$
- Radial Distance from Connection Centerline to Bolts = $32.25\ in$
- Distance from W to the point where the clamp lug joins the clamp body = $3.7\ in$
- Allowable Stress @ Design Temperature = $20000\ psi$
- Allowable Stress @ Ambient Temperature = $20000\ psi$

Hub

- Material = $SA-105$
- Inside Diameter = $18.0\ in$
- Pipe End Neck Thickness = $12.75\ in$
- Shoulder End Neck Thickness = $12.75\ in$
- Shoulder Thickness = $7.321\ in$
- Shoulder Height = $2.75\ in$
- Friction Angle = $5\ deg$
- Shoulder Transition Angle = $10\ deg$
- Allowable Stress @ Design Temperature = $20000\ psi$
- Allowable Stress @ ambient Temperature = $20000\ psi$

Bolt Data

- Material = $SA-193,\ Grade\ B7$
- Allowable Stress @ Design Temperature = $23000\ psi$
- Allowable Stress @ Gasket Temperature = $23000\ psi$
- Diameter = $1.75\ in$
- Number of Bolts = 2

- Root area = $1.980\ in^2$

Gasket Data

- Material = Self Energizing O-ring Type
- Gasket Reaction Location = $19.0\ in$
- Gasket Factor = 0
- Seating Stress = $0\ psi$

Design rules for clamped connections are provided in VIII-1 Mandatory Appendix 24. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.17. However, there are differences to be noted, including; a step-by-step design procedure and nomenclature with regard to operating and gasket seating bolt loads. Therefore, while the example problem will be presented for use with VIII-1, Appendix 24, references to VIII-2 paragraphs will be provided, as applicable.

Evaluate the clamp in accordance with VIII-1, Appendix 24.

Establish the design conditions and gasket reaction diameter.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

$$P = 3000\ psig\ at\ 200°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 2-5.1.

$$m = 0.0 \qquad for\ self-energized\ gaskets$$
$$y = 0.0$$

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_0, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.0 \qquad for\ self-energized\ gaskets$$

From Table 2-5.2, Facing Sketch Detail (not required because gasket is self-energized)

$$b_o = \frac{N}{2} = \frac{0.0}{2} = 0.0\ in$$

For $b_o \le 0.25\ in$,

$$b = b_o = 0.0\ in$$

Therefore, paragraph 2-3, the location of the gasket reaction is calculated as follows.

$$G = mean\ diameter\ of\ the\ gasket\ contact\ face$$
$$G = 19.0\ in$$

Paragraphs 24-3 and 24-4 – Calculate the design bolt loads for the operating and gasket seating conditions.

a) STEP 1 – Paragraph 24-3, determine the flange forces for the bolt load calculation.

$$H = 0.785G^2P = 0.785(19.0)^2(3000) = 850155.0\ lbs$$
$$H_p = 0.0 \qquad (for\ self-energized\ gaskets)$$
$$H_m = 0.0 \qquad (for\ self-energized\ gaskets)$$

b) STEP 2 – Paragraph 24-4(b)(1), determine the required bolt load for the operating condition.

$$W_{m1} = 0.637(H + H_p)\tan[\phi - \mu] = 0.637(850155 + 0)\cdot\tan[10 - 5] = 47379.3751\ lbs$$

c) STEP 3 – Paragraph 24-4(b)(2), determine the minimum required total bolt load for the gasket seating conditions, .

$$W_{m2} = 0.637H_m\tan[\phi + \mu] = 0.637(0.0)\cdot\tan[10 + 5] = 0.0\ lbs$$

d) STEP 4 – Paragraph 24-4(b)(3), determine the minimum required total bolt load for the assembly condition.

$$W_{m3} = 0.637(H + H_p)\tan[\phi + \mu] = 0.637(850155 + 0)\cdot\tan[10 + 5] = 145107.5462\ lbs$$

e) STEP 5 – Paragraph 24-4(c), determine the total required and actual bolt areas.

The total cross-sectional area of bolts A_{mL} required for the operating conditions, gasket seating, and assembly condition is determined as follows.

$$A_{mL} = \max\left[\frac{W_{m1}}{2S_b},\ \frac{W_{m2}}{2S_a},\ \frac{W_{m3}}{2S_a}\right] = \max[1.0300,\ 0.0,\ 3.1545] = 3.1545\ in^2$$

Where,

$$A_{m1} = \frac{W_{m1}}{2S_b} = \frac{47379.3751}{2(23000)} = 1.0300\ in^2$$
$$A_{m2} = \frac{W_{m2}}{2S_a} = \frac{0}{2(23000)} = 0.0\ in^2$$
$$A_{m3} = \frac{W_{m3}}{2S_a} = \frac{145107.5462}{2(23000)} = 3.1545\ in^2$$

The actual bolt area is calculated as follows (using two 1.75 in diameter bolts).

$$A_{bL} = (Number\ of\ bolts)(Root\ area\ of\ one\ bolt) = 2(1.980) = 3.96\ in^2$$

Verify that the actual bolt area is equal to or greater than the total required area.

$$\{A_{bL} = 3.96\ in^2\} = \{A_{mL} = 3.1545\ in^2\} \qquad True$$

f) STEP 6 – Paragraph 24-4(d), determine the clamp connection design bolt load, W .

Operating conditions.

$$W = W_{m1} = 47379.3751\ lbs$$

Assembly conditions.

$$W = (A_{mL} + A_{bL})S_a = (3.1545 + 3.96)23000 = 163633.5\ lbs$$

Alternatively, if controlled bolting (e.g., bolt tensioning or torque control) techniques are used to assemble the clamp, assembly design bolt load may be calculated as follows.

$$W = 2A_{mL} \cdot S_a = 2(3.1545)23000 = 145107.0\ lbs$$

Note: This calculation is shown for informational purposes only and will not be used in the example problem.

Commentary: VIII-1, Appendix 24 does not include an overall step-by-step procedure to design a clamp. However, an organized procedure of the steps taken when designing a clamp is presented in VIII-2, paragraph 4.17.5. The procedure is applicable to VIII-1, Appendix 24 and is presented in this example problem in an effort to assist the designer.

The procedure to design a clamp connection is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flange joint.

See above data.

b) STEP 2 – Determine an initial flange and clamp geometry see Figures 24-1 Sketch (a) and Sketch (c) and Figure E4.17.1 of this example.

c) STEP 3 – Determine the design bolt loads for operating condition, W, and the gasket seating and assembly condition, W, from VIII-1, paragraph 24-4(d).

$$W = 47379.3751\ lbs \qquad \textit{Operating Condition}$$

$$W = 163633.5\ lbs \qquad \textit{Assembly Condition}$$

d) STEP 4 – Determine the flange forces, H_D, H_G, and H_T from VIII-1, paragraph 24-3.

$$H_D = 0.785B^2P = 0.785(18.0)^2(3000) = 763020.0\ lbs$$

$$H_G = \frac{1.571W}{\tan[\phi+\mu]} - (H + H_p) = \frac{1.571(47379.3751)}{\tan[10+5]} - (850155.0 + 0)$$

$$H_G = -572367.2687\ lbs$$

$$H_T = H - H_D = 850155.0 - 763020.0 = 87135.0\ lbs$$

e) STEP 5 – Determine the flange moment for the operating condition paragraphs 24-3 and 24-5. In these equations, the moment arms h_D, h_G, and h_T and constants A, C, N, $\bar{h}$, and h_2 are determined from paragraph 24-3.

$$M_o = M_D + M_G + M_T + M_F + M_P + M_R$$

$$M_o = 5961093.75 + 0.0 + 1214444.063 + 0 + 25944.1891 + (-254998.8042)$$

$$M_o = 6946483.1980\ in-lbs$$

Where,

$$h_D = \left[\frac{C-(B+g_1)}{2}\right] = \left[\frac{46.375-(18.0+12.75)}{2}\right] = 7.8125\ in$$

$$h_G = 0.0 \qquad \textit{for full face contact geometries}$$

$$h_T = \frac{\left[C - \frac{(B+G)}{2}\right]}{2} = \left[\frac{\left[46.375 - \frac{(18.0+19.0)}{2}\right]}{2}\right] = 13.9375\ in$$

$$A = B + 2(g_1 + g_2) = 18.0 + 2(12.75 + 2.75) = 49.0\ in$$

$$C = \frac{(A + C_i)}{2} = \frac{(49.0 + 43.75)}{2} = 46.375\ in$$

$$N = B + 2g_1 = 18.0 + 2(12.75) = 43.5\ in$$

$$\bar{h} = \frac{T^2 g_1 + h_2^2 g_2}{2(Tg_1 + h_2 g_2)} = \frac{(7.321)^2\, 12.75 + (7.0785)^2\, 2.75}{2\left(7.321(12.75) + 7.0785(2.75)\right)} = 3.6396\ in$$

$$h_2 = T - \frac{g_2 \tan[\phi]}{2} = 7.321 - \frac{2.75 \tan[10]}{2} = 7.0786\ in$$

Therefore,

$$M_D = H_D h_D = 763020.0(7.8125) = 5961093.75\ lbs$$

$$M_G = H_G h_G = -572367.2687(0.0) = 0.0$$

$$M_T = H_T h_T = 87135.0(13.9375) = 1214444.063\ in-lbs$$

$$M_F = H_D\left(\frac{g_1 - g_0}{2}\right) = 763020.0\left(\frac{12.75 - 12.75}{2}\right) = 0.0$$

$$M_P = (3.14)PBT\left(\frac{T}{2} - \bar{h}\right)$$

$$M_P = (3.14)(3000)(18.0)(7.321)\left(\frac{7.321}{2} - 3.6396\right) = 25944.1891\ lbs$$

$$M_R = 1.571W\left(\bar{h} - T + \frac{(C-N)\tan[\phi]}{2}\right)$$

$$M_R = 1.571(47351.0941)\left(3.6396 - 7.321 + \frac{(46.375 - 43.5)\tan[10]}{2}\right)$$

$$M_R = -254998.8042\ lbs$$

f) STEP 6 – Determine the flange moment for the assembly condition paragraph 24-5.

$$M_o = \frac{0.785W(C-G)}{\tan[\phi+\mu]} = \frac{0.785(163633.5)(46.375-19.0)}{\tan[10+5]} = 13123314.95\ \ in-lbs$$

g) STEP 7 – Determine the hub factors, paragraph 24-3.

$$I_h = \frac{g_1 T^3}{3} + \frac{g_2 h_2^3}{3} - \left(g_2 h_2 + g_1 T\right)\bar{h}^2$$

$$I_h = \frac{12.75(7.321)^3}{3} + \frac{2.75(7.0786)^3}{3} - \left(2.75(7.0786) + 12.75(7.321)\right)(3.6396)^2$$

$$I_h = 498.4148 \; in^4$$

$$\bar{g} = \frac{Tg_1^2 + h_2 g_2\left(2g_1 + g_2\right)}{2\left(Tg_1 + h_2 g_2\right)} = \frac{7.321(12.75)^2 + 7.0786(2.75)\left(2(12.75) + 2.75\right)}{2\left(7.321(12.75) + 7.0786(2.75)\right)}$$

$$\bar{g} = 7.7123 \; in$$

h) STEP 8 – Determine the reaction moment, M_H and reaction shear force, Q at the hub neck for the operating condition, paragraph 24-3.

$$M_H = \frac{M_o}{1 + \frac{1.818}{\sqrt{Bg_1}}\left[T - \bar{h} + \frac{3.305 I_h}{g_1^2\left(0.5B + \bar{g}\right)}\right]}$$

$$M_H = \frac{6946483.1980}{1 + \frac{1.818}{\sqrt{18.0(12.75)}}\left[7.321 - 3.6396 + \frac{3.305(498.4148)}{(12.75)^2\left(0.5(18.0) + 7.7123\right)}\right]}$$

$$M_H = 4586492.859 \; in-lbs$$

$$Q = \frac{1.818 M_H}{\sqrt{Bg_1}} = \frac{1.818(4589492.859)}{\sqrt{18(12.75)}} = 550766.1394 \; lbs$$

i) STEP 9 – Determine the reaction moment, M_H and reaction shear force, Q at the hub neck for the assembly condition, paragraph 24-3.

$$M_H = \frac{M_o}{1 + \frac{1.818}{\sqrt{Bg_1}}\left[T - \bar{h} + \frac{3.305 I_h}{g_1^2\left(0.5B + \bar{g}\right)}\right]}$$

$$M_H = \frac{13123314.95}{1 + \frac{1.818}{\sqrt{18.0(12.75)}}\left[7.321 - 3.6396 + \frac{3.305(498.4148)}{(12.75)^2\left(0.5(18.0) + 7.7123\right)}\right]}$$

$$M_H = 8664814.783 \; in-lbs$$

$$Q = \frac{1.818 M_H}{\sqrt{Bg_1}} = \frac{1.818(8664814.783)}{\sqrt{18.0(12.75)}} = 1039828.742 \; lbs$$

j) STEP 10 – Determine the clamp factors paragraph 24-3.

$$A_c = A_1 + A_2 + A_3 = 97.2188 + 91.3389 + 38.5 = 227.0577 \; in^2$$

Where,

$$A_1 = (C_w - 2C_t)C_t = (28.0 - 2(7.625))7.625 = 97.2188\ in^2$$

$$A_2 = 1.571C_t^2 = 1.571(7.625)^2 = 91.3389\ in^2$$

$$A_3 = (C_w - C_g)l_c = (28.0 - 14.0)2.75 = 38.5\ \ in^2$$

$$e_b = B_c - \frac{C_i}{2} - l_c - X = 32.25 - \frac{43.75}{2} - 2.75 - 2.7009 = 4.9241\ in$$

Where,

$$X = \frac{\left(\frac{C_w}{2} - \frac{C_t}{3}\right)C_t^2 - 0.5(C_w - C_g)l_c^2}{A_c} = \frac{\left(\frac{28}{2} - \frac{7.625}{3}\right)(7.625)^2 - 0.5(28-14)(2.75)^2}{227.0577}$$

$$X = 2.7009\ in$$

$$I_c = \left(\frac{A_1}{3} + \frac{A_2}{4}\right)C_t^2 + \frac{A_3 l_c^2}{3} - A_c X^2$$

$$I_c = \left(\frac{97.2188}{3} + \frac{91.3389}{4}\right)(7.625)^2 + \frac{38.5(2.75)^2}{3} - 227.0577(2.7009)^2$$

$$I_c = 1652.4435\ in^4$$

k) STEP 11 – Determine the hub stress correction factor, f, based on g_1, g_0, h, and B using Fig. 2-7.6 and l_m using the following equation in paragraph 24-3.

$$X_g = \frac{g_1}{g_0} = \frac{12.75}{12.75} = 1.0$$

$$X_h = \frac{h}{h_o} = \frac{h}{\sqrt{Bg_o}} = \frac{0.0}{\sqrt{18(12.75)}} = 0.0$$

Since $X_g = 1.0$, $f = 1.0$ per Fig. 2-7.6.

$$l_m = l_c - 0.5(C - C_i) = 1652.4435 - 0.5(46.375 - 43.75) = 1.4375\ in$$

l) STEP 12 – Determine the hub and clamp stresses for the operating and assembly conditions using the equations in paragraphs 24-6 and 24-7.

<u>Operating Condition – Location: Hub</u>

Longitudinal Stress:

$$S_1 = f\left[\frac{PB^2}{4g_1(B+g_1)} + \frac{1.91M_H}{g_1^2(B+g_1)}\right]$$

$$S_1 = 1.0\left[\frac{3000(18.0)^2}{4(12.75)(18.0+12.75)} + \frac{1.91(4586492.859)}{(12.75)^2(18.0+12.75)}\right] = 2372.3\ psi$$

Hoop Stress:

$$S_2 = P\left(\frac{N^2+B^2}{N^2-B^2}\right) = 3000\left(\frac{(43.5)^2+(18.0)^2}{(43.5)^2-(18.0)^2}\right) = 4239.6\ psi$$

Axial Shear Stress:

$$S_3 = \frac{0.75W}{T(B+2g_1)\tan[\phi-\mu]} = \frac{0.75(47379.3751)}{7.321(18.0+2(12.75))\tan[10-5]} = 1275.4\ psi$$

Radial Shear Stress:

$$S_4 = \frac{0.477Q}{g_1(B+g_1)} = \frac{0.477(550766.1394)}{12.75(18.0+12.75)} = 670.1\ psi$$

Operating Condition – Location: Clamp

Longitudinal Stress:

$$S_5 = \frac{W}{2C\tan[\phi-\mu]}\left[\frac{1}{C_t} + \frac{3(C_t+2l_m)}{C_t^2}\right]$$

$$S_5 = \frac{47379.3751}{2\times 46.375\tan[10-5]}\left[\frac{1}{7.625} + \frac{3(7.635+2(1.4375))}{(7.625)^2}\right] = 3932.2\ psi$$

Tangential Stress:

$$S_6 = \frac{W}{2}\left[\frac{1}{A_c} + \frac{|e_b|\cdot(C_t-X)}{I_c}\right]$$

$$S_6 = \frac{47379.3751}{2}\left[\frac{1}{227.0577} + \frac{|4.9241|\cdot(7.625-2.7009)}{1652.4435}\right] = 451.9\ psi$$

Lip Shear Stress:

$$S_7 = \frac{1.5W}{(C_w-C_g)C\tan[\phi-\mu]} = \frac{1.5(47379.3751)}{(28.0-14.0)(46.375)\tan[10-5]} = 1251.2\ psi$$

Lug Bending Stress:

$$S_8 = \frac{3WL_a}{L_wL_h^2} = \frac{3(47379.3751)(3.7)}{28.0(15.0)^2} = 83.5\ psi$$

Bearing Stress at clamp-to-hub contact:

$$S_9 = \frac{W}{(A - C_i)C\tan[\phi - \mu]} = \frac{47379.3751}{(49.0 - 43.75)(46.375)\tan[10 - 5]} = 2224.3 \ psi$$

Assembly Condition – Location: Hub

Longitudinal Stress:

$$S_1 = f\left[\frac{1.91M_H}{g_1^2(B + g_1)}\right] = 1.0\left|\frac{1.91(8664814.783)}{(12.75)^2(18.0 + 12.75)}\right| = 3310.8 \ psi$$

Hoop Stress:

$$S_2 = 0.0$$

Axial Shear Stress:

$$S_3 = \frac{0.75W}{T(B + 2g_1)\tan[\phi + \mu]} = \frac{0.75(163633.5)}{7.321(18.0 + 2(12.75))\tan[10 + 5]} = 1438.2 \ psi$$

Radial Shear Stress:

$$S_4 = \frac{0.477Q}{g_1(B + g_1)} = \frac{0.477(1039828.742)}{12.75(18.0 + 12.75)} = 1265.1 \ psi$$

Gasket Seating Condition– Location: Clamp

Longitudinal Stress:

$$S_5 = \frac{W}{2C\tan[\phi + \mu]}\left[\frac{1}{C_t} + \frac{3(C_t + 2l_m)}{C_t^2}\right]$$

$$S_5 = \frac{163633.5}{2(46.375)\tan[10 + 5]}\left[\frac{1}{7.625} + \frac{3(7.625 + 2(1.4375))}{7.625^2}\right] = 4430.8 \ psi$$

Tangential Stress:

$$S_6 = \frac{W}{2}\left[\frac{1}{A_c} + \frac{|e_b| \cdot (C_t - X)}{I_c}\right]$$

$$S_6 = \frac{163633.5}{2}\left[\frac{1}{227.0577} + \frac{|4.9241| \cdot (7.625 - 2.7009)}{1652.4435}\right] = 1560.9 \ psi$$

Lip Shear Stress:

$$S_7 = \frac{1.5W}{(C_w - C_g)C\tan[\phi + \mu]} = \frac{1.5(163633.5)}{(28.0 - 14.0)(46.375)\tan[10 + 5]} = 1410.9 \ psi$$

Lug Bending Stress:

$$S_8 = \frac{3WL_a}{L_w L_h^2} = \frac{3(163633.5)(3.7)}{28.0(15.0)^2} = 288.3\ psi$$

Bearing Stress at clamp-to-hub contact:

$$S_9 = \frac{W}{(A - C_i)C\tan[\phi + \mu]} = \frac{163633.5}{(49.0 - 43.75)(46.375)\tan[10 + 5]} = 2508.3\ psi$$

m) STEP 13 – Check the flange stress acceptance criteria for the operating and gasket seating conditions are shown in Table 24-8.

<u>Operating Condition – Location: Hub</u>

$$\{S_1 = 2372.3\ psi\} \le \{1.5S_{OH} = 30000\ psi\} \qquad True$$

$$\{S_2 = 4239.6\ psi\} \le \{S_{OH} = 20000\ psi\} \qquad True$$

$$\{S_3 = 1275.4\ psi\} < \{0.8S_{HO} = 16000\ psi\} \qquad True$$

$$\{S_4 = 670.1\ psi\} \le \{0.8S_{OH} = 16000\ psi\} \qquad True$$

<u>Operating Condition – Location: Clamp</u>

$$\{S_5 = 3932.2\ psi\} \le \{1.5S_{OC} = 30000\ psi\} \qquad True$$

$$\{S_6 = 451.9\ psi\} \le \{1.5S_{OC} = 30000\ psi\} \qquad True$$

$$\{S_7 = 1251.2\ psi\} < \{0.8S_{OC} = 16000\ psi\} \qquad True$$

$$\{S_8 = 83.5\ psi\} \le \{S_{OC} = 20000\ psi\} \qquad True$$

$$\{S_9 = 2224.3\ psi\} \le \{1.6 \cdot \min[S_{OH}, S_{OC}] = 32000\ psi\} \qquad True$$

<u>Gasket Seating Condition – Location: Flange</u>

$$\{S_1 = 3310.8\ psi\} \le \{1.5S_{OH} = 30000\ psi\} \qquad True$$

$$\{S_2 = 0\ psi\} \le \{S_{OH} = 20000\ psi\} \qquad True$$

$$\{S_3 = 1438.2\ psi\} \le \{0.8S_{OH} = 16000\ psi\} \qquad True$$

$$\{S_4 = 1265.1\ psi\} \le \{0.8S_{OH} = 16000\ psi\} \qquad True$$

<u>Gasket Seating Condition – Location: Clamp</u>

$$\{S_5 = 4430.8\ psi\} \le \{1.5S_{OC} = 30000\ psi\} \qquad True$$

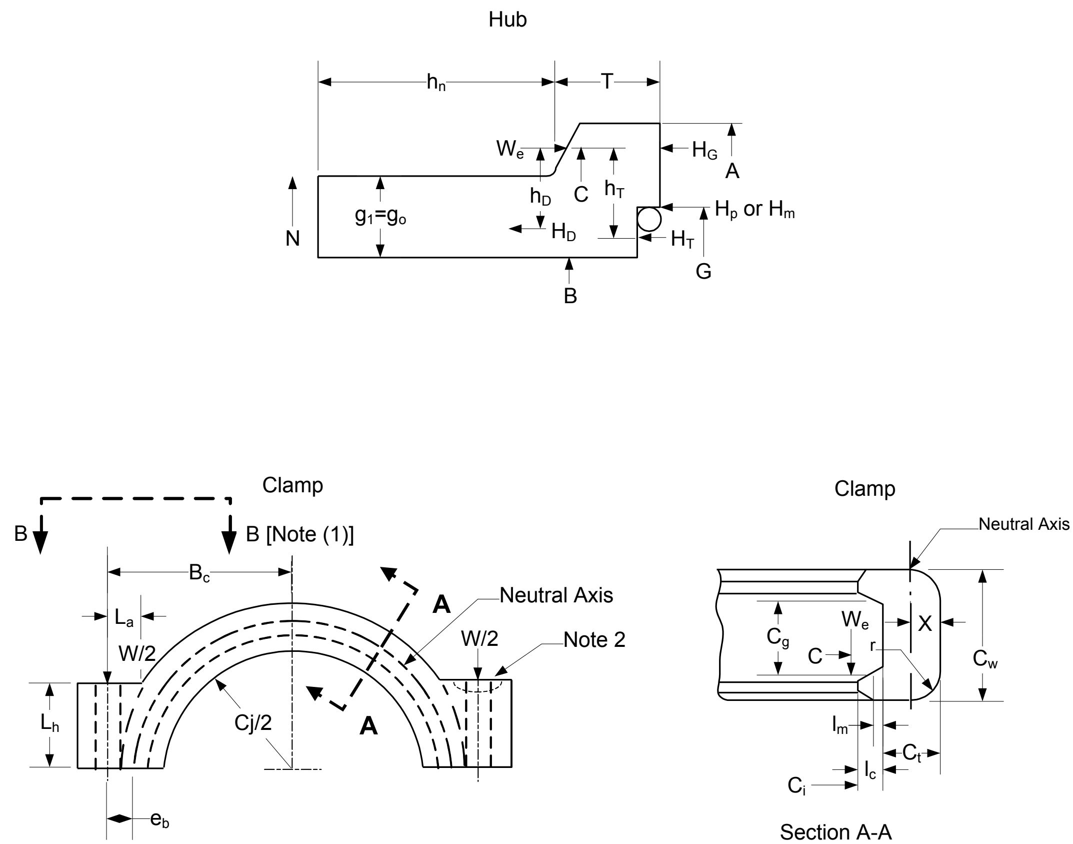
$$\{S_6 = 1560.9\ psi\} \le \{1.5S_{OC} = 30000\ psi\} \qquad True$$

$$\left\{S_7 = 1410.9\ psi\right\} \le \left\{0.8S_{OC} = 16000\ psi\right\} \qquad True$$

$$\left\{S_8 = 288.3\ psi\right\} < \left\{S_{OC} = 20000\ psi\right\} \qquad True$$

$$\left\{S_9 = 2508.3\ psi\right\} < \left\{1.6\min\left[S_{OH},\, S_{OC}\right] = 32000\ psi\right\} \qquad True$$

Since the acceptance criteria are satisfied, the design is complete.



Notes:
1) See Figure 4.17.2 for sction B-B
2) Clamp may have spherical depressions at bolt holes to facilitate the use of spherical nuts

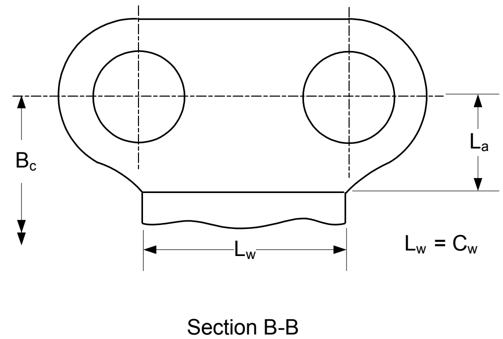


Figure E4.17.1 – Typical Hub and Clamp Configuration

4.18 Tubesheets in Shell and Tube Heat Exchangers

4.18.1 Example E4.18.1 - U-Tube Tubesheet Integral with Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration a as shown in VIII-1, Figure UHX-12.1, Configuration a.

- The shell side design conditions are -10 and 60 psig at 500°F.
- The tube side design conditions are -15 and 140 psig at 500°F.
- The tube material is SA-249 S31600 (Stainless Steel 316). The tubes are 0.75 in. outside diameter and 0.065 in. thick and are to be full-strength welded with no credit taken for expansion.
- The tubesheet material is SA-240 S31600 (Stainless Steel 316) with no corrosion allowance on the tube side and no pass partition grooves. The tubesheet outside diameter is 12.939 in. The tubesheet has 76 tube holes on a 1.0 in. square pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 2.25 in., and the radius to the outermost tube hole center is 5.438 in.
- The shell material is SA-312 S31600 (Stainless Steel 316) welded pipe. The shell inside diameter is 12.39 in. and the shell thickness is 0.18 in.
- The channel material is SA-240 S31600 (Stainless Steel 316). The channel inside diameter is 12.313 in. and the channel thickness is 0.313 in.

Data Summary

The data summary consists of those variables from the nomenclature (see VIII-1, paragraphs UHX-11.3 and UHX-12.3) that are applicable to this configuration.

The data for VIII-1, paragraph UHX-11.3 is:

$$c_t = 0\, in.$$

$$d_t = 0.75\, in.$$

$$E = 25.8E10^6\ psi \text{ from Table TM-1 of Section II, Part D at } 500^\circ\text{F}$$

$$E_t = 25.8E10^6\ psi \text{ from Table TM-1 of Section II, Part D at } 500^\circ\text{F}$$

$$h_g = 0\, in.$$

$$p = 1.0\, in.$$

$$r_o = 5.438\, in.$$

$$S = 18,000\ psi \text{ from Table 1A of Section II, Part D at } 500^\circ\text{F}$$

$$S_t = 18,000\ psi \text{ from Table 1A of Section II, Part D at } 500^\circ\text{F (for seamless tube, SA-213)}$$

$$t_t = 0.065\, in.$$

$$U_{L1} = 2.25\, in.$$

$$\rho = 0 \text{ for no tube expansion}$$

The data for VIII-1, paragraph UHX-12.3 is:

$$A = 12.939\, in.$$

$$D_c = 12.313\, in.$$

$$D_s = 12.39\, in.$$

$E = 25.8E10^6\ psi$ from Table TM-1 of Section II, Part D at 500°F

$E_c = 25.8E10^6\ psi$ from Table TM-1 of Section II, Part D at 500°F

$E_s = 25.8E10^6\ psi$ from Table TM-1 of Section II, Part D at 500°F

$P_{sd,\max} = 60\ psig$

$P_{sd,\min} = -10\ psig$

$P_{td,\max} = 140\ psig$

$P_{td,\min} = -15\ psig$

$S = 18,000\ psi$ from Table 1A of Section II, Part D at 500°F

$S_c = 18,000\ psi$ from Table 1A of Section II, Part D at 500°F

$S_s = 18,000\ psi$ from Table 1A of Section II, Part D at 500°F (for seamless pipe, SA-312)

$t_c = 0.313\ in.$

$t_s = 0.18\ in.$

$\nu_c = 0.3$

$\nu_s = 0.3$

Calculation Procedure

The calculation procedure for a U-tube heat exchanger tubesheet in given in VIII-1, paragraph UHX-12.5. The calculation results are shown for loading case 1 where $P_s = P_{sd,min} = -10\ psig$ and $P_t = P_{td,max} = 140\ psig$, since this case yields the greatest value of σ.

a) STEP 1 – Calculate D_o, μ, $\mu*$, and h'_g from VIII-1, paragraph UHX-11.5.1.

$$D_o = 11.626\ in.$$
$$L_{L1} = 11.6\ in.$$
$$A_L = 26.2\ in.^2$$
$$\mu = 0.25$$
$$d* = 0.75\ in.$$
$$p* = 1.15\ in.$$
$$\mu* = 0.349$$
$$h'_g = 0\ in.$$

b) STEP 2 – Calculate ρ_s, ρ_c, and M_{TS} for configuration a.

$$\rho_s = 1.07$$
$$\rho_c = 1.06$$
$$M_{TS} = -160\ in.-lb/in.$$

c) STEP 3 – Assume a value for the tubesheet thickness, h, and calculate h/p. Determine E^*/E and ν^* from VIII-1, paragraph UHX-11.5.2 and calculate E^*.

$$h = 0.521\,in.$$
$$h/p = 0.521$$
$$E^*/E = 0.445$$
$$\nu^* = 0.254$$
$$E^* = 11.5E10^6\ psi$$

d) STEP 4 – For configuration a, calculate $\beta_s,\ k_s,\ \lambda_s,\ \delta_s$, and ω_s for the shell and $\beta_c,\ k_c,\ \lambda_c,\ \delta_c$, and ω_c for the channel.

$$\beta_s = 1.21\,in.^{-1}$$
$$k_s = 33{,}300\,lb.$$
$$\lambda_s = 32.0\times10^6\ psi$$
$$\delta_s = 7.02\times10^{-6}\ in.^3/lb$$
$$\omega_s = 0.491\,in.^2$$
$$\beta_c = 0.914\,in.^{-1}$$
$$k_c = 132{,}000\,lb$$
$$\lambda_c = 110\times10^6\ psi$$
$$\delta_c = 3.99\times10^{-6}\ in.^3/lb$$
$$\omega_c = 0.756\,in.^2$$

e) STEP 5 – Calculate K and F for configuration a.

$$K = 1.11$$
$$F = 9.41$$

f) STEP 6 – Calculate M^* for configuration a.

$$M^* = -49.4\ \ in.-lb/in.$$

g) STEP 7 – Calculate $M_p,\ M_o$, and M.

$$M_p = 568\ in.-lb/in.$$
$$M_o = -463\ in.-lb/in.$$
$$M = 568\ in.-lb/in.$$

h) STEP 8 – Calculate σ and check the acceptance criterion.

$$\sigma = 36{,}000\,psi \le 2S = 36{,}000\,psi$$

i) STEP 9 – Calculate τ and check the acceptance criterion.

$$\tau = 3{,}350\,psi \le 0.8S = 14{,}400\,psi$$

j) STEP 10 – For configuration a, calculate $\sigma_{s,m}$, $\sigma_{s,b}$, and σ_s for the shell and $\sigma_{c,m}$, $\sigma_{c,b}$, and σ_c for the channel, and check the acceptance criterion. The shell thickness shall be 0.18 in. for a minimum length of 2.69 in. adjacent to the tubesheet and the channel thickness shall be 0.313 in. for a minimum length of 3.53 in. adjacent to the tubesheet.

$$\sigma_{s,m} = -170\ psi$$

$$\sigma_{s,b} = -17,600\ psi$$

$$\sigma_s = 17,700\ psi \le 1.5S_s = 27,000\ psi$$

$$\sigma_{c,m} = 1,340\ psi$$

$$\sigma_{c,b} = 25,300\ psi$$

$$\sigma_c = 26,600\ psi \le 1.5S_c = 27,000\ psi$$

The assumed value for the tubesheet thickness, h, is acceptable and the shell and channel stresses are within the allowable stresses; therefore, the calculation procedure is complete.

4.18.2 Example E4.18.2 - U-Tube Tubesheet Gasketed With Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration d as shown in VIII-1, Figure UHX-12.1, Configuration d.

