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ASME Section VIII – Division 3 Example Problem Manual

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ASME Section VIII Division 3 Example Problem Manual

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FOREWORD

In the 1980's, the Special Working Group on High Pressure Vessels was established for the purpose of creating a Standard dealing with the construction of "high pressure vessels" which are in general above 10,000 psi. This was based on recommendations made by the Operations, Applications, and Components Technical Committee of the ASME Pressure Vessel and Piping Division. "ASME Section VIII, Division 3 Alternative Rules for Construction of High Pressure Vessels" was first published in 1997. The Committee continues to refine and develop the Standard to this day.

Some of the innovative concepts which began with ASME Section VIII, Division 3 include:

- Use of elastic-plastic finite element analysis in design of pressure equipment
- One of the lowest design margins which was originally published at 2.0 and then lowered to 1.8
- Use of high strength materials for the pressure equipment used in manufacture of high pressure equipment
- Stringent requirements on fracture toughness for materials used in construction
- Complete volumetric and surface examination after hydrotest
- The use of fracture mechanics for evaluation of design life assessment in all cases where "Leak-Before-Burst" cannot be shown
- Consideration of beneficial residual stresses in the evaluation of the design life of vessels

ASME contracted with Structural Integrity Associates, Inc. to develop the ASME Section VIII, Division 3 Example Problem Manual. This publication is provided to illustrate some of the design calculations and methodologies used in the ASME B&PV Code, Section VIII, Division 3. It is recognized that many high pressure designs are unique and quite innovative and therefore, this example problem manual cannot cover all design aspects within the scope of Section VIII, Division 3. This is an attempt at covering some of the most common ones.

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PART 1
General Requirements

1 GENERAL REQUIREMENTS

1.1 Introduction

ASME B&PV Code, Section VIII, Division 3 contains mandatory requirements, specific prohibitions, and non-mandatory guidance for the design, materials, fabrication, examination, inspection, testing, and certification of high pressure vessels and their associated pressure relief devices. This manual is based on the 2011 edition of the code.

1.2 Scope

Example problems illustrating the use of the analysis methods in ASME B&PV Code, Section VIII, Division 3 are provided in this document.

1.3 Organization and Use

An introduction to the example problems in this document is described in Part 2 of this document. The remaining Parts of this document contain the example problems. The Parts 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13 in this document coincide with the Parts KM, KD-2 Elastic – Plastic analysis, KD-2 Elastic Analysis, KD-3 fatigue analysis, KD-4 fracture mechanics assessment, KD-5 evaluation of autofrettaged vessels, KD-6 evaluation of openings and closures, KD-8 evaluation of residual stresses in dual walled vessels, KT determination of limits on hydrostatic test pressure, Appendix E determination of bottom head dimensions for thick & thin walls and evaluation of thread load distributions in the ASME B&PV Code, Section VIII, Division 3, respectively. All paragraph references are to the ASME B&PV Code, Section VIII, Division 3 2010 edition with the 2011 Addenda [1].

The example problems in this manual follow the calculation procedures in ASME B&PV Code, Section VIII, Division 3. It is recommended that users of this manual obtain a copy of ”Criteria of the ASME Boiler and Pressure Vessel Code Section VIII, Division 3” [2] that contains criteria on the use of the Code.

It should be noted that VIII-3 requires the use of API 579-1/ASME FFS-1 [3] for some calculation procedures. When reviewing certain example problems in this manual, it is recommended that a copy be obtained of this standard.

PART 2

Example Problem Descriptions Part Contents

2 EXAMPLE PROBLEM DESCRIPTIONS PART CONTENTS

2.1 General

Example problems are provided for the following parts of the document;

- Part KM - Materials Requirements
- Part KD-2 - Design By Rule Requirements
- Part KD-2 – Elastic – Plastic Analysis
- Part KD-2- Elastic Analysis Requirements
- Part KD-3 – Life Assessment using Fatigue
- Part KD-4 – Life Assessment using Fracture Mechanics
- Part KD-5 – Evaluation of Residual Stress due to Autofrettage
- Part KD-6 – Design Assessment of Heads and Connections
- Part KD-8 – Evaluation of Residual Stress Due to Shrink Fitting
- Part KT – Determination of Hydrostatic Test Range
- Appendix E – Special Design by Rules for Closed Ends and Threads

A summary of the example problems provided is contained in Table 1.

2.2 Calculation Precision

The calculation precision used in the example problems is intended for demonstration proposes only; an intended precision is not implied. In general, the calculation precision should be equivalent to that obtained by computer implementation, rounding of calculations should only be performed on the final results.

2.3 Tables

Table 1 – Summary of Example Problems

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4	E-KD-2.2.1	Elastic Plastic Analysis
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Part	Example	Description
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9	E-KD-8.1.1	Dual Wall Cylindrical Vessel Stress Distribution
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11	E-AE-2.1.1	Blind End Dimensions and Corner Stresses in a Vessel without Detailed Stress Analysis – Thick Wall Pressure Vessel
11	E-AE-2.1.2	Blind End Dimensions and Corner Stresses in a Vessel without Detailed Stress Analysis – Thin Wall Pressure Vessel
11	E-AE-2.2.1	Thread Load Distribution

PART 3
Example Problems Materials

3 EXAMPLE PROBLEMS MATERIALS

3.1 Example Problem E-KM-2.1.1 – Evaluation of Test Locations for Cylindrical Forgings in Accordance with KM-2

This problem presents an evaluation of requirements for the minimum number, location and type of tests required for the following two cases:

<u>Material dimensions</u>

Case 1 - 12 in. diameter x 18 in. long and

Case 2 - 12 in. diameter x 15 ft. long (180 in.)

The forgings are SA-723 Grade 2 Class 2 material with a minimum specified yield strength of 120,000 psi and a minimum specified tensile strength of 135,000 psi. The forgings are all solid cylinders and all tests are to be taken from test material on the end of the cylinders, which when remove, will leave the size of material listed in each of the cases.

<u>**STEP 1** – Determine the thickness of the forging at heat treat (KM-201.2 / KM-211.2)</u>

The thickness of the forging is defined as:

Case 1 – This is a cylindrical forging which the thickness is equal to the diameter of the forging or 12 inches (T = 12 in).

Case 2 - This is a cylindrical forging which the thickness is equal to the diameter of the forging or 12 inches (T = 12 in).

<u>**STEP 2** – Determine the location of the "datum point" for the forgings (KM-211.2)</u>

The datum point is either the mid-point of the tension test specimen or the area under the notch of the impact test specimens. This datum will be used for all of the specimens including both the tension and Charpy specimens.

Case 1 – The datum point is located at a position of T/4 or 3 inches from the OD of the cylinder and 2T/3 or 8 inches from the end of the cylinder. Tensile samples shall be longitudinal and CVN samples shall be transverse.

Case 2 – The datum point is located at a position of T/4 or 3 inches from the OD of the cylinder and 2T/3 or 8 inches from the end of the cylinder with a 180° offset between the test locations on each end.

<u>**STEP 3** – Determine the minimum number test specimens required</u>

Case 1 – The overall dimensions of this forging at the time of heat treatment is:

Diameter = 15 in diameter

Length = 27 inches.

Therefore, the weight of the 15 inch diameter x 27 inch long forging is approximately 1400 lb. Per KM-231(b), this means that the piece will require one tension test and one set of three Charpy V-notch specimens per component.

Case 2 – The weight of this forging is in excess of 5000 lb and it is in excess of 80 inches long. Therefore, KM-231(c) requires that two tension tests and two sets of Charpy V-notch impact tests be taken at a datum point from each end of the forging 180 degrees apart. Therefore, for each forging, four tension tests (two from each end) and four sets of three impact specimens (two sets from each end) shall be taken from the forging.

STEP 4 – Supplementary Fracture Toughness Testing (KM-250)

It is the responsibility of the designer to specify to the material supplier if supplementary impact testing is required. This can be accomplished in several methods including:

Charpy V-Notch Impact Testing

Crack Tip Opening Displacement (CTOD) Fracture Toughness Testing

J_{Ic} Fracture Toughness Testing

K_{Ic} Fracture Toughness Testing

It is noted that if Charpy V-notch impact testing, CTOD, or J_{Ic} testing data is used for determination of the K_{Ic} value for use in fracture mechanics calculations, the Manufacturer is required to determine the appropriate conversion correlation to determine K_{Ic}.

3.2 Example Problem E-KM-2.1.2 – Calculation of Fracture Toughness based on Charpy Impact Tests (KM-251)

Determine the fracture toughness of a vessel made from SA-723 Grade 2 Class 2.

Vessel Data

Material – All Components = SA-723 Grade 2 Class 2 ($S_y = 120$ ksi)

Charpy Impact value used in the calculation is assumed to be the minimum required by KM-234.2(a):

Specimen Orientation	Number of Specimens	Energy (CVN), ft-lbf for Specified Min Yield Strength up to 135 ksi
Transverse	Average for 3	30 ft-lbf
	Minimum for 1	24 ft-lbf

The fracture toughness K_{Ic} is then found using the first equation in Appendix D-600 which has been re-written here as:

$$K_{Ic} = S_y \cdot \sqrt{5 \cdot \left(\frac{CVN}{S_y} - 0.05 \right)}$$

Which yields $K_{Ic} = 104$ ksi-in$^{0.5}$ in this case, conservatively based on the minimum for one value from the table above.

PART 4

Example Problems General Design Issues

4 EXAMPLE PROBLEMS GENERAL DESIGN ISSUES

4.1 Example Problem E-KD-2.1.1 – Determination of Design Pressure in Cylindrical Vessel – Monobloc Vessel

Determine the design pressure for a monobloc cylindrical vessel and associated stress distribution given the following data. Perform calculations for both open and closed-end vessels.

Vessel Data:

- Material = SA-705 Gr. XM-12 Condition H1100
- Design Temperature = 70°F
- Inside Diameter = 6.0 in
- Outside Diameter = 12.0 in
- Diameter Ratio (Y) = 2.0 [KD-250]
- Minimum Specified Yield Strength = 115,000 psi @ 100°F per Table Y-1 of Section II, Part D
- Minimum Specified Tensile Strength= 140,000 psi @ 100°F

Note, tensile strength is not given for the above material in Table U of Section II, Part D, therefore, a value of yield strength (S_y) is to be used in place of the actual tensile strength per KD-221.1.

It should be noted that this example is limited to the application of the equations in KD-220. An actual vessel requires evaluation of all of its features in accordance with all of the rules in Part KD.

Evaluate design pressure per KD-220 for an open-end cylindrical shell for Y ≤ 2.85.

$$Y = 2.0$$

$$P_D = \min\left[\left[2.5856 \cdot S_y \cdot \left(Y^{0.268} - 1\right)\right], \left[1.0773 \cdot \left(S_y + S_u\right) \cdot \left(Y^{0.268} - 1\right)\right]\right]$$

The design pressure is 50,581 *psi.*

Note that this calculation does not account for any loading in addition to internal pressure. If shell is subject to additional loading, the design shall be modified per KD-221.5 so that the collapse pressure is greater than or equal to 1.732 times the design pressure.

Evaluate the stress distribution for the open-end cylinder (KD-250)

The stresses for the cylinder can then be determined by using KD-250 equations (1) and (2).

$$\sigma_t = \frac{P}{Y^2 - 1}\left(1 + Z^2\right)$$

$$\sigma_r = \frac{P}{Y^2 - 1}\left(1 - Z^2\right)$$

Where $Y = D_O / D_I$ and $Z = D_O / D$. It is noted that the longitudinal stress in an open-end cylinder is zero ($s_l = 0$).

For the case of the open end cylinder, at the inside diameter at the design pressure, s_t = 84,302 psi and s_r = -50,581 psi. The stress distribution is shown in Figure E-KD-2.1.1-1.

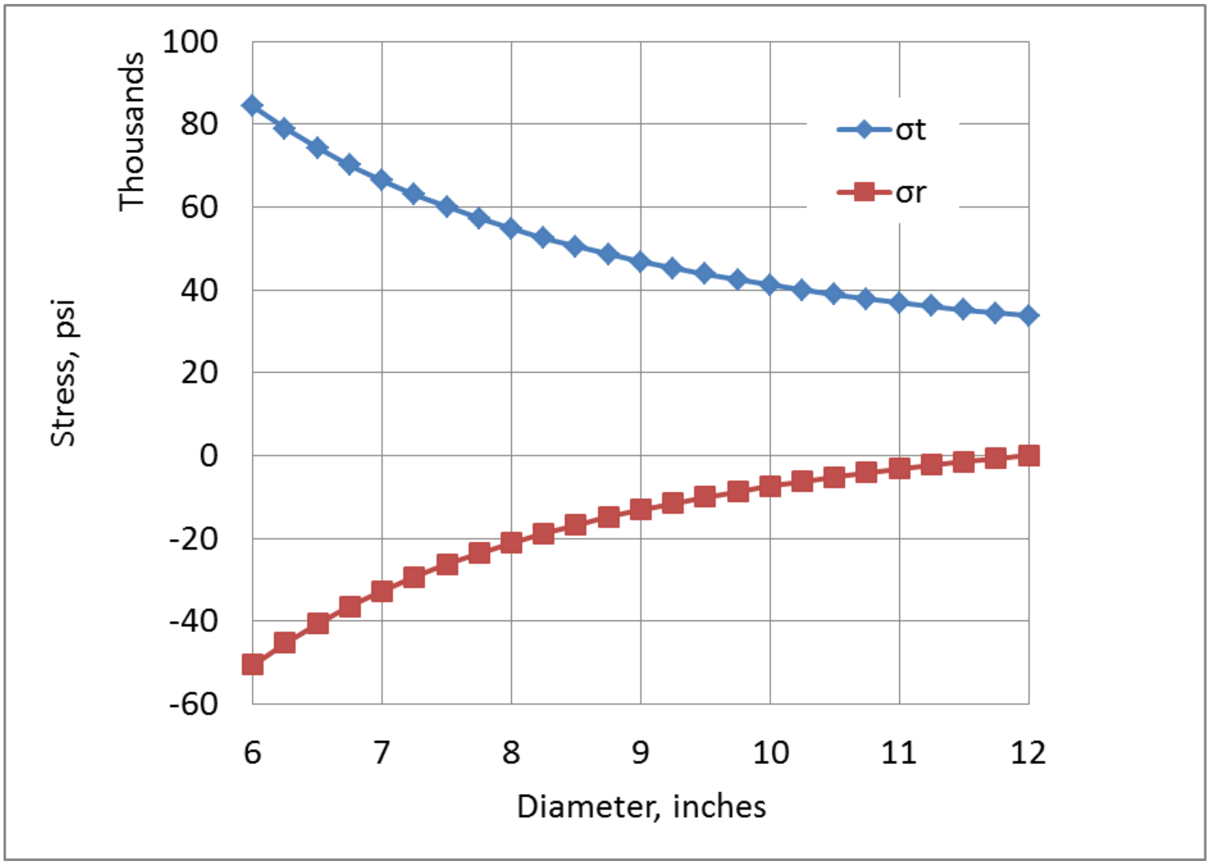

Figure 1 – E-KD-2.1.1-1 – Stress Distribution in Monoblock Open End Shell

Evaluate design pressure per KD-220 for a closed-end cylindrical shell.

$$P_D = \min\left[\left(\frac{1}{1.25}\cdot S_y\cdot \ln(Y)\right),\left[\frac{1}{3}\cdot\left(S_y + S_u\right)\ln(Y)\right]\right]$$

The design pressure is 53,141 *psi.*

Evaluate the stress distribution for the closed ended cylinder (KD-250)

The stresses for the cylinder can then be determined by using KD-250 equations (1) and (2), as discussed previously. The primary difference is that for the closed ended cylinder is the longitudinal stress. The longitudinal stress in a closed ended cylinder can be calculated using KD-250 equation (3):

$$\sigma_l = \frac{P}{Y^2 - 1}$$

For the case of the closed end cylinder, at the inside diameter at the design pressure, s_t = 88,568 psi, s_r = -53,141 psi, and s_l = 17,714 psi. The stress distribution is shown in Figure E-KD-2.1.1-2.