- The shell side design conditions are -15 and 10 psig at 300°F.
- The tube side design condition is 135 psig at 300°F.
- The tube material is SB-111 C44300 (Admiralty). The tubes are 0.625 in. outside diameter and 0.065 in. thick and are to be expanded for the full thickness of the tubesheet.
- The tubesheet material is SA-285, Grade C (K02801) with a 0.125 in. corrosion allowance on the tube side and no pass partition grooves. The tubesheet outside diameter is 20.0 in. The tubesheet has 386 tube holes on a 0.75 in. equilateral triangular pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 1.75 in., and the radius to the outermost tube hole center is 8.094 in.
- The diameter of the shell gasket load reaction is 19.0 in. and the shell flange design bolt load is 147,000 lb.
- The diameter of the channel gasket load reaction is 19.0 in. and the channel flange design bolt load is 162,000 lb.

<u>Data Summary</u>

The data summary consists of those variables from the nomenclature (see VIII-1, paragraphs UHX-11.3 and UHX-12.3) that are applicable to this configuration.

The data for VIII-1, paragraph UHX-11.3 is:

$c_t = 0.125\, in.$

$d_t = 0.625\, in.$

$E = 28.3E10^6\ psi$ from Table TM-1 of Section II, Part D at 300°F

$E_t = 15.4E10^6\ psi$ from Table TM-3 of Section II, Part D at 300°F

$h_g = 0\, in.$

$p = 0.75\, in.$

$r_o = 8.094\, in.$

$S = 15,700\ psi$ from Table 1A of Section II, Part D at 300°F

$S_t = 10,000\ psi$ from Table 1B of Section II, Part D at 300°F

$t_t = 0.065\, in.$

$U_{L1} = 1.75\, in.$

$\rho = 1.0$ for a full length tube expansion

The data for VIII-1, paragraph UHX-12.3 is:

$A = 20.0\, in.$

$E = 28.3E10^6\ psi$ from Table TM-1 of Section II, Part D at 300°F

$G_c = 19.0\, in.$

$G_s = 19.0\, in.$

$P_{sd,\max} = 10\ psig$

$P_{sd,\min} = -15\ psig$

$P_{td,\max} = 135\ psig$

$P_{td,\min} = 0\ psig$

$S = 15,700\ psi$ per Table 1A of Section II, Part D at 300°F

$W^* = 162,000\, lb$ from Table UHX-8.1

Calculation Procedure

The calculation procedure for a U-tube heat exchanger tubesheet in given in VIII-1, paragraph UHX-12.5. The calculation results are shown for loading case 1 where $P_s = P_{sd,min} = -15\ psig$ and $P_t = P_{td,max} = 135\ psig$ since this case yields the greatest value of σ.

a) STEP 1 – Calculate $D_o, \mu, \mu*,$ and h'_g from VIII-1, paragraph UHX-11.5.1.

$$D_o = 16.813\, in.$$
$$L_{L1} = 16.8\, in.$$
$$A_L = 29.4\ in.^2$$
$$\mu = 0.167$$
$$d* = 0.580\, in.$$
$$p* = 0.805\, in.$$
$$\mu* = 0.280$$
$$h'_g = 0\, in.$$

b) STEP 2 – Calculate $\rho_s, \rho_c,$ and M_{TS} for configuration d.

$$\rho_s = 1.13$$
$$\rho_c = 1.13$$
$$M_{TS} = -785\ in.\!-\!lb/in.$$

c) STEP 3 – Assume a value for the tubesheet thickness, h, and calculate h/p. Determine $E*/E$ and $\nu*$ from VIII-1, paragraph UHX-11.5.2 and calculate $E*$.

$$h = 1.28\, in.$$
$$h/p = 1.71$$
$$E*/E = 0.265$$
$$\nu* = 0.358$$
$$E* = 7.50E10^6\ psi$$

d) STEP 4 – For configuration d, skip STEP 4 and proceed to STEP 5.

e) STEP 5 – Calculate K and F for configuration d.

$$K = 1.19$$
$$F = 0.420$$

f) STEP 6 – Calculate M^* for configuration d.

$$M^* = -785\ in.\!-\!lb/in.$$

g) STEP 7 – Calculate $M_p, M_o,$ and M.

$$M_p = -160\ in.\!-\!lb/in.$$
$$M_o = -2,380\ in.\!-\!lb/in.$$
$$M = 2,380\ in.\!-\!lb/in.$$

h) STEP 8 – Calculate σ and check the acceptance criterion.

$$\sigma = 31,200\, psi \le 2S = 31,400\, psi$$

i) STEP 9 – Calculate τ and check the acceptance criterion.

$$\tau = 2,960\ psi \le 0.8S = 12,600\ psi$$

The assumed value for the tubesheet thickness, h, is acceptable and the calculation procedure is complete.

4.18.3 Example E4.18.3 - U-Tube Tubesheet Gasketed With Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration d as shown in VIII-1, Figure UHX-12.1 Configuration d.

- The shell side design condition is 375 psig at 500°F.
- The tube side design condition is 75 psig at 500°F.
- The tube material is SB-111 C70600 (90/10 copper-nickel). The tubes are 0.75 in. outside diameter and 0.049 in. thick and are to be expanded for one-half of the tubesheet thickness.
- The tubesheet material is SA-516, Grade 70 (K02700) with a 0.125 in. corrosion allowance on the tube side and a 0.1875 in. deep pass partition groove. The tubesheet outside diameter is 48.88 in. The tubesheet has 1,534 tube holes on a 0.9375 in. equilateral triangular pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 2.25 in., and the radius to the outermost tube hole center is 20.5 in.
- The diameter of the shell gasket load reaction is 43.5 in. and the shell flange design bolt load is 675,000 lb.
- The diameter of the channel gasket load reaction is 44.88 in. and the channel flange design bolt load is 584,000 lb.
- The tubesheet shall be designed for the differential design pressure.

Data Summary

The data summary consists of those variables from the nomenclature (see VIII-1, paragraphs UHX-11.3 and UHX-12.3) that are applicable to this configuration.

The data for VIII-1, paragraph UHX-11.3 is:

$$c_t = 0.125\,in.$$

$$d_t = 0.75\,in.$$

$$E = 27.1E10^6\ psi \text{ from Table TM-1 of Section II, Part D at } 500^\circ\text{F}$$

$$E_t = 16.6E10^6\ psi \text{ from Table TM-3 of Section II, Part D at } 500^\circ\text{F}$$

$$h_g = 0.1875\,in.$$

$$p = 0.9375\,in.$$

$$r_o = 20.5\,in.$$

$$S = 20,000\ psi \text{ from Table 1A of Section II, Par D at } 500^\circ\text{F}$$

$$S_t = 8,000\ psi \text{ from Table 1B of Section II, Par D at } 500^\circ\text{F}$$

$$t_t = 0.049\,in.$$

$$U_{L1} = 2.25\,in.$$

$$\rho = 0.5 \text{ for tubes expanded for one-half the tubesheet thickness}$$

The data for VIII-1, paragraph UHX-12.3 is:

$A = 48.88\,in.$

$E = 27.1E10^6\ psi$ from Table TM-1 of Section II, Part D at $500^\circ F$

$G_c = 44.88\,in.$

$G_s = 43.5\,in.$

$P_{sd,\max} = 375\ psig$

$P_{td,\max} = 75\ psig$

$S = 20,000\ psi$ from Table 1A of Section II, Part D at $500^\circ F$

$W^* = 675,000\,lb$ from Table UHX-8.1

Calculation Procedure

The calculation procedure for a U-tube heat exchanger tubesheet in given in VIII-1, paragraph UHX-12.5. Since differential pressure design is specified, the calculation results are shown for loading case 3.

a) STEP 1 – Calculate $D_o, \mu, \mu*$, and h'_g from VIII-1, paragraph UHX-11.5.1.

$$D_o = 41.75\,in.$$
$$L_{L1} = 41.8\,in.$$
$$A_L = 93.9\,in.^2$$
$$\mu = 0.2$$
$$d* = 0.738\,in.$$
$$p* = 0.971\,in.$$
$$\mu* = 0.240$$
$$h'_g = 0.0625\,in.$$

b) STEP 2 – Calculate ρ_s, ρ_c, and M_{TS} for configuration d.

$$\rho_s = 1.04$$
$$\rho_c = 1.07$$
$$M_{TS} = 2,250\,in.\text{–}lb/in.$$

c) STEP 3 – Assume a value for the tubesheet thickness, h, and calculate h/p. Determine $E*/E$ and $\nu*$ from VIII-1, paragraph UHX-11.5.2 and calculate $E*$.

$$h = 4.15\,in.$$
$$h/p = 4.43$$
$$E*/E = 0.204$$
$$\nu* = 0.407$$
$$E* = 5.54E10^6\ psi$$

d) STEP 4 – For configuration d, skip STEP 4 and proceed to STEP 5.

e) STEP 5 – Calculate K and F for configuration d.

$$K = 1.17$$
$$F = 0.458$$

f) STEP 6 – Calculate $M*$ for configuration d.

$$M* = 5800\ in.-lb/in.$$

g) STEP 7 – Calculate $M_p, M_o,\ and\ M$.

$$M_p = -1150\ in.-lb/in.$$
$$M_o = 26,700\ in.-lb/in.$$
$$M = 26,700\ in.-lb/in.$$

h) STEP 8 – Calculate σ and check the acceptance criterion.

$$\sigma = 39,900\ psi \le 2S = 40,000\ psi$$

i) STEP 9 – Calculate τ and check the acceptance criterion.

$$\tau = 3,770\ psi \le 0.8S = 16,000\ psi$$

The assumed value for the tubesheet thickness, h, is acceptable and the calculation procedure is complete.

4.18.4 Example E4.18.4 - U-Tube Tubesheet Gasketed With Shell and Integral with Channel, Extended as a Flange

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration as shown in VIII-1, Figure UHX-12.1, Configuration e.

- The shell side design condition is 650 psig at 400°F.
- The tube side design condition is 650 psig at 400°F.
- The tube material is SA-179 (K10200). The tubes are 0.75 in. outside diameter and 0.085 in. thick and are to be expanded for the full thickness of the tubesheet.
- The tubesheet material is SA-516, Grade 70 (K02700) with a 0.125 in. corrosion allowance on the tube side and no pass partition grooves. The tubesheet outside diameter is 37.25 in. The tubesheet has 496 tube holes on a 1.0 in. square pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 1.375 in., and the radius to the outermost tube hole center is 12.75 in.
- The diameter of the shell gasket load reaction is 32.375 in., the shell flange bolt circle is 35 in., and the shell flange design bolt load is 656,000 lb.
- The channel material is SA-516, Grade 70, (K02700). The channel inside diameter is 31 in. and the channel thickness is 0.625 in.

Data Summary

The data summary consists of those variables from the nomenclature (see VIII-1, paragraphs UHX-11.3 and UHX-12.3) that are applicable to this configuration.

The data for VIII-1, paragraph UHX-11.3 is:

$c_t = 0.125\,in.$

$d_t = 0.75\,in.$

$E = 27.7E10^6\ psi$ from Table TM-1 of Section II, Part D at 400°F

$E_t = 27.7E10^6\ psi$ from Table TM-1 of Section II, Part D at 400°F

$h_g = 0\,in.$

$p = 1.0\,in.$

$r_o = 12.75\,in.$

$S = 20,000\ psi$ from Table 1A of Section II, Part D at 400°F

$S_t = 13,400\ psi$ from Table 1A of Section II, Part D at 400°F

$t_t = 0.085\,in.$

$U_{L1} = 1.375\,in.$

$\rho = 1.0$ for full length tube expansion

The data for VIII-1, paragraph UHX-12.3 is:

$A = 37.25\,in.$

$C = 35\,in.$

$D_c = 31\,in.$

$E = 27.7E10^6\ psi$ from Table TM-1 of Section II, Part D at 400°F

$E_c = 27.7E10^6\ psi$ from Table TM-1 of Section II, Part D at 400°F

$G_s = 32.375\,in.$

$P_{sd,\max} = 650\ psig$

$P_{sd,\min} = 0\ psig$

$P_{td,\max} = 650\ psig$

$P_{td,\min} = 0\ psig$

$S = 20,000\ psi$ from Table 1A of Section II, Part D at 400°F

$S_c = 20,000\ psi$ from Table 1A of Section II, Part D at 400°F

$S_{y,c} = 32,500\ psi$ from Table Y-1 of Section II, Part D at 400°F

$S_{PS,c} = 65,000\ psi$ (MYS/UTS < 0.7; therefore use $2S_{y,c}$)

$t_c = 0.625\,in.$

$W^* = 656,000\,lb$

$\nu_c = 0.3$

Calculation Procedure

The calculation procedure for a U-tube heat exchanger tubesheet in given in VIII-1, paragraph UHX-12.5. The calculation results are shown for loading case 2 where $P_s = P_{sd,max} = 650\ psig$ and

$P_t = P_{td,min} = 0\,psig$, since this case yields the greatest value of σ .

a) STEP 1 – Calculate $D_o, \mu, \mu*$ and h'_g from VIII-1, paragraph UHX-11.5.1.

$$D_o = 26.25\,in.$$
$$L_{L1} = 26.3\,in.$$
$$A_L = 36.1\,in.^2$$
$$\mu = 0.25$$
$$d* = 0.636\,in.$$
$$p* = 1.04\,in.$$
$$\mu* = 0.385$$
$$h'_g = 0\,in.$$

b) STEP 2 – Calculate $\rho_s, \rho_c,$ and M_{TS} for configuration e.

$$\rho_s = 1.23$$
$$\rho_c = 1.18$$
$$M_{TS} = 16,500\ in.\text{–}lb/in.$$

c) STEP 3 – Assume a value for the tubesheet thickness, h, and calculate h/p. Determine $E*/E$ and $\nu*$ from VIII-1, paragraph UHX-11.5.2 and calculate $E*$.

$$h = 3.50\,in.$$
$$h/p = 3.50$$
$$E*/E = 0.441$$
$$\nu* = 0.318$$
$$E* = 12.2E10^6\ psi$$

d) STEP 4 – For configuration e, calculate β_c, $k_c, \lambda_c, \delta_c,$ and ω_c for the channel.

$$\beta_c = 0.409\,in.^{-1}$$
$$k_c = 506,000\,lb$$
$$\lambda_c = 7.59\times10^6\ psi$$
$$\delta_c = 1.18\times10^{-5}\ in.^3/lb$$
$$\omega_c = 7.01\,in.^2$$

e) STEP 5 – Calculate K and F for configuration e.

$$K = 1.42$$
$$F = 0.964$$

f) STEP 6 – Calculate $M*$ for configuration e.

$$M^* = 26,900\ in.-lb/in.$$

g) STEP 7 – Calculate $M_p, M_o,\ and\ M$.

$$M_p = 6830\ in.-lb/in.$$
$$M_o = 30,000\ in.-lb/in.$$
$$M = 30,000\ in.-lb/in.$$

h) STEP 8 – Calculate σ and check the acceptance criterion.

$$\sigma = 38,200\ psi \le 2S = 40,000\ psi$$

i) STEP 9 – Calculate τ and check the acceptance criterion.

$$\tau = 4880\ psi \le 0.8S = 16,000\ psi$$

j) STEP 10 – For configuration e, calculate $\sigma_{c,m}, \sigma_{c,b}$, and σ_c for the channel, and check the acceptance criterion. The channel thickness shall be 0.625 in. for a minimum length of 7.92 in. adjacent to the tubesheet.

$$\sigma_{c,m} = 0\ psi$$
$$\sigma_{c,b} = -57,000\ psi$$
$$\sigma_c = 57,000\ psi > 1.5S_c = 30,000\ psi$$

k) STEP 11 – Since the channel stress exceeds the allowable stress, the design must be reconsidered using one of three options.

- Option 1 requires that the tubesheet thickness be increased until the channel stresses calculated in STEP 9 are within the allowable stress for each loading case.
- Option 2 requires that the shell and/or channel thickness be increased until their respective stresses calculated in STEP 9 are within the allowable stress for each loading case.
- Option 3 permits one elastic-plastic calculation for each design. If the tubesheet stress is still within the allowable stress given in STEP 8, the design is acceptable and the calculation procedure is complete. If the tubesheet stress is greater than the allowable stress, the design shall be reconsidered by using Option 1 or 2.

Choose Option 3, configuration e. Since $\sigma_c \le S_{PS,c} = 65,000\ psi$ for all loading cases, this option may be used. The calculations for this option are only required for each loading case where $\sigma_c > 1.5S_c = 30,000\ psi$.

Calculate E_c^* for each loading case where $\sigma_c > 30,000\ psi$. For this example, E_c^* and the calculations for loading case 2 are shown.

$$E_c^* = 20.1E10^6\ psi$$

Recalculate k_c and λ_c given in STEP 4 using the applicable reduced effective modulus E_c.

$$k_c = 368,000\ lb$$
$$\lambda_c = 5.51E10^6\ psi$$

Recalculate F given in STEP 5.

$$F = 0.848$$

Recalculate $M_p, M_o,$ and M given in STEP 7.

$$M_p = 8{,}130\ in.\!-\!lb/in.$$
$$M_o = 31{,}400\ in.\!-\!lb/in.$$
$$M = 31{,}400\ in.\!-\!lb/in.$$

Recalculate σ given in STEP 8.

$$\sigma = 39{,}800\ psi \le 2S = 40{,}000\ psi$$

The assumed value for the tubesheet thickness, h, is acceptable and the calculation procedure is complete.

4.18.5 Example E4.18.5 - Fixed Tubesheet Exchanger, Configuration b, Tubesheet Integral with Shell, Extended as a Flange and Gasketed on the Channel Side

A fixed tubesheet heat exchanger is to be designed with the tubesheet construction in accordance with configuration b as shown in VIII-1, Figure UHX-13.1, Configuration b.

- For the Design Condition, the shell side design pressure is 150 psig at 700°F, and the tube side design pressure is 400 psig at 700°F.
- There is one operating condition. For Operating Condition 1, the shell side design pressure is 150 psig at 700°F, the tube side design pressure is 400 psig at 700°F, the shell mean metal temperature is 550°F, and the tube mean metal temperature is 510°F. For this example, the operating pressures and operating metal temperatures are assumed to be the same as the design values.
- The tube material is SA-214 welded (K01807). The tubes are 1 in. outside diameter, 0.083 in. thick and are to be expanded to 95% of the tubesheet thickness.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet outside diameter is 40.5 in. There are 649 tube holes on a 1.25 in. triangular pattern. There is no pass partition lane, and the outermost tube radius from the tubesheet center is 16.625 in. The distance between the outer tubesheet faces is 168 in. There is no corrosion allowance on the tubesheet.
- The shell material is SA-516, Grade 70 (K02700). The shell inside diameter is 34.75 in. and the thickness is 0.1875 in. There is no corrosion allowance on the shell. The shell contains an expansion joint that has an inside diameter of 38.5 in. and an axial rigidity of 11,388 lb/in. The efficiency of the shell circumferential welded joint (Category B) is 1.0.
- The diameter of the channel flange gasket load reaction is 36.8125 in., the bolt circle diameter is 38.875 in., the design bolt load is 512,937 lb, and the operating condition bolt load is 512,473 lb.

Data Summary - Tubesheet

Tube Layout: Triangular

$h = 3.0625\, in.$

$h_g = 0\, in.$

$c_t = 0\, in.$

$A = 40.5\, in.$

$r_o = 16.625\, in.$

$A_L = 0.0\, in.^2$

$N_t = 649$

$L_t = 168\, in.$

$p = 1.2500\, in.$

$T = 700^{\circ}\text{F}$

$T_a = 70^{\circ}\text{F}$

$S = 18{,}100\, psi$ at T from Table 1A of Section II, Part D

$S_y = 27{,}200\, psi$ at T

$S_{PS} = 54{,}400\, psi$ at T

$E = 25.5E6\, psi$ at T from TM-1 of Section II, Part D

$\nu = 0.3$

Data Summary - Tubes

$P_{td,\max} = 400\ psig$

$P_{td,\min} = 0\ psig$

$P_{to1} = 400\ psig$

$\ell_{tx} = 2.909\,in.$

$k = 1$

$\ell = 59\,in.$

$t_t = 0.083\,in.$

$d_t = 1\,in.$

$T_t = 700^{\circ}\mathrm{F}$

$T_{t,m} = 510^{\circ}\mathrm{F}$

$S_t = 10,500\ psi$ at T_t from Table 1A of Section II, Part D

$S_{y,t} = 18,600\ psi$ at T_t from Y-1of Section II, Part D

$S_{tT} = 10,500\ psi$ at T from Table 1A of Section II, Part D

$\alpha_{t,m} = 7.3E-06\,in./in.^{\circ}F$ at $T_{t,m}$

$E_t = 25,500,000\ psi$ at T_t from TM-1 of Section II, Part D

$E_{tT} = 25,500,000\ psi$ at T from TM-1 of Section II, Part D

$\upsilon_t = 0.3$

Note: Since the tubes are welded (SA-214), the tube allowable stresses S_t and S_{tT} can be divided by 0.85 per VIII-1, paragraph UHX-13.3. This results in adjusted values of $S_t = 12,353\ psi$ and $S_{tT} = 12,353\ psi$.

Data Summary - Shell

$P_{sd,\max} = 150\ psig$

$P_{sd,\min} = 0\ psig$

$P_{so1} = 150\ psig$

$t_s = 0.1875\,in.$

$D_s = 34.75\,in.$

$D_j = 38.5\,in.$

$K_j = 11,388\,lb/in.$

$T_s = 700^{\circ}\text{F}$

$T_{s,m} = 550^{\circ}\text{F}$

$S_s = 18,100\ psi$ at T_s from Table 1A of Section II, Part D

$E_{s,w} = 1.0$

$S_{y,s} = 27,200\ psi$ from Table Y-1 of Section II, Part D

$S_{PS,s} = 54,400\ psi$ at T see UG-23(e)

$E_s = 25,500,000\ psi$ from TM-1 of Section II, Part D

$\alpha_{s,m} = 7.3E-06\,in./in./^{\circ}\text{F}$ at $T_{s,m}$

$\nu_s = 0.3$

Data Summary - Channel Flange

$Gasket\ I.D. = 36.3125\,in.$

$Gasket\ O.D. = 37.3125\,in.$

$Mean\ Gasket\ Diameter, G = G_c = 36.8125\,in.$

$Gasket, m, Factor = 3.75$

$Gasket, y, Factor = 7,600\ psi$

$Flange\ Outside\ Diameter = 40.5\,in.$

$Bolt\ Circle, C = 38.875\,in.$

$Bolting\ Data = 68\,bolts, 0.75\,in.\,diameter$

$BoltingMaterial = \text{SA-193 B7}$

$Bolt\ Load, W = 512,937\,lb\ \ per\ VIII-1\ Appendix\ 2$

$Bolt\ Load, W_{m1} = 512,473\,lb\ \ per\ VIII-1\ Appendix\ 2$

$W^*\ from\ Table\ UHX-8.1$ (see Summary Table for Step 5)

$Gasket\ Monument\ Arm, h_g = \left(C - G_c\right)/2 = 1.03125\,in.$

Calculation Procedure

The tubesheet is extended as a flange. The calculation procedure for a tubesheet flanged extension is given in VIII-1, paragraph UHX-9.

The tubesheet flanged extension required thickness for the operating condition $(S = 18{,}100\ psi \text{ at } T)$ is:

$$h_r = 1.228\, in.$$

The tubesheet flanged extension required thickness for the gasket seating condition $(S = 20{,}000\ psi \text{ at } T_a)$ is:

$$h_r = 1.168\, in.$$

The calculation procedure for a Fixed Tubesheet heat exchanger is given in VIII-1, paragraph UHX-13.5. The following results are for the design and operating loading cases required to be analyzed (see paragraph UHX-13.4). This example illustrates the calculation of both the elastic and elastic-plastic solutions.

a) STEP 1 – Calculate D_o, μ, $\mu*$ and h'_g from VIII-1, paragraph UHX-11.5.1.

$$L = 161.875\, in.$$
$$D_o = 34.25\, in.$$
$$a_o = 17.125\, in.$$
$$\rho = 0.95$$
$$d* = 0.8924\, in.$$
$$\mu = 0.2000$$
$$\rho* = 1.2500\, in.$$
$$\mu* = 0.2861$$
$$\rho_s = 1.014598$$
$$\rho_c = 1.074818$$
$$x_s = 0.4467$$
$$x_t = 0.6152$$

b) STEP 2 – Calculate the shell axial stiffness, K_s, tube axial stiffness, K_t, stiffness factors, $K_{s,t}$ and J.

$$K_s = 3{,}241{,}928\, lb/in.$$
$$K_t = 37{,}666\, lb/in.$$
$$K_{s,t} = 0.13262$$
$$J = 0.0035$$

Calculate the shell coefficients β_s, k_s, λ_s, and δ_s.

$$\beta_s = 0.7102\, in.^{-1}$$
$$k_s = 21{,}866\, lb$$
$$\lambda_s = 879{,}437\ psi$$
$$\delta_s = 0.0000536694\, in.^3 / lb$$

Calculate the channel coefficients β_c, k_c, λ_c, and δ_c.

$$\beta_c = 0\, in.^{-1}$$
$$k_c = 0\, lb.$$
$$\lambda_c = 0\ psi$$
$$\delta_c = 0\, in.^3 / lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-1, paragraph UHX-11.5.2 and calculate E^*.

$$h/p = 2.45$$
$$E^*/E = 0.262993 \text{ from Table UHX-11.2}$$
$$\nu^* = 0.363967 \text{ from Table UHX-11.2}$$
$$E^* = 6{,}706{,}322\ psi$$

Calculate, X_a, and the parameters from VIII-1, Table UHX-13.1.

$$X_a = 3.9630$$
$$Z_d = 0.024609$$
$$Z_v = 0.064259$$
$$Z_m = 0.371462$$
$$Z_a = 6.54740$$
$$Z_w = 0.064259$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.1825$$
$$F = 0.4888$$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} *and* U.

$$\Phi = 0.6667$$
$$Q_1 = -0.022635$$
$$Q_{Z1} = 2.8556$$
$$Q_{Z2} = 6.888$$
$$U = 13.776$$

e) STEP 5 – Calculate γ, ω_s, ω_s^*, ω_c, ω_c^*, and γ_b

Summary Table for Step 5 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	γ	W*
1	0	400	0	512473
2	150	0	0	0
3	150	400	0	512473

Summary Table for Step 5 – Operating Condition 1				
Loading Case	P_s (psi)	P_t (psi)	γ	W*
1	0	400	-0.047	512937
2	150	0	-0.047	512937
3	150	400	-0.047	512937
4	0	0	-0.047	512937

$$\omega_s = 2.685\,in.^2$$
$$\omega^*_s = -2.6536\,in.^2$$
$$\omega_c = 0\,in.^2$$
$$\omega^*_c = 9.6816\,in.^2$$
$$\gamma_b = -0.06022$$

f) STEP 6 – For each loading case, calculate P_s', P_t', P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

Summary Table for STEP 6 – Design Condition							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	862,002	0	0	230.7	181.9	-399.4
2	-46,387	0	0	0	0	18.7	-21.5
3	-46,387	862,002	0	0	230.7	200.6	-420.9

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	862,002	-1,254	0	230.9	181.9	-400
2	-46,387	0	-1,254	0	230.9	18.7	-22
3	-46,387	862,002	-1,254	0	230.9	200.6	-421.5
4	0	0	-1,254	0	230.9	0	-0.5

g) STEP 7 – Elastic Iteration, calculate Q_2 and Q_3, the tubesheet bending stress, and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition								
Loading Case	Q_2 (lbs)	Q_3	F_m	h'_g (in)	$h-h'_g$ (in)	$\lvert\sigma_{elastic}\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	-7,040.7	0.0976	0.0975	0	3.0625	25,540	27,150	---
2	-319	0.0786	0.0901	0	3.0625	-1,269	27,150	---
3	-7,359.7	0.0966	0.0971	0	3.0625	26,809	27,150	---

Summary Table for STEP 7 – Operating Condition 1								
Loading Case	Q_2 (lbs)	Q_3	F_m	h'_g (in)	$h-h'_g$ (in)	$\lvert\sigma_{elastic}\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	-7,044.2	0.09746	0.09749	0	3.0625	25,569	---	54,400
2	-4,259.3	1.299	0.67047	0	3.0625	9,658	---	54,400
3	-7,363.3	0.09650	0.09711	0	3.0625	26,839	---	54,400
4	-3,940.3	56.627	28.409	0	3.0625	8,838	---	54,400

For Design Loading Cases 1-3 $\lvert\sigma_{elastic}\rvert < 1.5S$, and for Operating Cases 1-4 $\lvert\sigma_{elastic}\rvert < S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Elastic Iteration, calculate the tubesheet shear stress and the allowable tubesheet shear stress.

Summary Table for STEP 8 – Design Condition		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	5,584	14,480
2	300	14,480
3	5,884	14,480

Summary Table for STEP 8 – Operating Condition 1		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	5,591.8	14,480
2	307.1	14,480
3	5,892.3	14,480
4	6.6	14,480

For all Loading Cases $\lvert\tau\rvert < 0.8S$. The shear stress criterion for the tubesheet is satisfied.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.3255\ in.$$

$$F_t = 181.24\,in.$$

$$C_t = 164.5$$

Summary Table for STEP 9 – Design Condition				
Loading Case	F_t, min	$\sigma_{t,1}$ (psi)	F_t, max	$\sigma_{t,2}$ (psi)
1	-1.081	-4024	3.809	7,570
2	-1.011	269	3.658	865
3	-1.077	-3,755	3.801	8,435

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	F_t, min	$\sigma_{t,1}$ (psi)	F_t, max	$\sigma_{t,2}$ (psi)
1	-1.081	-4,028.8	3.807	7,580.9
2	-5.520	-322.2	13.334	2,137
3	-1.078	-3,760	3.8	8,445.5
4	-213.188	-600.4	451.8	1,272.4

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\lvert\sigma_{t,\min}\rvert$ (psi)	F_s	S_{tb} (psi)
1	7,570	12,353	4,024	1.346	5,693.9
2	865	12,353	0	0	0
3	8,435	12,353	3,755	1.349	5,677

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\lvert\sigma_{t,\min}\rvert$ (psi)	F_s	S_{tb} (psi)
1	7,580.9	24,706	4,028.8	1.346	5,690.9
2	2,137.0	24,706	322.2	1.250	6,129.4
3	8,445.5	24,706	3,760	1.350	5,674.9
4	1,272.4	24,706	600.4	1.250	6,129.4

For all Loading Cases $\lvert\sigma_{t,\max}\rvert < S_t$. The axial tension stress criterion for the tube is satisfied.

For all Loading Cases $|\sigma_{t,\min}| < S_{tb}$. The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Design Condition				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	26.1	18,100	---	---
2	-760	18,100	---	8,505
3	-738.7	18,100	---	8,508

Summary Table for STEP 10 – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	0.0579	---	36,200	…
2	-786.1	---	36,200	8,505
3	-764.8	---	36,200	8,505
4	-21.2	---	36,200	8,505

k) STEP 11 – For each loading case, calculate the stresses in the shell and/or channel when integral with the tubesheet.

Summary Table for STEP 11 – Design Condition					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	26.1	-42,440	42,466	27,150	---
2	-760	19,214	19,978	27,150	---
3	-738.7	-23,227	23,966	27,150	---

Summary Table for STEP 11 – Operating Condition 1					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	0.0579	-42,484	42,484	---	54,400
2	-786.1	8,633	9,419	---	54,400
3	-764.8	-23,271	24,035	---	54,400
4	-21.2	-10,581	10,602	---	54,400

For Design Loading Cases 1 and 3 $|\sigma_s| > 1.5S_s$, and for Operating Loading Cases 1-4

$|\sigma_s| < S_{PS,s}$. The stress criterion for the shell is not satisfied.

l) STEP 12 – The design shall be reconsidered by using one or a combination of the following options.

- Option 1 – Increase the tubesheet thickness and return to STEP 1.
- Option 2 – Increase the integral shell and/or channel thickness and return to STEP 1
- Option 3 – Perform the elastic-plastic calculation procedures as defined in VIII-1, paragraph UHX-13.7.

Choose Option 3. Since the total axial stress in the shell σ_s is between $1.5S_s$ and $S_{PS,s}$ for Design Condition Loading Case 1 and 3, the procedure of VIII-1, paragraph UHX-13.7 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

Summary Results for STEP 12, Elastic Plastic Iteration Results per VIII-1, paragraph UHX-13.7.3	
Design Condition Loading Case	1
S^*_s, psi	27,200
$fact_s$	0.776
E^*_s, psi	19,785,000
k_s, lb	16,965
λ_s	0.682E+06
F	0.470
ϕ	0.641
Q_1	-0.0215
Q_{Z1}	2.865
Q_{Z2}	6.941
U	13.882
P_W, psi	232.5
P_{rim}, psi	183.309
P_e, psi	-399.4
Q_2, lb	-7,095
Q_3	0.100
F_m	0.098
$\|\sigma\|, psi$	25,752

The final calculated tubesheet bending stress of 25,752 psi (Design Loading Case 1) is less than the allowable tubesheet bending stress of 27,150 psi. As such, this geometry meets the requirements of VIII-1, paragraph UHX. The intermediate results for the elastic-plastic calculation are shown above.

4.18.6 Example E4.18.6 - Fixed Tubesheet Exchanger, Configuration b, Tubesheet Integral with Shell, Extended as a Flange and Gasketed on the Channel Side

A fixed tubesheet heat exchanger is to be designed with the tubesheet construction in accordance with configuration b as shown in VIII-1, Figure UHX-13.1, Configuration b.

- For the Design Condition, the shell side design pressure is 335 psig at 675°F, and the tube side design pressure is 1040 psig at 650°F.
- There is one operating condition. For Operating Condition 1, the shell side design pressure is 335 psig at 675°F, the tube side design pressure is 1040 psig at 650°F, the shell mean metal temperature is 550°F, and the tube mean metal temperature is 490°F. For this example, the operating pressures and operating metal temperatures are assumed to be the same as the design values.
- The tube material is welded SA-214 (K01807). The tubes are 0.75 in. outside diameter, are 0.083 in. thick, and are to be expanded for a length of 4.374 in.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet outside diameter is 32.875 in. There are 434 tube holes on a 0.9375 in. triangular pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 10.406 in. The distance between the outer tubesheet faces is 144.375 in. There is a 0.125 in. corrosion allowance on both sides of the tubesheet.
- The shell material is SA-516, Grade 70 (K02700). The shell outside diameter is 24 in. and the thickness is 0.5 in. There is a 0.125 in. corrosion allowance on the shell. There is also a shell band 1.25 in. thick, 9.75 in. long with a 0.125 in. corrosion allowance. The shell and shell band materials are the same. The shell contains an expansion joint that has an inside diameter of 29.46 in. and an axial rigidity of 14,759 lb/in. The efficiency of shell circumferential welded joint (Category B) is 0.85.
- The diameter of the channel flange gasket load reaction is 25.625 in., the bolt circle diameter is 30.125 in., the design bolt load is 804,478 lb, and the operating condition bolt load is 804,456 lb.

Data Summary - Tubesheet

Tube Layout: Triangular

$h = 4.75\,in. - 0.125\,in. - 0.125\,in. = 4.5\,in.$

$h_g = 0\,in.$

$c_t = 0.125\,in.$

$A = 32.875\,in.$

$r_o = 10.406\,in.$

$A_L = 0.0\,in.^2$

$N_t = 434$

$L_t = 144.375\,in.$

$p = 0.9375\,in.$

$T = 675^{\circ} F$

$T_a = 70^{\circ} F$

$S = 18,450\,psi$ at T from Table 1A of Section II, Part D

$S_y = 27,700\,psi$ at T from Table Y-1 of Section II, Part D

$S_{PS} = 55,400\,psi$ at T

$E = 25,575,000\,psi$ at T from TM-1 of Section II, Part D

$\nu = 0.3$

Data Summary – Tubes

$P_{td,\max} = 1040\ psig$

$P_{td,\min} = 0\ psig$

$P_{to1} = 1040\ psig$

$\ell_{tx} = 4.374\ in.$

$k = 1$

$\ell = 34\ in.$

$t_t = 0.083\ in.$

$d_t = 0.75\ in.$

$T_t = 675^\circ F$

$T_{t,m} = 490^\circ F$

$S_t = 10,700\ psi$ from Table A1 of Section II, Part D

$S_{y,t} = 18,950\ psi$ from Y-1 of Section II, Part D

$S_{tT} = 10,700\ psi$ at T from Table 1A of Section II, Part D

$\alpha_{t,m} = 7.28E-06\ in./in./^\circ F$ at $T_{t,m}$

$E_t = 25,750,000\ psi$ at T_t from TM-1 of Section II, Part D

$E_{tT} = 25,750,000\ psi$ at T from TM-1 of Section II, Part D

$\nu_t = 0.3$

Since the tubes are welded (SA-214), the tube allowable stresses S_t and S_{tT} can be delivered by 0.85 per VIII-1, paragraph UHX-13.3. This results in adjusted values of $S_t = 12,588\ psi$ and $S_{tT} = 12,588\ psi$.

Data Summary - Shell

$P_{sd,\max} = 335\, psig$

$P_{sd,\min} = 0\, psig$

$P_{so1} = 335\, psig$

$t_s = 0.5\, in. - 0.125\, in. = 0.375\, in.$

$D_s = 23\, in. + 2(0.125)\, in. = 23.25\, in.$

$D_j = 29.46\, in.$

$K_j = 14,759\, lb / in.$

$T_s = 675^\circ F$

$T_{s,m} = 550^\circ F$

$E_s = 25,750,000\, psi$ from TM-1 of Section II, Part D

$\alpha_{s,m} = 7.3E-06\ in./in./^\circ F$ at $T_{s,m}$

$\nu_s = 0.3$

$t_{s,1} = 1.25\ in. - 0.125\, in. = 1.125\, in.$

$\ell_1 = 9.75\, in. + 0.125\, in. = 9.875\, in.$

$\ell_1' = 9.75\, in. + 0.125\, in. = 9.875\, in.$

$S_{s,1} = 18,450\, psi$ at T_s from Table 1A of Section II, Part D

$S_{y,s,1} = 27,700\, psi$ at T_s from Table Y-1 of Section II, Part D

$S_{PS,s,1} = 55,400\, psi$; see UG-23(e)

$E_{s,1} = 25,750,000\, psi$ from TM-1 of Section II, Part D

$E_{s,w} = 0.85$

$\alpha_{s,m,1} = 7.3E-06\, in./in./^\circ F$ at $T_{s,m}$

Data Summary - Channel Flange

$$Gasket\,I.D. = 25.125\,in.$$

$$Gasket\,O.D. = 26.125\,in.$$

$$Mean\,Gasket\,Diameter, G = G_c = 25.625\,in.$$

$$Gasket, m, Factor = 6.5$$

$$Gasket, y, Factor = 26,000\,psi$$

$$Flange\,Outside\,Diameter = 32.875\,in.$$

$$Bolt\,Circle, C = 30.125\,in.$$

$$Bolting\,Data = 28\,bolts, 1.375\,in.diameter, SA-193\,B7$$

$$Bolt\,Load, W = 808,478\,lb\;\; per\; VIII-1\;\; Appendix\;\; 2$$

$$Bolt\,Load, W_{m1} = 808,456\,lb\;\; per\; VIII-1\;\; Appendix\;\; 2$$

$$W^*\; from\; Table\; UHX-8.1\; \text{(see Summary Table for Step 5)}$$

$$Gasket\; Moment\; Arm, h_g = \left(C-G\right)/2 = 2.25\,in.$$

Calculation Procedure

The tubesheet is extended as a flange. The calculation procedure for a tubesheet flanged extension is given in VIII-1, paragraph UHX-9.

The tubesheet flanged extension required thickness for the operating condition $\left(S = 18,450\,psi \text{ at } T\right)$ is:

$$h_r = 2.704\,in.$$

The tubesheet flanged extension required thickness for the gasket seating condition $\left(S = 20,000\,psi \text{ at } T_a\right)$ is:

$$h_r = 2.597\,in.$$

The calculation procedure for a Fixed Tubesheet heat exchanger is given in VIII-1, paragraph UHX-13.5. The following results are for the design and operating loading cases required to be analyzed (see paragraph UHX-13.4). This example illustrates the calculation of both the elastic and elastic-plastic solutions.

a) STEP 1 – Calculate the parameters from VIII-1, paragraph UHX-11.5.1.

$$L = 134.875\,in. + 0.125\,in. + 0.125\,in. = 135.125\,in.$$
$$D_o = 21.562\,in.$$
$$a_o = 10.781\,in.$$
$$\rho = 0.972$$
$$d^* = 0.6392\,in.$$
$$\mu = 0.2$$
$$p^* = 0.9375\,in.$$
$$\mu^* = 0.3182$$
$$\rho_s = 1.078286$$
$$\rho_c = 1.188433$$
$$x_s = 0.4749$$
$$x_t = 0.6816$$

b) STEP 2 – Calculate the shell axial stiffness, K_s, tube axial stiffness, K_t, stiffness factors, $K_{s,t}$ and J.

$$K^*_s = 5{,}876{,}500\,lb/in.$$
$$K_t = 33{,}143\,lb/in.$$
$$K_{st} = 0.40854$$
$$J = 0.0025063$$

Calculate the shell coefficients β_s, k_s, λ_s, and δ_s.

$$\beta_s = 0.3471\,in.^{-1}$$
$$k_s = 2{,}331{,}037\,lb$$
$$\lambda_s = 13{,}497{,}065\,psi$$
$$\delta_s = 0.0000039653\,in.^3/lb$$

Calculate the channel coefficients β_c, k_c, λ_c, and δ_c.

$$\beta_c = 0\,in.^{-1}$$
$$k_c = 0\,lb$$
$$\lambda_c = 0\,psi$$
$$\delta_c = 0\,in.^3/lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-1, paragraph UHX-11.5.2 and calculate E^*.

$$h/p = 4.80$$
$$E^*/E = 0.305132 \text{ from Table UHX-11.2}$$
$$\nu^* = 0.342304 \text{ from Table UHX-11.2}$$
$$E^* = 7{,}803{,}761\ psi$$

Calculate, X_a, and the parameters from VIII-1, Table UHX-13.1.

$$X_a = 1.9955$$
$$Z_d = 0.174495$$
$$Z_m = 0.667867$$
$$Z_v = 0.160532$$
$$Z_a = 0.809161$$
$$Z_w = 0.160532$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.5247$$
$$F = 2.0466$$

Calculate $\Phi,\ Q_1,\ Q_{z1}, Q_{z2}\ and\ U$.

$$\Phi = 2.747$$
$$Q_1 = -0.128$$
$$Q_{Z1} = 1.2206$$
$$Q_{Z2} = 0.5952$$
$$U = 1.1904$$

e) STEP 5 – Calculate γ, ω_s, ω_s^*, ω_c, ω_c^*, and γ_b. The following results are those for the corroded condition, elastic solution.

Summary Table for Step 5 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	γ	*W**
1	0	1,040	0	808456
2	335	0	0	0
3	335	1040	0	808456

Summary Table for Step 5 – Operating Condition 1				
Loading Case	P_s (psi)	P_t (psi)	γ	*W**
1	0	1040	-0.060	808478
2	335	0	-0.060	808478
3	335	1040	-0.060	808478
4	0	0	-0.060	808478

$$\omega_s = 8.8648\, in.^2$$
$$\omega^*_s = -8.4947\, in.^2$$
$$\omega_c = 0\, in.^2$$
$$\omega^*_c = 8.6591\, in.^2$$
$$\gamma_b = -0.2087$$

f) STEP 6 – For each loading case, calculate P_s', P_t', P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

Summary Table for STEP 6 – Design Condition							
Loading Case	P'_s (psi)	P'_t (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	1,017,041	0	0	275	92.2	-1,039.2
2	-167,351	0	0	0	0	29.1	-170.7
3	-167,351	1,017,041	0	0	275	121.4	-1,210.2

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P'_s (psi)	P'_t (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	1,017,041	-2,376	0	275	92.2	-1,041.6
2	-167,351	0	-2,376	0	275	29.1	-173.2
3	-167,351	1,017,041	-2,376	0	275	121.4	-1,212.7
4	0	0	-2,376	0	275	0	-2.1

g) STEP 7 – Elastic Iteration, calculate Q_2 and Q_3, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition								
Loading Case	Q_2 (lbs)	Q_3	F_m	h'_g (in)	$h-h'_g$ (in)	$\lvert\sigma_{elastic}\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	-12,650	0.0815	0.19861	0	4.5	22,335.8	27,675	---
2	-1003.9	-0.027	0.1574	0	4.5	2913	27,675	---
3	-13,654	0.06617	0.1927	0	4.5	25,249.4	27,675	---

Summary Table for STEP 7 – Operating Condition 1								
Loading Case	Q_2 (lbs)	Q_3	F_m	h'_g (in)	$h-h'_g$ (in)	$\lvert\sigma_{elastic}\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	-12,650	0.08101	0.1984	0	4.5	22,367.1	---	55,400
2	-10,477	0.91305	0.5333	0	4.5	9,994.5	---	55,400
3	-13,654	0.06578	0.19264	0	4.5	25,280.7	---	55,400
4	-9,473	75.77	37.935	0	4.5	8,817.0	---	55,400

For Design Loading Cases 1-3 $\lvert\sigma_{elastic}\rvert < 1.5S$, and for Operating Cases 1-4 $\lvert\sigma_{elastic}\rvert < S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Elastic Iteration, calculate the tubesheet shear stress and the allowable tubesheet shear stress.

Summary Table for STEP 8 – Design Condition		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	6,224.3	14,760
2	1024	14,760
3	7,248.6	14,760

Summary Table for STEP 8 – Operating Condition 1		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	6,238.8	14,760
2	1,037.2	14,760
3	7,263.2	14,760
4	12.9	14,760

For all Loading Cases $\lvert\tau\rvert < 0.8S$. The shear stress criterion for the tubesheet is satisfied.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.2376\,in.$$

$$F_t = 143.07$$

$$C_t = 163.7755$$

Summary Table for STEP 9 – Design Condition				
Loading Case	F_t, min	$\sigma_{t,1}$ (psi)	F_t, max	$\sigma_{t,2}$ (psi)
1	0.459	-1,120.1	1.487	4,046.9
2	0.59	1258	1.349	1886
3	0.478	137.7	1.468	5,932.7

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	F_t, min	$\sigma_{t,1}$ (psi)	F_t, max	$\sigma_{t,2}$ (psi)
1	0.460	-1,111.8	1.487	4,061.2
2	-0.543	-314.9	2.545	2,902.1
3	0.478	146	1.467	5,947
4	-90.755	-942.9	97.817	1,016.3

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\lvert\sigma_{t,\min}\rvert$ (psi)	F_s	S_{tb} (psi)
1	4,046	12,588.2	1,120.1	2	5,336.3
2	1886	12,588.2	---	---	---
3	5,932	12,588.2	---	---	---

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\lvert\sigma_{t,\min}\rvert$ (psi)	F_s	S_{tb} (psi)
1	4,061	25,176.5	1,111.8	2	5,336.3
2	2,902	25,176.5	---	---	---
3	5,947	25,176.5	---	---	---
4	1,016	25,176.5	942.9	1.25	8,538.1

For all Loading Cases $\left|\sigma_{t,\max}\right| < S_t$. The axial tension stress criterion for the tube is satisfied.

For all Loading Cases $\left|\sigma_{t,\min}\right| < S_{tb}$. The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Main Shell – Design Condition				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	10.4	15,682.5	---	---
2	-1,525.1	15,682.5	----	10,802
3	-1,518.3	15,682.5	---	10,802

Summary Table for STEP 10 – Main Shell – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	-21.4	---	36,900	10,802
2	-1,556.9	---	36,900	10,802
3	-1,550.2	---	36,900	10,802
4	-28.2	---	36,900	10,802

Summary Table for STEP 10 – Shell Band – Design Condition				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	3.4	15,682.5	---	-41040
2	-492.7	15,682.5	---	618
3	-490.5	15,682.5	---	-40422

Summary Table for STEP 10 – Shell Band – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	-6.9	---	36,900	-41074
2	-503	---	36,900	-19412
3	-500.8	---	36,900	-40456
4	-9.1	---	36,900	-20030

k) STEP 11 – For each loading case, calculate the stresses in the shell and/or channel when integral with the tubesheet.

Summary Table for STEP 11 – Design Condition					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	S_s (psi)	$S_{PS,s}$ (psi)
1	3.4	-41,040	41,043	27,675	---
2	-492.7	618	1112	27,675	---
3	-490.5	-40,422	40,912	27,675	---
Summary Table for STEP 11 – Operating Condition 1					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	S_s (psi)	$S_{PS,s}$ (psi)
1	-6.9	-41,074	41,081	---	55,400
2	-503	-19,412	19,915	---	55,400
3	-500.8	-40,456	40,957	---	55,400
4	-9.1	-20,030	20,039	---	55,400

For Design Loading Cases 1 and 3 $|\sigma_s| > 1.5S_s$, and for Operating Loading Cases 1-4 $|\sigma_s| < S_{PS,s}$. The stress criterion for the shell is not satisfied.

l) STEP 12 – The design shall be reconsidered by using one or a combination of the following options.

- Option 1 – Increase the tubesheet thickness and return to STEP 1.
- Option 2 – Increase the integral shell and/or channel thickness and return to STEP 1
- Option 3 – Perform the elastic-plastic calculation procedures as defined in VIII-1, paragraph UHX-13.7.

Since the total axial stress in the shell σ_s is between $1.5S_{s,1}$ and $S_{PS,s,1}$ for Design Condition Loading Cases 1 and 3, the procedure of VIII-1, paragraph UHX-13.7 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

Summary Results for STEP 12, Elastic Plastic Iteration Results per VIII-1, paragraph UHX-13.7.3		
Design Condition Loading Case	1	3
S^*_s, psi	27,700	27,700
$fact_s$	0.807	0.820
E^*_s, psi	20.789E6	21.1E6
k_s, lb	1.88E6	1.91E6
λ_s	0.109E8	0.111E8
F	1.827	1.842
ϕ	2.453	2.472
Q_1	-0.1196	-0.1202
Q_{Z1}	1.231	1.230
Q_{Z2}	0.640	0.636
U	1.279	1.273
P_W, psi	295.5	294.1
P_{rim}, psi	99.099	129.78
P_e, psi	-1,039.2	-1,210.2
Q_2, lb	-13,592	-14,599
Q_3	0.105	0.087
F_m	0.208	0.201
$\|\sigma\|, psi$	23,358	26,304

The final calculated tubesheet bending stresses of 23,358 psi (Loading Case 1) and 26,304 psi (Loading Case 3) are less than the allowable tubesheet bending stress of 27,675 psi. As such, this geometry meets the requirements of VIII-1, paragraph UHX. The intermediate results for the elastic-plastic calculation are shown above.

4.18.7 Example E4.18.7 - Fixed Tubesheet Exchanger, Configuration a

A fixed tubesheet heat exchanger with the tubesheet construction in accordance with configuration a as shown in VIII-1, Figure UHX-13.1, Configuration a.

- For the Design Condition, the shell side design pressure is 325 psig at 400°F, and the tube side design pressure is 200 psig at 300°F.
- There is one operating condition. For Operating Condition 1, the shell side design pressure is 325 psig at 400°F, the tube side design pressure is 200 psig at 300°F, the shell mean metal temperature is 151°F, and the tube mean metal temperature is 113°F. For this example, the operating pressures and operating metal temperatures are assumed to be the same as the design values.
- The tube material is SA-249, Type 304L (S30403). The tubes are 1 in. outside diameter and are 0.049 in. thick.
- The tubesheet material is SA-240, Type 304L (S30403). The tubesheet outside diameter is 43.125 in. There are 955 tube holes on a 1.25 in. triangular pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 20.125 in. The distance between the outer tubesheet faces is 240 in. The option for the effect of differential radial expansion is not required. There is no corrosion allowance on the tubesheet.
- The shell material is SA-240, Type 304L (S30403). The shell inside diameter is 42 in. and the thickness is 0.5625 in. There is no corrosion allowance on the shell and no expansion joint in the shell. The efficiency of shell circumferential welded joint (Category B) is 0.85.
- The channel material is SA-516, Grade 70 (K02700). The inside diameter of the channel is 42.125 in. and the channel is 0.375 in. thick. There is no corrosion allowance on the channel.

<u>Data Summary - Tubesheet</u>

Tube Layout: Triangular

$$h = 1.375\,in.$$

$$h_g = 0\,in.$$

$$c_t = 0\,in.$$

$$A = 43.125\,in.$$

$$r_o = 20.125\,in.$$

$$A_L = 0.0\,in.^2$$

$$N_t = 955$$

$$L_t = 240\,in.$$

$$p = 1.25\,in.$$

$$T = 400^\circ F$$

$$T_a = 70^\circ F$$

$$S = 15{,}800\,psi \text{ at } T \text{ from Table 1A of Section II, Part D}$$

$$S_y = 17{,}500\,psi \text{ at } T$$

$$S_{PS} = 47{,}400\,psi \text{ at } T$$

$$E = 26{,}400{,}000\,psi \text{ at } T$$

$$\nu = 0.3$$

Data Summary – Tubes

$P_{td,\max} = 200\ psig$

$P_{td,\min} = 0\ psig$

$P_{to1} = 200\ psig$

$\ell_{tx} = 1.25$

$k = 1$

$\ell = 48\,in.$

$t_t = 0.049\,in.$

$d_t = 1\,in.$

$T_t = 300^\circ F$

$T_{t,m} = 113^\circ F$

$S_t = 14,200\ psi$ at T_t from Table 1A of Section II, Part D

$S_{y,t} = 19,200\ psi$ at T_t from Y-1 of Section II, Part D

$S_{tT} = 13,400\ psi$ at T from Y-1 of Section II, Part D

$\alpha_{t,m} = 8.65E-06\,in./in./^\circ F$ at $T_{t,m}$

$E_t = 27,000,000\ psi$ at T_t from TM-1 of Section II, Part D

$E_{tT} = 26,400,000\ psi$ at T from TM-1 of Section II, Part D

$\nu_t = 0.3$

Since the tubes are welded SA-249, Type 304L, the tube allowable stresses S_t and S_{tT} can be divided by 0.85 per VIII-1, paragraph UHX-13.3. This results in adjusted values of $S_t = 16,706\ psi$ and $S_{tT} = 15,765\ psi$.

Data Summary - Shell

Since there is no expansion joint in the shell, $J = 1$ and D_j and K_j need not be defined.

$P_{sd,\max} = 325\ psig$

$P_{sd,\min} = 0\ psig$

$P_{so1} = 325\ psig$

$t_s = 0.5625\, in.$

$D_s = 42\, in.$

$T_s = 400^\circ F$

$T_{s,m} = 151^\circ F$

$S_s = 15,800\ psi$ at T_s from Table 1A of Section II, Part D

$E_{s,w} = 0.85$

$S_{PS,s} = 47,400\ psi$ at T_s

$S_{y,s} = 17,500\ psi$ at T_s

$E_s = 26,400,000\ psi$ from TM-1 of Section II, Part D

$\alpha_{s,m} = 8.802E-06\, in./in./^\circ F$ at $T_{s,m}$

$\nu = 0.3$

Data Summary – Channel

$t_c = 0.375\, in.$

$D_c = 42.125\, in.$

$T_c = 300^\circ F$

$S_c = 20,000\ psi$ at T_c from Table 1A of Section II, Part D

$S_{y,c} = 33,600\ psi$ at T_c

$S_{PS,c} = 67,200\ psi$ at T_c

$E_c = 28,300,000\ psi$ at T_c from TM-1 of Section II, Part D

$\nu_c = 0.3$

Calculation Procedure

The calculation procedure for a Fixed Tubesheet heat exchanger is given in VIII-1, paragraph UHX-13.5. The following results are for the design and operating loading cases required to be analyzed (see VIII-1, paragraph UHX-13.4). This example illustrates the calculation of both the elastic and elastic-plastic solutions.

a) STEP 1 – Calculate the parameters from VIII-1, paragraph UHX-11.5.1.

$$L = 237.25\,in.$$
$$D_o = 41.25\,in.$$
$$a_o = 20.625\,in.$$
$$\rho = 0.9091$$
$$d^* = 0.9111\,in.$$
$$\mu = 0.2$$
$$p^* = 1.25\,in.$$
$$\mu^* = 0.2711$$
$$\rho_s = 1.0182$$
$$\rho_c = 1.0212$$
$$x_s = 0.4388$$
$$x_t = 0.5434$$

b) STEP 2 – Calculate the shell axial stiffness, K_s, tube axial stiffness, K_t, stiffness factors, $K_{s,t}$ and J.

$$K_s = 8,369,456\ lb/in.$$
$$K_t = 16,660\ lb/in.$$
$$K_{st} = 0.526$$
$$J = 1$$

Calculate the shell coefficients $\beta_s,\ k_s,\ \lambda_s,\ \text{and}\ \delta_s$.

$$\beta_s = 0.3715\,in.^{-1}$$
$$k_s = 319,712\ lb$$
$$\lambda_s = 50,867,972\ psi$$
$$\delta_s = 25.24E-6\ in.^3/lb$$

Calculate the channel coefficients $\beta_c,\ k_c,\ \lambda_c,\ \text{and}\ \delta_c$.

$$\beta_c = 0.4554\ in.^{-1}$$
$$k_c = 124,461\ lb$$
$$\lambda_c = 22,049,112\ psi$$
$$\delta_c = 35.532E-6\ in.^3/lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-1, paragraph UHX-11.5.2 and calculate E^*.

$$h/p = 1.10000$$

$$E^*/E = 0.274948 \text{ from Table UHX-11.2}$$

$$\nu^* = 0.340361 \text{ from Table UHX-11.2}$$

$$E^* = 7.26E6\ psi$$

Calculate, X_a, and the parameters from VIII-1, Table UHX-13.1.

$$X_a = 7.0155$$

$$Z_d = 0.00433$$

$$Z_v = 0.02064$$

$$Z_m = 0.2067$$

$$Z_a = 295.63$$

$$Z_w = 0.02064$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.0455$$

$$F = 6.7322$$

Calculate $\Phi,\ Q_1,\ Q_{z1},\ Q_{z2}\ and\ U$.

$$\Phi = 9.0236$$

$$Q_1 = -0.058647$$

$$Q_{Z1} = 3.7782$$

$$Q_{Z2} = 10.3124$$

$$U = 20.6248$$

e) STEP 5 – Calculate γ, ω_s, ω_s^*, ω_c, ω_c^*, and γ_b. The following results are those for the corroded condition, elastic solution

Summary Table for Step 5 – Design Condition			
Loading Case	P_s (psi)	P_t (psi)	γ
1	0	200	0
2	325	0	0
3	325	200	0

Summary Table for Step 5 – Operating Condition 1			
Loading Case	P_s (psi)	P_t (psi)	γ
1	0	200	-0.0809
2	325	0	-0.0809
3	325	200	-0.0809
4	0	0	-0.0809

$$\omega_s = 4.6123\ in.^2$$

$$\omega*_s = -4.5413\ in.^2$$

$$\omega_c = 3.344\ in.^2$$

$$\omega*_c = -2.6027\ in.^2$$

$$\gamma_b = 0$$

f) STEP 6 – For each loading case, calculate P_s', P_t', P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

Summary Table for STEP 6 – Design Condition							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	543.7	0	0	0	-25.2	-97
2	613.7	0	0	0	0	71.6	116.8
3	613.7	543.7	0	0	0	46.3	19.8

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	543.7	-963	0	0	-25.2	-261.1
2	613.7	0	-963	0	0	71.6	-47.3
3	613.7	543.7	-963	0	0	46.3	-144.3
4	0	0	-963	0	0	0	-164.1

g) STEP 7 – Elastic Iteration, calculate Q_2 and Q_3, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition								
Loading Case	Q_2 (lbs)	Q_3	F_m	h'_g (in)	$h-h'_g$ (in)	$\lvert\sigma_{elastic}\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	181.7	-0.0675	0.03373	0	1.375	16,286	23,700	---
2	-515.1	-0.0794	0.03969	0	1.375	23,084	23,700	---
3	-333.4	-0.138	0.06886	0	1.375	6,798	23,700	---

Summary Table for STEP 7 – Operating Condition 1								
Loading Case	Q_2 (lbs)	Q_3	F_m	h'_g (in)	$h-h'_g$ (in)	$\lvert\sigma_{elastic}\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	181.7	-0.0619	0.03096	0	1.375	40,253	---	47,400
2	-515.1	-0.00749	0.03210	0	1.375	7,566	---	47,400
3	-333.4	-0.0478	0.02389	0	1.375	17,169	---	47,400
4	0	-0.0587	0.02932	0	1.375	23,967	---	47,400

For Design Loading Cases 1-3 $\lvert\sigma_{elastic}\rvert < 1.5S$, and for Operating Cases 1-4 $\lvert\sigma_{elastic}\rvert < S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Elastic Iteration, calculate the tubesheet shear stress and the allowable tubesheet shear stress.

Summary Table for STEP 8 – Design Condition		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	3,636	12,640
2	4,380	12,640
3	744	12,640

Summary Table for STEP 8 – Operating Condition 1		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	9,792	12,640
2	1,775	12,640
3	5,412	12,640
4	6,155	12,640

For all Loading Cases $\lvert\tau\rvert < 0.8S$. The shear stress criterion for the tubesheet is satisfied.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.3367\,in.$$

$$F_t = 142.57$$

$$C_t = 166.6$$

Summary Table for STEP 9 – Design Condition				
Loading Case	F_t, min	$\sigma_{t,1}$ (psi)	F_t, max	$\sigma_{t,2}$ (psi)
1	-0.270	-1,289.5	3.558	2,259.3
2	-0.243	1,634.3	3.260	-2,276.6
3	-0.191	360.4	2.123	-78.2

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	F_t, min	$\sigma_{t,1}$ (psi)	F_t, max	$\sigma_{t,2}$ (psi)
1	-0.285	-1,751.2	3.696	8,187.4
2	-0.490	1,141.5	5.057	3,651.5
3	-0.329	-129.1	4.050	5,910.8
4	-0.295	-462.4	3.778	5,928.1

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\lvert\sigma_{t,\min}\rvert$ (psi)	F_s	S_{tb} (psi)
1	2,259	16,706	1,289.5	1.471	7,468
2	-2,277	16,706	2,276.6	1.620	6,781
3	360.4	16,706	78.2	2	5,493

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\lvert\sigma_{t,\min}\rvert$ (psi)	F_s	S_{tb} (psi)
1	8,187	33,412	1,751.2	1.402	7,836.5
2	3,652	33,412	---	---	---
3	5,911	33,412	129	1.25	8788
4	5,928	33,412	462.4	1.361	8,072

For all Loading Cases $\left|\sigma_{t,\max}\right| < S_t$. The axial tension stress criterion for the tube is satisfied.

For all Loading Cases $\left|\sigma_{t,\min}\right| < S_{tb}$. The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Main Shell – Design Condition				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	1,830.6	13,430	---	---
2	2,287.2	13,430	---	---
3	4,117.8	13,430	---	---

Summary Table for STEP 10 – Main Shell – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	-1,085.9	---	47,400	6,730
2	-629.3	---	47,400	6,730
3	1,201.3	---	47,400	---
4	-2,916.5	---	47,400	6,730

k) STEP 11 – For each loading case, calculate the stresses in the shell and/or channel when integral with the tubesheet.

Summary Table for STEP 11, Shell Results – Design Condition					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	1,830.6	-12,184	14,015	23,700	---
2	2,287.2	27,748	30,036	23,700	---
3	4,117.8	15,564	19,682	23,700	---

Summary Table for STEP 11, Shell Results – Operating Condition 1					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	-1,085.9	-38,418	39,504	---	47,400
2	-629.3	1,514	2,144	---	47,400
3	1,201.3	-10,670	11,871	---	47,400
4	-2,916.5	-26,234	29,150	---	47,400

For Design Loading Case 2 $|\sigma_s| > 1.5S_s$, and for Operating Cases 1-4 $|\sigma_s| < S_{PS,s}$. The stress criterion for the shell is not satisfied.

Summary Table for STEP 11, Channel Results – Design Condition					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	$S_{PS,c}$ (psi)
1	5,567	28,346	33,913	30,000	---
2	0	-8,492	8,492	30,000	---
3	5,567	19,854	25,420	30,000	---

Summary Table for STEP 11, Channel Results – Operating Condition 1					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	$S_{PS,c}$ (psi)
1	5,567	52,380	57,946	---	67,200
2	0	15,541	15,541	---	67,200
3	5,567	43,887	49,453	---	67,200
4	0	24,033	24,033	---	67,200

For Design Loading Case 1 $|\sigma_c| > 1.5S_c$, and for Operating Cases 1-4 $|\sigma_c| < S_{PS,c}$. The stress criterion for the channel is not satisfied.

l) STEP 12 – The design shall be reconsidered by using one or a combination of the following options.
   - Option 1 – Increase the tubesheet thickness and return to STEP 1.
   - Option 2 – Increase the integral shell and/or channel thickness and return to STEP 1
   - Option 3 – Perform the elastic-plastic calculation procedures as defined in VIII-1, paragraph UHX-13.7.

Since the total axial stress in the shell σ_s is between $1.5S_s$ and $S_{PS,s}$ for Design Condition Loading Case 2, the procedure of VIII-1, paragraph UHX-13.7 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

Since the total axial stress in the channel σ_c is between $1.5S_c$ and $S_{PS,c}$ for Design Condition Loading Case 1, the procedure of UHX-13.7 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the channel occurs. The results are not presented for Design Condition Loading Case 1, because the calculated values of $fact_s$ and $fact_c$ do not exceed 1.0 for this case and further plasticity calculations are not required.

Summary Results for STEP 12, Elastic Plastic Iteration Results per VIII-1, paragraphs UHX-13.7.3	
Design Condition Loading Case	1
S^*_s, psi	17,500
S^*_c, psi	33,600
$fact_s$	0.766
$fact_c$	1.000
E^*_s, psi	20.2E6
E^*_c, psi	28.3E6
k_s, lb	24.48E4
λ_s	0.390E+08
F	5.65
ϕ	7.572
Q_1	-0.0538
Q_{Z1}	3.898
Q_{Z2}	11.518
U	23.037
P_W, psi	0
P_{rim}, psi	79.9
P_e, psi	115.4
Q_2, lb	-575
Q_3	-0.0773
F_m	0.0386
$\lvert\sigma\rvert, psi$	22,204

The final calculated tubesheet bending stress is 22,204 psi, which is less than the Code allowable of 23,700 psi. As such, this geometry meets the requirement of VIII-1, paragraph UHX. The intermediate results for the elastic-plastic iteration are shown above.

4.18.8 Example E4.18.8 - Stationary Tubesheet Gasketed With Shell and Channel; Floating Tubesheet Gasketed, Not Extended as a Flange

A floating tubesheet exchanger with an immersed floating head is to be designed as shown in VIII-1, Figure UHX-14.1, Configuration a. The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in VIII-1, Figure UHX-14.2, sketch (d). The floating tubesheet is not extended as a flange in accordance with configuration C as shown in VIII-1, Figure UHX-14.3, sketch (c). There is no allowance for corrosion.