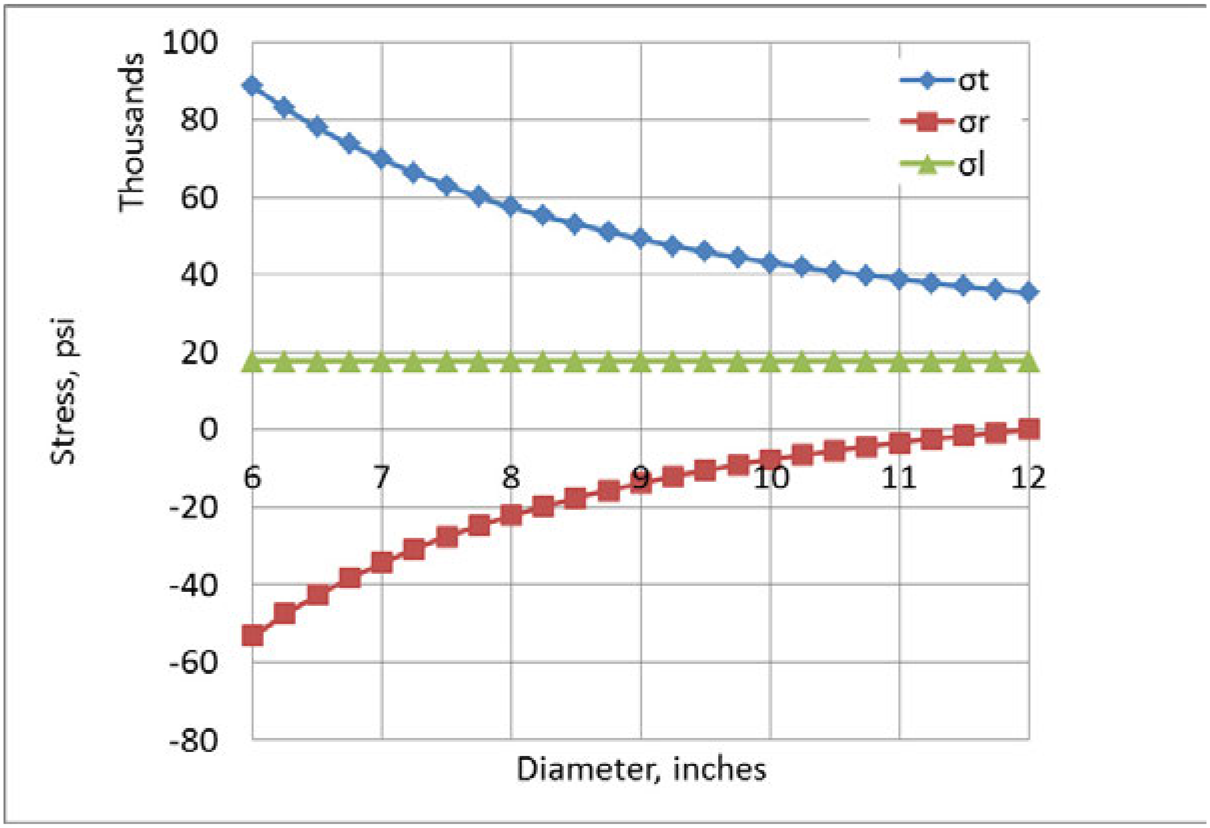

Figure 2 – E-KD-2.1.1-2 – Stress Distribution in Monoblock Closed End Shell

Table 2 – E-KD-2.1.1-1 – Tabulated Stresses from Figures E-KD-2.1.1-1 and -2 at Corresponding Design Pressure

	Figure E-KD-2.1.1-1 Open Ended		Figure E-KD-2.1.1-1 Closed Ended		
Do/D (Z)	σ_t	σ_r	σ_t	σ_r	σ_l
2.000	84302	-50581	88568	-53141	17714
1.920	79014	-45294	83013	-47586	17714
1.846	74325	-40604	78087	-42660	17714
1.778	70147	-36427	73698	-38270	17714
1.714	66409	-32688	69770	-34343	17714
1.655	63051	-29330	66242	-30815	17714
1.600	60023	-26302	63061	-27633	17714
1.548	57283	-23562	60182	-24755	17714
1.500	54796	-21075	57569	-22142	17714
1.455	52532	-18811	55191	-19763	17714
1.412	50464	-16744	53018	-17591	17714
1.371	48572	-14851	51030	-15602	17714
1.333	46834	-13114	49205	-13777	17714
1.297	45236	-11515	47525	-12098	17714
1.263	43762	-10041	45977	-10550	17714
1.231	42400	-8680	44546	-9119	17714
1.200	41139	-7419	43221	-7794	17714
1.171	39969	-6249	41992	-6565	17714
1.143	38882	-5161	40850	-5423	17714
1.116	37870	-4149	39786	-4359	17714
1.091	36926	-3205	38794	-3367	17714
1.067	36044	-2323	37868	-2441	17714
1.043	35219	-1498	37001	-1574	17714
1.021	34446	-725	36189	-762	17714
1.000	33721	0	35427	0	17714

4.2 Example Problem E-KD-2.1.2 – Determination of Design Pressure in Cylindrical Vessel – Dual Layered Vessel

Determine the design pressure for a dual wall cylindrical vessel given the following data. Perform calculations for both open and closed-end vessels.

Vessel Data:

- Liner Material = SA-705 Gr. XM-12 Condition H1100
  - Yield Strength = 115,000 psi @ 70°F per Table Y-1 of Section II, Part D

  - Tensile Strength = 140,000 psi @ 70°F
- Body Material = SA-723 Gr. 2 Class 2
  - Yield Strength = 120,000 psi @ 70°F per Table Y-1 of Section II, Part D
  - Tensile Strength = 135,000 psi @ 70°F per Table U of Section II, Part D
- Design Temperature = 70°F
- Liner Inside Diameter = 16.00 in
- Liner Outside Diameter = 24.00 in
- Outer Body Inside Diameter = 23.950 in
- Outer Body Outside Diameter = 50.0 in
- Overall Diameter Ratio (Y) = 3.125

Note that this calculation does not account for any loading in addition to internal pressure. If shell is subject to additional loading, the design shall be modified per KD-221.5 so that the collapse pressure is greater than or equal to 1.732 times the design pressure.

Evaluate design pressure per KD-220 for a closed-end cylindrical shell and for an open ended cylindrical shell with Y > 2.85.

$$P_D = \min\left[\sum_{j=1}^{2}\left(\frac{1}{1.25}\cdot S_{y_j}\cdot \ln\left(Y_j\right)\right), \sum_{j=1}^{2}\left[\frac{1}{3}\cdot\left(S_{y_j}+S_{u_j}\right)\ln\left(Y_j\right)\right]\right]$$

The design pressure is 97,029 *psi.*

4.3 Example Problem E-KD-2.2.1 – Elastic Plastic Analysis

Evaluate a monobloc vessel for compliance with respect to the elastic-plastic analysis criteria for plastic collapse provided in paragraph KD-231.

a) **STEP 1** – Develop a numerical model using finite element analysis of the component including all relevant geometry characteristics. The model used for the analysis shall accurately represent the component geometry, boundary conditions, and applied loads. In addition, the model shall be refined around areas of stress and strain concentrations to accurately assess local areas for the criteria to be satisfied in KD-230. The analysis of one or more numerical models may be required to ensure that an accurate description of the stress and strains in each component is achieved.

   The model geometry is depicted in Figure E-KD-2.2.1-1. The monobloc vessel model with the finite element mesh is shown in Figure E-KD-2.2.1-2.

Vessel Data

- Material – All Components = SA-723 Grade 2 Class 2
- Design Pressure = 45,000 psi at 150°F
- Operating Pressure = 40,000 psi at 100°F
- Elastic Modulus = 27.37×10^{6} ksi at 150°F (design condition), and 27.64×10^{6} ksi at 100°F(operating condition) ASME Section II Part D, Table TM-1, Material Group B

- Yield Strength = 117 ksi at 150°F and 120 ksi at 100°F, ASME Section II Part D, Table Y-1
- Ultimate Strength = 135 ksi at both 100°F and 150°F ASME Section II Part D, Table U
- Density = 0.280, ASME Section II Part D, Table PRD
- Poisson's Ratio = 0.3, ASME Section II Part D, Table PRD
- Minimum specified elongation – 14%
- Minimum specified reduction in area – 45%

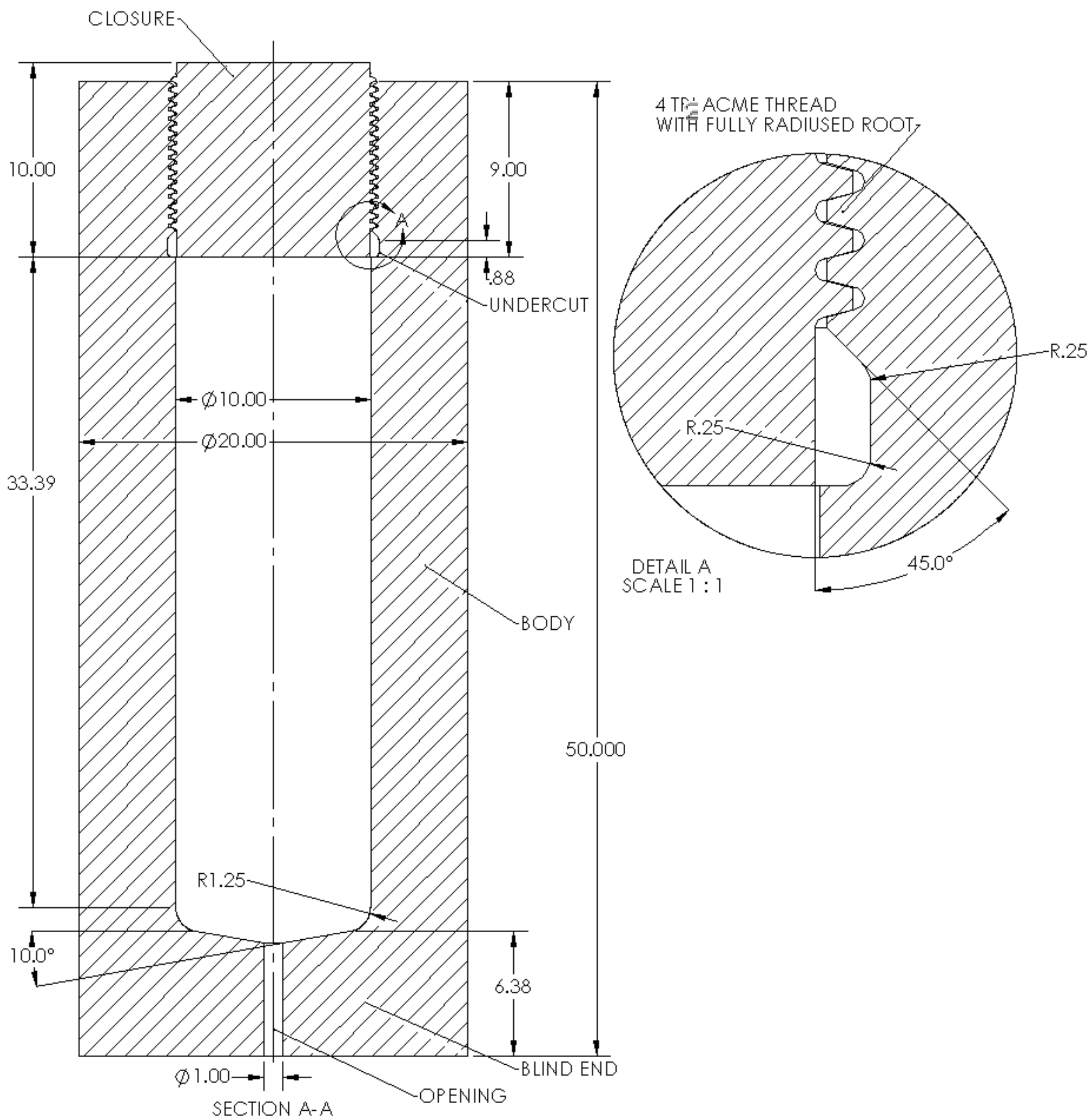

Figure 3 – E-KD-2.2.1-1 – ASME Section VIII Division 3 Monobloc Vessel Configuration (Y = 2.0) with 2 TPI ACME thread with full radius root

Note: Dimensions are in inches unless otherwise specified.

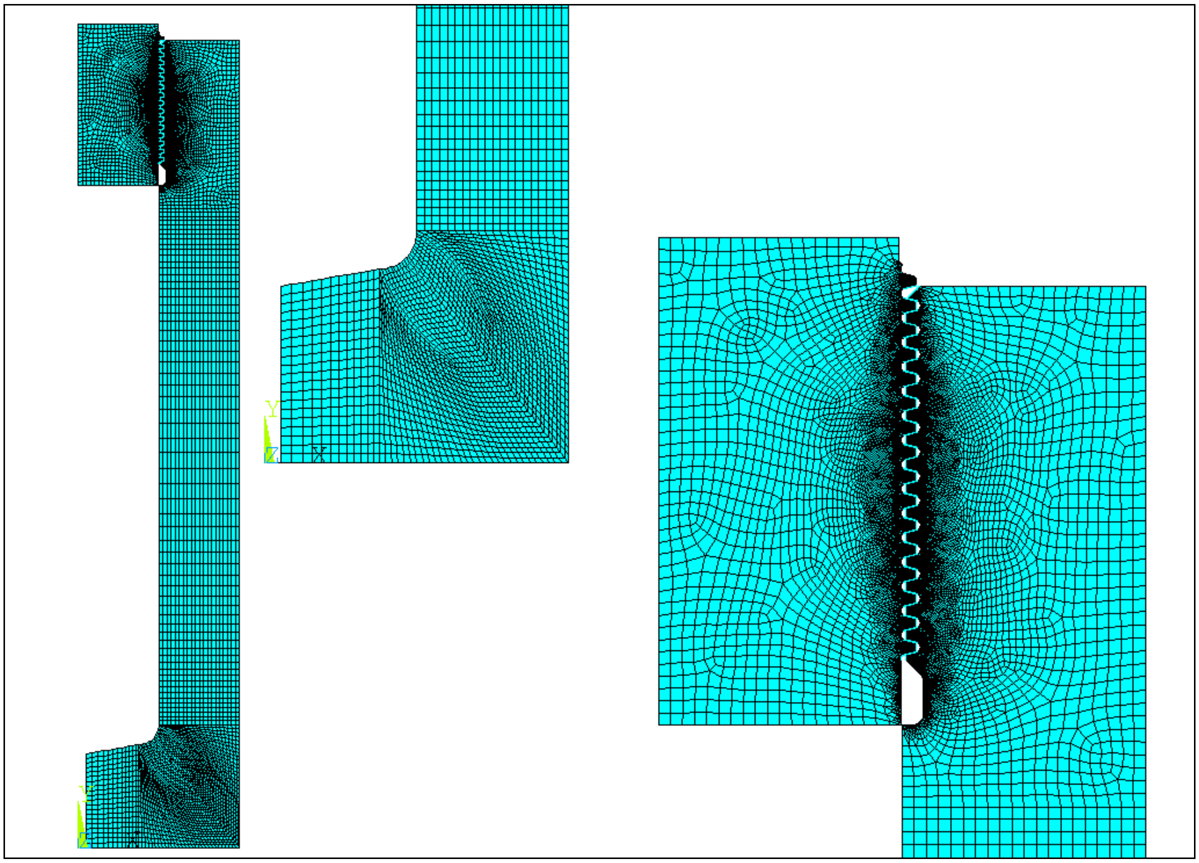

Figure 4 – E-KD-2.2.1-2 – Mesh of the Monobloc Vessel with Detailed Views of the Blind End, Closure and Body Threaded Connection

b) **STEP 2** – Define all relevant loads and applicable load cases. The loads to be considered in the analysis shall include, but not be limited to, those given in Table KD-230.1.

The primary loads to be considered are internal pressure and dead weight factored according to Table KD-230.4, in this example two load cases are analyzed that are as shown below:

Load case	Criteria	Load Combination[1, 2]
LC # 1	Global Design Condition	$1.8\ (P_D + D)$
LC # 2	Global Hydrostatic Test Condition	$P_t + D$ [3]

Notes:

1. P_D refers to design pressure of 45,000 psi, P_T refers to hydrostatic test pressure of $1.25{*}P_D$ multiplied by ratio of yield strength at test temperature to the yield strength at design temperature (120 ksi / 117 ksi) which is equal to $1.28{*}P_D$ (57,600 psi), and D refers to dead weight applied as acceleration ($1g$) in the finite element model.
2. The static head in the vessel is negligible compared to the pressure of the vessel.

3. Note that the S_y/S_u for this case is in excess of 0.72, so per KD-231.2(d) the hydrostatic testing criterion is non-mandatory. However, it was evaluated for demonstration purposes of this problem. It did not need to be completed.

The boundary conditions and loading applied on the model are as shown in Figure E-KD-2.2.1-3. It should be noted that the edge of the blind end at the opening is fixed vertically through a distance of 1 inch, as shown in the figure below, to simulate a threaded connection in that region. Frictionless contact was applied between the cover and body threads

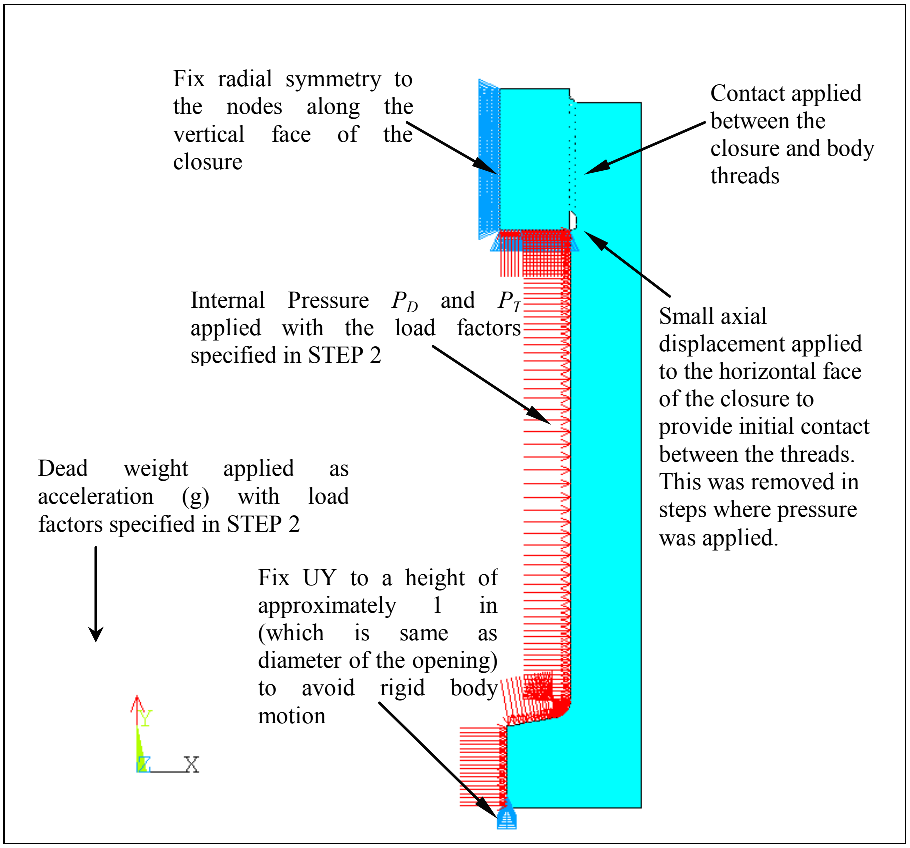

Figure 5 – E-KD-2.2.1-3 – Load and Boundary Conditions on the Monobloc Model

c) **STEP 3** – An elastic-plastic material model shall be used in the analysis for LC # 1 global design condition. The von Mises yield function and associated flow rule should be utilized if plasticity is anticipated. A material model that includes hardening or softening, or an elastic perfectly plastic model may be utilized. A true stress-strain curve model that includes temperature dependent hardening behavior is provided in KD-231.4. When using this material model, the hardening behavior shall be included up to the true ultimate stress and perfect plasticity behavior (i.e., the slope of the stress-strain curves is zero) beyond this limit. The effects of nonlinear geometry shall be considered in the analysis.