<u>Data Summary - Data Common to Both Tubesheets</u>

$$P_{sd,\max} = 250\ psig$$
$$P_{sd,\min} = 0\ psig$$
$$P_{td,\max} = 150\ psig$$
$$P_{td,\min} = 0\ psig$$

<u>Data Summary – Tubesheet</u>

The tube layout pattern is triangular with one centerline pass lane

$$N_t = 466$$
$$p = 1\ in.$$
$$r_o = 12.5\ in.$$
$$\rho = 0.8$$
$$U_{L1} = 2.5\ in.$$
$$c_t = 0\ in.$$
$$\nu = 0.31$$
$$E = 27.0E10^6\ psi$$
$$S = 19,000\ psi$$

<u>Data Summary – Tubes</u>

$$d_t = 0.75\ in.$$
$$t_t = 0.083\ in.$$
$$L_t = 256\ in.$$
$$\ell_t = 15.375\ in.$$
$$\nu_t = 0.31$$
$$E_t = 27.0 \times 10_6\ psi$$
$$S_t = 13,350\ psi$$
$$S_{y,t} = 20,550\ psi$$

Data Summary – Stationary Tubesheet Data Summary

$$W^* = 211,426\,lb \text{ from Table UHX-8.1}$$
$$A = 33.071\,in.$$
$$h = 1.75\,in.$$
$$G_s = 29.375\,in.$$
$$a_s = 14.6875\,in.$$
$$G_c = 29.375\,in.$$
$$a_c = 14.6875\,in.$$
$$C = 31.417\,in.$$
$$h_g = 0.197\,in.$$

Data Summary – Floating Tubesheet Data Summary

$$W^* = 26,225\,lb \text{ from Table UHX-8.1}$$
$$A = 26.89\,in.$$
$$h = 1.75\,in.$$
$$G_1 = 26.496\,in.$$
$$G_c = 26.496\,in.$$
$$a_c = 13.248\,in.$$
$$a_s = 13.248\,in.$$
$$C = 27.992\,in.$$
$$h_g = 0\,in.$$

Calculation Procedure – Stationary Tubesheet

The following results are for the 3 load cases required to be analyzed (see VIII-1, paragraph UHX-14.4).

a) STEP 1 – Calculate the parameters from VIII-1, paragraph UHX-11.5.1.

$$D_o = 25.75\, in.$$
$$L_{L1} = 25.8\, in.$$
$$A_L = 64.4\, in.^2$$
$$d^* = 0.6567\, in.$$
$$\mu = 0.250$$
$$\mu^* = 0.385$$
$$h'_g = 0.197\, in.$$
$$a_o = 12.875\, in.$$
$$\rho_s = 1.14$$
$$\rho_c = 1.14$$
$$x_s = 0.605$$
$$x_t = 0.760$$

b) STEP 2 – Calculate the shell coefficients $\beta_s, k_s, \lambda_s,$ and δ_s and the channel coefficients $\beta_c, k_c, \lambda_c,$ and δ_c.

$$\beta_s = 0\, in.^{-1}$$
$$k_s = 0\, lb$$
$$\lambda_s = 0\, psi$$
$$\delta_s = 0\, in.^3 / lb$$
$$\beta_c = 0\, in.^{-1}$$
$$k_c = 0\, lb$$
$$\lambda_c = 0\, psi$$
$$\delta_c = 0\, in.^3 / lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-1, paragraph UHX-11.5.2 and calculate E^*.

$$h/p = 1.75$$
$$E^*/E = 0.404$$
$$\nu^* = 0.308$$
$$E^* = 10.91E6\;\; psi$$

Calculate, X_a, and the parameters from VIII-1, Table UHX-13.1.

$$X_a = 3.61$$
$$Z_d = 0.0328$$
$$Z_v = 0.0787$$
$$Z_m = 0.421$$
$$Z_w = 0.0787$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.28$$
$$F = 0.429$$

Calculate Φ and Q_1.

$$\Phi = 0.561$$
$$Q_1 = 0.0782$$

e) STEP 5 – Calculate ω_s, ω^*_s, ω_c, ω^*_c, γ_b, P^*_s and P^*_c.

$$\omega_s = 0\, in.^2$$
$$\omega^*_s = 1.758\, in.^2$$
$$\omega_c = 0\ in.^2$$
$$\omega^*_c = 1.758\ in.^2$$
$$\gamma_b = 0$$
$$P^*_s = 0\ psi$$
$$P^*_c = 0\ psi$$

f) STEP 6 – Calculate

Summary Table for STEP 6	
Loading Case	P_e (psi)
1	-150
2	250
3	100

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\|\sigma\|$ (psi)	$1.5S$ (psi)
1	-213	0.0953	0.102	16,400	28,500
2	356	0.0953	0.102	27,400	28,500
3	142	0.0953	0.102	10,900	28,500

For all loading cases $|\sigma| \le 1.5S$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Calculate the tubesheet shear stress and the allowable tubesheet shear stress.

Summary Table for STEP 8		
Loading Case	$\|\tau\|$ (psi)	$0.8S$ (psi)
1	2210	15200
2	3680	15200
3	1470	15200

For all loading cases $|\tau| \le 0.8S$. The shear stress criterion for the tubesheet is satisfied.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.238\,in.$$
$$F_t = 64.7$$
$$C_t = 161$$

Summary Table for STEP 9					
Loading Case	F_s	$\sigma_{t,1}$ (psi)	$\sigma_{t,2}$ (psi)	S_{tb} (psi)	S_t (psi)
1	1.54	-1,716	2,564	10,700	13350
2	1.54	2,609	-4,524	10,700	13350
3	1.54	894	-1,959	10,700	13350

Determine $\sigma_{t,\max} = \max\left[|\sigma_{t,1}|,|\sigma_{t,2}|\right]$ and $\sigma_{t,\min} = \min\left[\sigma_{t,1},\sigma_{t,2}\right]$

For all loading cases $\sigma_{t,\max} < S_t$. The axial tension stress criterion for the tube is satisfied.

For all loading cases $|\sigma_{t,\min}| < S_{tb}$. The buckling criterion for the tube is satisfied.

Calculation Procedure – Floating Tubesheet

The following results are for the 3 load cases required to be analyzed (see VIII-1, paragraph UHX-14.4).

a) STEP 1 – Calculate the parameters from VIII-1, paragraph UHX-11.5.1.

$$D_o = 25.75\, in.$$
$$L_{L1} = 25.8\, in.$$
$$A_L = 64.4\, in.^2$$
$$d^* = 0.6567\, in.$$
$$\mu = 0.250$$
$$\mu^* = 0.385$$
$$h'_g = 0\, in.$$
$$a_o = 12.875\, in.$$
$$\rho_s = 1.03$$
$$\rho_c = 1.03$$
$$x_s = 0.605$$
$$x_t = 0.760$$

b) STEP 2 – Calculate the shell coefficients β_s, k_s, λ_s, and δ_s and the channel coefficients β_c, k_c, λ_c, and δ_c.

$$\beta_s = 0\, in.^{-1}$$
$$k_s = 0\, lb$$
$$\lambda_s = 0\, psi$$
$$\delta_s = 0\, in.^3 / lb$$
$$\beta_c = 0\, in.^{-1}$$
$$k_c = 0\, lb$$
$$\lambda_c = 0\, psi$$
$$\delta_c = 0\, in.^3 / lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν from VIII-1, paragraph UHX-11.5.2 and calculate E^*.

$$h/p = 1.75$$
$$E^*/E = 0.404$$
$$\nu^* = 0.308$$
$$E^* = 10.91E6\ psi$$

Calculate, X_a, and the parameters from VIII-1, Table UHX-13.1.

$$X_a = 3.61$$
$$Z_d = 0.0328$$
$$Z_v = 0.0787$$
$$Z_m = 0.421$$
$$Z_w = 0.0787$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.04$$
$$F = 0.0742$$

Calculate Φ and Q_1.

$$\Phi = 0.0971$$
$$Q_1 = 0.0205$$

e) STEP 5 – Calculate ω_s, ω^*_s, ω_c, ω^*_c, γ_b, P^*_s and P^*_c.

$$\omega_s = 0\ in.^2$$
$$\omega^*_s = 7.06 \times 10^{-2}\ in.^2$$
$$\omega_c = 0\ in.^2$$
$$\omega^*_c = 7.06 \times 10^{-2}\ in.^2$$
$$\gamma_b = 0$$
$$P^*_s = 0\ psi$$
$$P^*_c = 0\ psi$$

f) STEP 6 – Calculate P_e.

Summary Table for STEP 6	
Loading Case	P_e (psi)
1	-150
2	250
3	100

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\|\sigma\|$ (psi)	$1.5S$ (psi)
1	-10.2	0.0213	0.0751	9,500	28,500
2	16.9	0.0213	0.0751	15,800	28,500
3	6.78	0.0213	0.0751	6,330	28,500

For all loading cases $|\sigma| < 1.5S$. The bending stress criterion for the tubesheet is satisfied.

The calculation procedure is complete and the unit geometry is acceptable for the given design conditions.

4.18.9 Example E4.18.9 - Stationary Tubesheet Gasketed With Shell and Channel; Floating Tubesheet Integral

A floating tubesheet exchanger with an externally sealed (packed) floating head is to be designed as shown in VIII-1, Figure UHX-14.1, Configuration b. The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in VIII-1, Figure UHX-14.2, sketch (d). The floating tubesheet is integral with the head in accordance with configuration A as shown in VIII-1, Figure UHX-14.3, sketch (a). There is no allowance for corrosion.

<u>Data Summary - Data Common to Both Tubesheets</u>

$$P_{sd,\max} = 150\ psig$$
$$P_{sd,\min} = 0\ psig$$
$$P_{td,\max} = 30\ psig$$
$$P_{td,\min} = 0\ psig$$

Data Summary – Tubesheet

The tube layout pattern is triangular with no pass lanes

$N_t = 1189$

$p = 1.25\, in.$

$r_o = 22.605\, in.$

$\rho = 0.958$

$h_g = 0\, in.$

$c_t = 0\, in.$

$\nu = 0.32$

$E = 14.8E10^6\ psi$

$S = 11,300\ psi$

$S_y = 31,600\ psi$

$S_{PS} = 33,900\ psi$ (MYS/UTS $>$ 0.7; therefore use 3S)

Data Summary – Tubes

$d_t = 1.0\ in.$

$t_t = 0.049\, in.$

$L_t = 144\, in.$

$\ell_t = 16\, in.$

$\nu_t = 0.32$

$E_t = 14.8E10^6\ psi$

$S_t = 11,300\ psi$

$S_{y,t} = 31,600\ psi$

Data Summary – Stationary Tubesheet

$W^* = 288,910\, lb$

$A = 51\, in.$

$h = 1.375\, in.$

$G_s = 49.71\, in.$

$a_s = 24.9\, in.$

$G_c = 49.616\, in.$

$a_c = 24.8\, in.$

$C = 49.5\, in.$

Data Summary – Floating Tubesheet

$$P_{so1} = 150\, psig$$
$$P_{to1} = 30\, psig$$
$$W^* = 0\, lb$$
$$T' = 200^\circ F$$
$$T_c' = 235^\circ F$$
$$A = 47.625\, in.$$
$$h = 1.375\, in.$$
$$\alpha' = 4.8E-6 \;\; in./in./^\circ F$$
$$D_c = 47\, in.$$
$$a_c = 23.5\, in.$$
$$a_s = 23.5\, in.$$
$$t_c = 0.3125\, in.$$
$$\nu_c = 0.32$$
$$E_c = 14.8E6\, psi$$
$$S_c = 11,300\, psi$$
$$S_{y,c} = 31,600\, psi$$
$$S_{PS,c} = 33,900\, psi \text{ (MYS/UTS } > \text{ 0.7; therefore use } 3S\text{)}$$
$$\alpha_c' = 4.8E-6 \;\; in./in./^\circ F$$

Calculation Procedure – Stationary Tubesheet

The following results are for the 3 load cases required to be analyzed (see VIII-1, paragraph UHX-14.4).

a) STEP 1 – Calculate the parameters from VIII-1, paragraph UHX-11.5.1.

$$D_o = 46.21\, in.$$
$$A_L = 0\, in.^2$$
$$\mu = 0.200$$
$$\mu^* = 0.275$$
$$h_g' = 0\, in.$$
$$a_o = 23.105\, in.$$
$$\rho_s = 1.08$$
$$\rho_c = 1.07$$
$$x_s = 0.443$$
$$x_t = 0.547$$

b) STEP 2 – Calculate the shell coefficients β_s, k_s, λ_s, and δ_s and the channel coefficients β_c, k_c, λ_c, and δ_c.

$$\beta_s = 0\,in.^{-1}$$
$$k_s = 0\,lb$$
$$\lambda_s = 0\,psi$$
$$\delta_s = 0\,in.^3 / lb$$
$$\beta_c = 0\,in.^{-1}$$
$$k_c = 0\,lb$$
$$\lambda_c = 0\,psi$$
$$\delta_c = 0\,in.^3 / lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-1, paragraph UHX-11.5.2 and calculate E^*.

$$h/p = 1.10$$
$$E^*/E = 0.280$$
$$\nu^* = 0.337$$
$$E^* = 4.149E6\ psi$$

Calculate, X_a, and the parameters from VIII-1, Table UHX-13.1.

$$X_a = 8.84$$
$$Z_a = 3161.6$$
$$Z_d = 0.00214$$
$$Z_v = 0.0130$$
$$Z_m = 0.163$$
$$Z_w = 0.013$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.10$$
$$F = 0.233$$

Calculate Φ and Q_1.

$$\Phi = 0.312$$
$$Q_1 = 0.0682$$

e) STEP 5 – Calculate ω_s, ω^*_s, ω_c, ω^*_c, γ_b, P^*_s and P^*_c.

$$\omega_s = 0\,in.^2$$

$$\omega^*_s = 1.59\,in.^2$$

$$\omega_c = 0\,in.^2$$

$$\omega^*_c = 0.961\,in.^2$$

$$\gamma_b = -2.03\times10^{-3}$$

$$P^*_s = 0\,psi$$

$$P^*_c = 0\,psi$$

f) STEP 6 – Calculate P_e.

Summary Table for STEP 6	
Loading Case	P_e (psi)
1	-30
2	-23.6
3	-53.6

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-116	0.0828	0.0594	11,000	16950
2	138	0.0463	0.0442	6,420	16950
3	110	0.0605	0.0499	16,500	16950

For all loading cases $\lvert\sigma\rvert \le 1.5S$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Calculate the tubesheet shear stress and the allowable tubesheet shear stress.

Summary Table for STEP 8		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	1260	9040
2	991	9040
3	2250	9040

For all loading cases $\lvert\tau\rvert \le 0.8S$. The shear stress criterion for the tubesheet is satisfied.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.3367\, in.$$
$$F_t = 47.5235$$
$$C_t = 96.1507$$

Summary Table for STEP 9					
Loading Case	F_s	$\sigma_{t,1}$ (psi)	$\sigma_{t,2}$ (psi)	S_{tb} (psi)	S_t (psi)
1	1.25	-525	2,685	11300	11300
2	---	424	2,546	11300	11300
3	1.25	-72	5,104	11300	11300

Determine $\sigma_{t,\max} = \max\left[\lvert\sigma_{t,1}\rvert, \lvert\sigma_{t,2}\rvert \right]$ and $\sigma_{t,\min} = \min\left[\sigma_{t,1}, \sigma_{t,2} \right]$

For all loading cases $\sigma_{t,\max} < S_t$. The axial tension stress criterion for the tube is satisfied.

For all loading cases $\lvert \sigma_{t,\min} \rvert < S_{tb}$. The buckling criterion for the tube is satisfied.

Calculation Procedure – Floating Tubesheet

The following results are for the design and operating loading cases required to be analyzed (see VIII-1, paragraph UHX-14.4).

a) STEP 1 – Calculate the parameters from VIII-1, paragraph UHX-11.5.1.

$$D_o = 46.2\,in.$$
$$A_L = 0\,in.^2$$
$$\mu = 0.200$$
$$\mu^* = 0.275$$
$$h'_g = 0\,in.$$
$$a_o = 23.1\,in.$$
$$\rho_s = 1.02$$
$$\rho_c = 1.02$$
$$x_s = 0.443$$
$$x_t = 0.547$$

b) STEP 2 – Calculate the shell coefficients β_s, k_s, λ_s, and δ_s and the channel coefficients β_c, k_c, λ_c, and δ_c.

$$\beta_s = 0\,in.^{-1}$$
$$k_s = 0\,lb$$
$$\lambda_s = 0\,psi$$
$$\delta_s = 0\,in.^3/lb$$

$$\beta_c = 0.471\,in.^{-1}$$
$$k_c = 39,500\,lb$$
$$\lambda_c = 7.96\times10^6\,psi$$
$$\delta_c = 1.00\times10^{-4}\,in.^3/lb$$

c) STEP 3 – Calculate h/p. Determine $E*/E$ and ν from VIII-1, paragraph UHX-11.5.2 and calculate $E*$.

$$h/p = 1.10$$
$$E*/E = 0.280$$
$$\nu^* = 0.337$$
$$E^* = 4.149E6\ psi$$

Calculate, X_a, and the parameters from VIII-1, Table UHX-13.1.

$$X_a = 8.84$$
$$Z_a = 3161.6$$
$$Z_d = 0.00214$$
$$Z_v = 0.0130$$
$$Z_m = 0.163$$
$$Z_w = 0.013$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.03$$
$$F = 1.34$$

Calculate Φ and Q_1.

$$\Phi = 1.80$$
$$Q_1 = -4.83 \times 10^{-3}$$

e) STEP 5 – Calculate ω_s, ω^*_s, ω_c, ω^*_c, γ_b, T_r, T^*_c, P^*_s and P^*_c.

$$\omega_s = 0\, in.^2$$
$$\omega^*_s = 7.87 \times 10^{-2}\, in.^2$$
$$\omega_c = 3.13\, in.^2$$
$$\omega^*_c = -3.05\, in.^2$$
$$\gamma_b = 0$$
$$T_r = 217.5^\circ F$$
$$T^*_c = 226.25^\circ F$$
$$P^*_s = 0\, psi$$
$$P^*_c = 8.27\, psi$$

f) STEP 6 – Calculate P_e.

Summary Table for STEP 6 – Design Condition	
Loading Case	P_e (psi)
1	-30
2	-5.17
3	-35.2

Summary Table for STEP 6 – Operating Condition	
Loading Case	P_e (psi)
1	-30
2	-5.17
3	-35.2
4	0

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\|\sigma\|$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	70.8	-0.0137	0.0228	4,210	16950	---
2	9.12	-0.0114	0.0235	748	16950	---
3	79.9	-0.0133	0.0229	4,950	16950	---

Summary Table for STEP 7 – Operating Condition						
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\|\sigma\|$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	90.8	-0.0162	0.0220	4,070	---	33900
2	29.1	-0.0259	0.0193	615	---	33900
3	99.9	-0.0155	0.0222	4,810	---	33900
4	20.0	---	---	231	---	33900

For Design Loading Cases 1-3 $|\sigma| \le 1.5S$ and for Operating Cases 1-4 $|\sigma| \le S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEPS 8 and 9 – For configuration A, skip STEPS 8 and 9 and proceed to STEP 10.

i) STEP 10 – Calculate the stresses in the shell and/or integral channel with the tubesheet.

Summary Table for STEP 10 – Design Condition					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	S_{PS_c} (psi)
1	1,110	9,750	10,900	16950	---
2	0	1,120	1,120	16950	---
3	1,110	10,900	12,000	16950	---

Summary Table for STEP 10 – Operating Condition					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	S_{PS_c} (psi)
1	1,110	10,600	11,800	---	33900
2	0	2,010	2,010	---	33900
3	1,110	11,800	12,900	---	33900
4	0	890	890	---	33900

For Design Loading Cases 1-3 $\sigma_c \leq 1.5S_c$ and for Operating Cases 1-4 $\sigma_c \leq S_{PS,c}$. The stress criterion for the shell and/or integral channel with tubesheet is satisfied.

The calculation procedure is complete and the unit geometry is acceptable for the given design conditions.

4.18.10 Example E4.18.10 - Stationary Tubesheet Gasketed With Shell and Channel; Floating Tubesheet Internally Sealed

A floating tubesheet exchanger with an internally sealed floating head is to be designed as shown in VIII-1, Figure UHX-14.1, sketch (c). The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in VIII-1, Figure UHX-14.2, sketch (d). The floating tubesheet is packed and sealed on its edge in accordance with configuration D as shown in VIII-1, Figure UHX-14.3, sketch (d). There is no allowance for corrosion.

<u>Data Summary - Data Common to Both Tubesheets</u>

$$P_{sd,\max} = 150\ psig$$
$$P_{sd,\min} = 0\ psig$$
$$P_{td,\max} = 175\ psig$$
$$P_{td,\min} = 0\ psig$$

Data Summary – Tubesheets

The tube layout pattern is triangular with no pass lanes

$N_t = 1066$

$p = 0.9375\, in.$

$r_o = 15.563\, in.$

$\rho = 0.88$

$h_g = 0\, in.$

$c_t = 0\, in.$

$\nu = 0.31$

$E = 26.5E6\, psi$

$S = 15,800\, psi$

Data Summary - Tubes

$d_t = 0.75\, in.$

$t_t = 0.065\, in.$

$L_t = 155.875\, in.$

$\ell_t = 20.75\, in.$

$\nu_t = 0.31$

$E_t = 26.5E6\, psi$

$S_t = 15,800\, psi$

$S_{y,t} = 17,500\, psi$

Data Summary - Stationary Tubesheet

$W^* = 290,720\, lb$ from Table UHX-8.1

$A = 39.875\, in.$

$h = 1.188\, in.$

$G_s = 39.441\, in.$

$a_s = 19.7\, in.$

$G_c = 39.441\, in.$

$a_c = 19.7\, in.$

$C = 41.625\, in.$

Data Summary - Floating Tubesheet

$W^{*} = 0\,lb$

$A = 36.875\,in.$

$a_c = 18.4375\,in.$

$a_s = 18.4375\,in.$

$h = 1.188\,in.$

Calculation Procedure – Stationary Tubesheet

The following results are for the 3 load cases required to be analyzed (see VIII-1, paragraph UHX-14.4).

a) STEP 1 – Calculate the parameters from VIII-1, paragraph UHX-11.5.1.

$D_o = 31.876\,in.$

$A_L = 0\,in.^2$

$\mu = 0.200$

$\mu^* = 0.322$

$h'_g = 0\,in.$

$a_o = 15.938\,in.$

$\rho_s = 1.24$

$\rho_c = 1.24$

$x_s = 0.410$

$x_t = 0.597$

b) STEP 2 – Calculate the shell coefficients $\beta_s,\ k_s,\ \lambda_s, \text{ and } \delta_s$ and the channel coefficients $\beta_c,\ k_c,\ \lambda_c, \text{ and } \delta_c$.

$\beta_s = 0\,in.^{-1}$

$k_s = 0\,lb$

$\lambda_s = 0\,psi$

$\delta_s = 0\,in.^3/lb$

$\beta_c = 0\,in.^{-1}$

$k_c = 0\,lb$

$\lambda_c = 0\,psi$

$\delta_c = 0\,in.^3/lb$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-1, paragraph UHX-11.5.2 and calculate E^*.

$h/p = 1.27$

$E^*/E = 0.338$

$\nu^* = 0.316$

$E^* = 8.947E6\ psi$

Calculate, X_a, and the parameters from VIII-1, Table UHX-13.1.

$X_a = 7.40$

$Z_a = 482.2$

$Z_d = 0.00369$

$Z_v = 0.0186$

$Z_m = 0.197$

$Z_w = 0.0186$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$K = 1.25$

$F = 0.454$

Calculate Φ and Q_1.

$\Phi = 0.597$

$Q_1 = 0.202$

e) STEP 5 – Calculate ω_s, ω^*_s, ω_c, ω^*_c, γ_b, P^*_s and P^*_c.

$\omega_s = 0\ in.^2$

$\omega^*_s = 8.00\ in.^2$

$\omega_c = 0\ in.^2$

$\omega^*_c = 8.00\ in.^2$

$\gamma_b = 0$

$P^*_s = 0\ psi$

$P^*_c = 0\ psi$

f) STEP 6 – Calculate P_e.

Summary Table for STEP 6	
Loading Case	P_e (psi)
1	92.9
2	-79.6
3	13.3

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\|\sigma\|$ (psi)	$1.5S$ (psi)
1	-1,250	0.0962	0.0702	21,900	23700
2	1,070	0.0962	0.0702	18,800	23700
3	-179	0.0962	0.0702	3,130	23700

For all loading cases $|\sigma| \le 1.5S$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – Calculate the tubesheet shear stress and the allowable tubesheet shear stress.

Summary Table for STEP 8		
Loading Case	$\|\tau\|$ (psi)	$0.8S$ (psi)
1	3120	12640
2	2670	12640
3	445	12640

For all loading cases $|\tau| \le 0.8S$. The shear stress criterion for the tubesheet is satisfied.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.243\,in.$$

$$F_t = 85.3$$

$$C_t = 173$$

Summary Table for STEP 9					
Loading Case	F_s	$\sigma_{t,1}$ (psi)	$\sigma_{t,2}$ (psi)	S_{tb} (psi)	S_t (psi)
1	1.25	2	-4,647	10,550	15800
2	1.25	-152	3,833	10,550	15800
3	1.25	-150	-814	10,550	15800

Determine $\sigma_{t,\max} = \max\left[|\sigma_{t,1}|, |\sigma_{t,2}| \right]$ and $\sigma_{t,\min} = \min\left[\sigma_{t,1}, \sigma_{t,2} \right]$.

For all loading cases $\sigma_{t,\max} < S_t$. The axial tension stress criterion for the tube is satisfied.

For all loading cases $|\sigma_{t,\min}| < S_{tb}$. The buckling criterion for the tube is satisfied.

Calculation Procedure – Floating Tubesheet

The following results are for the 3 load cases required to be analyzed (see VIII-1, paragraphs UHX-14.4).

a) STEP 1 – Calculate the parameters from VIII-1, paragraph UHX-11.5.1.

$$D_o = 31.876\,in.$$
$$A_L = 0\,in.^2$$
$$\mu = 0.200$$
$$\mu^* = 0.322$$
$$h'_g = 0\,in.$$
$$a_o = 15.938\,in.$$
$$\rho_s = 1.16$$
$$\rho_c = 1.16$$
$$x_s = 0.410$$
$$x_t = 0.597$$

b) STEP 2 – Calculate the shell coefficients $\beta_s, k_s, \lambda_s,$ and δ_s and the channel coefficients $\beta_c, k_c, \lambda_c,$ and δ_c.

$\beta_s = 0\,in.^{-1}$

$k_s = 0\,lb$

$\lambda_s = 0\,psi$

$\delta_s = 0\,in.^3 / lb$

$\beta_c = 0\,in.^{-1}$

$k_c = 0\,lb$

$\lambda_c = 0\,psi$

$\delta_c = 0\,in.^3 / lb$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from VIII-1, paragraph UHX-11.5.2 and calculate E^*.

$h/p = 1.27$

$E^*/E = 0.338$

$\nu^* = 0.316$

$E^* = 8.947E6\ psi$

Calculate, X_a, and the parameters from VIII-1, Table UHX-13.1.

$X_a = 7.40$

$Z_a = 482.2$

$Z_d = 0.00369$

$Z_v = 0.0186$

$Z_m = 0.197$

$Z_w = 0.0186$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$K = 1.16$

$F = 0.295$

Calculate Φ and Q_1.

$\Phi = 0.388$

$Q_1 = 0.139$

e) STEP 5 – Calculate ω_s, ω^*_s, ω_c, ω^*_c, γ_b, T_r, T^*_c, P^*_s and P^*_c.

$\omega_s = 0\,in.^2$

$\omega^*_s = 3.37\,in.^2$

$\omega_c = 0\,in.^2$

$\omega^*_c = 3.37\,in.^2$

$\gamma_b = 0$

$P^*_s = 0\,psi$

$P^*_c = 0\,psi$

f) STEP 6 – Calculate P_e.

Summary Table for STEP 6	
Loading Case	P_e (psi)
1	59.2
2	-50.7
3	8.46

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-548	0.0661	0.0575	11,400	23700
2	469	0.0661	0.0575	9,780	23700
3	-78.2	0.0661	0.0575	1,630	23700

For loading cases $\lvert\sigma\rvert \le 1.5S$. The bending stress criterion for the tubesheet is satisfied.

The calculation procedure is complete and the unit geometry is acceptable for the given design conditions.

4.19 Bellows Expansion Joints

4.19.1 Example E4.19.1 – U-Shaped Un-reinforced Bellows Expansion Joint and Fatigue Evaluation

Check the acceptability of a U-shaped unreinforced bellows expansion joint for the given design conditions in accordance with Section VIII, Division 1.

<u>Design Conditions:</u>

- Pressure (Internal) = $50\ psig\ @\ 650°F$
- Axial Movements in Compression and Extension = $Independent$
- Axial Movement (Compression) = $4.5\ in$
- Axial Movement (Extension) = $0.375\ in$
- Angular Deflection = $None$
- Lateral Deflection = $None$
- Number of Cycles Required = 1000

<u>Bellows:</u>

- Material = $SA-240,\ Type\ 321$
- Allowable Stress = $17900\ psi$
- Yield Strength = $19800\ psi$
- Modulus of Elasticity at Design Temperature = $25.04E+06\ psi$
- Modulus of Elasticity at Room Temperature = $28.26E+06\ psi$
- Inside Diameter of Convolution = $48.0\ in$
- Outside Diameter of Convolution = $52.0\ in$
- Number of Convolutions = 12
- Number of Plies = 1
- Nominal Ply Thickness = $0.048\ in$
- Convolution Pitch = $1.0\ in$
- Mean Radius of Convolution = $0.25\ in$
- Crest Convolution Inside Radius = $0.226\ in$
- Root Convolution Inside Radius = $0.226\ in$
- End Tangent Length = $1.25\ in$
- Installed without Cold Spring = Yes
- Circumferential welds = No

The bellows was formed with a mandrel from a cylinder with an inside diameter of $48.0\ in$ and preformed 100% to the outside of the cylinder. The bellows is in as-formed condition.

<u>Collar:</u>

- Collar = $None$

<u>Cylindrical shell on which the bellows is attached:</u>

- Inside Diameter of Shell = $47.25\ in$

- Thickness of Shell = $0.375\ in$
- Minimum Length of Shell on each Side of the Bellows = $10.5\ in$

Design rules for bellows expansion joints are provided in Mandatory Appendix 26. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.19. The design procedures in VIII-2, paragraph 4.19 are used in this example problem with substitute references made to VIII-1 Mandatory Appendix 26 paragraphs.

Evaluate per VIII-2, paragraph 4-19.

a) STEP 1 – Check applicability of design rules per paragraph 4.19.2 (VIII-1, paragraph 26-2).

1) Bellows length must satisfy $Nq \le 3D_b$:

$$\left\{Nq = 12(1.0) = 12\right\} \le \left\{3D_b = 3(48.0) = 144\right\} \qquad (True)$$

2) Bellows thickness must satisfy $nt \le 0.2\,in$:

$$\left\{nt = 1(0.048) = 0.048\right\} \le \left\{0.2\right\} \qquad (True)$$

3) Number of plies must satisfy $n \le 5$:

$$\left\{n = 1\right\} \le \left\{5\right\} \qquad (True)$$

4) Displacement shall be essentially axial.

No angular or lateral deflection is specified, so the condition is satisfied

5) Design temperature is below the creep domain.

The material is $SA-240,\ Type\ 321$ is an austenitic stainless steel, the design temperature is $650°F$ which is less than the time-dependent value of $800°F$; therefore, the condition is satisfied.

6) The length of the cylindrical shell on each side of the bellows shall not be less than $1.8\sqrt{D_s t_s}$.

$$\left\{10.5\right\} \ge \left\{1.8\sqrt{D_s t_s} = 1.8\sqrt{(47.25)(0.375)} = 7.577\right\} \qquad (True)$$

b) STEP 2 – Check the applicability of paragraph 4.19.5.2 (VIII-1, paragraph 26-6.2)

1) Check that the following condition is satisfied.

$$\left\{0.9r_{ir} = 0.9(0.226) = 0.203\right\} \le \left\{r_{ic} = 0.226\right\} \le \left\{1.1r_{ir} = 1.1(0.226) = 0.249\right\} \quad (True)$$

2) Torus radius shall satisfy $r_i \ge 3t$.

$$\left\{r_i = \frac{r_{ic} + r_{ir}}{2} = \frac{0.226 + 0.226}{2} = 0.226\ in\right\} \ge \left\{3t = 3(0.048) = 0.144\ in\right\} \qquad (True)$$

3) Sidewall offset angle shall meet $-15\ deg \le \alpha \le 15\ deg$.

$$\alpha = \text{atan}\left[\left(\frac{q}{2} - 2r_m\right) / \left(w - 2r_m\right)\right] = \text{atan}\left[\left(\frac{(1.0)}{2} - 2(0.25)\right) / \left((2.0) - 2(0.25)\right)\right] = 0\ rad$$

$$-15\ deg \le \{\alpha = 0\} \le 15\ deg \qquad (True)$$

4) Convolution height shall meet $w \le D_b / 3$.

$$\left\{ w = \left(\frac{D_o}{2} - \frac{D_b}{2} \right) = \left(\frac{52.0}{2} - \frac{48.0}{2} \right) = 2.0\ in \right\} \le \left\{ \frac{D_b}{3} = \frac{48.0}{3} = 16.0\,in \right\} \qquad (True)$$

c) STEP 3 – Check stresses in bellows at design conditions per paragraph 4.19.5.3 (VIII-1, paragraph 26-6.3). Since the bellows are subject to internal pressure, calculations and acceptability criteria are per Table 4.19.1. The following values are calculated.

$$D_m = D_b + w + nt = 48.0 + 2.0 + 1(0.048) = 50.048\ in$$

$$t_p = t\sqrt{\frac{D_b}{D_m}} = 0.048\sqrt{\frac{48.0}{50.048}} = 0.047\ in$$

$$k = \min\left[\left(\frac{L_t}{1.5\sqrt{D_b t}}\right), 10\right] = \min\left[\left(\frac{1.25}{1.5\sqrt{48.0(0.048)}}\right), 1.0\right] = 0.549$$

$$A = \left[2\pi r_m + 2\sqrt{\left\{\frac{q}{2} - 2r_m\right\}^2 + \{w - 2r_m\}^2} \right] n t_p$$

$$A = \left[2\pi(0.25) + 2\sqrt{\left\{\frac{(1.0)}{2} - 2(0.25)\right\}^2 + \{(2.0) - 2(0.25)\}^2} \right] (1)(0.047) = 0.215\ in^2$$

Table 4.19.2 (VIII-1, Fig. 26-4, paragraph 26-15) is used to determine C_p. The following values are calculated.

$$C_1 = \frac{2r_m}{w} = \frac{2(0.25)}{(2.0)} = 0.250 \text{ with } 0.0 \le C_1 \le 1.0$$

$$C_2 = \frac{1.82\,r_m}{\sqrt{D_m t_p}} = \frac{1.82(0.25)}{\sqrt{(50.048)(0.047)}} = 0.297 \text{ with } 0.2 \le C_2 \le 4.0$$

The coefficients, $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are interpolated.

$$\alpha_0 = 1.000 \qquad \alpha_3 = 0.711$$
$$\alpha_1 = -0.587 \qquad \alpha_4 = 0.662$$
$$\alpha_2 = -0.589 \qquad \alpha_5 = -0.646$$

$$C_p = \begin{pmatrix} \alpha_0 + \alpha_1 C_1 + \alpha_2 C_1^2 + \\ \alpha_3 C_1^3 + \alpha_4 C_1^4 + \alpha_5 C_1^5 \end{pmatrix}$$

$$C_p = \begin{pmatrix} 1.000 + (-0.587)(0.25) + (-0.589)(0.25)^2 + \\ (0.711)(0.25)^3 + (0.662)(0.25)^4 + (-0.646)(0.25)^5 \end{pmatrix} = 0.830$$

Calculate Stresses

Circumferential Membrane stress in bellows tangent due to pressure (S_1).

$$S_1 = \frac{P(D_b + nt)^2 L_t E_b k}{2\left[nt(D_b + nt)L_t E_b + t_c D_c L_c E_c k\right]}$$

$$S_1 = \frac{50(48.0 + 1(0.048))^2 (1.25)(25.04E+06)(0.549)}{2\left[1(0.048)(48.0 + 1(0.048))(1.25)(25.04E+06) + 0\right]} = 13738.7\ psi$$

Circumferential Membrane stress in bellows end convolutions due to pressure $(S_{2,E})$.

$$S_{2,E} = \frac{P\left[qD_m + L_t(D_b + nt)\right]}{2(A + nt_p L_t + t_c L_c)} = \frac{50\left[1.0(50.048) + 1.25(48.0 + 1(0.048))\right]}{2(0.215 + 1(0.047)(1.25) + 0)} = 10055.5\ psi$$

Circumferential Membrane stress in bellows intermediate convolutions due to pressure $(S_{2,I})$.

$$S_{2,I} = \frac{PqD_m}{2A} = \frac{50(1.0)(50.048)}{2(0.215)} = 5819.5\ psi$$

Meridional Membrane stress in bellows due to pressure (S_3).

$$S_3 = \frac{Pw}{2nt_p} = \frac{50(2.0)}{2(1(0.047))} = 1063.8\ psi$$

Meridional Bending stress in bellows due to pressure (S_4).

$$S_4 = \frac{PC_p}{2n}\left(\frac{w}{t_p}\right)^2 = \frac{50(0.830)}{2(1)}\left(\frac{2.0}{0.047}\right)^2 = 37573.6\ psi$$

Acceptance Checks

$$\{S_1 = 13738.7\ psi\} \le \{S = 17900\ psi\} \qquad (True)$$

$$\{S_{2,E} = 10055.5\ psi\} \le \{S = 17900\ psi\} \qquad (True)$$

$$\{S_{2,I} = 5819.5\ psi\} \le \{S = 17900\ psi\} \qquad (True)$$

$$\{(S_3 + S_4) = (1063.8 + 37573.6) = 38637.4\ psi\} \le \{K_m S = (3.0)(17900) = 53700\ psi\} \quad (True)$$

The bellows meet internal pressure acceptance criteria at design conditions.