The material model for the hydrostatic test pressure case was an elastic-perfectly plastic model.

The true stress-strain curve from KD-231.4 was used for the analysis. The material keywords used in the ANSYS input file are shown below. See problem E-KD-2.2.4 for an example of the generation of a typical stress-strain curve using this method.

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TBPT,,1.0,120000
```

d) **STEP 4** – Perform an elastic-plastic analysis for each of the load cases defined in STEP 2. If a converged solution is achieved with the application of the full load, the component is acceptable for a given load case. If convergence is not achieved, the model of the vessel should be investigated to determine the cause of the non-convergence, and the design (i.e. thickness) should be modified or applied loads reduced and the analysis repeated until convergence is achieved.

The von Mises and equivalent plastic strain results for the two load cases evaluated are as shown in Figures E-KD-2.2.1-4 through E-KD-2.2.1-7, convergence was achieved therefore the vessel satisfies the global criteria for these load cases.

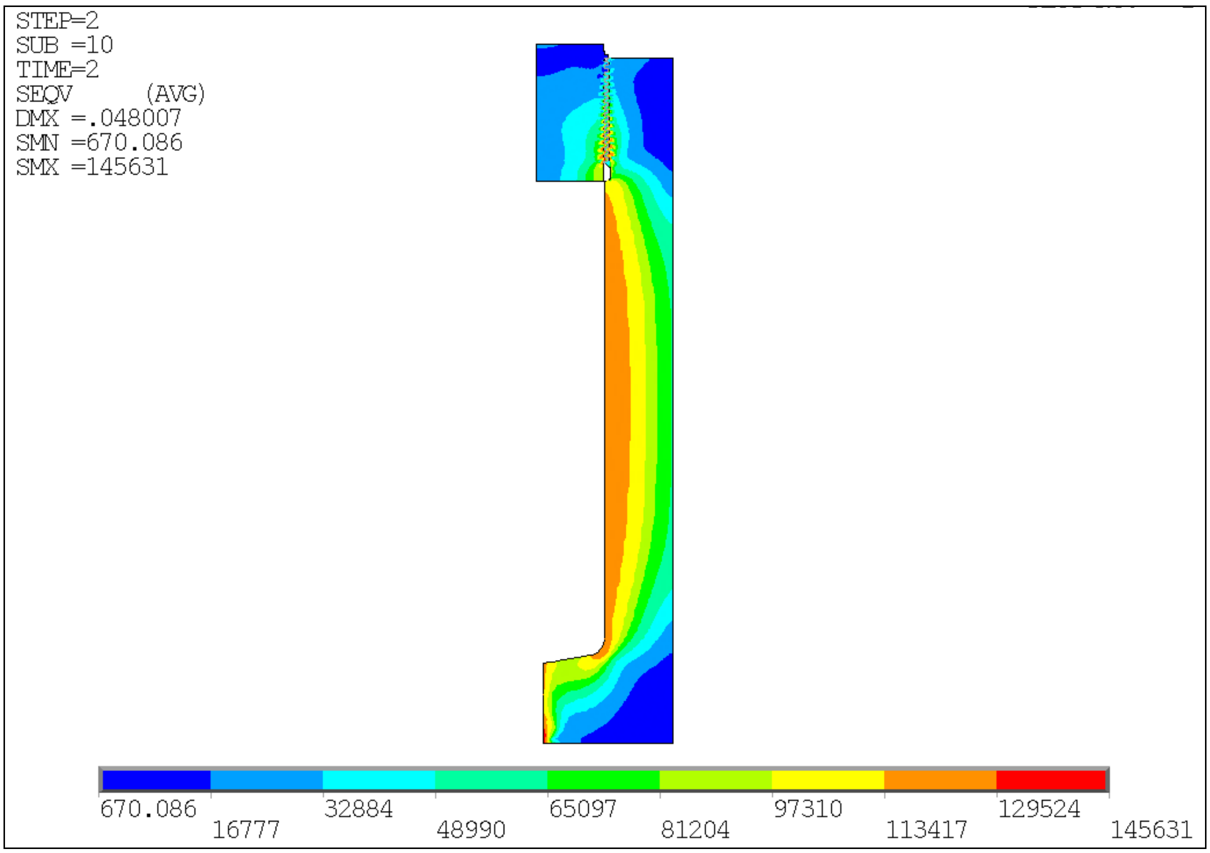

Figure 6 – E-KD-2.2.1-4 – Results of the Elastic-Plastic Analysis for LC #1 at a Factored Load of 81,000 psi and acceleration of 1.8 g; von Mises Stress

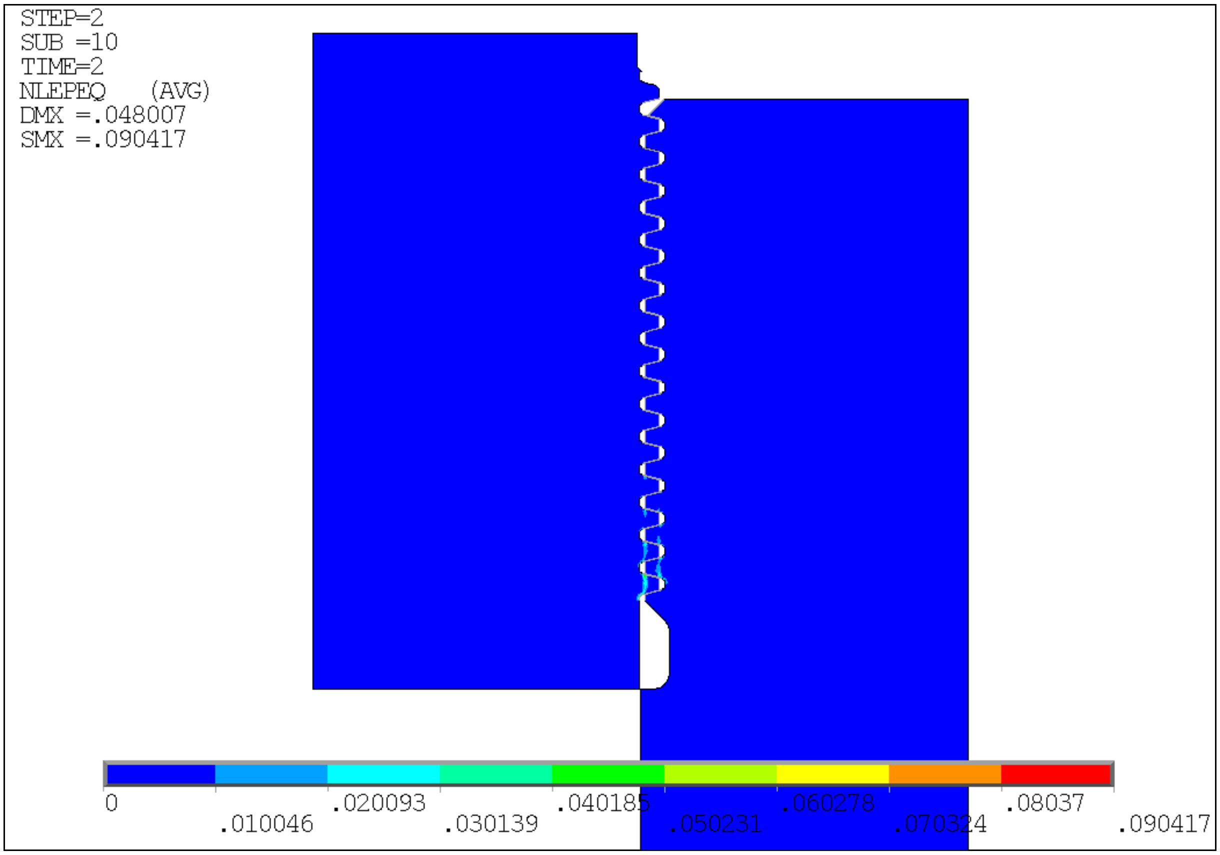

Figure 7 – E-KD-2.2.1-5 – Results of the Elastic-Plastic Analysis for LC #1 at a Factored Load of 81,000 psi and acceleration of 1.8g; Equivalent Plastic Strain

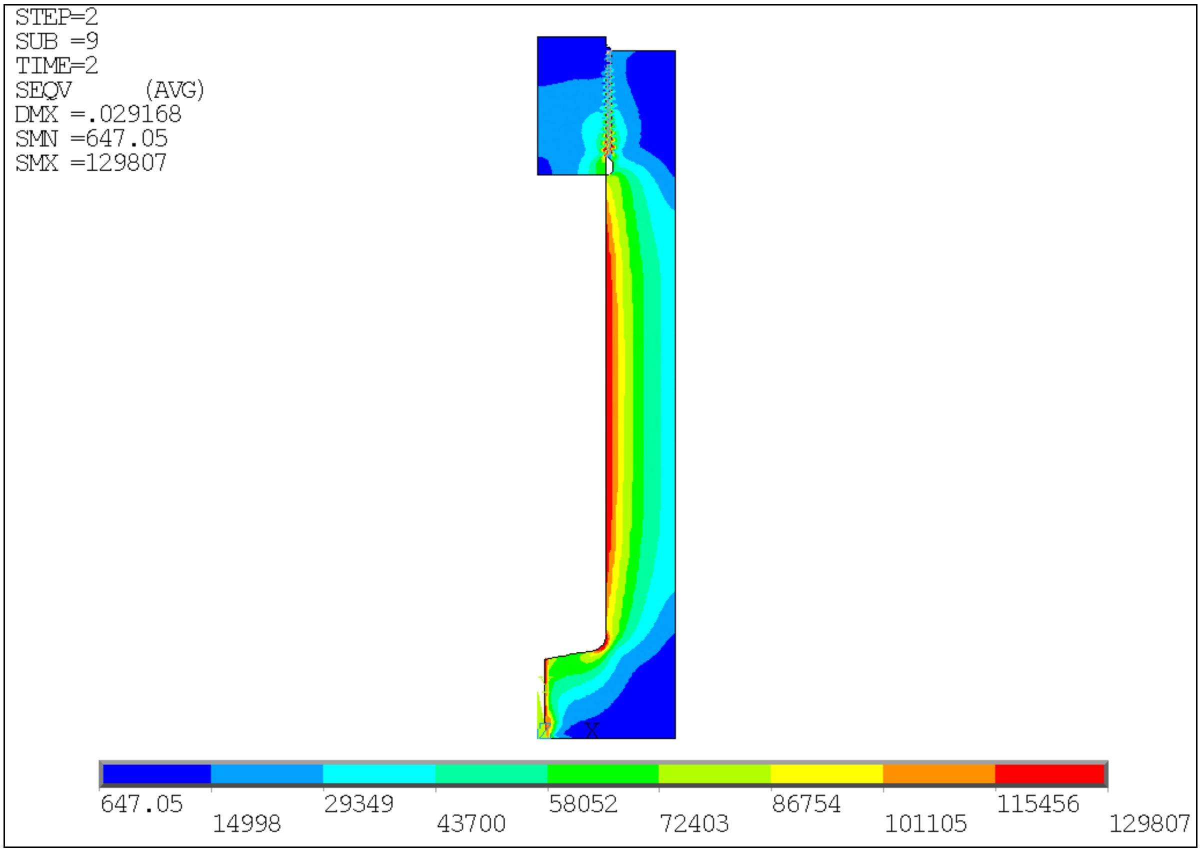

Figure 8 – E-KD-2.2.1-6 – Results of the Elastic-Plastic Analysis for LC #2 at a Factored Load of 57,600 psi and gravitational load of 1.0 g; von Mises Stress

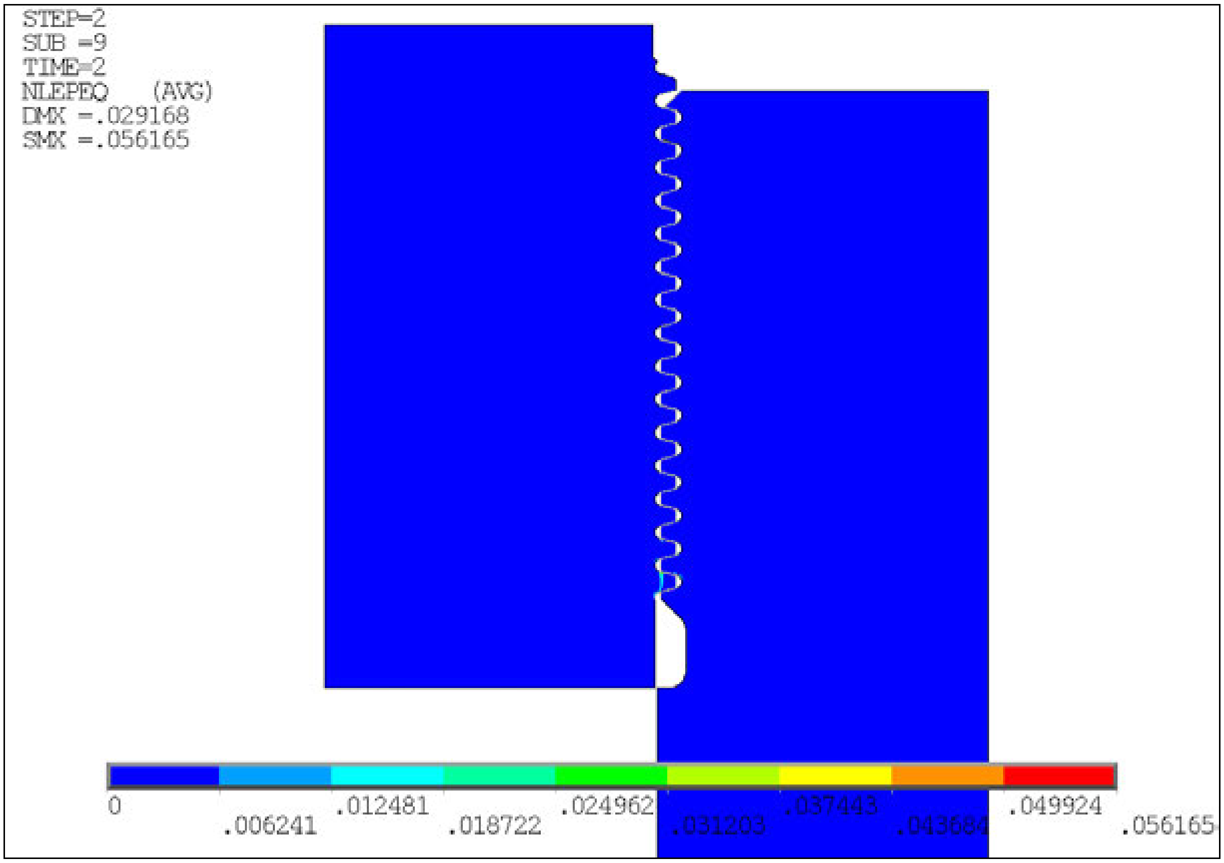

Figure 9 – E-KD-2.2.1-7 – Results of the Elastic-Plastic Analysis for LC #2 at a Factored Load of 57,600 psi and gravitational load of 1.0 g; Equivalent Plastic Strain

4.4 Example Problem E-KD-2.2.2 – Protection Against Local Failure (Elastic-Plastic Analysis)

The following procedure shall be used to evaluate protection against local failure for a sequence of applied loads.

a) **STEP 1** – Perform an elastic-plastic stress analysis based on the load case combinations for the local criterion given in Table KD-230.4. The effects of non-linear geometry shall be considered in the analysis.

   The same model and material conditions were used as an Example Problem E-KD-2.2.1. The only load to be considered is internal pressure factored according to Table KD-230.4 for the local criterion, i.e., $1.28(P_D + D)$ where P_D equals 45,000 psi and D is dead weight applied as acceleration due to gravity (1g).

b) **STEP 2** – For a location in the component subject to evaluation, determine the principal stresses, σ_1, σ_2, σ_3 the equivalent stress, σ_e, using Equation (13) of paragraph KD-232.1 and the total equivalent plastic strain ε_{peq}.

Values for the principal stresses, equivalent stress and total equivalent plastic strain for each point in the model were extracted from the Ansys results file. The following example calculation is for one integration point in the model. The full model (all integration points) will be evaluated using customized output from Ansys.

The principal stresses to be evaluated are shown below:

s_1 = - 4010 psi

s_2 = - 76,062 psi

s_3 = - 153,622 psi

s_e = 129,597 psi

c) **STEP 3** – Determine the limiting triaxial strain ε_L, using equation below, where ε_{Lu}, m_2, and m_5 are determined from the coefficients given in Table KD-230.5.