Factor K_m is calculated by:

$$K_m = 1.5Y_{sm} = 1.5(2.0) = 3.0 \qquad (for\ As-Formed\ Bellows)$$

The forming stain ε_f for bellows formed 100% to the outside of the initial cylinder is:

$$\varepsilon_f = \sqrt{\left[\ln\left(1+\frac{2w}{D_b}\right)\right]^2 + \left[\ln\left(1+\frac{nt_p}{2r_m}\right)\right]^2}$$

$$\varepsilon_f = \sqrt{\left[\ln\left(1+\frac{2(2.0)}{(48.0)}\right)\right]^2 + \left[\ln\left(1+\frac{(1)(0.047)}{2(0.25)}\right)\right]^2} = 0.120$$

The forming method factor K_f for forming with expanding mandrel is:

$$K_f = 1.0$$

Since material $SA-240,\ Type\ 321$ is an austenitic stainless steel, the yield strength multiplier Y_{sm} is:

$$Y_{sm} = 1 + 9.94(K_f\varepsilon_f) - 7.59(K_f\varepsilon_f)^2 - 2.4(K_f\varepsilon_f)^3 + 2.21(K_f\varepsilon_f)^4$$

$$Y_{sm} = 1 + 9.94((1.0)(0.120)) - 7.59((1.0)(0.120))^2 - 2.4((1.0)(0.120))^3 + 2.21((1.0)(0.120))^4$$

$$Y_{sm} = 2.083$$

If Y_{sm} is greater than 2.0, then $Y_{sm} = 2.0$

d) STEP 4 – Check column instability due to internal pressure per paragraph 4.19.5.4 (VIII-1, paragraph 26-6.4.1).

$$P_{sc} = \frac{0.34\pi K_b}{Nq} = \frac{0.34\pi(1648.7)}{12(1.0)} = 146.8\ psi$$

The axial stiffness, K_b, is calculated using Equation 4.19.17 (VIII-1, paragraph 26-6.7).

$$K_b = \frac{\pi E_b D_m}{2\left(1-\nu_b^{\,2}\right)C_f}\left(\frac{n}{N}\right)\left(\frac{t_p}{w}\right)^3 = \frac{\pi\left(25.04E+06\right)\left(50.048\right)}{2\left(1-0.3^2\right)\left(1.419\right)}\left(\frac{1}{12}\right)\left(\frac{0.047}{2.0}\right)^3$$

$$K_b = 1648.7\ lb/in$$

C_f is calculated using the method described in Table 4.19.3 (VIII-1, Fig. 26-5, paragraph 26-15). With $C_2 = 0.297$, interpolate for the coefficients $\alpha_0,\alpha_1,\alpha_2,\alpha_3,\alpha_4,\alpha_5$.

$$\alpha_0 = 1.006 \qquad \alpha_3 = 5.719$$

$$\alpha_1 = 2.106 \qquad \alpha_4 = -5.501$$

$$\alpha_2 = -2.930 \qquad \alpha_5 = 2.067$$

$$C_f = \begin{pmatrix} \alpha_0 + \alpha_1 C_1 + \alpha_2 C_1^2 + \\ \alpha_3 C_1^3 + \alpha_4 C_1^4 + \alpha_5 C_1^5 \end{pmatrix}$$

$$C_f = \begin{pmatrix} 1.006 + \left(2.106\right)\left(0.25\right) + \left(-2.930\right)\left(0.25\right)^2 + \\ \left(5.719\right)\left(0.25\right)^3 + \left(-5.501\right)\left(0.25\right)^4 + \left(2.067\right)\left(0.25\right)^5 \end{pmatrix} = 1.419$$

$$\left\{P = 50\ psi\right\} \le \left\{P_{sc} = 146.7\ psi\right\} \qquad \left(True\right)$$

The bellows meet columns instability criteria at design conditions.

e) STEP 5 – Check in-plane instability due to internal pressure per paragraph 4.19.5.5 (VIII-1, paragraph 26-6.4.2).

$$P_{si} = \frac{A S_y^*\left(\pi - 2\right)}{D_m q\left[1 + 2\delta^2 + \left(1 - 2\delta^2 + 4\delta^4\right)^{0.5}\right]^{0.5}}$$

$$P_{si} = \left(\frac{0.215\left(45540\right)\left(\pi - 2\right)}{50.048\left(1.0\right)\left[1 + 2\left(2.15\right)^2 + \left(1 - 2\left(2.15\right)^2 + 4\left(2.15\right)^4\right)^{0.5}\right]^{0.5}}\right) = 51.2\ psi$$

$$S_y^* = 2.3 S_y = 2.3\left(19800\right) = 45540\ psi \qquad \left(for\ as-formed\ \ bellows\right)$$

$$\delta = \frac{S_4}{3S_{2,I}} = \frac{37573.6}{3\left(5819.5\right)} = 2.15$$

$$\left\{P = 50\ psi\right\} \le \left\{P_{si} = 51.2\ psi\right\} \qquad \left(True\right)$$

The bellows meet in-plane instability criteria at design conditions.

f) STEP 6 – Perform a fatigue evaluation per paragraph 4.19.5.7 (VIII-1, paragraph 26-6.6)

Calculate the equivalent axial displacement range

The axial displacement range, Δq, is calculated using the procedure shown in paragraph 4.19.8 (VIII-1, paragraph 26-9).

$$\left\{\begin{matrix} x_e = 0.375\ in \\ x_c = -4.5\ in \end{matrix}\right\} \rightarrow \quad See\ design\ data$$

$$\Delta q_{x,e} = \frac{x_e}{N} = \frac{0.375}{12} = 0.031\ in$$

$$\Delta q_{x,c} = \frac{x_c}{N} = \frac{-4.5}{12} = -0.375\ in$$

Since no lateral or angular movement:

$$\Delta q_{e,1} = \Delta q_{x,e} = 0.031\ in$$

$$\Delta q_{c,1} = \Delta q_{x,c} = -0.375\ in$$

Since the bellows was installed without cold spring and the axial movements in extension and compression are independent (as opposed to concurrent):

$$\begin{array}{ll} Initial\ Position & Final\ Position \\ \quad \Delta q_{e,0} = 0.0\ in & \Delta q_{e,1} = +0.031\ in \\ \quad \Delta q_{c,0} = 0.0\ in & \Delta q_{c,1} = -0.375\ in \end{array}$$

$$\Delta q = \max\left[\left|\Delta q_{e,1}\right|, \left|\Delta q_{c,1}\right|\right] = \max\left[\left|0.031\right|, \left|-0.375\right|\right] = 0.375\ in$$

C_d is calculated using the method described in Table 4.19.4 (VIII-1, Fig. 26-6, paragraph 26-15)
With $C_2 = 0.297$, interpolate for the coefficients $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$.

$$\begin{array}{ll} \alpha_0 = 1.000 & \alpha_3 = -3.441 \\ \alpha_1 = 1.228 & \alpha_4 = 3.453 \\ \alpha_2 = 1.309 & \alpha_5 = -1.190 \end{array}$$

$$C_d = \begin{pmatrix} \alpha_0 + \alpha_1 C_1 + \alpha_2 C_1^2 + \\ \alpha_3 C_1^3 + \alpha_4 C_1^4 + \alpha_5 C_1^5 \end{pmatrix}$$

$$C_d = \begin{pmatrix} 1.000 + (1.228)(0.25) + (1.309)(0.25)^2 + \\ (-3.441)(0.25)^3 + (3.453)(0.25)^4 + (-1.190)(0.25)^5 \end{pmatrix} = 1.347$$

Calculate stresses due to equivalent axial displacement range:

Meridional membrane (S_5).

$$S_5 = \frac{E_b t_p^2 \Delta q}{2w^3 C_f} = \frac{(25.04E+06)(0.047)^2(0.375)}{2(2.0)^3(1.419)} = 913.6\ psi$$

Meridional bending (S_6).

$$S_6 = \frac{5E_b t_p \Delta q}{3w^2 C_d} = \frac{5(25.04E+06)(0.047)(0.375)}{3(2.0)^2(1.347)} = 136516.3\ psi$$

Total stress range due to cyclic displacement (S_t)

$$S_t = 0.7[S_3 + S_4] + [S_5 + S_6]$$
$$S_t = 0.7(1063.8 + 37573.6) + (913.6 + 136516.3) = 164476.1\ psi$$

Calculate allowable number of cycles, N_{alw}, using the equations from Table 4.19.5 (VIII-1, paragraph 26-6.6.3.2).

$$K_g\left(\frac{E_o}{E_b}\right)S_t = \left\{1.0\left(\frac{28.26E+06}{25.04E+06}\right)(164476.1) = 185626.78\right\} \geq 65000$$

$$N_{alw} = \left(\frac{5.2E+06}{K_g\left(\frac{E_0}{E_b}\right)S_t - 38300}\right)^2 = \left(\frac{5.2E+06}{185626.78 - 38300}\right)^2 = 1246\,cycles$$

$$\{N_{alw} = 1246\ cycles\} \geq \{N_{spe} = 1000\ cycles\}$$

The bellows meets fatigue design criteria at design conditions.

The bellows meets all of the design requirements of paragraph 4.19 (VIII-1, Mandatory Appendix 26) at design conditions.

4.19.2 Example E4.19.2 - Toroidal Bellows Expansion Joint and Fatigue Evaluation

Check the acceptability of a toroidal bellows for the given design conditions in accordance with Section VIII, Division 1.

Design Conditions:

- Pressure (Internal) = $400\ psig\ @\ 650°F$
- Axial Movements in Compression and Extension = $Independent$
- Axial Displacement (Compression) = $0.25\ in$
- Axial Displacement (Extension) = $0.745\ in$
- Angular Deflection = $None$
- Lateral Deflection = $None$
- Number of Cycles Required = 1000

Bellows:

- Material = $SA-240,\ Type\ 321$
- Allowable Stress = $17900\ psi$
- Modulus of Elasticity at Design Temperature = $25.04E+06\ psi$
- Modulus of Elasticity at @ Room Temperature = $28.26E+06\ psi$
- Inside Diameter of Bellows = $36.0\ in$
- Mean Diameter of Bellows = $40.0\ in$
- Number of Convolutions = 2
- Convolution Pitch = $4.000\ in$
- Mean Radius of Convolutions = $1.5\ in$
- Number of Plies = 1
- Ply Thickness = $0.078\ in$
- Installed without Cold Spring = Yes
- Circumferential welds = No

The bellows is attached to the shell externally on both sides.

Reinforcing and Tangent Collars:

- Material = $SA-240,\ Type\ 304$
- Allowable Stress = $16200\ psi$
- Modulus of Elasticity at Design Temperature = $25.04E+06\ psi$

Tangent Collars:

- Tangent Collar Joint Efficiency = 1.0
- Tangent Collar Thickness = $0.75\ in$
- Cross Sectional Metal Area of one Tangent Collar = $1.034\ in^2$
- Length from Attachment Weld to the Center of the First Convolution = $2.000\ in$

Reinforcing Collars:

- Reinforcing Collar Joint Efficiency = 1.0

- Reinforcing Collar Thickness = $0.75\ in$
- Overall Length of one Reinforcing Collar = $3.094\ in$
- Cross Sectional Metal Area of one Reinforcing Collar based on Overall Length = $2.068\ in^2$

<u>Cylindrical shell on which the bellows is attached:</u>

- Inside Diameter of Shell = $35.0\ in$
- Thickness of Shell = $0.50\ in$
- Minimum Length of Shell on each Side of the Bellows = $10.5\ in$

Design rules for bellows expansion joints are provided in Mandatory Appendix 26. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.19. The design procedures in VIII-2, paragraph 4.19 are used in this example problem with substitute references made to VIII-1 Mandatory Appendix 26 paragraphs.

Evaluate per VIII-2, paragraph 4-19.

a) STEP 1 – Check applicability of design rules per paragraph 4.19.2 (VIII-1, paragraph 26-2).

1) Bellows length must satisfy $Nq \le 3D_b$.

$$\left\{(N-1)q+2L_d=(2-1)(4.0)+2(2.0)=8\ in\right\} \le \left\{3D_b=3(36)=108\ in\right\} \qquad (True)$$

2) Bellows thickness must satisfy $nt \le 0.2\,in$.

$$\left\{nt=1(0.078)=0.078\right\} \le \left\{0.2\right\} \qquad (True)$$

3) Number of plies must satisfy $n \le 5$.

$$\left\{n=1\right\} \le \left\{5\right\} \qquad (True)$$

4) Displacement shall be essentially axial.

No angular or lateral deflection is specified, so the condition is satisfied

5) Design temperature is below the creep domain.

The material is $SA-240,\ Type\ 321$ is an austenitic stainless steel, the design temperature is $650^\circ F$ which is less than the time-dependent value of $800^\circ F$; therefore, the condition is satisfied.

6) The length of the cylindrical shell on each side of the bellows shall not be less than $1.8\sqrt{D_s t_s}$.

$$\left\{10.5\right\} \ge \left\{1.8\sqrt{D_s t_s}=1.8\sqrt{(35.0)(0.50)}=7.530\right\} \qquad (True)$$

b) STEP 2 – Check the applicability of paragraph 4.19.7.2 (VIII-1, paragraph 26-8.2)

1) The type of attachment to the shell shall be the same on both sides.

The bellows is attached to the shell externally on both sides, so the condition is satisfied.

2) Distance L_g shall be less than $0.75r$ in the maximum extended position.

The distance across the inside opening in the neutral position is calculated

$$L_{g0} = q - (L_{rt} + 2nt) = (4.000) - ((3.094) + 2(1)(0.78)) = 0.750\ in$$

The only movement is in the axial direction. The maximum opening corresponds to the maximum extension

$$L_g = L_{g0} + \Delta q_{x,e} = L_{g0} + \frac{x_e}{N} = (0.750) + \left(\frac{0.745}{2}\right) = 1.1225\ in$$

$$\left\{L_g = 1.1225\right\} < \left\{0.75r = 0.75(1.5) = 1.1250\right\} \qquad (True)$$

3) The third condition applies to internally attached bellows.

Not applicable

c) STEP 3 – Check stresses in bellows at design conditions per paragraph 4.19.7.3 (VIII-1, paragraphs 26-8.3). Since the bellows are subject to internal pressure, calculations and acceptability criteria are per Table 4.19.8.

$$D_c = D_b + 2nt + t_c = 36.0 + 2(1)(0.078) + 0.75 = 36.906\ in$$

$$D_r = D_b + 2nt + t_r = 36.0 + 2(1)(0.078) + 0.75 = 36.906\ in$$

$$t_p = t\sqrt{\frac{D_b}{D_m}} = 0.078\sqrt{\frac{36.0}{40.0}} = 0.074\ in$$

<u>Calculate stresses</u>

Circumferential Membrane stress in end tangent due to internal pressure (S_1).

$$S_1 = \frac{P(D_b + nt)^2 L_d E_b}{2 D_c E_c A_{tc}}$$

$$S_1 = \frac{(400)(36.0 + (1)(0.078))^2 (2.0)(25.04E+06)}{2(36.906)(25.04E+06)(1.034)} = 13643.6\ psi$$

Circumferential Membrane stress in tangent collar due to internal pressure $(S_1^{'})$.

$$S_1^{'} = \frac{P D_c L_d}{2A_{tc}} = \frac{(400)(36.906)\ (2.0)}{2(1.034)} = 14277.0\ psi$$

Circumferential Membrane stress in bellows due to internal pressure (S_2).

$$S_2 = \frac{Pr}{2nt_p} = \frac{(400)(1.5)}{2(1)(0.074)} = 4054.1\ psi$$

Meridional membrane stress in bellows due to internal pressure (S_3).

$$S_3 = \frac{Pr}{nt_p}\left(\frac{D_m - r}{D_m - 2r}\right) = \frac{(400)(1.5)}{(1)(0.074)}\left(\frac{40.0-1.5}{40.0-2(1.5)}\right) = 8436.8\ psi$$

Circumferential Membrane stress in reinforcing collar due to internal pressure (S_2').

Since $\{L_{rt} = 3.094\} \le \left\{\frac{2}{3}\sqrt{D_r t_r} = \frac{2}{3}\sqrt{(36.906)(0.75)} = 3.507\right\}$

$$S_2' = \frac{D_r\,(L_{rt} + L_g + 2nt)}{2\,A_{rt}}\ P$$

$$S_2' = \frac{(36.906)(3.094 + 1.1225 + 2(1)(0.078))}{2(2.068)}\ (400) = 15606.6\ psi$$

Acceptance Checks

$$\{S_1 = 13643.6\ psi\} \le \{S = 17900\ psi\} \qquad (True)$$

$$\{S_1' = 14277.0\ psi\} \le \{C_{wc}S_c = (1.0)(16200) = 16200\ psi\} \qquad (True)$$

$$\{S_2 = 4054.1\ psi\} \le \{S = 17900\ psi\} \qquad (True)$$

$$\{S_3 = 8437.1\ psi\} \le \{S = 17900\ psi\} \qquad (True)$$

$$\{S_2' = 15606.6\ psi\} \le \{C_{wr}S_r = (1.0)(16200) = 16200\ psi\} \qquad (True)$$

Therefore, bellows meets internal pressure stress acceptance criteria at design conditions.

d) STEP 4 – Check column instability due to internal pressure per paragraph 4.19.7.4 (VIII-1, paragraph 26-8.4.

The following values are calculated using the procedure shown in Table 4.19.9 (VIII-1, Table 26-8).

$$C_3 = \frac{6.61r^2}{D_m t_p} = \frac{6.61(1.5)^2}{40.0(0.074)} = 5.024$$

$$B_3 = \left(\frac{\begin{array}{c} 0.99916 - 0.091665C_3 + 0.040635C_3^2 - \\ 0.0038483C_3^3 + 0.00013392C_3^4 \end{array}}{\begin{array}{c} 1 - 0.1527C_3 + 0.013446C_3^2 - \\ 0.00062724C_3^3 + 1.4374(10)^{-5} C_3^4 \end{array}} \right)$$

$$B_3 = \left(\frac{\begin{array}{c} 0.99916 - 0.091665(5.024) + 0.040635(5.024)^2 - \\ 0.0038483(5.024)^3 + 0.00013392(5.024)^4 \end{array}}{\begin{array}{c} 1 - 0.1527(5.024) + 0.013446\left((5.024)^2\right) - \\ 0.00062724(5.024)^3 + 1.4374(10)^{-5}(5.024)^4 \end{array}} \right) = 2.315$$

The axial stiffness, K_b, is calculated using Equation 4.19.28 (VIII-1, paragraph 26-8.7).

$$K_b = \frac{E_b D_m B_3}{12\,(1-\nu_b{}^2)} \left(\frac{n}{N}\right)\left(\frac{t_p}{r}\right)^3 = \frac{\left(2.504(10)^7\right)(40.0)(2.315)}{12\left(1-(0.3)^2\right)}\left(\frac{1}{2}\right)\left(\frac{0.074}{1.5}\right)^3$$

$$K_b = 12747.2\ lb/in$$

Calculate allowable internal pressure:

$$P_{sc} = \frac{0.15\pi K_b}{Nr} = \frac{0.15\pi(12747.2)}{2(1.5)} = 2002\ psi$$

Acceptance criteria

$$\{P = 400\ psi\} \le \{P_{sc} = 2002\ psi\} \qquad (True)$$

The bellows meets column instability criteria at design conditions.

e) STEP 5 – Perform a fatigue evaluation per paragraph 4.19.7.7 (VIII-1, paragraph 26-8.6).

The axial displacement range, Δq, is calculated using the procedure shown in paragraph 4.19.8 (VIII-1, paragraph 26-9).

$$\left\{\begin{array}{l} x_e = 0.745\ in \\ x_c = -0.25\ in \end{array}\right\} \rightarrow \quad See\ design\ data$$

$$\Delta q_{x,e} = \frac{x_e}{N} = \frac{0.745}{2} = 0.3725\ in$$

$$\Delta q_{x,c} = \frac{x_c}{N} = \frac{-0.25}{2} = -0.125\ in$$

Since no angular or lateral displacement:

$$\Delta q_{e,1} = \Delta q_{x,e} = 0.3725\ in$$

$$\Delta q_{c,1} = \Delta q_{x,c} = -0.125\ in$$

Since the bellows was installed without cold spring and the axial movements in extension and compression are independent (as opposed to concurrent):

$$\Delta q = \max\left[\left|\Delta q_{c,1}\right|, \left|\Delta q_{e,1}\right|\right] = \max\left[\left|0.3725\right|, \left|-0.125\right|\right] = 0.3725\ in$$

Calculate coefficient B_1, B_2 from VIII-2, Table 4.19.9 (VIII-1, Table 26-8):

$$B_1 = \frac{\begin{pmatrix} 1.00404 + 0.028725C_3 + \\ 0.18961C_3^2 - 0.00058626C_3^3 \end{pmatrix}}{\begin{pmatrix} 1 + 0.14069C_3 - 0.0052319C_3^2 + \\ 0.00029867C_3^3 - 6.2088(10)^{-6} C_3^4 \end{pmatrix}}$$

$$B_1 = \frac{\begin{pmatrix} 1.00404 + 0.028725(5.024) + \\ 0.18961(5.024)^2 - 0.00058626(5.024)^3 \end{pmatrix}}{\begin{pmatrix} 1 + 0.14069(5.024) - 0.0052319(5.024)^2 + \\ 0.00029867(5.024)^3 - 6.2088(10)^{-6}(5.024)^4 \end{pmatrix}} = 3.643$$

$$B_2 = \frac{\begin{pmatrix} 0.049198 - 0.77774C_3 - \\ 0.13013C_3^2 + 0.080371C_3^3 \end{pmatrix}}{\begin{pmatrix} 1 - 2.81257C_3 + \\ 0.63815C_3^2 + 0.0006405C_3^3 \end{pmatrix}}$$

$$B_2 = \frac{\begin{pmatrix} 0.049198 - 0.77774(5.024) - \\ 0.13013(5.024)^2 + 0.080371(5.024)^3 \end{pmatrix}}{\begin{pmatrix} 1 - 2.81257(5.024) + \\ 0.63815(5.024)^2 + 0.0006405(5.024)^3 \end{pmatrix}} = 0.997$$

Calculate meridional stresses due to axial displacement range:

$$S_5\left(membrane\right) = \frac{E_b t_p^2 B_1 \Delta q}{34.3r^3} = \frac{(25.04E+06)(0.074)^2(3.643)(0.3725)}{34.3(1.5)^3} = 1607.4\ psi$$

$$S_6\left(bending\right) = \frac{E_b t_p B_2 \Delta q}{5.72r^2} = \frac{(25.04E+06)(0.074)(0.997)(0.3725)}{5.72(1.5)^2} = 53469.9\ psi$$

Calculate total cycle stress range due to displacement:

$$S_t = 3S_3 + S_5 + S_6 = 3(8436.8) + 1607.4 + 53469.9 = 80387.7 \ psi$$

Calculate the allowable number of cycles, N_{alw}, per Table 4.19.10.

$$\left\{ K_g \left(\frac{E_o}{E_b} \right) S_t = 1.0 \left(\frac{28.26E+06}{25.04E+06} \right) (80387.7) = 90725.1 \ psi \right\} \geq \{65000 \ psi\}$$

$$N_{alw} = \left(\frac{5.2E+06}{\left(K_g \left(\frac{E_o}{E_b} \right) S_t - 38300 \right)} \right)^2 = \left(\frac{5.2E+06}{(90725.1 - 38300)} \right)^2 = 9838 \ cycles$$

$$\{ N_{alw} = 9838 \ cycles \} \geq \{ N_{spe} = 1000 \ cycles \}$$

The bellows meets fatigue design criteria at design conditions.

The bellows meets all of the design requirements of paragraph 4.19 (VIII-1, Mandatory Appendix 26) at design conditions.

4.20 Tube-To-Tubesheet Welds

4.20.1 Example E4.20.1 – Full Strength Welds

Determine the size and allowable axial load of full strength tube-to-tubesheet welds for each of the joint types shown in Fig. UW-20.1 (see Figure E4.20.1).

Design Data:

- Design Temperature = $600°F$
- Tube Outside Diameter = $1.0\ in$
- Tube Thickness = $0.065\ in$
- Tube Material = $SB-338,\ Grade\ 3$
- Tubesheet Material = $SB-265,\ Grade\ 2$
- Corrosion Allowance = $0.0\ in$
- Tube Allowable Stress = $7400\ psi$
- Tubesheet Allowable Stress = $6500\ psi$

Design rules for tube-to-tubesheet welds are provided in UW-20. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.18.10.

Evaluate per UW-20.

UW-20.2 Definitions – UW-20.2(a) Full Strength Welds, A full strength tube-to-tubesheet weld is one in which the design strength is equal to or greater than the axial tube strength, F_t. When the weld in a tube-to-tubesheet joint meets the requirements of UW-20.4, it is a full strength weld and the joint does not require qualification by shear load testing. Such a weld also provides tube joint leak tightness.

Based on the allowable stresses of the tubesheet and tubes and the determination that full strength welds are to be evaluated, the following parameters are determined from UW-20.3.

Allowable Stress of the welds:

$$S_w = \min\left[S_a,\ S_t\right] = \min\left[7400,\ 6500\right] = 6500\ psi$$

Weld Strength Factor:

$$f_w = \frac{S_a}{S_w} = \frac{7400}{6500} = 1.1385$$

Ratio of Design Strength to Tube Strength:

$$f_d = \frac{F_d}{F_t} = 1.0 \quad for\ \ full\ \ strength\ \ welds$$

Joint Types of Fig. UW–20.1 Sketch (a):

UW–20.4(a), the size of a full strength weld shall be determined in accordance with UW–20.6.

UW–20.6 Weld Size Design Formulas – The size of tube–to–tubesheet strength welds shown in Fig. UW–20.1 shall conform to the following requirements.

UW–20.6(a), for fillet welds shown in sketch (a), the minimum required weld length is calculated as follows.

$$a_r = \sqrt{(0.75d_o)^2 + 2.73t(d_o - t)f_w f_d} - 0.75d_o$$

$$a_r = \sqrt{(0.75(1.0))^2 + 2.73(0.065)(1.0 - 0.065)(1.1385)(1.0)} - 0.75(1.0)$$

$$a_r = 0.1168\ in$$

UW–20.6(a)(1), for full strength welds, the fillet weld leg, a_f shall not be less than the greater of a_r or t.

$$a_f \geq \{\max[a_r, t] = \max[0.1168,\ 0.065] = 0.1168\ in\}$$

UW–20.4(b), the maximum allowable axial load in either direction on a tube-to-tubesheet joint with a full strength weld shall be determined as follows.

UW-20.4(b)(1), for loads due to pressure-induced axial forces,

$$L_{max} = F_t = \pi t(d_o - t)S_a = \pi(0.065)(1.0 - 0.065)(7400) = 1412.9\ lbs$$

UW-20.4(b)(2), for loads due to thermally-induced or pressure plus thermally-induced axial forces,

UW-20.4(b)(2)(a), $L_{max} = F_t$ for welded only tube-to-tubesheet joints where the thickness through the weld throat is less than the nominal tube thickness.

For a fillet weld, the weld throat is calculated as follows.

$$throat = 0.707a_f = 0.707(0.1168) = 0.0826\ in$$

$$\{throat = 0.0826\ in\} > \{t = 0.065\ in\}$$

Therefore, UW-20.4(b)(2)(a) is not applicable.

UW-20.4(b)(2)(b), $L_{max} = 2F_t$ for all other welded tube-tubesheet joints.

$$L_{max} = 2F_t = 2(1412.9) = 2825.8\ lbs$$

<u>Joint Types of Fig. UW–20.1 Sketch (b):</u>

UW–20.4(a), the size of a full strength weld shall be determined in accordance with UW–20.6.

UW–20.6 Weld Size Design Formulas – The size of tube–to–tubesheet strength welds shown in Fig. UW–20.1 shall conform to the following requirements.

UW–20.6(b), for groove welds shown in sketch (b), the minimum required weld length is calculated as follows.

$$a_r = \sqrt{(0.75d_o)^2 + 1.76t(d_o - t)f_w f_d} - 0.75d_o$$

$$a_r = \sqrt{(0.75(1.0))^2 + 1.76(0.065)(1.0 - 0.065)(1.1385)(1.0)} - 0.75(1.0)$$

$$a_r = 0.0772\ in$$

UW–20.6(b)(1), for full strength welds, the groove weld leg, a_g shall not be less than the greater of a_r or t.

$$a_g \ge \left\{\max\left[a_r,\, t\right] = \max\left[0.0772,\, 0.065\right] = 0.0772\ in\right\}$$

UW–20.4(b), the maximum allowable axial load in either direction on a tube-to-tubesheet joint with a full strength weld shall be determined as follows.

UW-20.4(b)(1), for loads due to pressure-induced axial forces,

$$L_{max} = F_t = \pi t\left(d_o - t\right)S_a = \pi\left(0.065\right)\left(1.0 - 0.065\right)\left(7400\right) = 1412.9\ lbs$$

UW-20.4(b)(2), for loads due to thermally-induced or pressure plus thermally-induced axial forces,

UW-20.4(b)(2)(a), $L_{max} = F_t$ for welded only tube-to-tubesheet joints where the thickness through the weld throat is less than the nominal tube thickness.

For a groove weld, the weld throat is the length of the weld,

$$throat = a_g = 0.0772\ in$$

$$\left\{throat = 0.0772\ in\right\} > \left\{t = 0.065\ in\right\}$$

Therefore, UW-20.4(b)(2)(a) is not applicable.

UW-20.4(b)(2)(b), $L_{max} = 2F_t$ for all other welded tube-tubesheet joints.

$$L_{max} = 2F_t = 2\left(1412.9\right) = 2825.8\ lbs$$

Joint Types of Fig. UW–20.1 Sketch (c):

UW–20.4(a), the size of a full strength weld shall be determined in accordance with UW–20.6.

UW–20.6 Weld Size Design Formulas – The size of tube–to–tubesheet strength welds shown in Fig. UW–20.1 shall conform to the following requirements.

UW–20.6(c), for combined groove and fillet welds shown in sketch (c), where $a_f = a_g$, the minimum required weld lengths are calculated as follows.

$$a_r = 2\left(\sqrt{\left(0.75d_o\right)^2 + 1.07t\left(d_o - t\right)f_w f_d} - 0.75d_o\right)$$

$$a_r = 2\left(\sqrt{\left(0.75\left(1.0\right)\right)^2 + 1.07\left(0.065\right)\left(1.0 - 0.065\right)\left(1.1385\right)\left(1.0\right)} - 0.75\left(1.0\right)\right)$$

$$a_r = 0.0957\ in$$

UW–20.6(c)(1), for full strength welds, the length of the combined weld legs measured parallel to the longitudinal axis of the tube at its outside diameter, a_c shall not be less than the greater of a_r or t.

$$a_c \ge \left\{\max\left[a_r,\, t\right] = \max\left[0.0957,\, 0.065\right] = 0.0957\ in\right\}$$

Therefore, the fillet weld and groove weld lengths are calculated as follows.

$$a_f = a_g = \frac{a_c}{2} = \frac{0.0957}{2} = 0.0479\ in$$

UW–20.4(b), the maximum allowable axial load in either direction on a tube-to-tubesheet joint with a full strength weld shall be determined as follows.