$$\varepsilon_L = \varepsilon_{Lu} \exp\left[\frac{-m_5}{1+m_2}\left(\frac{\sigma_1+\sigma_2+\sigma_3}{3\sigma_e}-\frac{1}{3}\right)\right]$$

The strain limit parameters are shown below:

R = 0.867

m_2 = 0.08

m_3 =0.262

m_4 = 0.5978

m_5 =2.2

e_{Lu} = 0.5978

The computed limit strain is:

ε_L = 4.011

d) **STEP 4** – Determine the forming strain ε_{cf} based on the material and fabrication method in accordance with Part KF. If heat treatment is performed in accordance with Part KF, the forming strain may be assumed to be zero. The forming strain is:

$$\varepsilon_{cf} = 0$$

e) **STEP 5** – Determine if the strain limit is satisfied. The location in the component is acceptable for the specified load case if equation below is satisfied.

$$\varepsilon_{peq} + \varepsilon_{cf} \le \varepsilon_L$$

The total equivalent plastic strain is:

ε_{peq} = 0.0185

i.e., $\varepsilon_{peq} \le \varepsilon_L$ since $\varepsilon_{cf} = 0$, the strain at this integration point passes the Elastic-Plastic criterion.

A full model contour plot of the strain limit is shown in Figure E-KD-2.2.2-1.

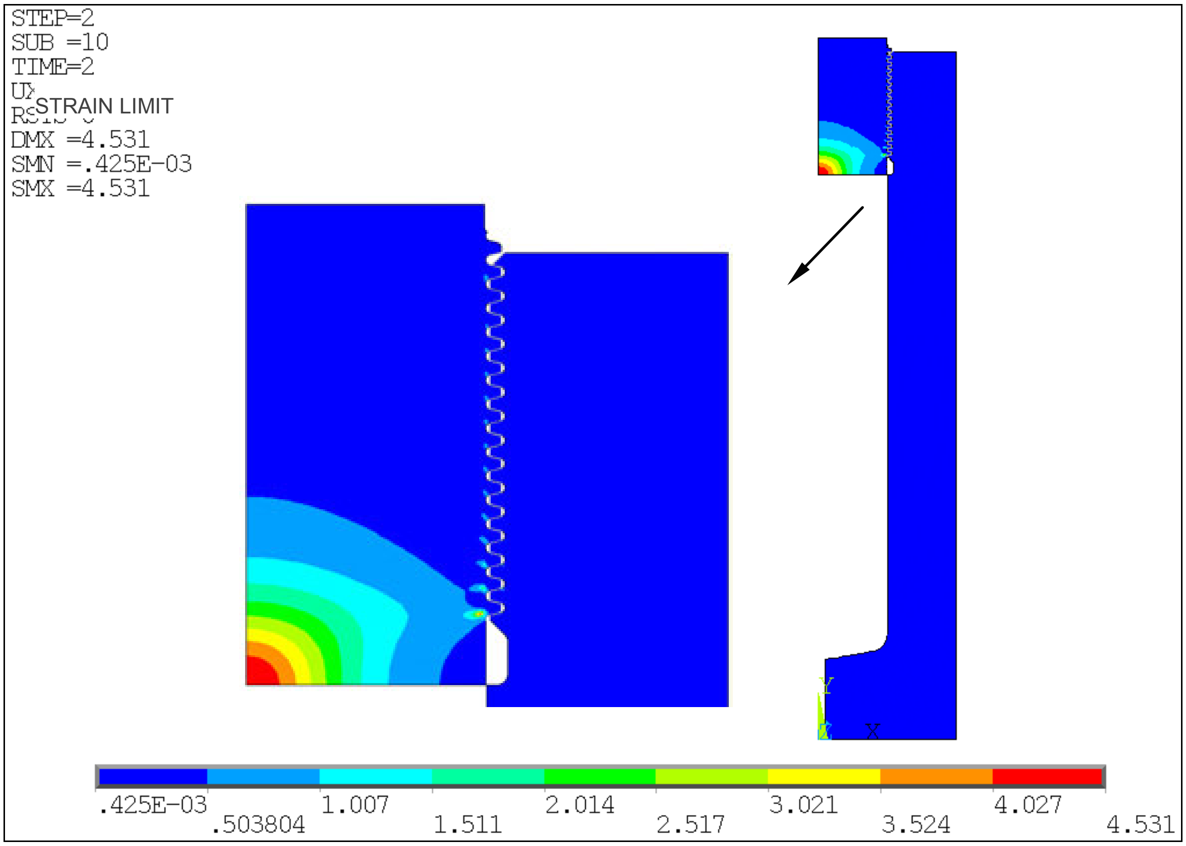

Figure 10 – E-KD-2.2.2-1 – Contour Plot of the Strain Limit, ε_L

A full model contour plot of the equivalent plastic strain is shown in Figure E-KD-2.2.2-2.

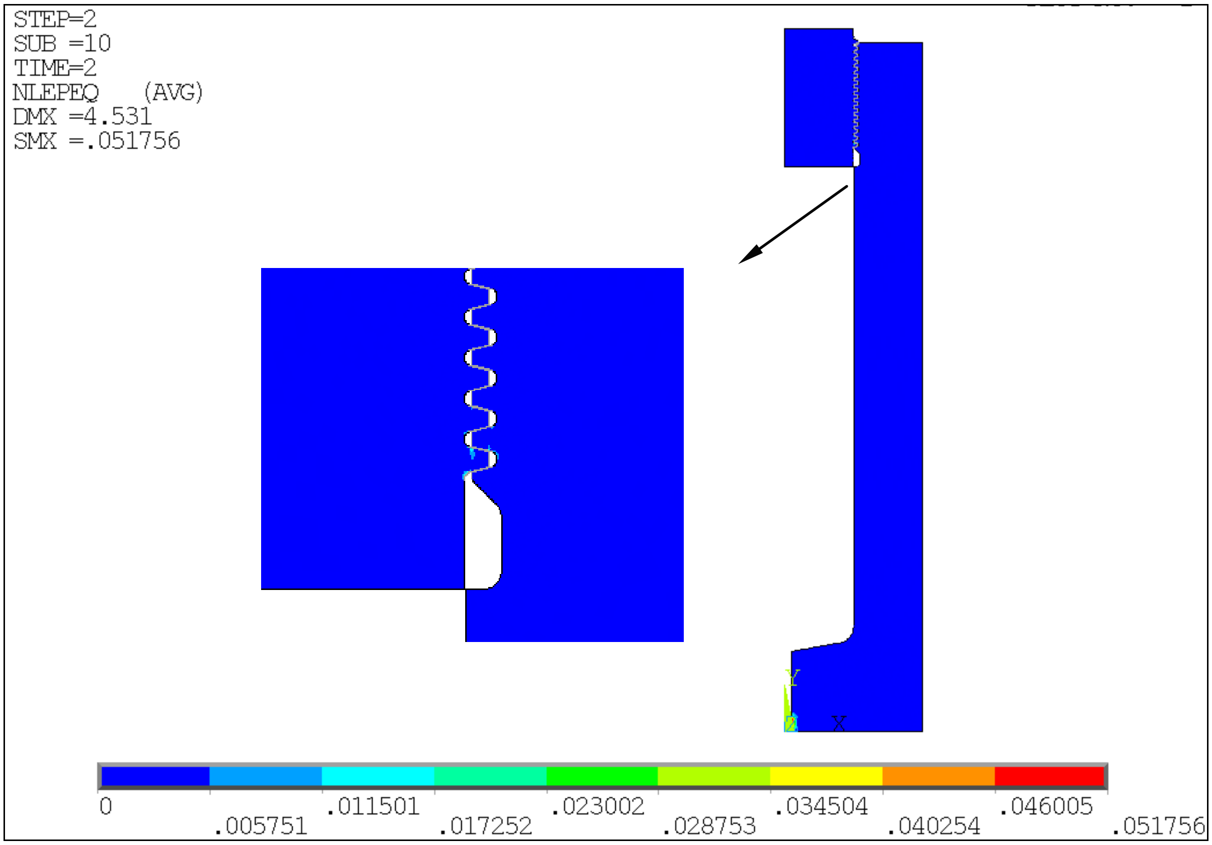

Figure 11 – E-KD-2.2.2-2 – Contour Plot of Equivalent Plastic Strain, ε_{peq} - Local Criteria

Full model evaluation of the Elastic-Plastic criterion is shown in Figure E-KD-2.2.2-3.

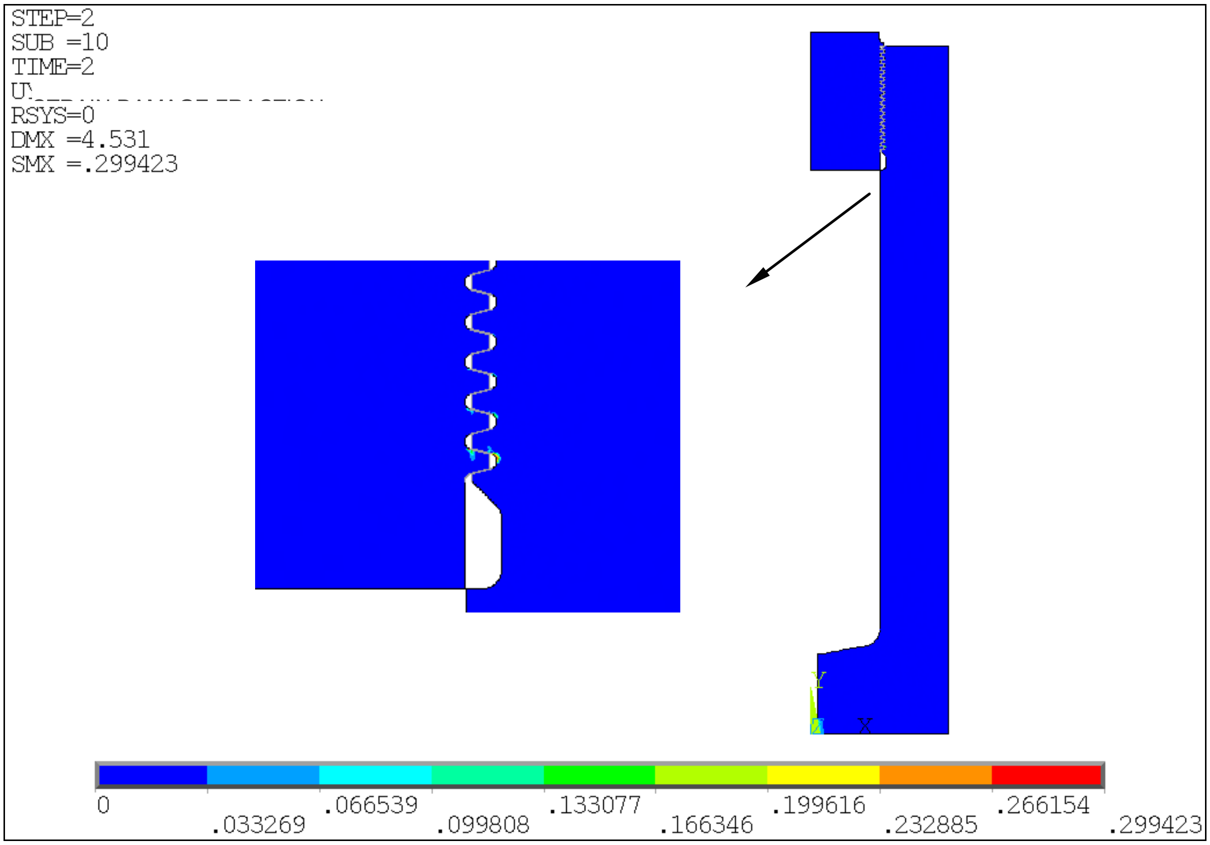

Figure 12 – E-KD-2.2.2-3 – Elastic-Plastic Strain Limit Ratio Results for Local Failure Analysis Results at 57,600 psi

Full model evaluation indicates that all integration points meet the criterion of $\varepsilon_{peq} + \varepsilon_{cf} \leq \varepsilon_L$. The maximum strain limit ratio $[(\varepsilon_{peq} + \varepsilon_{cf}) / \varepsilon_L]$ for this model is 0.2994, as indicated in Figure E-KD-2.2.2-3. Since this value is less than 1.0 the model passes the elastic-plastic local strain analysis.

4.5 Example Problem E-KD-2.2.3 – Ratcheting Assessment Elastic-Plastic Stress Analysis

Evaluate the monobloc vessel shown in Example Problem E-KD-2.2.1 for compliance with respect to the elastic-plastic ratcheting criteria provided in paragraph KD-234.

a) **STEP 1** – Develop a numerical model of the vessel components. The axisymmetric finite element model geometry was taken from Example E-KD-2.2.1 (see Figures E-KD-2.2.1-1 and E-KD-2.2.1-2). The boundary conditions and relevant internal pressure load applied as shown in Figure E-KD-2.2.3-1.

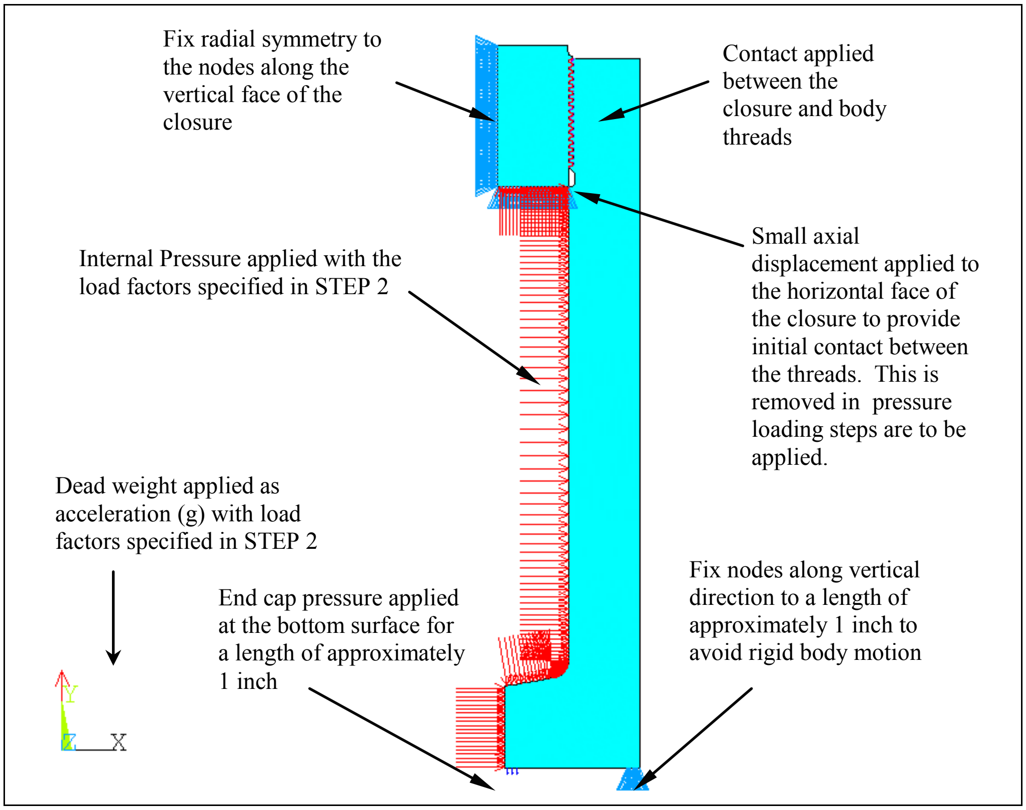

Figure 13 – E-KD-2.2.3-1 – Loads and Boundary Conditions on the Monobloc Model for Ratcheting Assessment

b) **STEP 2** – Define all relevant loads and applicable load cases. The loads are considered in accordance with Table KD-230.1 are internal pressure and dead weight. First the vessel is ramped up to hydrostatic test pressure ($1.28P_D$ = 57,600 psig) and then cycled with internal pressure between 0 psig and operating pressure 40,000 psig for three cycles.

c) **STEP 3** – Modify the material model option used in ANSYS for Example Problem E-KD-2.2.1 from multilinear isotropic hardening (MISO) to multilinear kinematic hardening (KINH). The material model used here is an elastic-perfectly plastic material model. The effects of nonlinear geometry shall be considered. The true stress-strain data for SA-723, Grade 2, Class 2 material at operating temperature of 100°F is developed in accordance with paragraph KD-231.4 and the material keywords used in ANSYS are as shown below:

```
/COM,*************************************************************************
/COM, Material Properties
/COM,*************************************************************************

MPTEMP,      1, 100
MP, EX,      1, 27.64e6
MP, DENS,    1, 0.280
MP, NUXY,    1, 0.3

/COM,**************************************************************
/COM, True Stress-True Strain Data Elastic-Perfectly Plastic Model
/COM,**************************************************************

! SA-723 Gr 2 Class 2

TB,KINH,1, ,3                                        ! Activate a data
table
TBTEMP,100                                           ! Temperature = 20.0
TBPT,,0.0,0.0                                        ! Strain = 0.0, Stress
= 0.0
TBPT,,0.00434153,120000                              ! Strain = 0.00434153,
Stress = 120000
TBPT,,1.0,120000                              ! Strain = 1.0, Stress =
120000
```

d) **STEP 4** – Perform an elastic-plastic analysis using the applicable loading from STEP 2. The elastic-plastic analysis was performed using the 57,600 psig hydrostatic test pressure and 40,000 psig operating pressure and the elastic-perfectly plastic material model from STEP 3. A plot of the von Mises stress and equivalent plastic strain under these loads are shown in Figures E-KD-2.2.3-2 through E-KD-2.2.3-5.

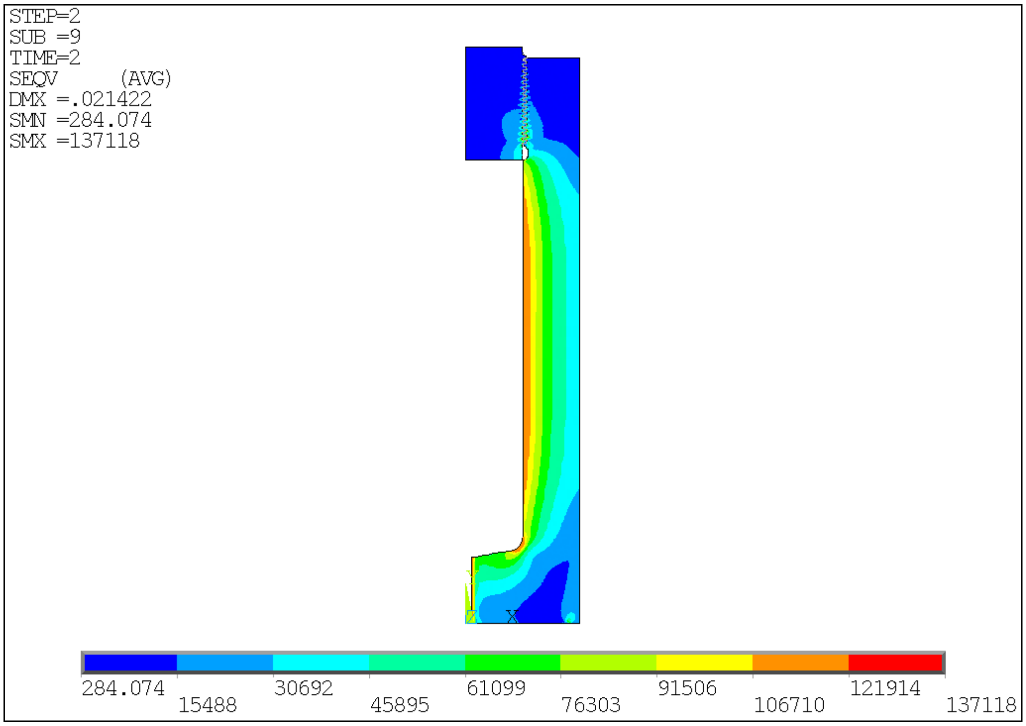

Figure 14 – E-KD-2.2.3-2 – von Mises Stress Plot for Hydrostatic Test Pressure of 57,600 psig

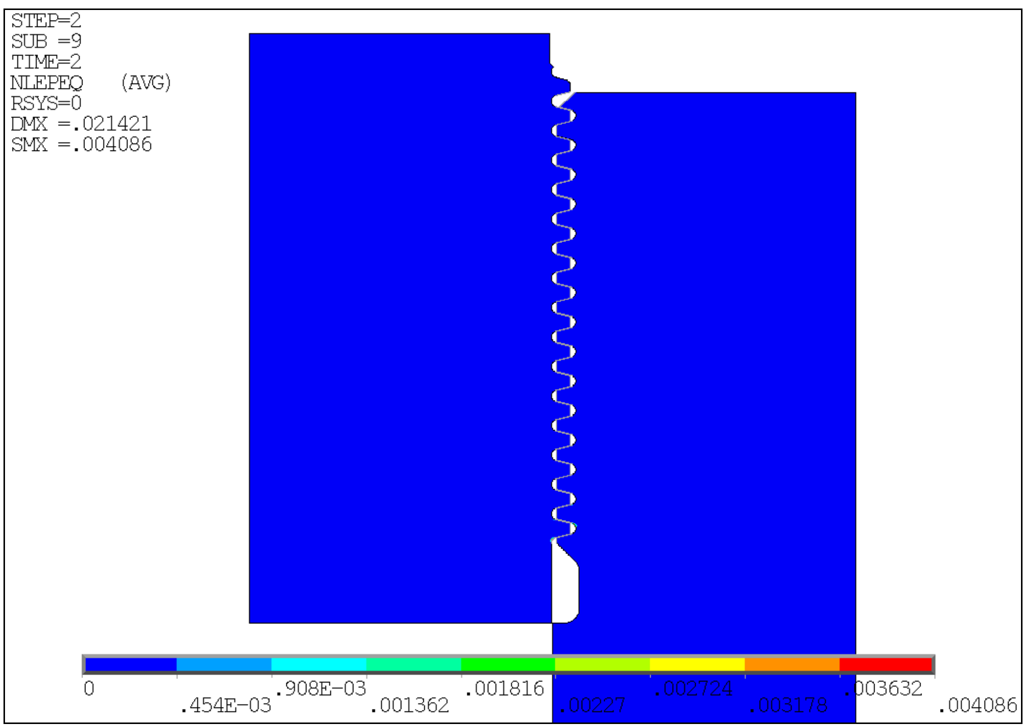

Figure 15 – E-KD-2.2.3-3 – Equivalent Plastic Strain for Hydrostatic Test Pressure of 57,600 psig

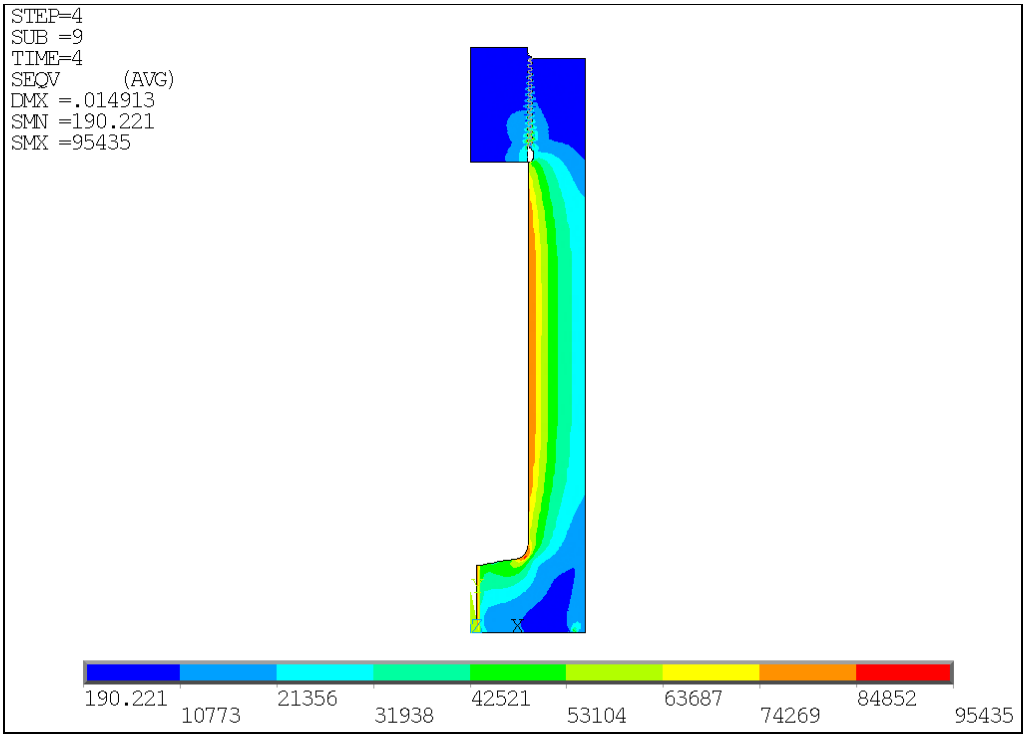

Figure 16 – E-KD-2.2.3-4 – von Mises Stress Plot for Operating Pressure of 40,000 psig, 1st cycle

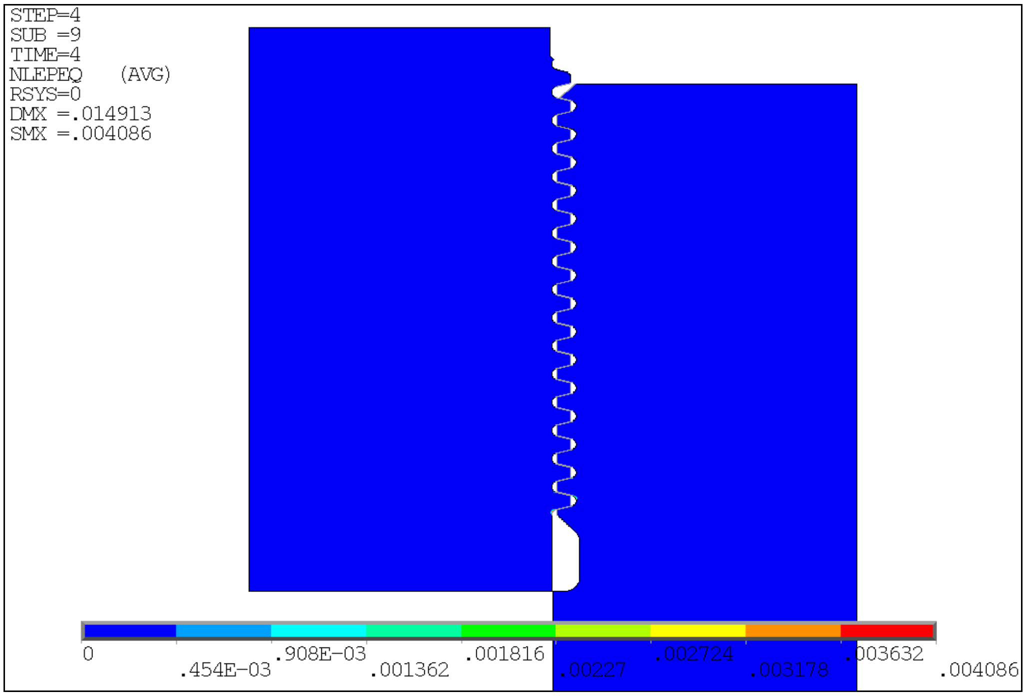

Figure 17 – E-KD-2.2.3-5 – Equivalent Plastic Strain for Operating Pressure of 40,000 psig, 1st cycle

e) **STEP 5** – Evaluate the ratcheting criteria in paragraph KD-234.1 STEP 5 at the end of the third cycle. E-KD-2.2.3-6 and E-KD-2.2.3-7 shows the von Mises stress and equivalent plastic strain in the model following the completion of the third cycle.

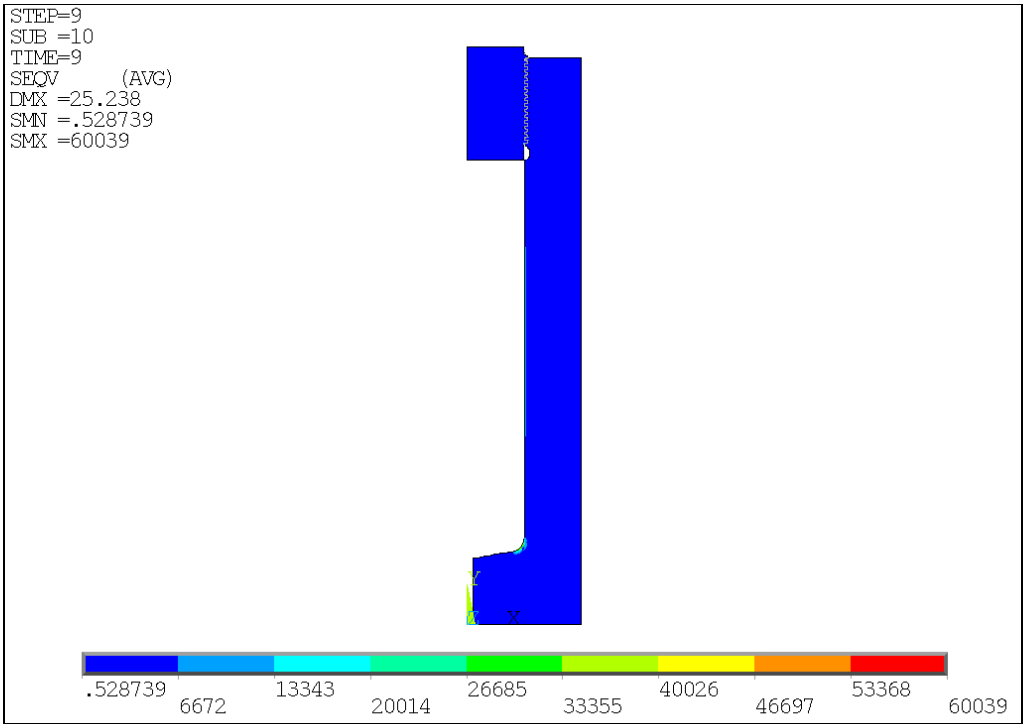

Figure 18 – E-KD-2.2.3-6 – von Mises Stress Plot for Operating Pressure of 40,000 psig, End of the 3rd cycle

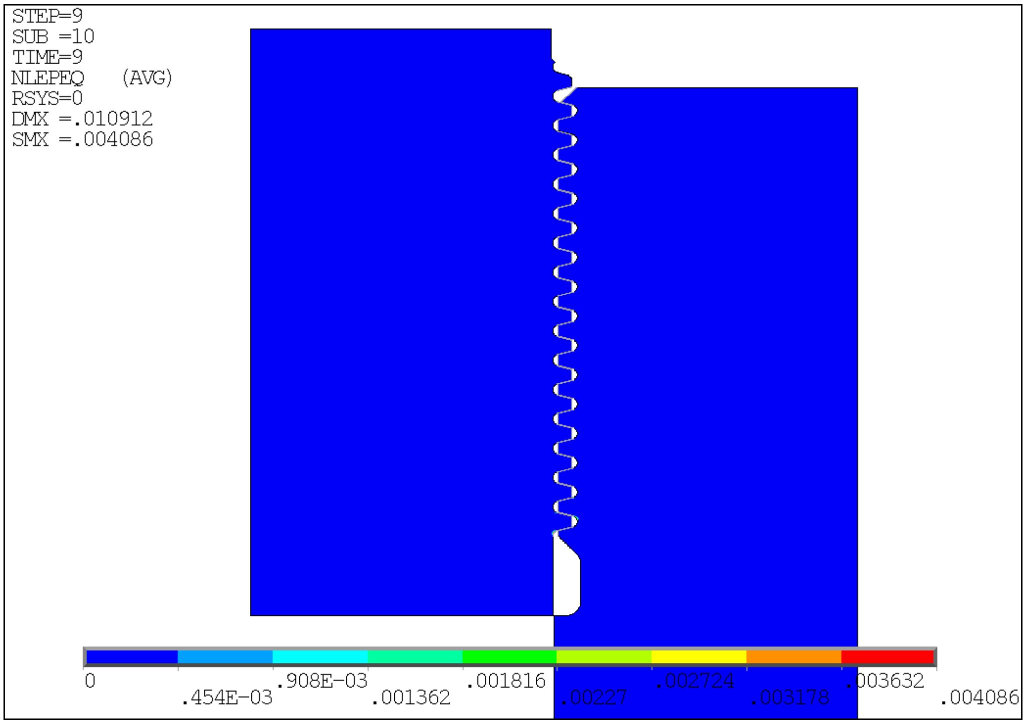

Figure 19 – E-KD-2.2.3-7 – Equivalent Plastic Strain for Operating Pressure of 40,000 psig, End of the 3rd cycle

It can be seen from Figures E-KD-2.2.3-3, E-KD-2.2.3-5 and E-KD-2.2.3-7 that the equivalent plastic strain does not change among these cases. In other words, the zero plastic strains have been incurred in the closure, body, and blind end of the monobloc vessel from 1st cycle to the end of 3rd cycle. Thus, these components meet the condition detailed in KD-234.1 STEP 5 (a) and the ratcheting criteria are satisfied. The vessel components, therefore, are acceptable per the elastic-plastic ratcheting criteria for an operating pressure cycle between 0 *psig* and 40,000 *psig*.

f) **STEP 6** – A full model contour plot of the strain limit is shown in Figure E-KD-2.2.3-8. The strain limit is determined using the equation given in KD-232.1 (c) built into an Ansys macro. The total accumulated damage at the end of the third operating cycle is shown in Figure E-KD-2.2.3-9. In the whole model the accumulated damage is less than 1.0 therefore satisfying the KD-232.1 (i) criteria.

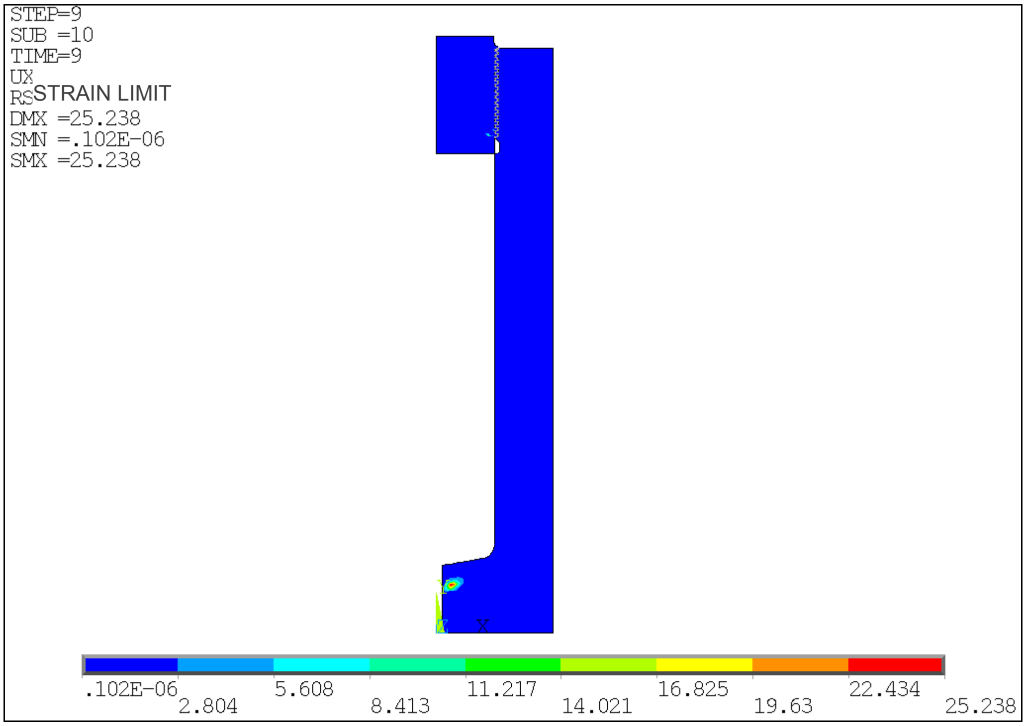

Figure 20 – E-KD-2.2.3-8 – Contour Plot of the Strain Limit, ε_L in the overall model – Ratcheting Criteria

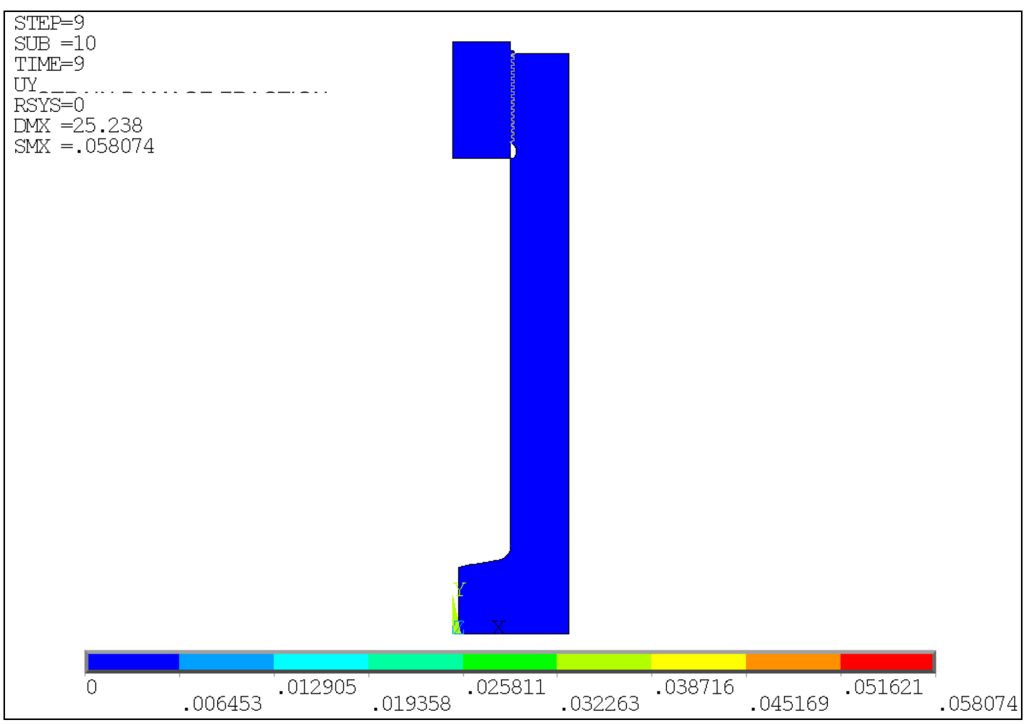

Figure 21 – E-KD-2.2.3-9 – Contour Plot of the Total Accumulated Damage, $D_{\varepsilon t}$ – End of 3rd Operating cycle

4.6 Example Problem E-KD-2.2.4 – Generate a Stress-Strain Curve for Use in Elastic-Plastic Finite Element Analysis

Generate a true stress – true strain curve for use in elastic-plastic finite element analysis. Generate this curve for SA-723 Grade 2 Class 2 material at 150°F.

Material Data:

- Engineering Yield Strength (0.2% Offset)(S_y) = 117,000 psi @ 150°F per Section II-D Table Y-1
- Engineering Tensile Strength (S_u) = 135,000 psi @ 150°F per Section II-D Table U
- Modulus of Elasticity (E_y) = 27,371 ksi per Section II Part D
- Material Parameter (ε_p) = 2×10^{-5}

STEP 1 – Constants Generation

The first step in the evaluation is to determine the constants required from Table KD-230.5 and paragraph KD-231.4. SA-723 is a ferritic steel.

$$m_1 = \frac{\left[\ln(R_{rat}) + (\varepsilon_p - \varepsilon_{ys})\right]}{\ln\left(\frac{\ln(1+\varepsilon_p)}{\ln(1+\varepsilon_{ys})}\right)} \qquad m_1 = 3.15 \times 10^{-2}$$

$$A_1 = \frac{\sigma_{ys}(1+\varepsilon_{ys})}{\left(\ln(1+\varepsilon_{ys})\right)^{m_1}} \qquad A_1 = 143\,\text{ksi}$$

$$R = S_y / S_u = 0.867$$

$$m_2 = 0.6(1 - R) = 0.080$$

$$A_2 = \frac{\sigma_{uts}\exp(m_2)}{m_2^{\,m_2}} \qquad A_2 = 178.991\,\text{ksi}$$

$K = 1.5R^{1.5} - 0.5R^{2.5} - R^{3.5} = 0.255$

The end result is to determine the true stress vs. true strain for the material over the range from zero plastic strain to the value corresponding to the ultimate tensile stress converted to true stress.

The true strain is calculated as a function of true stress (σ_t) using equation 1 of KD-231.4 as:

$$\varepsilon_{ts} = \frac{\sigma_t}{E_y} + \gamma_1 + \gamma_2$$

Where:

$$\gamma_1 = \frac{\varepsilon_1}{2}\left[1.0 - \tanh(H)\right]$$

$$\gamma_2 = \frac{\varepsilon_2}{2}\left[1.0 + \tanh(H)\right]$$

And:

$$\varepsilon_1 = \left(\frac{\sigma_t}{A_1}\right)^{\frac{1}{m_1}}$$

$$\varepsilon_2 = \left(\frac{\sigma_t}{A_2}\right)^{\frac{1}{m_2}}$$

$$H = \frac{2\left(\sigma_t - \left[\sigma_{ys} + K\left(\sigma_{uts} - \sigma_{ys}\right)\right]\right)}{K\left(\sigma_{uts} - \sigma_{ys}\right)}$$

STEP 2 – Determination of the Proportional Limit

The proportional limit is determined by evaluating the point at which the true plastic strain is equal to zero. This is determined through an iterative procedure. Note that the true stress (σ_{ts}) is equal to the true elastic stress (σ_{es}) plus the true plastic stress (σ_{ps}), i.e.:

$$\sigma_{ts} = \sigma_{es} + \sigma_{ps}$$

Also, at the proportional limit, there is no plasticity, therefore,

$$\varepsilon_{ts} = \varepsilon_{es} = \frac{\sigma_t}{E_y}$$

And:

$$g_1 + g_2 = 0$$

An iterative procedure is used to determine the value of σ_t = 80 ksi and ε_{ts} = 0.002923.

STEP 3 – Determine the End of the True Stress – True Strain Curve

The maximum stress on the true stress – true strain curve is limited to the tensile stress (S_u) converted in terms of true stress. This conversion is:

$$\sigma_{True\,UTS} = S_U\, e^{m_2} = 146.2\ ksi$$

A plot of the true stress – true strain curve for SA-723 Grade 2 Class 2 is shown in Figure E-KD-2.2.4-1.

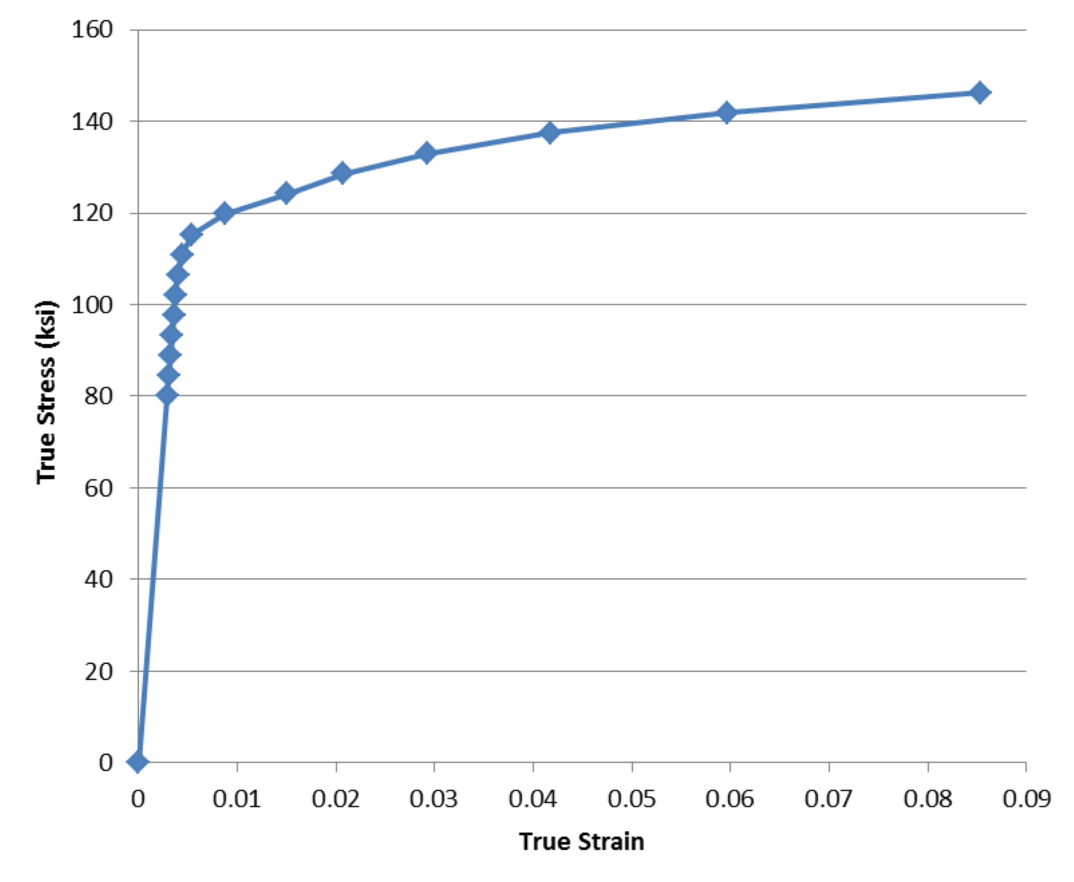

Figure 22 – E-KD-2.2.4-1 – True Stress – True Strain Curve for SA-723 Grade 2 Class 2

4.7 Example Problem E-KD-2.3.1 – Linear Elastic Stress Analysis

Evaluate a monobloc vessel body for compliance with ASME Section VIII, Division 3 according to the elastic stress analysis criteria provided in paragraph KD-240. Load due to internal pressure and the load acting on the body threads due to internal pressure applied on the closure inside surface are the loads that need to be considered. Relevant design data, geometry and nomenclature of the vessel body are provided below in Figure E-KD-2.3.1-1.

<u>Vessel Data</u>

- Material – All Components = SA-723 Grade 2 Class 2
- Design Pressure = 11,000 psig at 150°F
- Operating Pressure = 90% of Design Pressure = 9,900 psig at 150°F
- Elastic Modulus = 27.37×10^6 ksi at 150°F, ASME Section II Part D, Table TM-1, Material Group B
- Density = 0.280, ASME Section II Part D, Table PRD
- Poisson's Ratio = 0.3, ASME Section II Part D, Table PRD

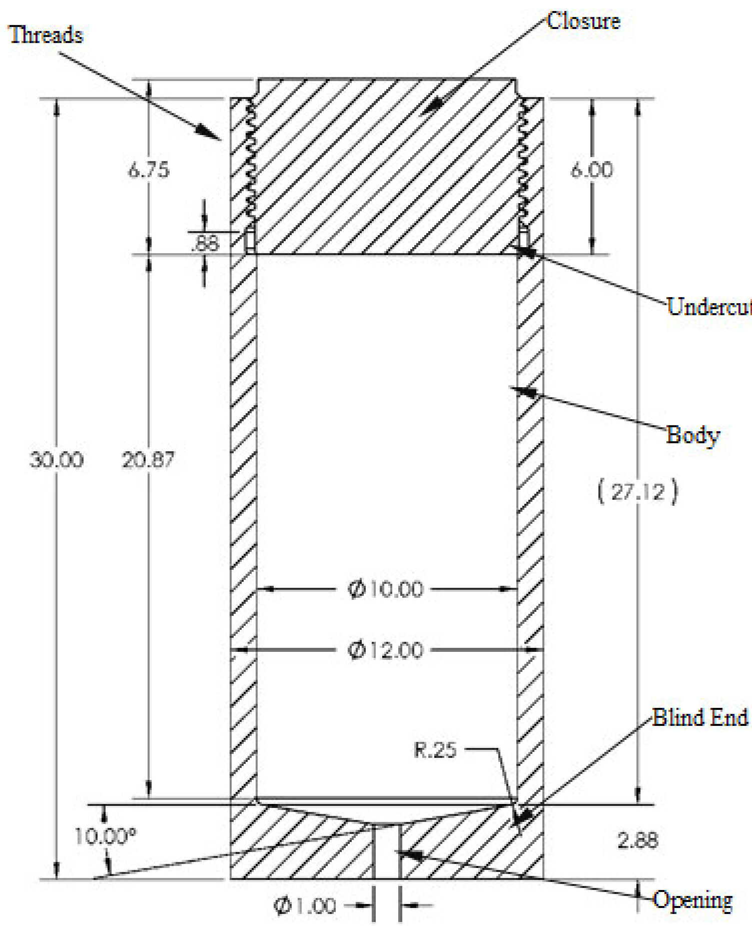

Figure 23 – E-KD-2.3.1-1 – ASME Section VIII Division 3 Monobloc Vessel Configuration with 2 TPI ACME thread with Full Radius Root

Note: Dimensions are in inches unless otherwise specified.

a) **STEP 1** – Determine that the vessel being analyzed has appropriate wall ratio for Linear Elastic Analysis per KD-200. KD-200(d) states that the wall ratio (Y=OD/ID) must be less than 1.25 to use the Linear Elastic Analysis method. The wall ratio of this vessel is Y=12 inches / 10 inches = 1.2.

b) **STEP 2** – Determine the types of loads acting on the component. In general, separate load cases are analyzed to evaluate “load-controlled” loads such as pressure and “strain-controlled” loads resulting from imposed displacements. The load analyzed is internal design/operating pressure. The resulting load on the body threads due to the internal pressure acting on the closure is also considered. Since distribution of the load is not uniform on all the threads, the load distribution on each thread is calculated per Appendix E-221 continuous load distribution equations as shown in example problem E-AE-2.2.1.

c) **STEP 3** – Develop the finite element model.

   1) Due to symmetry in geometry and loading, an axisymmetric solid model is generated. The axisymmetric model consists of the body shell, including the blind end with a centrally located opening and body threads. The closure component is not modeled and the pressure

load acting on the closure is modeled by transferring the load on to the body threads. The FE model is illustrated in Figure E-KD-2.3.1-2. The model was generated with the ANSYS 11.0 SP1 commercial FEA program.

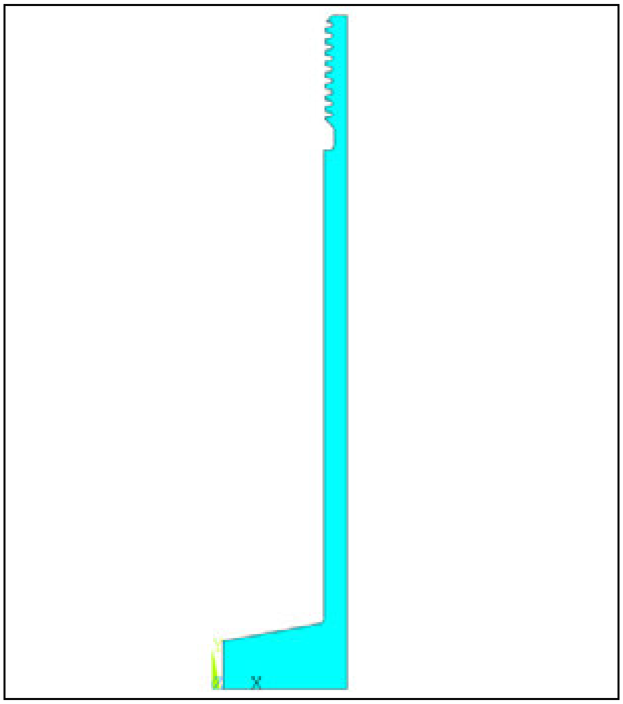

Figure 24 – E-KD-2.3.1-2 – Axisymmetric FE Model

2) Generate mesh. ANSYS 8-noded structural solid element (Plane 82) with axisymmetric key option is specified for the analysis. The mesh is illustrated in Figure E-KD-2.3.1-3.

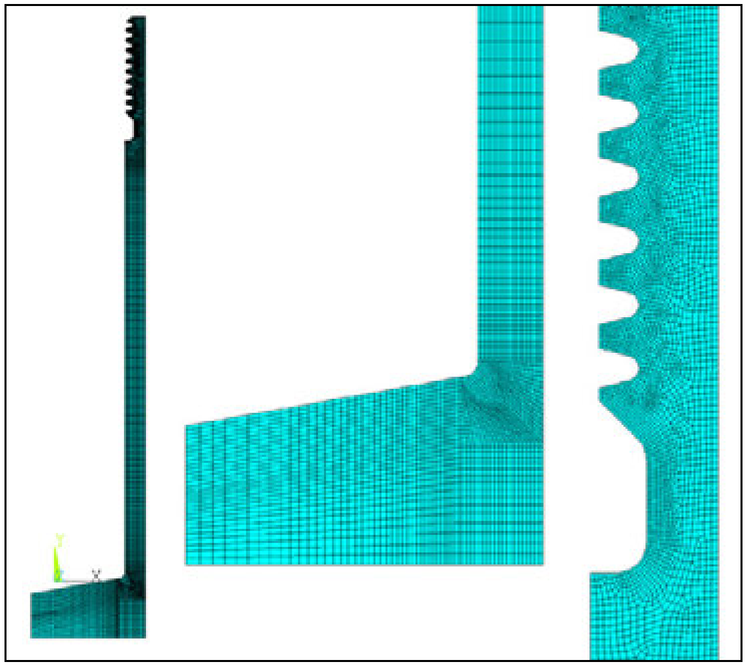

Figure 25 – E-KD-2.3.1-3 – Mesh of the Monobloc Vessel with Detailed Views of the Blind End and Body Thread Components

3) Apply the material properties given below to all the components in the monobloc vessel.

Component	Material	Modulus of Elasticity (psi)	Poisson Ratio
All	SA-723 Grade 2 Class 2	27.37E+06	0.3

4) Apply the internal pressure load to the pressure boundaries of the body shell and the blind end. Also, transfer the internal pressure acting on the closure on to the body threads by applying the loads on the body threads. The load applied on the threads is not equally distributed among all the threads. The first thread takes most of the load while the last thread takes a small portion of the total load applied. The percentage of the total load applied on individual threads is calculated using the equations given in E-221 and Table E-222.1. The actual percentage load applied on the individual treads is calculated in example problem E-AE-2.2.1 and shown in Figure E-KD-2.3.1-4. Apply the appropriate boundary conditions to the body as per the figure. The edge of blind end at the opening is fixed vertically through a distance of 1 inch, as shown in the figure below, to simulate a threaded connection in that region.

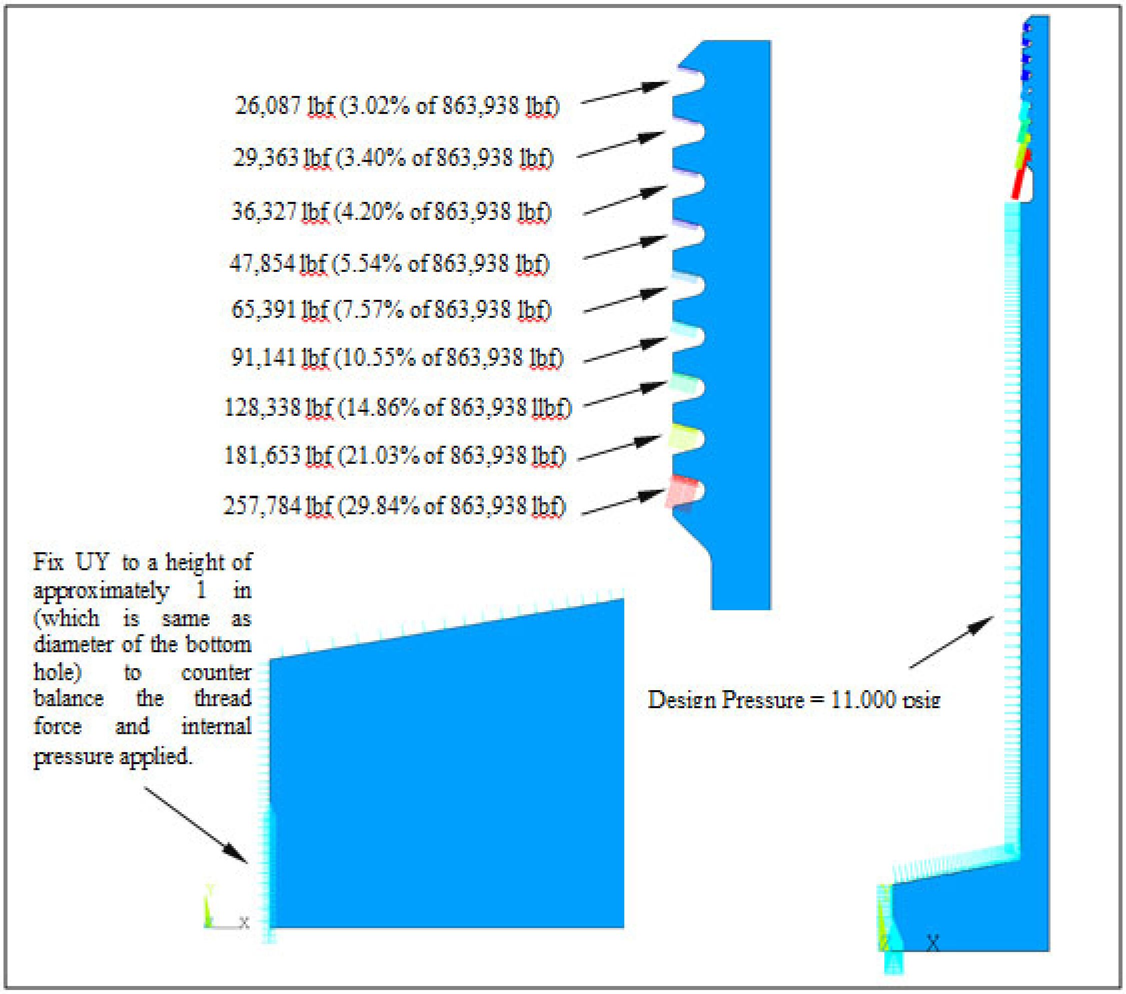

Figure 26 – E-KD-2.3.1-4 – Load and Boundary Conditions for the FE Model

Note: Thread load distribution on each individual body threads is calculated as shown in example problem E-AE-2.2.1

d) **STEP 4** – Run analysis and review results. Evaluate the displacements and compare calculated reaction force values to hand calculated values.

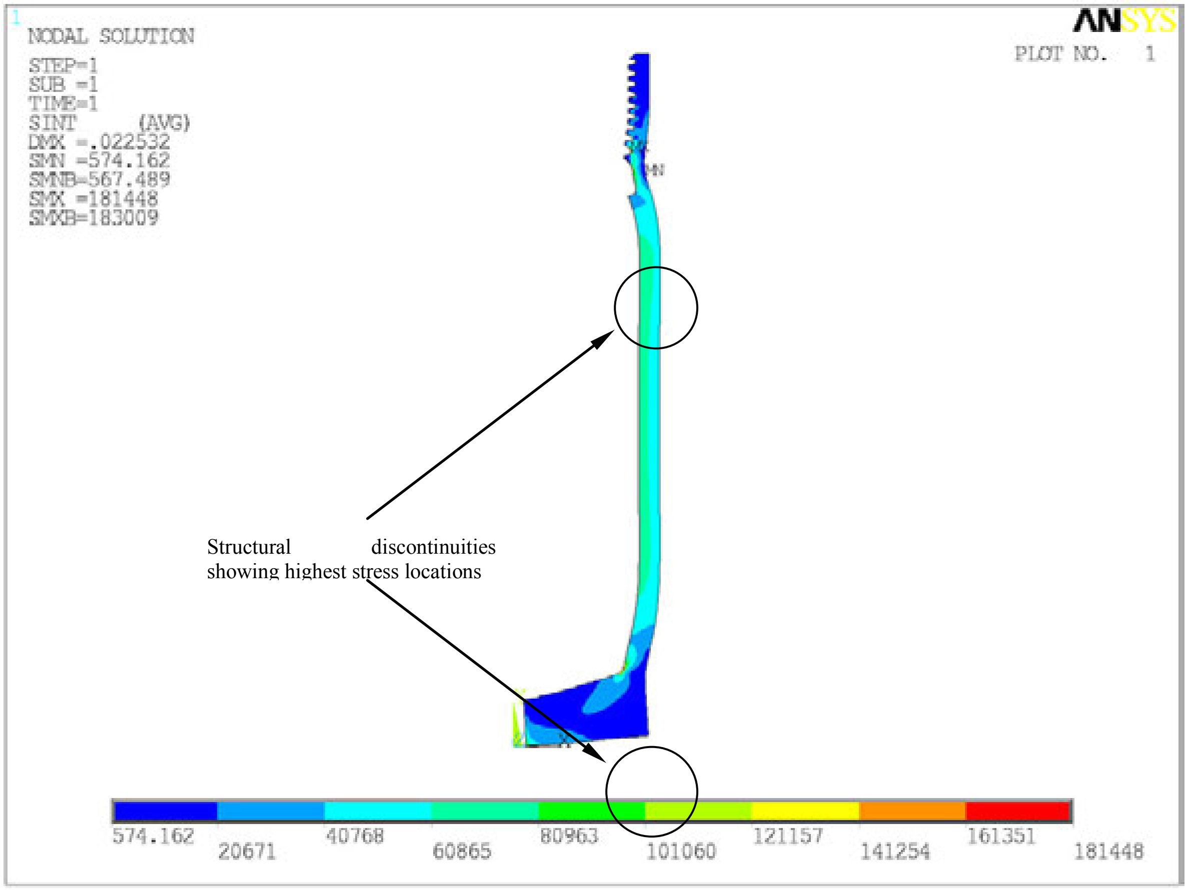

Figure 27 – E-KD-2.3.1-5 – Results of Elastic Analysis, Stress Intensity in Deformed State for Design Pressure and the Critical Locations through the Vessel Requiring Stress Evaluation

Reaction Force (y-direction)

Calculated (ANSYS)	8640.5 lbf
Hand Calculations	8639.4 lbf

Note: Results for Steps 4, 5, and 6 were calculated automatically by analysis routines contained in the ANSYS 11.0 SP1 commercial FEA program. Through-wall stress linearization was conducted at critical areas around the pressure boundary to provide data for the routines. The resultant stress intensities for P_m, P_L, and P_b stress categories are summarized in Tables E-KD-2.3.1-1 and E-KD-2.3.1-2 for design and operating pressures, respectively.

Note that per L-311 Step 2 (a), bending stresses are calculated only for the local hoop and meridional (normal) component stresses, and not for the local component stress parallel to the SCL or in-plane shear stress.