UW-20.4(b)(1), for loads due to pressure-induced axial forces,

$$L_{max} = F_t = \pi t\left(d_o - t\right)S_a = \pi\left(0.065\right)\left(1.0 - 0.065\right)\left(7400\right) = 1412.9\ lbs$$

UW-20.4(b)(2), for loads due to thermally-induced or pressure plus thermally-induced axial forces,

UW-20.4(b)(2)(a), $L_{max} = F_t$ for welded only tube-to-tubesheet joints where the thickness through the weld throat is less than the nominal tube thickness.

For a fillet weld, the weld throat is calculated as follows.

$$throat_f = 0.707a_f = 0.707\left(0.0479\right) = 0.0339\ in$$

For a groove weld, the weld throat is the length of the weld,

$$throat_g = a_g = 0.0479\ in$$

The combined fillet and groove weld throat is calculated as follows.

$$throat = throat_f + throat_g = 0.0339 + 0.0479 = 0.0818\ in$$

$$\left\{throat = 0.0818\ in\right\} > \left\{t = 0.065\ in\right\}$$

Therefore, UW-20.4(b)(2)(a) is not applicable.

UW-20.4(b)(2)(b), $L_{max} = 2F_t$ for all other welded tube-tubesheet joints.

$$L_{max} = 2F_t = 2\left(1412.9\right) = 2825.8\ lbs$$

Joint Types of Fig. UW–20.1 Sketch (d):

UW–20.4(a), the size of a full strength weld shall be determined in accordance with UW–20.6.

UW–20.6 Weld Size Design Formulas – The size of tube–to–tubesheet strength welds shown in Fig. UW–20.1 shall conform to the following requirements.

UW–20.6(d), for combined groove and fillet welds shown in sketch (d), where $a_f \neq a_g$, the minimum required fillet weld length is calculated as follows.

This procedure is iterative in that the calculation requires the designer to select an initial groove weld length, a_g, in order to determine the minimum required fillet weld length, a_f. If the combined fillet weld and groove weld length, a_c, is not adequate, then a new value of a_g is selected and the procedure is repeated.

With an initial value of $a_g = 0.0300\ in$, calculate the following parameters.

Axial tube strength, F_t:

$$F_t = \pi t\left(d_o - t\right)S_a = \pi\left(0.065\right)\left(1.0 - 0.065\right)\left(7400\right) = 1412.9\ lbs$$

Groove weld strength, F_g:

$$F_g = \min\left[\left\{0.85\pi a_g\left(d_o + 0.67a_g\right)S_w\right\},\ F_t\right]$$
$$F_g = \min\left[\left\{0.85\pi\left(0.0300\right)\left(1.0 + 0.67\left(0.0300\right)\right)6500\right\},\ 1412.9\right] = 531.2\ lbs$$

Ratio of the fillet weld strength to the design strength, f_f:

$$f_f = 1 - \frac{F_g}{\left(f_d F_t\right)} = 1 - \left(\frac{531.2}{\left(1.0\right)\left(1412.9\right)}\right) = 0.6240$$

The minimum required fillet weld length is calculated as follows

$$a_r = \sqrt{\left(0.75d_o\right)^2 + 2.73t\left(d_o - t\right)f_w f_d f_f} - 0.75d_o$$
$$a_r = \sqrt{\left(0.75\left(1.0\right)\right)^2 + 2.73\left(0.065\right)\left(1.0 - 0.065\right)\left(1.1385\right)\left(1.0\right)\left(0.6240\right)} - 0.75\left(1.0\right)$$
$$a_r = 0.0748\ in$$

UW–20.6(d)(1), for full strength welds, the length of the combined weld legs measured parallel to the longitudinal axis of the tube at its outside diameter, a_c shall not be less than the greater of $\left(a_r + a_g\right)$ or t.

$$a_c \geq \left\{\max\left[\left(a_r + a_g\right),\ t\right] = \max\left[\left(0.0748 + 0.0300\right),\ 0.065\right] = 0.1048\ in\right\}$$

Therefore, the fillet weld length is calculated as follows.

$$a_f = a_c - a_g = 0.1048 - 0.0300 = 0.0748\ in$$

UW–20.4(b), the maximum allowable axial load in either direction on a tube-to-tubesheet joint with a full strength weld shall be determined as follows.

UW-20.4(b)(1), for loads due to pressure-induced axial forces,

$$L_{max} = F_t = 1412.9\ lbs$$

UW-20.4(b)(2), for loads due to thermally-induced or pressure plus thermally-induced axial forces,

UW-20.4(b)(2)(a), $L_{max} = F_t$ for welded only tube-to-tubesheet joints where the thickness through the weld throat is less than the nominal tube thickness.

For a fillet weld, the weld throat is calculated as follows.

$$throat_f = 0.707a_f = 0.707(0.0748) = 0.0529\ in$$

For a groove weld, the weld throat is the length of the weld,

$$throat_g = a_g = 0.0300\ in$$

The combined fillet and groove weld throat is calculated as follows.

$$throat = throat_f + throat_g = 0.0529 + 0.0300 = 0.0829\ in$$

$$\{throat = 0.0829\ in\} > \{t = 0.065\ in\}$$

Therefore, UW-20.4(b)(2)(a) is not applicable.

UW-20.4(b)(2)(b), $L_{max} = 2F_t$ for all other welded tube-tubesheet joints.

$$L_{max} = 2F_t = 2(1412.9) = 2825.8\ lbs$$

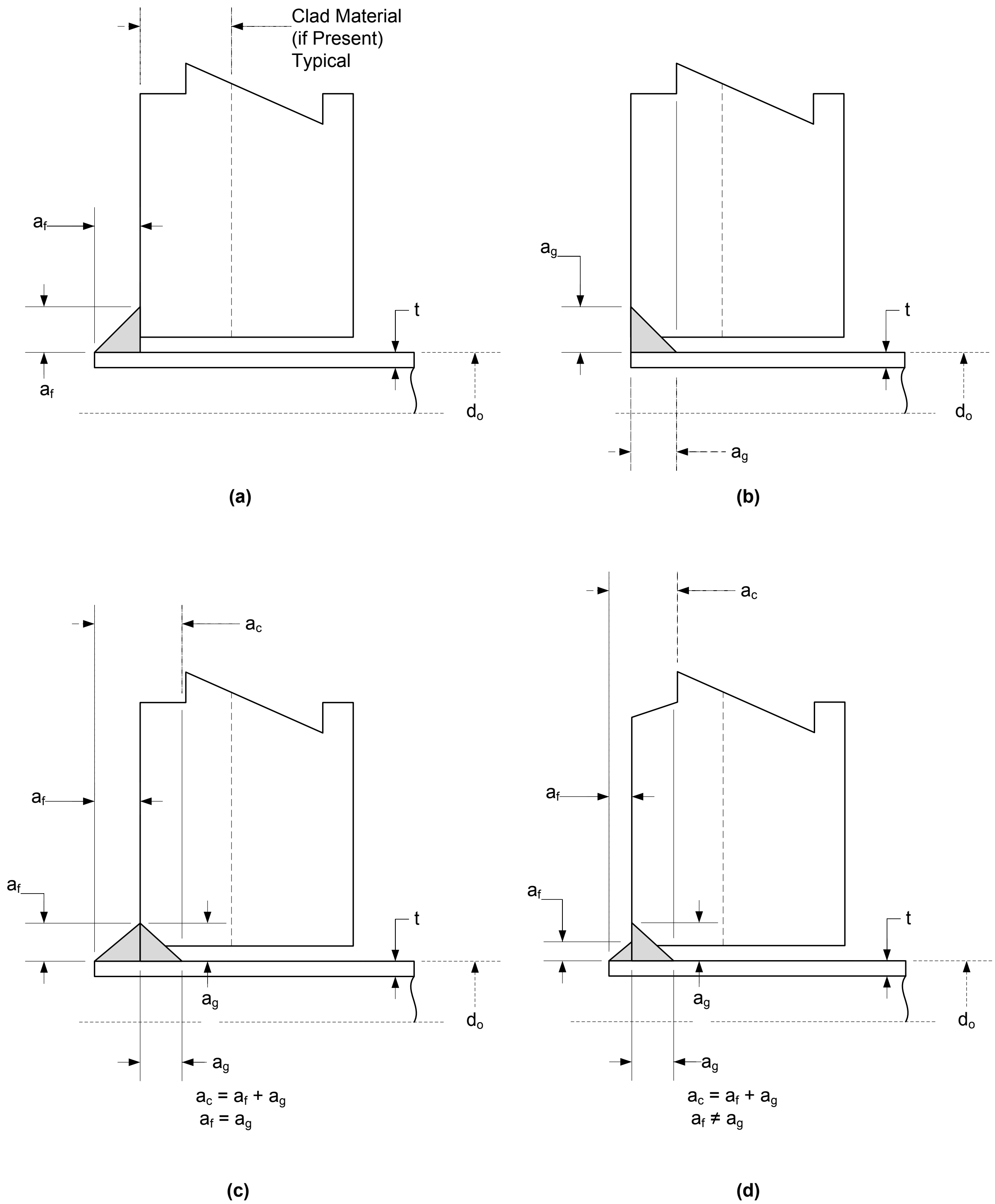


Figure E4.20.1 – Tube-to-Tubesheet Welds

4.20.2 Example E4.20.2 – Partial Strength Welds

Determine the size and allowable axial load of partial strength tube-to-tubesheet welds for each of the joint types shown in Fig. UW-20.1 (see Figure E4.20.1).

<u>Design Data:</u>

- Design Temperature = $600°F$
- Tube Outside Diameter = $1.0\ in$
- Tube Thickness = $0.065\ in$
- Tube Material = $SB-338,\ Grade\ 3$
- Tubesheet Material = $SB-265,\ Grade\ 2$
- Corrosion Allowance = $0.0\ in$
- Tube Allowable Stress = $7400\ psi$
- Tubesheet Allowable Stress = $6500\ psi$

Section VIII, Division 1 Solution Design rules for tube-to-tubesheet welds are provided in UW-20. The rules in this paragraph are the same as those provided in VIII-2, paragraph 4.18.10.

Evaluate per UW-20.

UW-20.2 Definitions – UW-20.2(b) Partial Strength Welds: A partial strength tube-to-tubesheet weld is one in which the design strength is based on the mechanical and thermal axial tube loads (in either direction) that are determined from the actual design conditions. The maximum allowable axial load of this weld may be determined in accordance with UW-20.5, Appendix A, or UW-18(d). When the weld in a tube-to-tubesheet joint meets the requirements of UW-20.5 or UW-18(d), it is a partial strength weld and the joint does not require qualification by shear load testing. Such a weld also provides tube joint leak tightness.

Based on the allowable stresses of the tubesheet and tubes and the determination that partial strength welds are to be evaluated, the following parameters are determined from UW-20.3.

Allowable Stress of the welds:

$$S_w = \min\left[S_a,\ S_t\right] = \min\left[7400,\ 6500\right] = 6500\ psi$$

Weld Strength Factor:

$$f_w = \frac{S_a}{S_w} = \frac{7400}{6500} = 1.1385$$

Axial tube Strength, F_t:

$$F_t = \pi t\left(d_o - t\right)S_a = \pi\left(0.065\right)\left(1.0 - 0.065\right)\left(7400\right) = 1412.9\ lbs$$

Design Strength, F_d: Based on anticipated mechanical and thermal axial tube loads,

$$F_d = \min\left[design\ strength,\ F_t\right] = \min\left[800.0,\ 1412.9\right] = 800.0\ lbs$$

Ratio of Design Strength to Tube Strength:

$$f_d = \frac{F_d}{F_t} = \frac{800.0}{1412.9} = 0.5662$$

<u>Joint Types of Fig. UW–20.1 Sketch (a):</u>

UW–20.5(a), the size of a partial strength weld shall be determined in accordance with UW–20.6.

UW–20.6 Weld Size Design Formulas – The size of tube–to–tubesheet strength welds shown in Fig. UW–20.1 shall conform to the following requirements.

UW–20.6(a), for fillet welds shown in sketch (a), the minimum required weld length is calculated as follows.

$$a_r = \sqrt{(0.75d_o)^2 + 2.73t(d_o - t)f_w f_d} - 0.75d_o$$

$$a_r = \sqrt{(0.75(1.0))^2 + 2.73(0.065)(1.0 - 0.065)(1.1385)(0.5662)} - 0.75(1.0)$$

$$a_r = 0.0682\ in$$

UW–20.6(a)(2), for partial strength welds, the fillet weld leg, a_f shall not be less than a_r.

$$a_f \geq \{a_r = 0.0682\ in\}$$

UW–20.5(b), the maximum allowable axial load in either direction on a tube-to-tubesheet joint with a partial strength weld shall be determined as follows.

UW-20.5(b)(1), for loads due to pressure-induced axial forces,

$$L_{max} = \min\left[\left(F_f + F_g\right),\ F_t\right] = \min\left[(800.0 + 0),\ 1412.9\right] = 800.0\ lbs$$

Where, F_f is the fillet weld strength and F_g is the groove weld strength,

$$F_g = 0.0\ lbs \qquad for\ no\ groove\ weld$$

$$F_f = \min\left[\left(F_d - F_g\right),\ F_t\right] = \min\left[(800.0 - 0.0),\ 1412.9\right] = 800.0\ lbs$$

UW-20.5(b)(2), for loads due to thermally-induced or pressure plus thermally-induced axial forces,

UW-20.5(b)(2)(a), $L_{max} = \min\left[\left(F_f + F_g\right),\ F_t\right]$ for welded only tube-to-tubesheet joints where the thickness through the weld throat is less than the nominal tube thickness.

For a fillet weld, the weld throat is calculated as follows.

$$throat = 0.707a_f = 0.707(0.0682) = 0.0482\ in$$

$$\{throat = 0.0482\ in\} < \{t = 0.065\ in\}$$

Therefore,

$$L_{max} = \min\left[\left(F_f + F_g\right),\ F_t\right] = \min\left[(800.0 + 0),\ 1412.9\right] = 800.0\ lbs$$

UW-20.5(b)(2)(b), $L_{max} = \min\left[2\left(F_f + F_g\right),\ 2F_t\right]$ for all other welded tube-tubesheet joints.

$$Not\ Required$$

Joint Types of Fig. UW–20.1 Sketch (b):

UW–20.5(a), the size of a partial strength weld shall be determined in accordance with UW–20.6.

UW–20.6 Weld Size Design Formulas – The size of tube–to–tubesheet strength welds shown in Fig. UW–20.1 shall conform to the following requirements.

UW–20.6(b), for groove welds shown in sketch (b), the minimum required weld length is calculated as follows.

$$a_r = \sqrt{\left(0.75d_o\right)^2 + 1.76t\left(d_o - t\right)f_w f_d} - 0.75d_o$$

$$a_r = \sqrt{\left(0.75(1.0)\right)^2 + 1.76(0.065)(1.0 - 0.065)(1.1385)(0.5662)} - 0.75(1.0)$$

$$a_r = 0.0446\ in$$

UW–20.6(b)(1), for partial strength welds, the groove weld leg, a_g shall not be less than a_r

$$a_g \geq \left\{a_r = 0.0446\ in\right\}$$

UW–20.5(b), the maximum allowable axial load in either direction on a tube-to-tubesheet joint with a partial strength weld shall be determined as follows.

UW-20.5(b)(1), for loads due to pressure-induced axial forces,

$$L_{max} = \min\left[\left(F_f + F_g\right),\ F_t\right] = \min\left[\left(0.0 + 797.3\right),\ 1412.9\right] = 797.3\ lbs$$

Where, F_f is the fillet weld strength and F_g is the groove weld strength,

$$F_g = \min\left[\left\{0.85\pi a_g\left(d_o + 0.67a_g\right)S_w\right\},\ F_t\right]$$

$$F_g = \min\left[\left\{0.85\pi(0.0446)\left(1.0 + 0.67(0.0446)\right)6500\right\},\ 1412.9\right] = 797.3\ lbs$$

$$F_f = 0.0\ lbs \qquad for\ no\ fillet\ weld$$

UW-20.5(b)(2), for loads due to thermally-induced or pressure plus thermally-induced axial forces,

UW-20.5(b)(2)(a), $L_{max} = \min\left[\left(F_f + F_g\right),\ F_t\right]$ for welded only tube-to-tubesheet joints where the thickness through the weld throat is less than the nominal tube thickness.

For a groove weld, the weld throat is the length of the weld, i.e.,

$$throat = a_g = 0.0446\ in$$

$$\left\{throat = 0.0446\ in\right\} < \left\{t = 0.065\ in\right\}$$

Therefore,

$$L_{max} = \min\left[\left(F_f + F_g\right),\ F_t\right] = \min\left[\left(0.0 + 797.3\right),\ 1412.9\right] = 797.3\ lbs$$

UW-20.5(b)(2)(b), $L_{max} = \min\left[2\left(F_f + F_g\right),\ 2F_t\right]$ for all other welded tube-tubesheet joints.

$$Not\ Required$$

<u>Joint Types of Fig. UW–20.1 Sketch (c):</u>

UW–20.5(a), the size of a partial strength weld shall be determined in accordance with UW–20.6.

UW–20.6 Weld Size Design Formulas – The size of tube–to–tubesheet strength welds shown in Fig. UW–20.1 shall conform to the following requirements.

UW–20.6(c), for combined groove and fillet welds shown in sketch (c), where $a_f = a_g$, the minimum required weld lengths are calculated as follows.

$$a_r = 2\left(\sqrt{\left(0.75d_o\right)^2 + 1.07t\left(d_o - t\right)f_w f_d} - 0.75d_o\right)$$

$$a_r = 2\left(\sqrt{\left(0.75\left(1.0\right)\right)^2 + 1.07\left(0.065\right)\left(1.0 - 0.065\right)\left(1.1385\right)\left(0.5662\right)} - 0.75\left(1.0\right)\right)$$

$$a_r = 0.0549\ in$$

UW–20.6(c)(1), for partial strength welds, the length of the combined weld legs measured parallel to the longitudinal axis of the tube at its outside diameter, a_c shall not be less than a_r.

$$a_c \geq \left\{a_r = 0.0549\ in\right\}$$

Therefore, the fillet weld and groove weld lengths are calculated as follows.

$$a_f = a_g = \frac{a_c}{2} = \frac{0.0549}{2} = 0.0275\ in$$

UW–20.5(b), the maximum allowable axial load in either direction on a tube-to-tubesheet joint with a full strength weld shall be determined as follows.

UW-20.5(b)(1), for loads due to pressure-induced axial forces,

$$L_{max} = \min\left[\left(F_f + F_g\right),\ F_t\right] = \min\left[\left(313.9 + 486.1\right),\ 1412.9\right] = 800.0\ lbs$$

Where, F_f is the fillet weld strength and F_g is the groove weld strength,

$$F_g = \min\left[\left\{0.85\pi a_g\left(d_o + 0.67a_g\right)S_w\right\},\ F_t\right]$$

$$F_g = \min\left[\left\{0.85\pi\left(0.0275\right)\left(1.0 + 0.67\left(0.0275\right)\right)6500\right\},\ 1412.9\right] = 486.1\ lbs$$

$$F_f = \min\left[\left(F_d - F_g\right),\ F_t\right] = \min\left[\left(800.0 - 486.1\right),\ 1412.9\right] = 313.9\ lbs$$

UW-20.5(b)(2), for loads due to thermally-induced or pressure plus thermally-induced axial forces,

UW-20.5(b)(2)(a), $L_{max} = \min\left[\left(F_f + F_g\right), F_t\right]$ for welded only tube-to-tubesheet joints where the thickness through the weld throat is less than the nominal tube thickness.

For a fillet weld, the weld throat is calculated as follows.

$$throat_f = 0.707 a_f = 0.707(0.0275) = 0.0194 \ in$$

For a groove weld, the weld throat is the length of the weld, i.e.,

$$throat_g = a_g = 0.0275 \ in$$

The combined fillet and groove weld throat is calculated as follows.

$$throat = throat_f + throat_g = 0.0194 + 0.0275 = 0.0469 \ in$$

$$\{throat = 0.0469 \ in\} < \{t = 0.065 \ in\}$$

Therefore,

$$L_{max} = \min\left[\left(F_f + F_g\right), F_t\right] = \min\left[(313.9 + 486.1), 1412.9\right] = 800.0 \ lbs$$

UW-20.5(b)(2)(b), $L_{max} = \min\left[2\left(F_f + F_g\right), 2F_t\right]$ for all other welded tube-tubesheet joints.

Not Required

Joint Types of Fig. UW–20.1 Sketch (d):

UW–20.5(a), the size of a full strength weld shall be determined in accordance with UW–20.6.

UW–20.6 Weld Size Design Formulas – The size of tube–to–tubesheet strength welds shown in Fig. UW–20.1 shall conform to the following requirements.

UW–20.6(d), for combined groove and fillet welds shown in sketch (d), where $a_f \neq a_g$, the minimum required fillet weld length is calculated as follows.

This procedure is iterative in that the calculation requires the designer to select an initial groove weld length, a_g, in order to determine the minimum required fillet weld length, a_f. If the combined fillet weld and groove weld length, a_c, is not adequate, then a new value of a_g is selected and the procedure is repeated.

With an initial value of $a_g = 0.0300 \ in$, calculate the following parameters.

Axial tube strength, F_t:

$$F_t = \pi t\left(d_o - t\right)S_a = \pi(0.065)(1.0 - 0.065)(7400) = 1412.9 \ lbs$$

Groove weld strength, F_g:

$$F_g = \min\left[\left\{0.85\pi a_g\left(d_o + 0.67a_g\right)S_w\right\},\ F_t\right]$$

$$F_g = \min\left[\left\{0.85\pi(0.0300)\left(1.0 + 0.67(0.0300)\right)6500\right\},\ 1412.9\right] = 531.2\ lbs$$

Ratio of the fillet weld strength to the design strength, f_f :

$$f_f = 1 - \frac{F_g}{\left(f_d F_t\right)} = 1 - \left(\frac{531.2}{(0.5662)(1412.9)}\right) = 0.3360$$

The minimum required fillet weld length is calculated as follows

$$a_r = \sqrt{\left(0.75d_o\right)^2 + 2.73t\left(d_o - t\right)f_w f_d f_f} - 0.75d_o$$

$$a_r = \sqrt{\left(0.75(1.0)\right)^2 + 2.73(0.065)(1.0 - 0.065)(1.1385)(0.5662)(0.3360)} - 0.75(1.0)$$

$$a_r = 0.0236\ in$$

UW–20.6(d)(1), for partial strength welds, the length of the combined weld legs measured parallel to the longitudinal axis of the tube at its outside diameter, a_c shall not be less than $\left(a_r + a_g\right)$.

$$a_c \geq \left\{\left(a_r + a_g\right) = (0.0236 + 0.0300) = 0.0536\ in\right\}$$

Therefore, the fillet weld length is calculated as follows.

$$a_f = a_c - a_g = 0.0536 - 0.0300 = 0.0236\ in$$

UW–20.5(b), the maximum allowable axial load in either direction on a tube-to-tubesheet joint with a full strength weld shall be determined as follows.

UW-20.5(b)(1), for loads due to pressure-induced axial forces,

$$L_{max} = \min\left[\left(F_f + F_g\right),\ F_t\right] = \min\left[(268.8 + 531.2),\ 1412.9\right] = 800.0\ lbs$$

Where,

$$F_g = \min\left[\left\{0.85\pi a_g\left(d_o + 0.67a_g\right)S_w\right\},\ F_t\right]$$

$$F_g = \min\left[\left\{0.85\pi(0.0300)\left(1.0 + 0.67(0.0300)\right)6500\right\},\ 1412.9\right] = 531.2\ lbs$$

$$F_f = \min\left[\left(F_d - F_g\right),\ F_t\right] = \min\left[(800.0 - 531.2),\ 1412.9\right] = 268.8\ lbs$$

UW-20.5(b)(2), for loads due to thermally-induced or pressure plus thermally-induced axial forces, UW-20.5(b)(2)(a), $L_{max} = \min\left[\left(F_f + F_g\right), F_t\right]$ for welded only tube-to-tubesheet joints where the thickness through the weld throat is less than the nominal tube thickness.

For a fillet weld, the weld throat is calculated as follows.

$$throat_f = 0.707a_f = 0.707(0.0236) = 0.0167\ in$$

For a groove weld, the weld throat is the length of the weld, i.e.,

$$throat_g = a_g = 0.0300\ in$$

The combined fillet and groove weld throat is calculated as follows.

$$throat = throat_f + throat_g = 0.0167 + 0.0300 = 0.0467\ in$$

$$\{throat = 0.0467\ in\} < \{t = 0.065\ in\}$$

Therefore,

$$L_{max} = \min\left[\left(F_f + F_g\right), F_t\right] = \min\left[(268.8 + 531.2), 1412.9\right] = 800.0\ lbs$$

UW-20.5(b)(2)(b), $L_{max} = \min\left[2\left(F_f + F_g\right), 2F_t\right]$ for all other welded tube-tubesheet joints.

$$Not\ Required$$

4.21 Nameplates

4.21.1 Example E4.21.1 – Single Chamber Pressure Vessel

A Manufacturer has constructed a single chamber pressure vessel with the details as shown below. The required markings and certification of pressure vessels built to Section VIII Division 1 are provided in UG-115 through UG-120. For the vessel data provided, the applicable markings are shown in Figure E4.21.1.a and Figure E4.21.1.b.

Vessel Data:

- Maximum Allowable Working Pressure = $150\ psi\ and\ Full\ Vacuum @ 300°F$
- Minimum Design Metal Temperature = $-20°F\ @\ MAWP$
- Manufacturer = $ACME\ Manufacturing$
- Manufacturer's Serial Number = 135642
- Year Built = 2013
- Type of Construction = $Arc/Gas\ Welded$
- Special Service = $None$
- Radiographic Examination = $Full\ Radiography, UW-11(a)$
- Post Weld Heat Treatment = Yes
- Inspection = $UG-90\ through\ UG-97$

Section VIII, Division 1 Solution

Rule for the required markings for pressure vessels are provided in UG-116; methods of markings are provided in UG-118; and nameplate requirements are provided in UG-119.

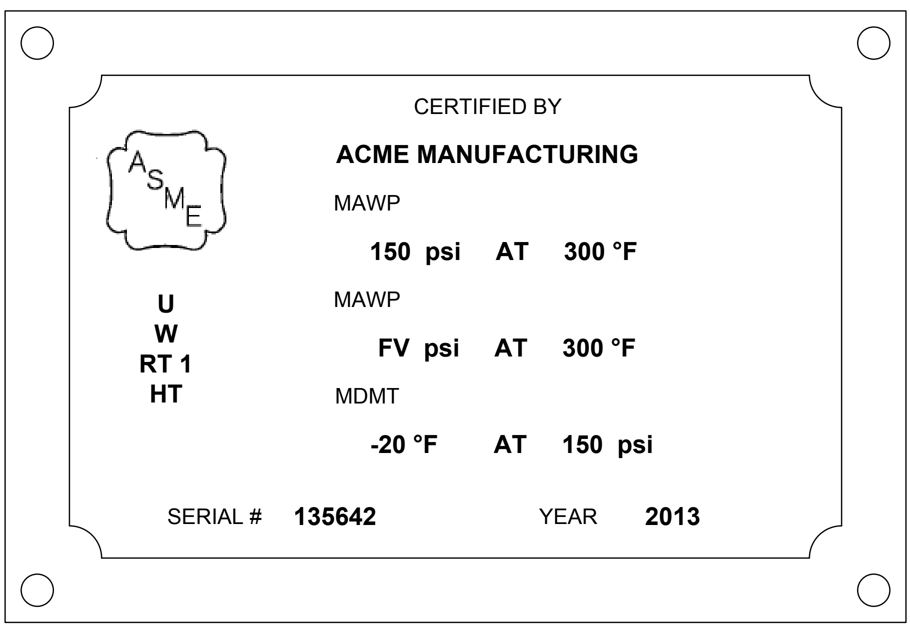


Figure E4.21.1.a – Nameplate Detail

Alternatively, since the MAWP's (internal and external) have the same designated coincident temperature, then these values can be listed on the same lines of the nameplate.

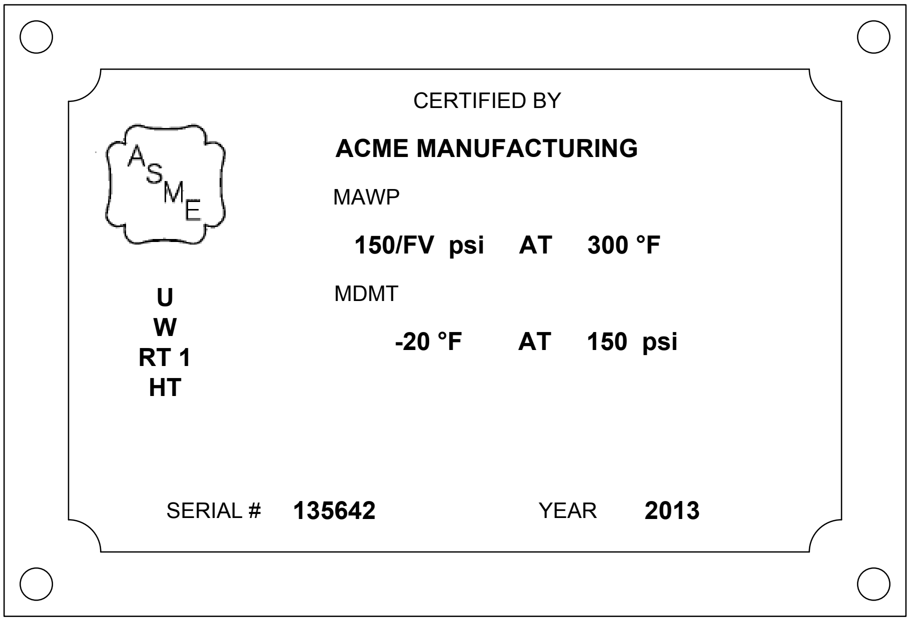


Figure E4.21.1.b – Nameplate Detail

4.21.2 Example E4.21.2 – Single Chamber Pressure Vessel

A Manufacturer has constructed a single chamber pressure vessel with the details as shown below. The required markings and certification of pressure vessels built to Section VIII Division 1 are provided in UG-115 through UG-120. For the vessel data provided, the applicable markings are shown in Figure E4.21.2.

<u>Vessel Data:</u>

- Maximum Allowable Working Pressure = $150\ psi\ @\ 300°F\ (internal)$
- Maximum Allowable Working Pressure = $-12\ psi\ @\ 100°F\ (external)$
- Minimum Design Metal Temperature = $-20°F\ @\ MAWP$
- Manufacturer = $ACME\ Manufacturing$
- Manufacturer's Serial Number = 135642
- Year Built = 2013
- Type of Construction = $Arc / Gas\ Welded$
- Special Service = $None$
- Radiographic Examination = $\begin{Bmatrix} Full\ Radiography: \\ UW-11(a)(5),\ UW-11(a)(5)(b) \end{Bmatrix}$
- Post Weld Heat Treatment = $None$
- Inspection = $UG-90\ through\ UG-97$

<u>Section VIII, Division 1 Solution</u>

Rule for the required markings for pressure vessels are provided in UG-116; methods of markings are provided in UG-118; and nameplate requirements are provided in UG-119.

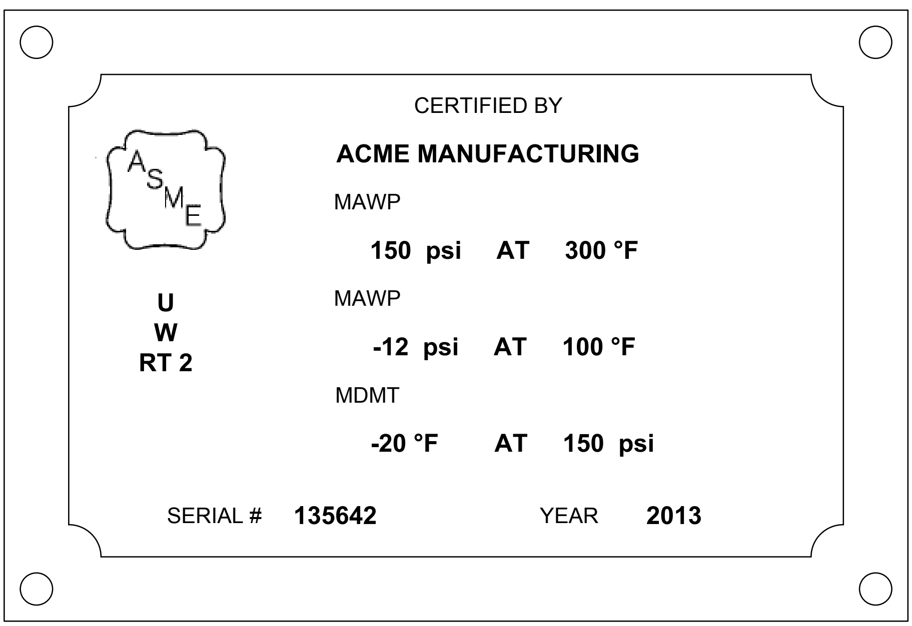


Figure E4.21.2 – Nameplate Detail

4.21.3 Example E4.21.3 – Shell and Tube Heat Exchanger

A Manufacturer has constructed a shell and tube heat exchanger utilizing fixed tubesheets, operating as independent chambers, with the details as shown below. The required markings and certification of pressure vessels built to Section VIII Division 1 are provided in UG-115 through UG-120 and UHX-19. For the vessel data provided, the applicable markings are shown in Figure E4.21.3.