e) **STEP 5** – At the point on the vessel that is being investigated, calculate the stress tensor (six unique components of stress) for each type of load. Assign each of the computed stress tensors to one or to a group of the categories defined below. Assistance in assigning each stress tensor to an appropriate category for a component can be obtained by using Figure KD-240. Note that the stress intensities Q and F do not need to be determined to evaluate protection against plastic collapse. However, these components are needed for fatigue and shakedown/ratcheting assessment of the structure based on elastic stress analysis. Note that the $2*S_y$ limit placed on sum of Primary Local Membrane (P_m) plus Primary Bending (P_L) plus Secondary Membrane Plus Bending (Q) has been placed at a level to ensure shakedown to elastic action after a few repetitions of the stress cycle. See paragraph KD-3 for the evaluation of fatigue analysis.

General primary membrane stress intensity –P_m

Local primary membrane stress intensity –P_L

Primary bending stress intensity –P_b

Secondary stress intensity –Q

Additional equivalent stress produced by a stress concentration or a thermal stress over and above the nominal ($P+Q$) stress level –F

The stress intensity categories are determined for the SCLs depicted in Figures E-KD-2.3.1-6 through E-KD-2.3.*1*-8.

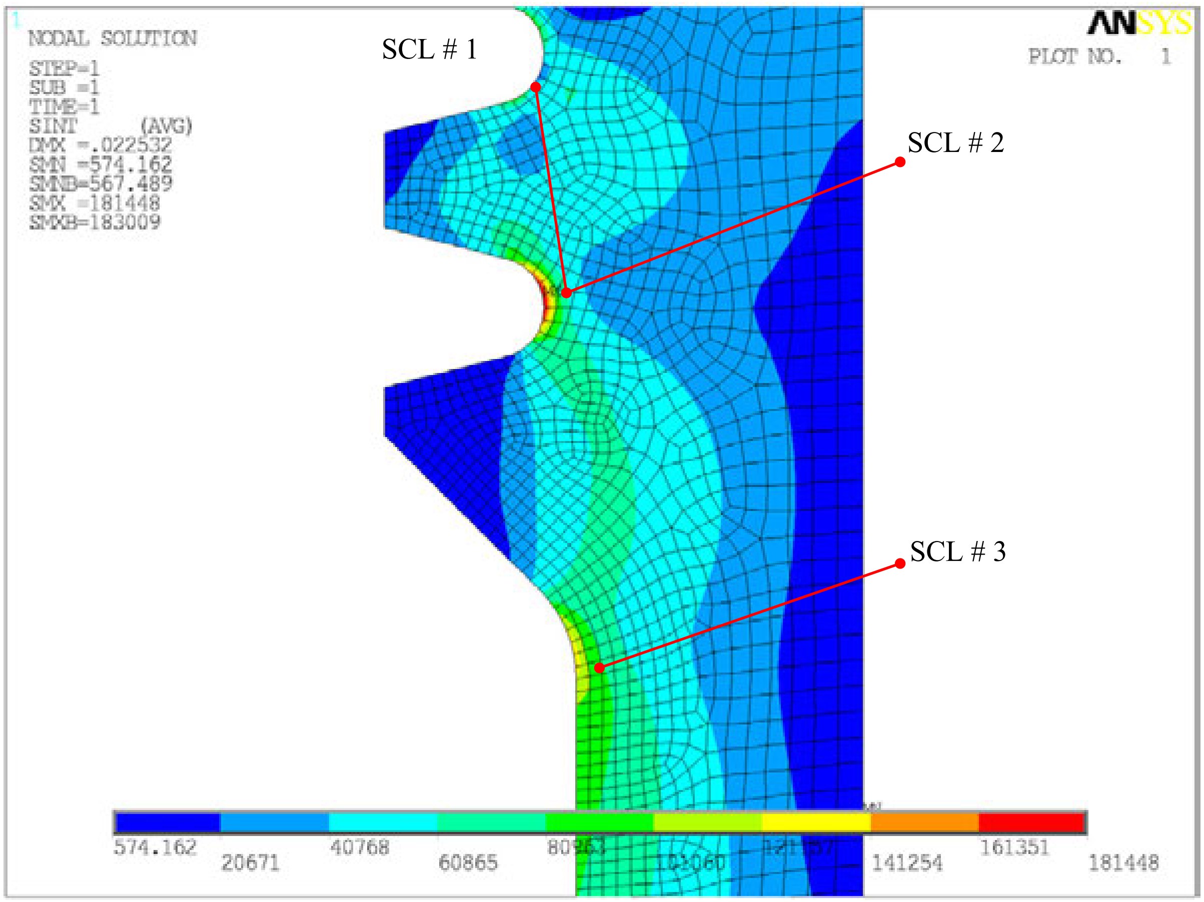

Figure 28 – E-KD-2.3.1-6 –Stress Classification Lines (SCLs) in the First Thread and Undercut Regions – Stress Intensity (psi)

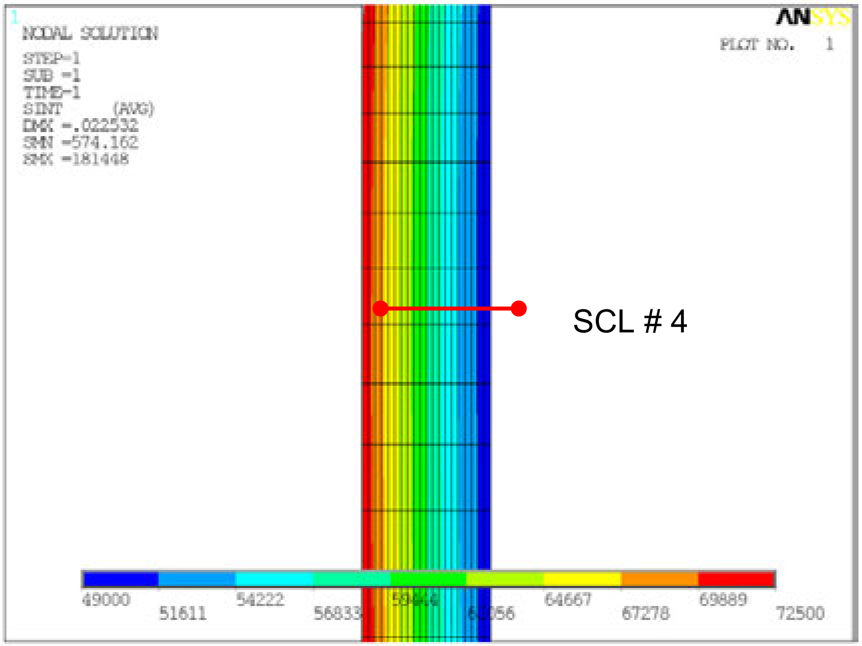

Figure 29 – E-KD-2.3.1-7 –Stress Classification Lines (SCLs) in the Body Shell Region Away from Discontinuities – Stress Intensity (psi)

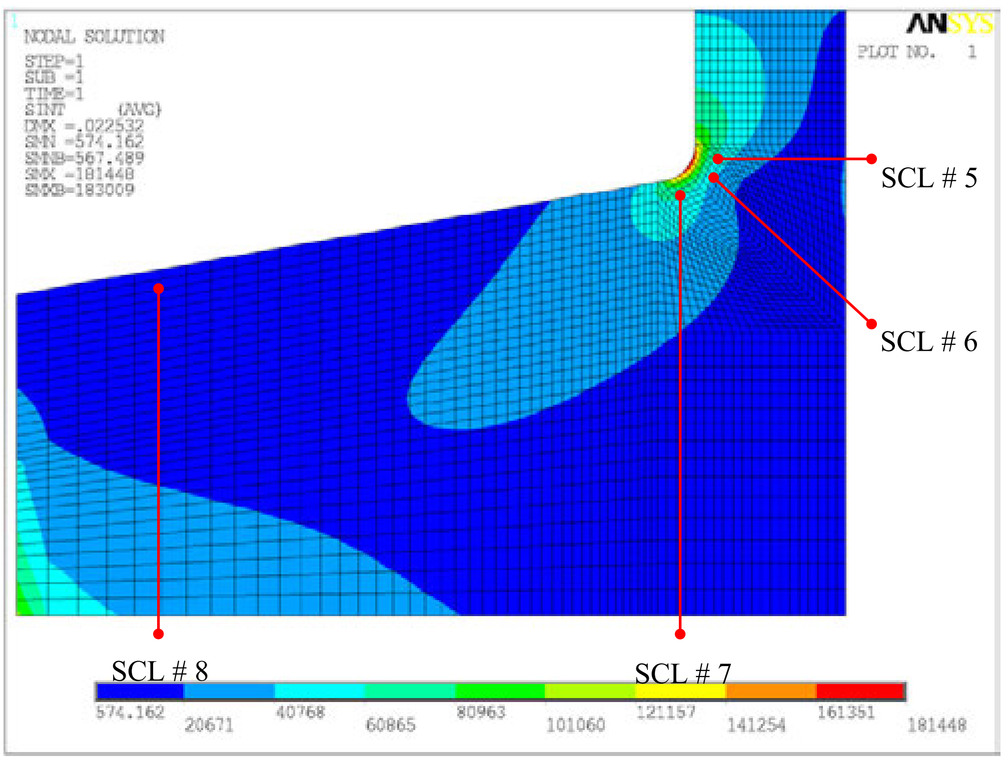

Figure 30 – E-KD-2.3.1-8 –Stress Classification Lines (SCLs) in the Blind End Region – Stress Intensity (psi)

f) **STEP 6** – Sum the stress tensors (stresses are added on a component basis) assigned to each stress intensity category. The final result is a stress tensor representing the effects of all the loads assigned to each stress intensity category. A detailed stress analysis performed using a numerical method such as finite element analysis typically provides a combination of $P_L + P_b$ and $P_L + P_b + Q + F$ directly.

1) If a load case is analyzed that includes only "load-controlled" loads (e.g. pressure and weight effects), the computed stress intensity shall be used to directly represent the P_m, $P_L + P_b$, and $P_L + P_b + Q$. For example, for a vessel subjected to internal pressure with an elliptical head; P_m stress intensity occurs away from the head to shell junction, P_L and $P_L + P_b + Q$ stress intensities occur at the junction.

2) If a load case is analyzed that includes only "strain-controlled" loads (e.g. thermal gradients), the computed stress intensity represents Q alone; the combination $P_L + P_b + Q$ shall be derived from load cases developed from both "load-controlled" and "strain-controlled" loads.

3) If the stress in category F is produced by a stress concentration, the quantity F is the additional stress produced by the stress concentration in excess of the nominal membrane plus bending stress. For example, if a plate has a nominal stress intensity of S_{int}, and has a stress concentration factor K, then: $P_m = S_{int}$, $P_b = 0$, $Q = 0$, and $F = P_m(K-1)$. The total stress intensity equals $P_m + P_m(K-1)$.

g) **STEP 7** – Determine the principal stresses of the sum of the stress tensors assigned to the stress intensity categories, and compute the stress intensity using KD-241(e) equations (1) through (3).

h) **STEP 8** – To evaluate protection against plastic collapse compare the computed stress intensities to their corresponding allowable values (see paragraph KD-242). See Tables E-KD-2.3.1-1 and E-KD-2.3.1-2 below for evaluation results.

$$P_m \le S_y/1.5$$

$$P_L \le S_y$$

$P_L + P_b \le \alpha S_y/1.5$, where α is the shape factor equal to 1.5 (see KD-210 (o))

i) **STEP 9** – To evaluate shakedown/ratcheting, compare the computed equivalent stress to their corresponding allowable values (see paragraph KD-242). See Tables E-KD-2.3.1-1 and E-KD-2.3.1-2 below for evaluation results.

$$P_L + P_b + Q \le 2S_y$$

Table 3 – E-KD-2.3.1-1 – Results of the Elastic Analysis Using Criterion from Figure KD-240 of the 2010 Section VIII, Division 3, KD-240 ASME Code – Design Pressure

SCL No.	Location Note (1)	Linearized Stress Intensities					Stress Evaluation			
		P_m	P_L	P_b	Q	F	$P_m \le S_y/1.5$ (78,000 psi)	$P_L \le S_y$ (117,000 psi)	$P_L + P_b \le \alpha S_y/1.5$ (117,000 psi) Note (2)	$P_L + P_b + Q \le 2S_y$ (234,000 psi)
1	First Thread	N/A	63,190	49,360	N/A	75930	N/A	PASS	PASS	N/A
2	First thread notch section	N/A	31,650	27,070	N/A	129,600	N/A	PASS	PASS	N/A
3	Tapered thread to undercut transition	N/A	38,820	44,100	N/A	57,440	N/A	PASS	PASS	N/A
4	Body shell (away from discontinuities)	59,910	N/A	N/A	N/A	1079	PASS	N/A	N/A	N/A
5	Body shell to blind end transition	N/A	32,460	N/A	53510	66800	N/A	PASS	PASS	PASS
6	Through blind end radius	N/A	16150	N/A	44220	120900	N/A	PASS	PASS	PASS
7	Blind end to body shell transition	18270	N/A	23730	N/A	61650	PASS	N/A	PASS	PASS
8	Blind end close to the opening	16780	N/A	20730	N/A	7422	PASS	N/A	PASS	PASS

Notes:

1) The linearized stress intensities are determined at operating conditions by scaling the linearized stresses computed at design temperature in the above table with a multiplication factor of 0.9.
2) The material at all the locations is SA-723 Gr.2 CL.2 and yield strength at 150°F is 117,000 psi, ASME Section II, Part D, 2010.
3) α is the shape factor equal to 1.5 (see KD-210 (o)).

4.8 Example Problem E-KD-2.3.2 – Elastic Stress Analysis Protection Against Local Failure KD-247

Evaluate the triaxial stress criteria of KD-247 for protection against local failure for the pressure vessel in problem E-KD-2.3.1. The procedure for this is to determine the algebraic sum of the three principal stresses for the eight paths given in Figures E-KD-2.3.1-6 through E-KD-2.3.1-8 and compare to the triaxial stress criteria given in KD-247 ($\sigma_1 + \sigma_2 + \sigma_3 \le 2.5S_y$).

The sum of the principal stresses was evaluated along each of the eight critical stress classification lines from problem E-KD-2.3.1. The peak value of the sum of the principal stresses along the SCL's are reported here.

Table 4 – E-KD-2.3.1-2 – KD-247 Triaxial Stress Criteria