Vessel Data:

- Shell Side MAWP = $300/FV\ \ psi\ @\ 500°F$
- Tube Side MAWP = $150\ \ psi\ @\ 300°F$
- Shell Side MDMT = $-20°F\ @\ MAWP$
- Tube Side MDMT = $-20°F\ @\ MAWP$
- Manufacturer = $ACME\ \ Manufacturing$
- Manufacturer's Serial Number = 135642
- Year Built = 2013
- Type of Construction = $Arc/Gas\ Welded$
- Special Service = $None$
- Radiographic Examination = $Full\ \ Radiography: UW-11(a)$
- Post Weld Heat Treatment = $None$
- Inspection = $UG-90\ through\ UG-97$

Section VIII, Division 1 Solution

Rule for the required markings for pressure vessels are provided in UG-116 and UHX-19; methods of markings are provided in UG-118; and nameplate requirements are provided in UG-119.

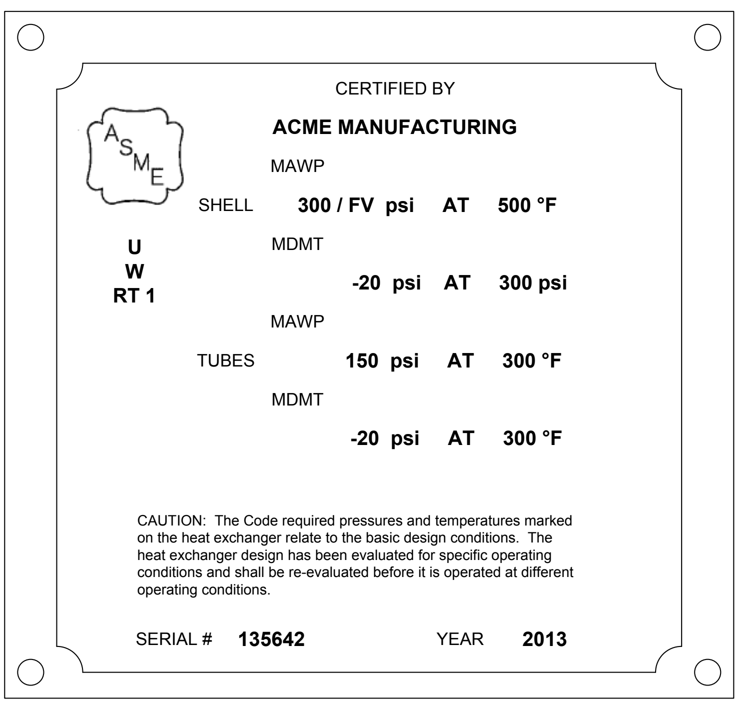


Figure E4.21.3 – Nameplate Detail

PART 5

DESIGN BY ANALYSIS REQUIREMENTS

5.1 Design-By-Analysis for Section VIII, Division 1

Section VIII, Division 1 does not explicitly incorporate design-by-analysis requirements. However, design by analysis is often used in the design of VIII-1 vessels. Specific examples include:

a) Stress analysis of nozzles subject to externally applied forces and moments – the user is warned in footnote 22 for Openings and Reinforcements that design rules for external loadings such as those due to thermal expansion or unsupported weight of connected piping are not provided and that attention is required in unusual designs or under the conditions of cyclic loadings. In practice, most user specifications require evaluation of externally applied loads. Localized stresses at nozzle locations due to externally applied loads may be evaluated using one of the methods shown below. For each method, the acceptance criteria in VIII-2, Part 5 may be use.
    1) Nozzles in cylindrical shells – stress calculations shall be in accordance with WRC 107, WRC 297, or WRC 497.
    2) Nozzles in formed heads – stress calculations shall be in accordance with WRC 107.
    3) For all configurations, the stress calculations may be performed using a numerical analysis such as the finite element method.
b) Fatigue analysis – UG-22(e) stipulates that loadings to be considered in the design of a vessel shall include cyclic and dynamic reactions due to pressure or thermal variations, equipment mounted on the vessel, and mechanical loadings. The time dependency implied in this requirement indicates that a fatigue analysis must be considered in the design. Rules for fatigue screening to determine if a fatigue analysis is required are given in VIII-2. If a fatigue analysis is required, fatigue rules are also provided in VIII-2, Part 5.
c) Flanged-and-Flued or Flanged-Only Expansion Joints – The design rules in Mandatory Appendix 5 essentially stipulate a design-by-analysis. An acceptance criterion is provided in VIII-1, 5-3; however, limited advice is provided for the stress analysis procedure, see 5-3(b). VIII-2, Part 5 may be used to establish guidelines for the stress calculation.

5.2 Paragraph U-2(g) – Design-By-Analysis Provision without Procedures

Paragraph U-2(g) states that VIII-1 does not contain rules to cover all details of design and construction. When complete details are not given in VIII-1, it is intended that the Manufacturer, subject to the acceptance of the Inspector, shall provide details of design and construction which will be as safe as those provided by the rules of VIII-1. Paragraph U-2(g) essentially permits the engineer to design components in the absence of rules in VIII-1 with the criterion that the design margin in VIII-1 be maintained. As discussed below, this requirement may be satisfied by using VIII-2, Part 4 or Part 5, as applicable, in conjunction with the allowable stress and weld joint efficiency from VIII-1.

The requirement "be as safe as" is difficult to quantify because the actual design margin for a particular failure mode is not provided in VIII-1. Design rules in construction codes are typically based on: analytical derivations, test results, and experience based on evidence of satisfactory performance. In addition, when design rules are developed, ease of use is balanced with required accuracy.

- Accuracy – Effort and time to implement may be prohibitive.
- Ease of use – Over conservatism may make assessment rules useless or expense.

Design margins in construction codes for pressurized equipment are typically set to prevent ductile rupture or plastic collapse. For example, in VIII-1 a design margin is used to determine the allowable stress. For time-independent behavior this margin is the minimum of the ultimate strength divided by 3.5 and the yield strength divided by 1.5; the ultimate and yield strength at room temperature and at the design temperature are considered in establishing this margin. The factors, such as 3.5 applied to the ultimate strength and 1.5 applied to the yield strength, represent the design margin to prevent ductile rupture or plastic collapse. However, to ensure "safe operation", other the failure modes such as those shown below are addressed in VIII-1 directly or indirectly by rules.

- Brittle Fracture
- Local Strain
- Structural Instability (elastic or plastic buckling)
- Fatigue
- Incremental Collapse (ratcheting)
- Creep Rupture
- Creep Buckling
- Creep-Fatigue Interaction

Design margins exist in these rules as well as the calculation procedures incorporated into VIII-1. Many of these rules are made conservative for ease of application, are based on test results with variability, or are service experience based. Since the rules are used in conjunction with an allowable stress (that already contains a margin), the exact design margin for a component is difficult to establish. Therefore, design margins may be considered as follows:

- Explicit Design Margins – Margins that can be directly identified, e.g. margins placed on material strength parameters, and
- Implicit Design Margins – Margins that result from design rule conservatism for ease of application, and from design margin requirements imposed because of test results and service experience

Many of the design-by-rule requirements in VIII-2, Part 4 represent a next generation of those rules in VIII-1, see Table E1.1 in Part 1 of this document, and many of the implicit margins in VIII-1 design procedures have been maintained in VIII-2. In addition, the design-by-analysis provisions in VIII-2, Part 5 represent a next generation to the design rules in VIII-2 up to the 2004 Edition, 2006 Addenda. Therefore if the VIII-1 explicit margins on the strength parameters, i.e. the allowable stresses from VIII-1, are used in conjunction with the design-by-rule or design-by-analysis procedures in VIII-2, then vessels constructed in accordance with this practice are judged to be "be as safe as" those constructed to VIII-1. To support this notion, VIII-1 Code Case 2695 permits the use of VIII-2 design rules with VIII-1 allowable stresses with some limitations.

Some of the paragraphs in VIII-1 where U-2(g) is invoked are shown below. In all instances, the design-by-rule or design-by-analysis procedures in VIII-2, Part 4 and Part 5, respectively, in conjunction with the allowable stresses and weld joint efficiency in accordance with VIII-1 may be used to qualify a component to meet the requirements of U-2(g).

a) UG-19, special shapes, see UG-19(f).
b) UG-29, cautionary note cover lateral buckling of stiffening rings.
c) UG-34, Footnote 20 covering special consideration that shall be given to the design of shells, nozzle necks, or flanges to which noncircular head are attached.
d) UG-36, reverse curve reducers, see UG-36(e)(5)(c).
e) UG-39, reinforcement required for openings in flat heads, see UG-39(c)(3)
f) Mandatory Appendix 1-5 covering rules for conical reducer sections and conical heads under internal pressure see 1-5(g).
g) Mandatory Appendix 1-6 covering dished covers, see 1-6(h).
h) Mandatory Appendix 1-7 covering large openings in cylindrical and conical shells, see 1-7(b)(1)(c).

i) Mandatory Appendix 1-8 covering rules for conical reducer sections and conical heads under external pressure see 1-8(b), 1-8(c), and 1-8(e).

j) Mandatory Appendix 1-10 covering the alternative design method for design of reinforcement for large openings in pressure vessels under internal pressure, see 1-10(d).

k) Mandatory Appendix 9 covering jacketed vessels, see 9-1(e).

l) Mandatory Appendix 13 covering vessels of noncircular cross section, see 13-1(c), 13-1(d).

m) Mandatory Appendix 24 covering design rules for clamps, see 24-1(e).

n) Mandatory Appendix 26 covering design rules for bellows expansion joints, see 26-4.1(f).

PART 6

FABRICATION REQUIREMENTS

6.1 Example E6.1 – Postweld Heat Treatment of a Pressure Vessel

Establish the postweld heat treatment (PWHT) requirements for a process tower considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516, Grade\ 70$
- Design Conditions = $1650\ psig @ 600°F$
- Liquid Head = $60\ ft$
- Liquid Specific Gravity = 0.89
- Inside Diameter = $96.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $19400\ psi$
- P Number and Group = $P-No.1, Group\ 2$
- Weld Joint Efficiency = 1.0
- Tangent-to-Tangent Vessel Length = $80\ ft$
- Top and Bottom Heads = $Hemispherical$

Evaluate the requirements of PWHT per paragraph UCS-56.

The design pressure used to establish the wall thickness for the bottom head and cylindrical shell section must be adjusted for the liquid head in accordance with paragraph UG-22.

Adjusted pressure for the cylindrical shell:

$$Design\ Pressure = Specified\ Design\ Pressure + \gamma gh$$

$$Design\ Pressure = 1650 + \frac{0.89(62.4)(56)}{144} = 1671.597\ psig$$

Adjusted pressure for the bottom hemispherical head:

$$Design\ Pressure = Specified\ Design\ Pressure + \gamma gh$$

$$Design\ Pressure = 1650 + \frac{0.89(62.4)(60)}{144} = 1673.140\ psig$$

Section VIII, Division 1 Solution

In accordance with paragraph UG-32(f), determine the required thickness of the top head.

$$L = \frac{D}{2} + Corrosion\ \ Allowance = \frac{96.0}{2} + 0.125 = 48.125\ in$$

$$t = \frac{PL}{2SE - 0.2P} = \frac{1650(48.125)}{2(19400)(1.0) - 0.2(1650)} = 2.0641\ in$$

$$t = 2.0641 + Corrosion\ \ Allowance = 2.0641 + 0.125 = 2.1891\ in$$

The required thickness of the top head is $2.1891\ in$.

In accordance with paragraph UG-32(f), determine the required thickness of the bottom head.

$$L = \frac{D}{2} + Corrosion\ \ Allowance = \frac{96.0}{2} + 0.125 = 48.125\ in$$

$$t = \frac{PL}{2SE - 0.2P} = \frac{1673.140(48.125)}{2(19400)(1.0) - 0.2(1673.140)} = 2.0933\ in$$

$$t = 2.0933 + Corrosion\ \ Allowance = 2.0933 + 0.125 = 2.2183\ in$$

The required thickness of the bottom head is $2.2183\ in$.

In accordance with paragraph UG-27(c)(1), determine the required thickness of the cylindrical shell.

$$R = \frac{D}{2} + Corrosion\ \ Allowance = \frac{96.0}{2} + 0.125 = 48.125\ in$$

$$t = \frac{PR}{SE - 0.6P} = \frac{1671.597(48.125)}{(19400)(1.0) - 0.6(1671.597)} = 4.3727\ in$$

$$t = 4.3727 + Corrosion\ \ Allowance = 4.3727 + 0.125 = 4.4977\ in$$

The required thickness of the cylindrical shell is $4.4977\ in$.

<u>Required Thickness Summary:</u>

Top Head = $2.1891\ in$

Bottom Head = $2.2183\ in$

Cylindrical Shell = $4.4977\ in$

The requirements for postweld heat treatment are found in paragraph UCS-56. Material specification SA-516, Grade 70 is a P-No. 1 Group No. 2 material. Therefore, the PWHT requirements are provided in Table UCS-56, Postweld Heat Treatment Requirements for Carbon and Low Alloy Steels. The definition of nominal thickness governing PWHT is provided in paragraph UW-40(f). For pressure vessels or parts of pressure vessels being postweld heat treated in a furnace charge, the nominal thickness is the greatest weld thickness in any vessel or vessel part which has not previously been postweld heat treated if the shell is fabricated from plate material. Therefore, the governing nominal thickness is that of the cylindrical shell, $4.4977\ in$.

The procedures for postweld heat treatment are found in paragraph UW-40. It is preferable that PWHT of the vessel be performed by heating the vessel as a whole in a closed furnace in accordance with paragraph UW-40(a)(1).

TABLE UCS-56
POSTWELD HEAT TREATMENT REQUIREMENTS FOR CARBON AND LOW ALLOY STEELS

Material	Normal Holding Temperature, °F (°C), Minimum	Minimum Holding Time at Normal Temperature for Nominal Thickness [See UW-40(f)]		
		Up to 2 in. (50 mm)	Over 2 in. to 5 in. (50 mm to 125 mm)	Over 5 in. (125 mm)
P-No. 1 Gr. Nos. 1, 2, 3	1,100 (595)	1 hr/in. (25 mm), 15 min minimum	2 hr plus 15 min for each additional inch (25 mm) over 2 in. (50 mm)	2 hr plus 15 min for each additional inch (25 mm) over 2 in. (50 mm)
Gr. No. 4	NA	None	None	None

NOTES:
(1) When it is impractical to postweld heat treat at the temperature specified in this Table, it is permissible to carry out the postweld heat treatment at lower temperatures for longer periods of time in accordance with Table UCS-56.1.
(2) Postweld heat treatment is mandatory under the following conditions:
(a) for welded joints over 1½ in. (38 mm) nominal thickness;
(b) for welded joints over 1¼ in. (32 mm) nominal thickness through 1½ in. (38 mm) nominal thickness unless preheat is applied at a minimum temperature of 200°F (95°C) during welding;
(c) for welded joints of all thicknesses if required by UW-2, except postweld heat treatment is not mandatory under the conditions specified below:
(1) for groove welds not over ½ in. (13 mm) size and fillet welds with a throat not over ½ in. (13 mm) that attach nozzle connections that have a finished inside diameter not greater than 2 in. (50 mm), provided the connections do not form ligaments that require an increase in shell or head thickness, and preheat to a minimum temperature of 200°F (95°C) is applied;
(2) for groove welds not over ½ in. (13 mm) in size or fillet welds with a throat thickness of ½ in. (13 mm) or less that attach tubes to a tubesheet when the tube diameter does not exceed 2 in. (50 mm). A preheat of 200°F (95°C) minimum must be applied when the carbon content of the tubesheet exceeds 0.22%.
(3) for groove welds not over ½ in. (13 mm) in size or fillet welds with a throat thickness of ½ in. (13 mm) or less used for attaching nonpressure parts to pressure parts provided preheat to a minimum temperature of 200°F (95°C) is applied when the thickness of the pressure part exceeds 1¼ in. (32 mm);
(4) for studs welded to pressure parts provided preheat to a minimum temperature of 200°F (95°C) is applied when the thickness of the pressure part exceeds 1¼ in. (32 mm);
(5) for corrosion resistant weld metal overlay cladding or for welds attaching corrosion resistant applied lining (see UCL-34) provided preheat to a minimum temperature of 200°F (95°C) is maintained during application of the first layer when the thickness of the pressure part exceeds 1¼ in. (32 mm).

NA = not applicable

Based on Table UCS-56, PWHT is mandatory due to the governing nominal thickness of $4.4977\ in$. The minimum holding temperature and time based on the nominal thickness within the range of $2\ in < t_n < 5\ in$ is 1100°F for 2 hours plus 15 minutes for each additional inch over 2 inches, respectively. For the vessel in question, the holding time is calculated as follows:

$$Holding\ time = 120\ min + \left(\frac{15\ min}{in}\right)\left(4.4977\ in - 2\ in\right) = 157\ min$$

The requirements for operation of PWHT are provided in paragraph UCS-56(d). The operation of postweld heat treatment shall be carried out by one of the procedures given in paragraph UW-40 in accordance with the following requirements.

a) When post weld heat treatment is performed in a furnace (see paragraph UW-40(a)(1)), the temperature of the furnace shall not exceed 800°F at the time the vessel or part is placed in it.
b) Above 800°F, the rate of heating shall be not more than 400°F/hr divided by the maximum metal thickness of the shell or head plate in inches, but in no case more than 400°F/hr, and in no case need it be less than 100°F/hr. During the heating period there shall not be a greater variation in temperature throughout the portion of the vessel being heated than 250°F within any 15 ft interval of length.

$$Maximum\ \ Heating\ \ Rate = \frac{400°F/hr}{4.4977\ in} = 89°F/hr$$

c) The vessel or vessel part shall be held at or above the temperature specified in Table UCS-56 for the period of time specified in this table. During the holding period, there shall not be a difference greater than 150°F between the highest and lowest temperatures throughout the portion of the vessel being heated, except where the range is further limited in Table UCS-56.

d) When post weld heat treatment is performed in a furnace (see paragraph UW-40(a)(1)), during the heating and holding periods, the furnace atmosphere shall be so controlled as to avoid excessive oxidation of the surface of the vessel. The furnace shall be of such design as to prevent direct impingement of the flame on the vessel.

e) Above 800°F, cooling shall be done at a rate not greater than 500°F/hr divided by the maximum metal thickness of the shell or head plate in inches, but in no case need it be less than 100°F/hr. From 800°F, the vessel may be cooled in still air.

$$Maximum\ \ Cooling\ \ Rate = \frac{500°F/hr}{4.4977\ in} = 111°F/hr$$

6.2 Example E6.2 – Out-of-Roundness of a Cylindrical Forged Vessel

A vessel is being fabricated using forged cylindrical shell segments. During fabrication, tolerances were checked and it was noted that out-of-roundness of one of the cylindrical shell segments is present that exceed tolerance limits specified in paragraph UF-27(a). In order to establish a plan of action, it was decided to use the provisions in paragraph UF-27(b) that permit a reduced permissible operating pressure be determined for cylindrical shells with general out-of-roundness characterized by a major and minor diameter. Establish the reduced permissible operating pressure requirements considering the following design conditions.

<u>Vessel Data:</u>

- Material = $SA-372,\ Grade\ C$
- Design Conditions = $2800\ \ psig@400°F$
- Inside Diameter = $112.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $25700\ \ psi$
- Weld Joint Efficiency = 1.0
- Modulus of Elasticity = $27.9E+06\ psi$

Evaluate the special requirements for forged fabrication per paragraph UF-27.

In accordance with paragraph UG-27(c)(1), determine the required thickness of the cylindrical shell.

$$R = \frac{D}{2} + Corrosion\ \ Allowance = \frac{112.0}{2} + 0.125 = 56.125\ in$$

$$t = \frac{PR}{SE-0.6P} = \frac{2800(56.125)}{(25700)(1.0)-0.6(2800)} = 6.5425\ in$$

$$t = 6.5425 + Corrosion\ \ Allowance = 6.5425 + 0.125 = 6.6675\ in$$

The required thickness of the cylindrical shell is $6.6675\ in$; therefore a forging with a wall thickness of $6.6875\ in$ will be used.

During fabrication of a section of the cylindrical shell with a nominal inside diameter of $112.0\ in$, the following tolerance readings were taken.

Maximum Inside Diameter = $113.0\ in$

Minimum Inside Diameter = $110.0\ in$

The shell tolerance limits, provided in paragraph UF-27(a), stipulate that the difference between the maximum and minimum inside diameters at any cross section shall not exceed 1% of the nominal diameter at the cross section under consideration.

$$\frac{Max\ Diameter - Min\ Diameter}{Nominal\ Diameter} = \frac{113.0 - 110.0}{112.0} = 2.7\%$$

In accordance with paragraph UF-27(b), if the out-of-roundness exceeds the limit in paragraph UF-27(a) and the condition cannot be corrected, then the forging shall be rejected, except that if the out-of-roundness does not exceed 3%, the forging may be certified for a reduced pressure, $P^{'}$, calculated using the following equations. The measurements used in these equations shall be corrected for the specified corrosion allowance.

With,

$$\left\{S_b = 13529.3\ psi\right\} \geq \left\{0.25S = 0.25(25700) = 6425\ psi\right\}$$

$$P^{'} = P\left(\frac{1.25}{S_b/S+1}\right) = 2800\left(\frac{1.25}{(13529.3/25700)+1}\right) = 2292.9\ psi$$

Where,

$$S_b = \frac{1.5PR_1t(D_1 - D_2)}{t^3 + 3\left(\frac{P}{E_y}\right)(R_1R_a^2)} = \frac{1.5(2800)(55.875)(6.5625)(113.25 - 110.25)}{(6.5625)^3 + 3\left(\frac{2800}{27.9E+06}\right)(55.875)(59.1563)^2}$$

$$S_b = 13529.3\ psi$$

And,

$$D_1 = 113.0 + 2(Corrosion\ Allowance) = 113.0 + 2(0.125) = 113.25\ in$$

$$D_2 = 110.0 + 2(Corrosion\ Allowance) = 110.0 + 2(0.125) = 110.25\ in$$

$$t = 6.6875 - Corrosion\ Allowance = 6.6875 - 0.125 = 6.5625\ in$$

$$R_1 = \frac{D_1 + D_2}{4} = \frac{113.25 + 110.25}{4} = 55.875\ in$$

$$R_a = R_1 + \frac{t}{2} = 55.875 + \frac{6.5625}{2} = 59.1563\ in$$

Therefore, with the current out-of-roundness in place, the maximum operating pressure of the vessel would be limited to $2292.9\ psi$, which represents a 18% reduction in pressure. If this reduction in pressure is unacceptable for the planned operation of the vessel, then the condition needs to be corrected. It is determined that the condition cannot be corrected, the forging shall be rejected and a new cylindrical forging shall be fabricated.

PART 7

INSPECTION AND EXAMINATION REQUIREMENTS

7.1 Inspection and Examination Rules Commentary

Introduction of Joint Efficiency, Welded Joint Category and Joint Types

In Subsection B of VIII-1, with particular reference to Part UW, Requirements for Pressure Vessels Fabricated by Welding, the concept of joint efficiencies is introduced. Specifically, in paragraph UW-12 with reference to Table UW-12, the joint efficiency, E, to be used in the equations of VIII-1 for joints completed by an arc or gas welding process is provided. With the exception of paragraph UW-11(a)(5), the joint efficiency depends only on the type of joint and on the degree of examination of the joint and does not depend on the degree of examination of any other joint. The types of welded joints, or Joint Types, are defined in Table UW-12 and are comprised of butt joints, fillet lap joints, corner joints, and angle joints. The Joint Types specified in VIII-1 and their definitions are summarized in Table 7.1.1.

Paragraph UW-3 introduces the term “Category” which defines the location of a joint in a vessel, but not the type of joint. The Categories are used for specifying special requirements regarding joint type and degree of inspection for certain welded pressure joints. The joints included in each category are designated as joints of Category A, B, C, and D. The typical joint locations included in each category are shown in Figure 7.1.1 (Fig, UW-3). Each of the categories specified in VIII-1 and their definitions are summarized in Table 7.1.2.

Paragraph UW-2 provides requirements on the Category and Joint Type of welds based upon service restrictions including; lethal service, operation below certain temperatures or impact tests of the material or weld metal, unfired steam boilers with design pressures exceeding 50 psi, and pressure vessels or parts subject to direct firing.

Radiographic and Ultrasonic Examination

Paragraph UW-11 provides rules and requirements based upon the type of radiography employed. The extent of radiography required of welded butt joints is determined based on vessel service, material thicknesses, or welding processes. When radiography is not mandatory, the degree of radiography is optional, and the amount of radiography must be determined by the user/designated agent/Manufacturer. Full Radiography shall be performed in accordance with paragraph UW-51 and spot radiography in accordance with paragraph UW-52.

Joint Efficiencies

Whether radiography is mandatory or optional, the amount of radiography preformed on each butt-welded joint together with the Joint Type will determine the joint efficiency to be applied in the various design equations, as described in paragraph UW-12. The longitudinal and circumferential directions of stress are investigated separately to determine the most restrictive condition governing stresses in the vessel. In terms of the application of joint efficiencies, each weld joint is considered separately, and the joint efficiency for that weld joint is then applied in the appropriate design equation for the component under consideration.

Required Markings

Paragraph UG-116(e) states when radiographic or ultrasonic examination has been performed on a vessel in accordance with UW-11, the pressure vessel shall be appropriately marked in conjunction with the amount of radiography performed. Additionally, the extent of radiography and the applicable joint efficiencies shall be noted on the Manufacturer's Data Report.

The following logic diagrams have been developed to provide guidance for determining the required degree of examination and the associated joint efficiencies for vessel components. See Figure 7.1.2, Figure 7.1.3 and Figure 7.1.4. Generally, the following points should be considered when designing a vessel or vessel part:

a) Is radiography mandatory due to service, material thickness, material specification, or welding process?
b) If radiography is not mandatory, the user/designated agent/Manufacturer shall determine the extent of radiography to be performed. How will the selection of radiography affect the thickness and cost of the vessel?
c) Is the Joint Type used appropriate for the Joint Category? i.e., UW-2 restricts the weld Joint Type to (1) or (2) for Category A and B welds.
d) Does the degree of radiography of a particular weld joint affect the degree of radiography and resulting joint efficiency on an intersecting weld joint?
e) Based on the noted exemptions from radiographic examination of welds in nozzles and communicating chambers, what are the benefits or consequences?
f) Other user/designated agent/Manufacturer requirements.

Commentary: The example problems provided in Part 7 are intended only to demonstrate the available options of radiographic examination, welded joint categories, weld joint types, and their resulting weld joint efficiencies. An arbitrary vessel sketch is provided and used for each of the example problems to help draw a comparison to the differences in weld joint types and associated weld joint efficiencies resulting from the selected degree of radiographic examination. The components that make up the vessel used in these examples problems, i.e. head types, nozzle identifiers and nominal sizes, and weld joint types are for illustration purposes and are not intended to replace good engineering judgment.

Table 7.1.1– Definition Of Weld Joint Types, Per Table UW-12

Weld Joint Type	Description
1	Butt joints as attained by double-welding or by other means which will obtain the same quality of deposited weld metal on the inside and outside weld surfaces to agree with the requirements of UW-35. Welds using backing strips which remain in place are excluded.
2	Single-welded butt joint with backing strip other than those included under (1).
3	Single-welded butt joint without use of backing strip.
4	Double full fillet lap joint.
5	Single full fillet lap joints with plug welds conforming to UW-17.
6	Single full fillet lap joints without plug welds.
7	Corner joints, full penetration, partial penetration, and/or fillet welded.
8	Angle joints.

Notes:

1. See Table UW-12 for Weld Joint Type limitations based upon Weld Joint Category.
2. Additional definitions for angle, butt, and corner joints are provided in Appendix 3.

Table 7.1.2 – Weld Joint Categories, per paragraph UW-3

Weld Category	Description
A	• Longitudinal and spiral welded joints within the main shell, communicating chambers (1), transitions in diameter, or nozzles • Any welded joint within a sphere, within a formed or flat head, or within the side plates (2) of a flat-sided vessel • Circumferential welded joints connecting hemispherical heads to main shells, to transitions in diameter, to nozzles, or to communicating chambers.
B	• Circumferential welded joints within the main shell, communicating chambers (1), nozzles or transitions in diameter including joints between the transition and a cylinder at either the large or small end • Circumferential welded joints connecting formed heads other than hemispherical to main shells, to transitions in diameter, to nozzles, or to communicating chambers.
C	• Welded joints connecting flanges, Van Stone laps, tubesheets or flat heads to main shell, to formed heads, to transitions in diameter, to nozzles, or to communicating chambers (1) • Any welded joint connecting one side plate (2) to another side plate of a flat-sided vessel.
D	• Welded joints connecting communicating chambers (1) or nozzles to main shells, to spheres, to transitions in diameter, to heads, or to flat-sided vessels • Welded joints connecting nozzles to communicating chambers (1) (for nozzles at the small end of a transition in diameter see Category B).

Notes:

1. Communicating chambers are defined as appurtenances to the vessel that intersect the shell or heads of a vessel and form an integral part of the pressure containing enclosure, e.g., sumps.
2. Side plates of a flat-sided vessel are defined as any of the flat plates forming an integral part of the pressure containing enclosure.

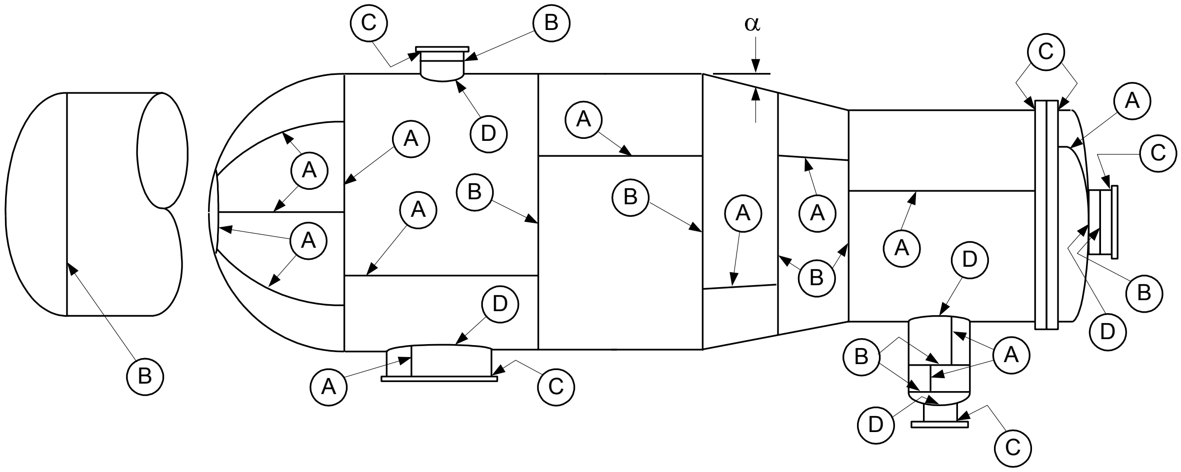


Figure 7.1.1 – UW-3 Illustration of Welded Joint Locations: Categories A, B, C, and D

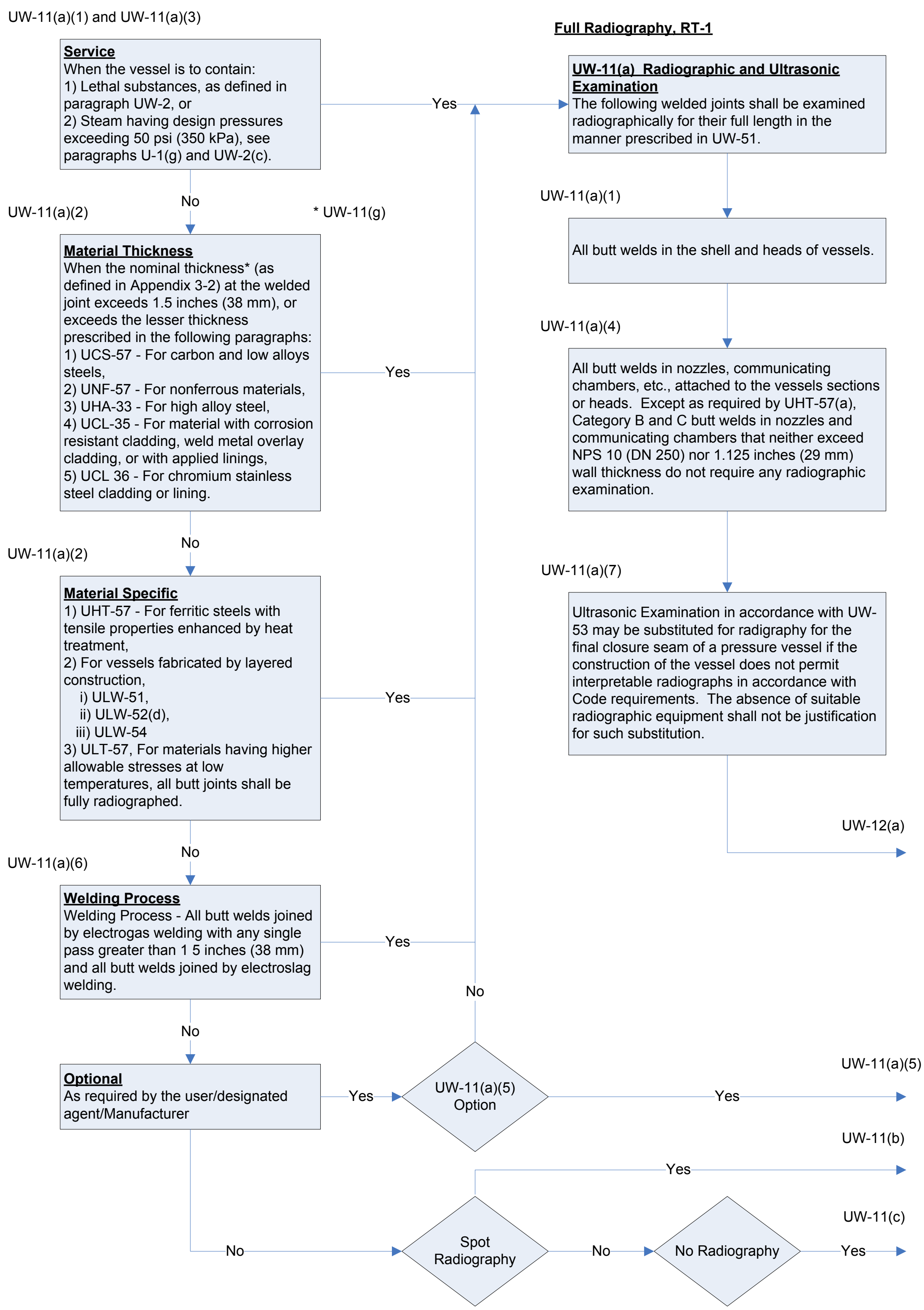


Figure 7.1.2 – Logic Diagram for UW-11(a)

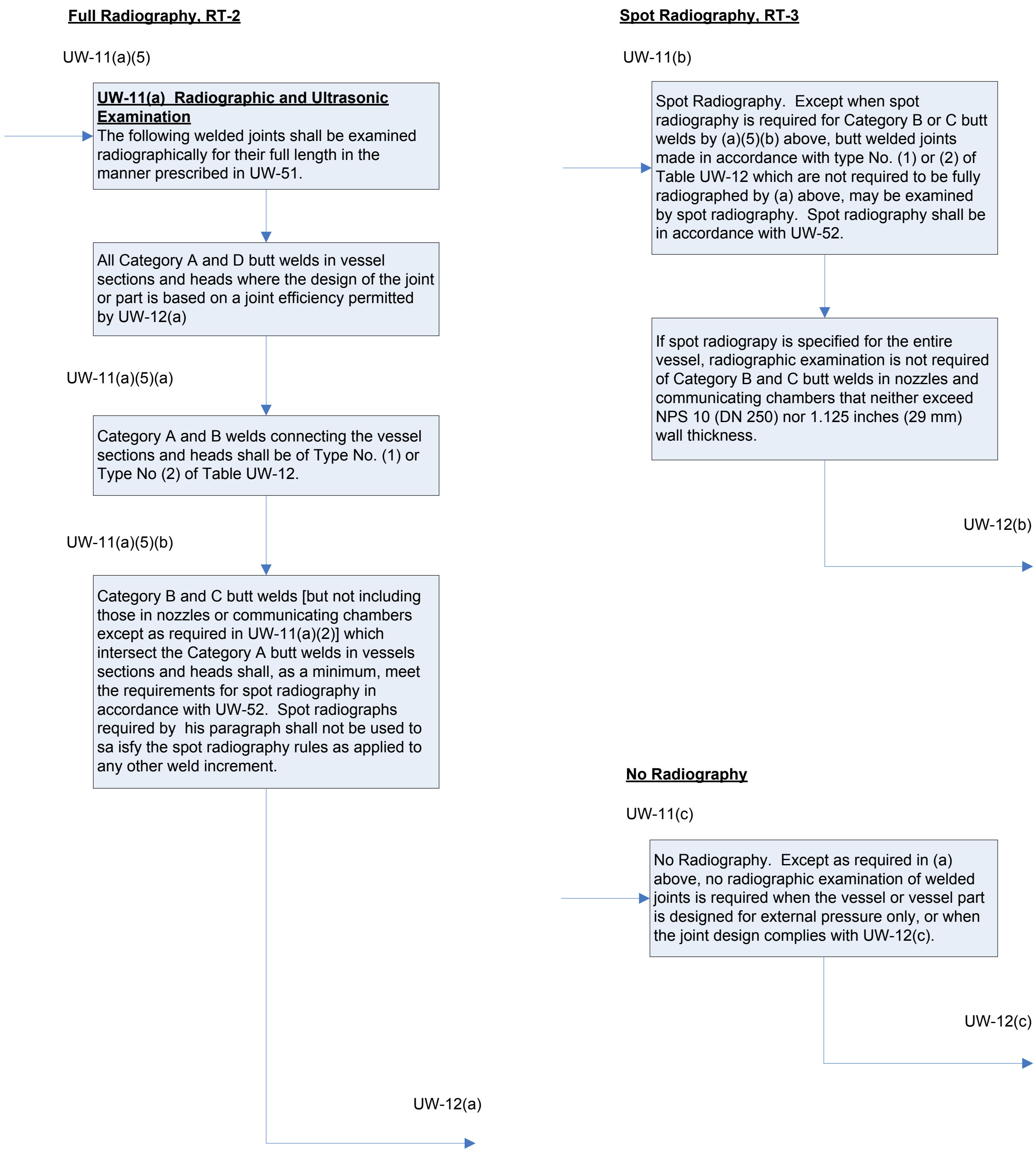


Figure 7.1.3 – Logic Diagram for UW-11(a)(5). UW-11(b) and UW-11(c)

UW-12(a)

A value of E not greater than that given in column (a) of Table UW-12 shall be used in the design calculations for fully radiographed butt joints [see UW-11(a)]. Except when the requirements of UW-11(a)(5) are not met, a value of E not greater than that given in column (b) of Table UW-12 shall be used.

UW-12(b)

A value of E not greater than that given in column (b) of Table UW-12 shall be used in the design calculations for spot radiographed butt welded joints [see UW-11(b)].

UW-12(c)

A value of E not greater than that given in column (c) of Table UW-12 shall be used in the design calculations for welded joints that are neither fully radiographed nor spot radiographed [see UW-11(c)].

UW-12(d)

Seamless vessel sections or heads shall be considered equivalent to welded parts of the same geometry in which all Category A welds are Type No. (1). For calculations involving circumferential stress in seamless vessel sections of for thickness of seamless heads, E=1.0 when the spot radiography requirements of UW-11(a)(5)(b) are met. E=0.85 when the spot radiography requirements of UW-11(a)(5)(b) are not met, or when the category A or B welds connecting seamless vessel sections or heads are Type No. 3, 4, 5, or 6 of Table UW-12.

UW-12(e)

Welded pipe or tubing shall be treated in the same manner as seamless, but with the allowable stensile stress taken from the welded product values of the stress tables, and the requirements of UW-12(d) applied.

Figure 7.1.4 – Logic Diagram for UW-12

7.2 Example E7.1 – NDE: Establish Joint Efficiencies, RT-1

An engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 1 (VIII-1). Based on the process service description, anticipated design data, materials of construction, and welding process, the engineer verifies that full radiography is required in accordance with paragraphs UW-11(a) and UW-51 for the entire vessel. A sketch of the vessel showing nozzle sizes, orientation, and weld seams is shown in Figure E7.1.

To assist with fabrication and inspection of the vessel, the engineer developed a table to summarize the NDE requirements and joint efficiencies applicable to each welded joint of the vessel based on the full radiography requirement of paragraph UW-11(a). Table E7.1 is a sub-set of the original table and only addresses the weld joint identifiers referenced on the vessel sketch in Figure E7.1.

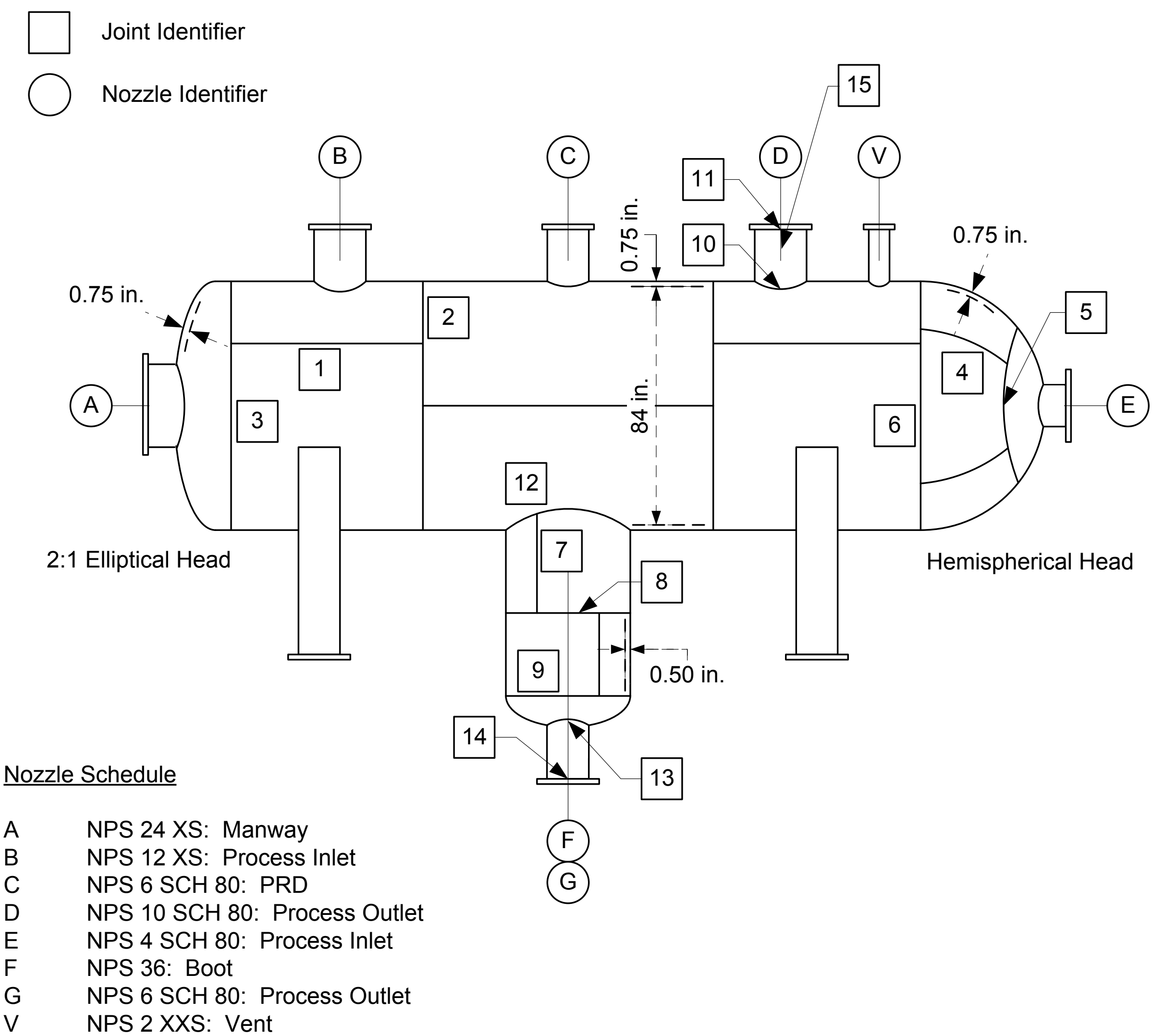


Figure E7.1 – Vessel Sketch

Table E7.1 – NDE Weld Joint Requirements

Service Description		Lethal	RT Nameplate Marking		RT–1
Welding Process		SMAW	Radiographic Examination Designation		UW-11(a)
Joint Identifier	**Joint Category**	**Joint Type**	**Limitations**	**Degree of Radiographic Examination**	**Joint Efficiency**
1	A	1	None	Full	1.0
2	B	1	None	Full	1.0
3	B	1	None	Full	1.0
4	A	1	None	Full	1.0
5	A	1	None	Full	1.0
6	A	1	None	Full	1.0
7	A	1	None	Full	1.0
8	B	2	None	Full	0.90
9	B	2	None	Full	0.90
10	D	7	Table UW-12, Note 5	—	1.0
11	C	2	None	Exempt UW-11(a)(4)	0.65
12	D	7	Table UW-12, Note 5	—	1.0
13	D	7	Table UW-12, Note 5	—	1.0
14	C	2	None	Exempt UW-11(a)(4)	0.65
15	A	1	None	—	1.0

COMMENTARY:

a) Since the Service Description was listed as “Lethal”, paragraph UW-11(a)(1) mandated that the vessel be subject to full radiography in accordance with paragraph UW-51. Additionally, paragraph UW-2(a)(1) requires Category A joints to be Type (1), Category B and C joints to be Type (1) or (2), and Category D joints shall be full penetration welds extending through the entire thickness of the vessel wall.
b) Joint Efficiencies were assigned per Table UW-12, based upon Joint Category and Joint Type.
c) All Category A, B, and C butt welds were radiographically examined for their full length.
d) The nozzles are attached to the vessel with Category D weld joints. The nozzles are set-in type nozzles forming corner joints. As referenced in Table UW-12, Note 5, there is no joint efficiency for these corner joints, and a joint efficiency of no greater than 1.0 may be used.
e) The Category B butt welds in the boot section (communicating chamber – Joint Identifiers 8 and 9) were radiographically examined for their full length. Therefore, a joint efficiency of 0.90 is applied to these circumferential weld joints per Table UW-12.
f) Joint Identifier 11 and 14 are Category C, single-welded butt joints with backing strips, Joint Type (2), and attach the flanges to nozzle necks in Nozzle D and G, respectively. These welds are exempt from radiography per UW-11(a)(4) with an applicable joint efficiency of 0.65 per Table UW-12.

g) Joint Identifier 15 represents the hypothetical Category A seam in a seamless pipe of Nozzle D. The resulting Category A joint efficiency to use for the seamless nozzle is 1.0 per Table UW-12.
h) The vessel is marked with an RT-1 designation per UG-116(e) because the complete vessel satisfies the requirements of full radiography requirements of UW-11(a) for their full length.

7.3 Example E7.2 – NDE: Establish Joint Efficiencies, RT-2

An engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with VIII-1. Based on the process service description, anticipated design data, materials of construction, and welding process, the engineer verifies that full radiography is not required. However, the engineer would like to apply full radiography in accordance with paragraphs UW-11(a) and UW-51, but would like to take advantage of the exemption from full radiography of the Category B and C butt welds in vessel sections and heads as permitted in paragraph UW-11(a)(5). A sketch of the vessel showing nozzle sizes, orientation, and weld seams is shown in Figure E7.1.

To assist with fabrication and inspection of the vessel, the engineer developed a table to summarize the NDE requirements and joint efficiencies applicable to each welded joint of the vessel based on the full radiography requirement of paragraph UW-11(a)(5). Table E7.2 is a sub-set of the original table and only addresses the weld joint identifiers referenced on the vessel sketch in Figure E7.1.

Table E7.2 – NDE Weld Joint Requirements

Service Description		General	RT Nameplate Marking		RT–2
Welding Process		SMAW	Radiographic Examination Designation		UW-11(a)(5)
Joint Identifier	Joint Category	Joint Type	Limitations	Degree of Radiographic Examination	Joint Efficiency
1	A	1	None	Full	1.0
2	B	1	None	Spot UW-11(a)(5)(b)	0.70
3	B	1	None	Spot UW-11(a)(5)(b)	0.70
4	A	1	None	Full	1.0
5	A	1	None	Full	1.0
6	A	1	None	Full	1.0
7	A	2	None	Full UW-11(a)(4)	0.90
8	B	2	None	Full UW-11(a)(4)	0.90
9	B	2	None	Full UW-11(a)(4)	0.90
10	D	7	Table UW-12, Note 5	——	1.0
11	C	2	None	Exempt UW-11(a)(5)(b)	0.65
12	D	7	Table UW-12, Note 5	——	1.0
13	D	7	Table UW-12, Note 5	——	1.0
14	C	3	Circ. Joint Only $t \le 0.625\ in.$ $OD \le 24\ in.$	Exempt UW-11(a)(5)(b)	0.60
15	A	1	None	——	1.0

COMMENTARY:

a) The Service Description was listed as “General”, and the requirements of UW-11(a)(1), (a)(2), (a)(3) were not applicable for this example. However, the engineer desired the vessel be subject to full radiography in accordance with UW-11(a)(5) and paragraph UW-51.
b) Joint Efficiencies were assigned per Table UW-12, based upon Joint Category and Joint Type.
c) All Category A butt welds were radiographically examined for their full length. The Category B butt welds in the vessel section (Joint Identifiers 2 and 3) were spot radiographed per paragraph UW-11(a)(5)(b). The spot radiographs of these Category B welds were performed only to achieve a joint efficiency of 1.0 for the Category A welds which they intersected. Therefore, a joint efficiency of 0.70 is applied to these circumferential weld joints per Table UW-12. The spot radiograph per paragraph UW-11(a)(5)(b) on Joint Identifier 3 also enables a joint efficiency of 1.0 be applied to the seamless 2:1 ellipsoidal head required

thickness calculation per UW-12(d).

d) The Category B butt welds in the boot section (communicating chamber – Joint Identifiers 8 and 9) were fully radiographed per paragraph UW-11(a)(4). Therefore, a joint efficiency of 0.90 is applied to these circumferential weld joints per Table UW-12.
e) The nozzles are attached to the vessel with Category D weld joints. The nozzles are set-in type nozzles forming corner joints. As referenced in Table UW-12, Note 5, there is no joint efficiency for these corner joints, and a joint efficiency of no greater than 1.0 may be used.
f) Joint Identifier 11 is a Category C, single-welded butt joint with backing strip, Joint Type (2), and attaches the flange-to-nozzle neck in Nozzle D. This weld is exempt from radiography per UW-11(a)(4) with an applicable joint efficiency of 0.65 per Table UW-12.
g) Joint Identifier 14 is a Category C, single-welded butt joint without the use of a backing strip, Joint Type (3), and attaches the flange-to-nozzle neck in Nozzle G. This weld is exempt from radiography per UW-11(a)(4) with an applicable joint efficiency of 0.60 per Table UW-12.
h) Joint Identifier 15 represents the hypothetical Category A seam in a seamless pipe of Nozzle D. The resulting Category A joint efficiency to use for the seamless nozzle is 1.0 per Table UW-12.
i) The vessel is marked with an RT-2 designation per UG-116(e) because the complete vessel satisfies the requirements of UW-11(a)(5) and when the spot radiography requirements of UW-11(a)(5)(b) have been applied.

7.4 Example E7.3 – NDE: Establish Joint Efficiencies, RT-3

An engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with VIII-1. Based on the process service description, anticipated design data, materials of construction, and welding process, the engineer verifies that full radiography is not required in accordance with paragraph UW-11(a) and decides that spot radiography in accordance with paragraphs UW-11(b) and UW-52 is adequate. A sketch of the vessel showing nozzle sizes, orientation, and weld seams is shown in Figure E7.1.

To assist with fabrication and inspection of the vessel, the engineer developed a table to summarize the NDE requirements and joint efficiencies applicable to each welded joint of the vessel based on the spot radiography requirement of paragraph UW-11(b). Table E7.3 is a sub-set of the original table and only addresses the weld joint identifiers referenced on the vessel sketch in Figure E7.1.

Table E7.3 – NDE Weld Joint Requirements

Service Description		General	RT Nameplate Marking		RT–3
Welding Process		SMAW	Radiographic Examination Designation		UW-11(b)
Joint Identifier	**Joint Category**	**Joint Type**	**Limitations**	**Degree of Radiographic Examination**	**Joint Efficiency**
1	A	1	None	Spot	0.85
2	B	1	None	None	0.85
3	B	1	None	Spot	0.85
4	A	1	None	Spot	0.85
5	A	1	None	None	0.85
6	A	1	None	Spot	0.85
7	A	2	None	None	0.80
8	B	2	None	None	0.80
9	B	2	None	None	0.80
10	D	7	Table UW-12, Note 5	—	1.0
11	C	2	None	Exempt UW-11(b)	0.80
12	D	7	Table UW-12, Note 5	—	1.0
13	D	7	Table UW-12, Note 5	—	1.0
14	C	3	Circ. Joint Only $t \le 0.625\ in.$ $OD \le 24\ in.$	Exempt UW-11(b)	0.60
15	A	1	None	—	1.0

COMMENTARY:

a) Since the Service Description was listed as “General”, and the requirements of UW-11(a)(1), (a)(2), (a)(3) were not applicable for this example, the vessel is subject to spot radiography in accordance with paragraph UW-52.

b) Joint Efficiencies were assigned per Table UW-12, based upon Joint Category and Joint Type.

c) Per UW-52, one spot radiograph must be taken for every 50 foot increment of weld or fraction thereof for which a joint efficiency from column (b) of Table UW-12 is selected. Based on an estimated 160 linear feet of weld for the vessel section and head welds seams, the Inspector chose four spot radiography locations: Joint Identifiers 1, 3, 4, and 6. These welds are Category A and B butt welds. The length of each spot was 6 inches, as provided in paragraph UW-52. A joint efficiency of 0.85 is applied to all weld Joint Type (1) and 0.80 is applied to all weld Joint Type (2), per Table UW-12.

d) A joint efficiency of 0.85 is applied to the seamless 2:1 ellipsoidal head required thickness calculation per UW-12(d) since the spot radiography requirements of UW-11(a)(5)(b) have not been met for the Category B circumferential weld seam, Joint Identifier 3.

e) The nozzles are attached to the vessel with Category D weld joints. The nozzles are set-in type nozzles forming corner joints. As referenced in Table UW-12, Note 5, there is no joint efficiency for these corner joints, and a joint efficiency of no greater than 1.0 may be used.

f) Joint Identifier 11 is a Category C, single-welded butt joint with backing strip, Joint Type (2), and attaches the flange-to-nozzle neck in Nozzle D. This weld is exempt from radiography per UW-11(b) with an applicable joint efficiency of 0.80 per Table UW-12.

g) Joint Identifier 14 is a Category C, single-welded butt joint without the use of a backing strip, Joint Type (3), and attaches the flange-to-nozzle neck in Nozzle G. This weld is exempt from radiography per UW-11(b) with an applicable joint efficiency of 0.60 per Table UW-12.

h) Joint Identifier 15 represents the hypothetical Category A seam in a seamless pipe of Nozzle D. The resulting Category A joint efficiency to use for the seamless nozzle is 1.0 per Table UW-12.

i) The vessel is marked with an RT-3 designation per UG-116(e) because the complete vessel satisfies the spot radiography requirements of UW-11(b).

7.5 Example E7.4 – NDE: Establish Joint Efficiencies, RT-4

An engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with VIII-1. Based on the process service description, anticipated design data, materials of construction, and welding process, the engineer verifies that full radiography is not required. However, similar to Example 7.2, the engineer would like to apply full radiography in accordance with paragraphs UW-11(a) and UW-51, but would like to take advantage of the exemption from full radiography of the Category B and C butt welds in vessel sections and heads as permitted in paragraph UW-11(a)(5). However, the joint efficiency assigned to the Category B welds in Example 7.2 was too low and it was decided to perform additional spot radiography of the Category B welds in accordance with paragraph UW-52. A sketch of the vessel showing nozzle sizes, orientation, and weld seams is shown in Figure E7.1.

To assist with fabrication and inspection of the vessel, the engineer developed a table to summarize the NDE requirements and joint efficiencies applicable to each welded joint of the vessel based on the full radiography requirement of paragraph UW-11(a)(5) and the spot radiography requirement of paragraph UW-52. Table E7.4 is a sub-set of the original table and only addresses the weld joint identifiers referenced on the vessel sketch in Figure E7.1.

Table E7.4 – NDE Weld Joint Requirements

Service Description		General	RT Nameplate Marking		RT–4
Welding Process		SMAW	Radiographic Examination Designation		UW-11(a)(5) and UW-52
Joint Identifier	Joint Category	Joint Type	Limitations	Degree of Radiographic Examination	Joint Efficiency
1	A	1	None	Full	1.0
2	B	1	None	Spot UW-11(a)(5) Spot UW-52	0.85
3	B	1	None	Spot UW-11(a)(5) Spot UW-52	0.85
4	A	1	None	Full	1.0
5	A	1	None	Full	1.0
6	A	1	None	Full	1.0
7	A	2	None	Full UW-11(a)(4)	0.90
8	B	2	None	Full UW-11(a)(4)	0.90
9	B	2	None	Full UW-11(a)(4)	0.90
10	D	7	Table UW-12, Note 5	——	1.0
11	C	2	None	Exempt UW-11(a)(5)(b)	0.80
12	D	7	Table UW-12, Note 5	——	1.0
13	D	7	Table UW-12, Note 5	——	1.0
14	C	3	Circ. Joint Only $t \le 0.625\ in.$ $OD \le 24\ in.$	Exempt UW-11(a)(5)(b)	0.60
15	A	1	None	——	1.0

COMMENTARY:

a) The Service Description was listed as “General”, and the requirements of UW-11(a)(1), (a)(2), (a)(3) were not applicable for this example. However, the engineer desired the vessel be subject to full radiography in accordance with UW-11(a)(5) and paragraph UW-51.

b) Joint Efficiencies were assigned per Table UW-12, based upon Joint Category and Joint Type.

c) All Category A butt welds were radiographically examined for their full length. The Category B butt welds in the vessel section (Joint Identifiers 2 and 3) were spot radiographed per paragraph UW-11(a)(5)(b). The spot radiographs of these Category B welds were performed per paragraph UW-11(a)(5)(b) to achieve a joint efficiency of 1.0 for the Category A welds which they intersected. However, one additional spot radiograph was taken in accordance with UW-52 on both Joint Identifier 2 and 3. Therefore, a joint efficiency of 0.85 is applied to these circumferential weld joints per Table UW-12. It should be noted, per

UW-52, one spot radiograph must be taken for every 50 foot increment of weld or fraction thereof for which a joint efficiency from column (b) of Table UW-12 is selected. For the 84 inch diameter vessel under consideration, each Category B circumferential weld seam consists of approximately 22 feet of weld. For the three Category B weld seams this accounts for 66 feet of weld and therefore, requires two spot radiographs be taken to achieve the a joint efficiency of 0.85 for each circumferential weld seam. Additionally, the spot radiographed per paragraph UW-11(a)(5)(b) on Joint Identifier 3 also enables a joint efficiency of 1.0 be applied to the seamless 2:1 ellipsoidal head required thickness calculation per UW-12(d).

d) The Category B butt welds in the boot section (communicating chamber – Joint Identifiers 8 and 9) were fully radiographed per paragraph UW-11(a)(4). Therefore, a joint efficiency of 0.90 is applied to these circumferential weld joints per Table UW-12.

e) The nozzles are attached to the vessel with Category D weld joints. The nozzles are set-in type nozzles forming corner joints. As referenced in Table UW-12, Note 5, there is no joint efficiency for these corner joints, and a joint efficiency of no greater than 1.0 may be used.

f) Joint Identifier 11 is a Category C, single-welded butt joint with backing strip, Joint Type (2), and attaches the flange-to-nozzle neck in Nozzle D. This weld is exempt from radiography per UW-11(a)(4) with an applicable joint efficiency of 0.65 per Table UW-12.

g) Joint Identifier 14 is a Category C, single-welded butt joint without the use of a backing strip, Joint Type (3), and attaches the flange-to-nozzle neck in Nozzle G. This weld is exempt from radiography per UW-11(a)(4) with an applicable joint efficiency of 0.60 per Table UW-12.

h) Joint Identifier 15 represents the hypothetical Category A seam in a seamless pipe of Nozzle D. The resulting Category A joint efficiency to use for the seamless nozzle is 1.0 per Table UW-12.

i) The vessel is marked with an RT-4 designation per UG-116(e) because none of the markings “RT-1”, “RT-2”, or “RT-3” are applicable.

PART 8

PRESSURE TESTING REQUIREMENTS

8.1 Example E8.1 – Determination of a Hydrostatic Test Pressure

Establish the hydrostatic test pressure for a process tower considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516, \ Grade \ 70$
- MAWP = $1650 \ psig @ 600°F$
- Liquid Head = $60 \ ft$
- Liquid Specific Gravity = 0.89
- Inside Diameter = $96.0 \ in$
- Corrosion Allowance = $0.125 \ in$
- Allowable Stress = $19400 \ psi$
- Allowable Stress at Ambient Conditions = $20000 \ psi$
- Yield Stress at Ambient Conditions = $38000 \ psi$
- Weld Joint Efficiency = 1.0
- Tangent-to-Tangent Vessel Length = $80 \ ft$
- Top and Bottom Heads = $Hemispherical$

Evaluate the requirements of hydrostatic testing per paragraph UG-99.

Paragraph UG-99(a), a hydrostatic test shall be conducted on all vessels after:

a) All fabrication has been completed, except for operations which could not be performed prior to the test such as a weld end preparation, cosmetic grinding on the base material which does not affect the required thickness; and
b) All examinations have been performed, except those required after the test.

Paragraph UG-99(b), except as otherwise permitted, vessels designed for internal pressure shall be subjected to a hydrostatic test pressure that at every point in the vessel is at least equal to the following equation:

$$P_T = 1.3 \cdot MAWP \cdot \left(\frac{S_T}{S}\right)$$

$$P_T = 1.3(1650)\left(\frac{20000}{19400}\right) = 2211 \ psi$$

a) The ratio S_T/S shall be the lowest ratio for the pressure-boundary materials. In this example problem, the MAWP is taken as the design pressure and the ratio of S_T/S is based on the shell material; however, all pressure part stress ratios should be checked.

b) Bolting shall not be included in the determination of the lowest stress ratio, except when

$$1.3(S_T/S)(S_B) > 0.90(S_y)$$

where,

$$S_B = bolt\ allowable\ stress\ at\ design\ temperature,\ psi$$
$$S_y = bolt\ minimum\ specified\ yield\ strength\ at\ test\ temperature,\ psi$$

c) All loadings that may exist during this test shall be given consideration.

Paragraph UG-99(c), a hydrostatic test based on a calculated pressure may be used by agreement between the user and the Manufacturer. When this pressure is used, the Inspector shall reserve the right to require the Manufacturer or the designer to furnish the calculations used for determining the hydrostatic test pressure for any part of the vessel.

Paragraph UG-99(d), the requirements of UG-99(b) represent the minimum standard hydrostatic test pressure required by this Division. The requirements of UG-99(c) represent a special test based on calculations. Any intermediate value of pressure may be used. This Division does not specify an upper limit for hydrostatic test pressure. However, if the hydrostatic test pressure is allowed to exceed, either intentionally or accidentally, the value determined as prescribed in UG-99(c) to the degree that the vessel is subjected to visible permanent distortion, the Inspector shall reserve the right to reject the vessel.

Paragraph UG-99(h), any non-hazardous liquid at any temperature may be used for the hydrostatic test if below its boiling point. Combustible liquids having a flash point less than 110°F, such as petroleum distillates, may be used only for near atmospheric temperature tests. It is recommended that the metal temperature during hydrostatic test be maintained at least 30°F above the minimum design metal temperature, but need not exceed 120°F, to minimize the risk of brittle fracture.

Paragraph UG-99(k)(1), unless permitted by the user or his designated agent, pressure retaining welds of vessels shall not be painted or otherwise coated either internally or externally prior to the pressure test.

Paragraph UG-99(k)(2), when painting or coating prior to the hydrostatic test is permitted, or when internal linings are to be applied, the pressure retaining welds shall first be leak tested in accordance with ASME Section V, Article 10. Such test may be waived with the approval of the user or his designated agent.

Paragraph UG-99(k)(3), vessels for lethal service (see UW-2(a)) shall not be painted or otherwise coated or lined either internally or externally prior to the hydrostatic pressure test.

8.2 Example E8.2 – Determination of a Pneumatic Test Pressure

Establish the pneumatic test pressure for a vessel considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

<u>Vessel Data:</u>

- Material = $SA-516,\ Grade\ 70$
- MAWP = $1650\ psig@300°F$

- Inside Diameter = $240.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20000\ psi$
- Allowable Stress at Ambient Conditions = $20000\ psi$
- Yield Stress at Ambient Conditions = $38000\ psi$
- Weld Joint Efficiency = 1.0
- Tangent-to-Tangent Vessel Length = $80\ ft$
- Top and Bottom Heads = $Hemispherical$

Evaluate the requirements of pneumatic testing per paragraph UG-100.

Paragraph UG-100(a), subject to the provisions of UG-99(a)(1) and (a)(2), a pneumatic test prescribed in this paragraph may be used in lieu of the standard hydrostatic test prescribed in UG-99 for vessels:

a) That are so designed and/or supported that they cannot be safely filled with water;

b) Not readily dried, that are to be used in services where traces of the testing liquid cannot be tolerated and the parts of which have, where possible, been previously tested by hydrostatic pressure to the pressure required in UG-99.

Paragraph UG-100(b), except for enameled vessels, for which the pneumatic test shall be at least equal to, but not exceed, the maximum allowable working pressure to be marked on the vessel, the pneumatic test pressure at every point in the vessel is at least equal to the following equation:

$$P_T = 1.1 \cdot MAWP \cdot \left(\frac{S_T}{S}\right)$$

$$P_T = 1.1(1650)\left(\frac{20000}{20000}\right) = 1815\ psi$$

c) The ratio S_T/S shall be the lowest ratio for the pressure-boundary materials. In this example problem, the MAWP is taken as the design pressure and the ratio of S_T/S is based on the shell material; however, all pressure part stress ratios should be checked.

d) Bolting shall not be included in the determination of the lowest stress ratio, except when

$$1.1(S_T/S)(S_B) > 0.90(S_y)$$

where,

$$S_B = bolt\ allowable\ stress\ at\ design\ temperature,\ psi$$

$$S_y = bolt\ minimum\ specified\ yield\ strength\ at\ test\ temperature,\ psi$$

e) All loadings that may exist during this test shall be given consideration. In no case shall the pneumatic test pressure exceed 1.1 times the basis for the calculated test pressure, as defined in Appendix 3-2.

Paragraph UG-100(c), the metal temperature during pneumatic test shall be maintained at least 30°F above the minimum design metal temperature, to minimize the risk of brittle fracture.

Paragraph UG-100(d), the pressure in the vessel shall be gradually increased to not more than one-half of the test pressure. Thereafter, the test pressure shall be increased in steps of approximately one-tenth of the test pressure until the required test pressure has been reached.

Paragraph UG-100(e)(1), unless permitted by the user or his designated agent, pressure retaining welds of vessels shall not be painted or otherwise coated either internally or externally prior to the pneumatic pressure test.

Paragraph UG-100(e)(2), when painting or coating prior to the pneumatic test is permitted, or when internal linings are to be applied, the pressure retaining welds shall first be leak tested in accordance with ASME Section V, Article 10. Such test may be waived with the approval of the user or his designated agent.

Paragraph UG-100(e)(3), vessels for lethal service (see UW-2(a)) shall not be painted or otherwise coated or lined either internally or externally prior to the pneumatic pressure test.

ASME PTB-4-2013

ISBN 978-0-7918-6889-8

9 780791 868898

A22913