Path Numbers	σ_1	σ_2	σ_3	Summation of the Principal Stresses	Criteria Evaluation
1	184,000	53,100	2444	239,484	PASS
2	184,000	53,100	2444	239,484	PASS
3	137,000	38,500	521	176,221	PASS
4	61,000	25,400	-11,000	75,380	PASS
5	118,000	35,300	-11,000	142,080	PASS
6	171,000	49,000	-11,000	209,490	PASS
7	47,500	6907	-11,000	40,627	PASS
8	39,100	22,500	2.869	61,513	PASS

Notes:

1) The material at all the locations is SA-723 Gr.2 CL.2 and Yield strength Sy at 150°F is 117,000 psi, ASME Section II, Part D, 2010.
2) σ_1, σ_2, σ_3 are the three principal stresses.
3) All stresses are shown in psi.
4) 2.5 Sy = 292,500 psi

PART 5

Example Problems Fatigue Assessment

5 EXAMPLE PROBLEMS FATIGUE ASSESSMENT

5.1 Example Problem E-KD-3.1.1 – Evaluation of Leak-Before-Burst in Cylindrical Vessel – Monobloc Vessel

Determine if the mode of failure for a crack in the wall of a pressure vessel is "Leak-Before-Burst" for the case of the open ended pressure vessel found in problem E-KD-2.1.1. This evaluation of is to be in accordance with paragraph KD-141(a) criteria. This evaluation is necessary for determination if KD-3 fatigue assessment or KD-4 fracture mechanics assessment is to be used for a particular failure mode of a vessel.

This problem assumes that this is a design without documented experience within industry.

The failure mode to be analyzed is a semi-elliptical surface connected flaw in the ID of the wall in the radial- axial plane.

Vessel Data:

- Material = SA-705 Gr. XM-12 Condition H1100
- Design Temperature = 70°F
- Critical Stress Intensity Factor (K_{Ic}) = 104 ksi-in$^{0.5}$ (based on minimum fracture toughness and specification minimum yield strength – see methodology in problem E-KM-2.1.2)
- Inside Diameter = 6.0 in
- Outside Diameter = 12.0 in
- Diameter Ratio (Y) = 2.0 [KD-250]
- Design Pressure = 50,581 psi (problem E-KD-2.1.1)
- Yield Strength = 115,000 psi @ 70°F per Table Y-1 of Section II, Part D
- Tensile Strength = 140,000 psi @ 70°F
- Assumed Crack Aspect Ratio (2c/a) = 3:1 per KD-410(b)

The stress in the wall of this pressure vessel is a combination of the pressure stress and the residual stresses induced during autofrettage. The residual stresses were calculated in E-KD-5.1.1. The pressure stress distribution was also calculated here using the methods of KD-250. The principal of superposition was used to combine the two for the total stress at design conditions. Figure E-KD-3.1.1-1 shows a plot of these stresses at the design condition.

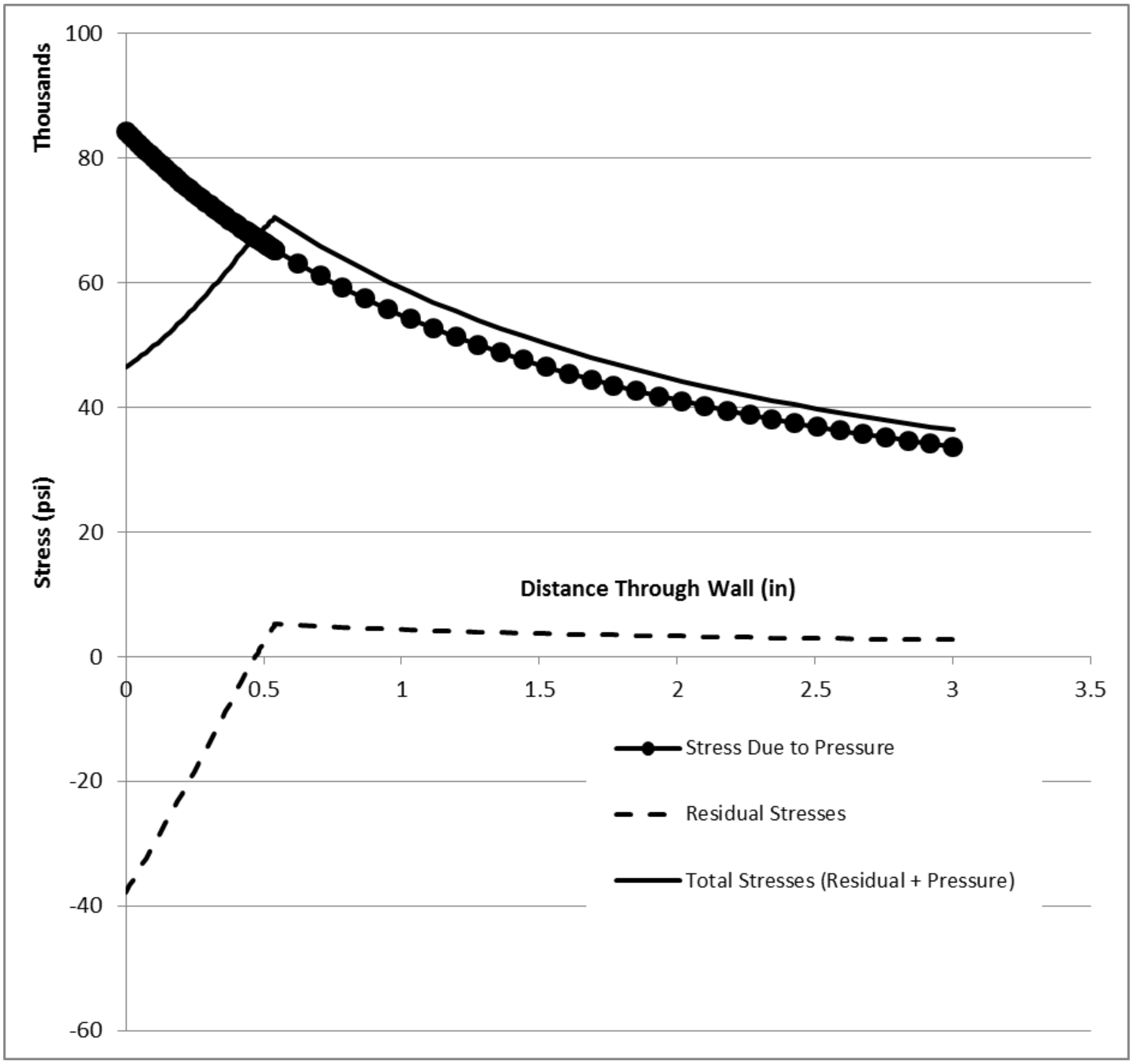

Figure 31 – E-KD-3.1.1-1 – Stress Distribution in Vessel Wall

STEP 1 – Determine if the stress intensity factor for a crack at 80% of the wall thickness will result in brittle failure

Many of the available methods for calculating stress intensity factors are not accurate beyond 80% of the wall.

$$0.8\frac{D_O - D_I}{2} = 2.4\ inches$$

The stress intensity factor at this depth must be less than K_{Ic}.

The stress intensity factor is to be calculated in accordance with the methods found in API 579-1 / ASME FFS-1 per KD-420(a).

The stress intensity factor solutions are found in Appendix C. Specifically, C.5.10 has a solution for "Cylinder – Surface Crack, Longitudinal Direction – Semi-Elliptical Shape, Internal Pressure (KCSCLE1)". Figure E-KD-3.1.1-2 shows the crack being analyzed.

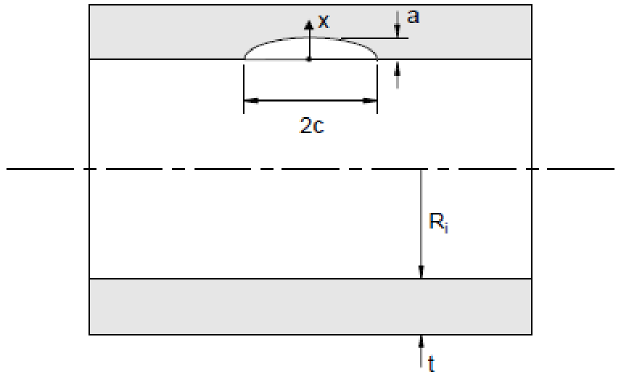

Figure 32 – E-KD-3.1.1-2 – Cylinder – Surface Crack, Longitudinal Direction Semi-Elliptical Shape (API 579-1 / ASME FFS-1 Figure C.15)

Paragraph C.5.10.1 is for a Mode I Stress Intensity Factor for an inside surface , including pressure in the crack face. Equation C.186 gives:

$$K_I = \frac{P * R_O}{R_O^2 - R_I^2}\left[2G_0 - 2G_1\left(\frac{a}{R_i}\right) + 3G_2\left(\frac{a}{R_i}\right)^2 - 4G_3\left(\frac{a}{R_i}\right)^3 + 5G_4\left(\frac{a}{R_i}\right)^4\right]\sqrt{\frac{\pi a}{Q}}$$

Where the influence coefficients, G_0 and G_1 are given by:

$$G_0 = A_{0,0} + A_{1,0}\beta + A_{2,0}\beta^2 + A_{3,0}\beta^3 + A_{4,0}\beta^4 + A_{5,0}\beta^5 + A_{6,0}\beta^6$$

$$G_1 = A_{0,1} + A_{1,1}\beta + A_{2,1}\beta^2 + A_{3,1}\beta^3 + A_{4,1}\beta^4 + A_{5,1}\beta^5 + A_{6,1}\beta^6$$

Where Table C.12 provides the $A_{i,j}$ coefficients and equation C.96 is used for the value of β as:

$$\beta = \frac{2\varphi}{\pi}$$

Influence coefficients G2, G3, and G4 are then determined by the methods found in paragraph C.14.3 or C.14.4, typically using the weight function approach. The value of Q is determined with equation C.15:

$$Q = 1.0 + 1.464\left(\frac{a}{c}\right)^{1.65} \quad for\ \ a/c \le 1.0$$

Using this methodology, the stress intensity factor for a crack with a depth of 2.4 inches is 285,100 psi-in$^{0.5}$. Therefore, the criterion is not satisfied.

STEP 2 – Evaluate if the remaining ligament (distance from the crack tip at the deepest point) to the free surface that the crack is approaching) is less than $(K_{Ic}/S_y)^2$

The limiting distance is $(K_{Ic}/S_y)^2 = 0.818$ inches. The remaining distance is 0.6 inches.

This criterion is not satisfied.

The requirement is for both of these criteria to be satisfied and in this case neither of the criteria is satisfied. Therefore, the vessel is not Leak-Before-Burst.

5.2 Example Problem E-KD-3.1.2 – Evaluation of Leak-Before-Burst in Cylindrical Vessel – Dual Layered Vessel

Determine if the mode of failure for a crack in the wall of a pressure vessel is "Leak-Before-Burst" for the case of the open end pressure vessel found in problem E-KD-2.1.2. This evaluation of is to be in accordance with paragraph KD-141(c) and KD-810(f) criteria. This evaluation is necessary for determination if KD-3 fatigue assessment or KD-4 fracture mechanics assessment is to be used for a particular failure mode of a vessel.

This problem assumes that this is a design without documented experience within industry.

This problem assumes that the closures will remain in place and not be ejected in the event of a failure.

This problem assumes that the fast fracture of either of the inner layer will not result in ejection of parts or fragments and the outer layer will remain intact.

The failure mode to be analyzed is a semi-elliptical surface connected flaw in the ID of the wall of each of the layers in the axial-radial plane.

Each of the materials meet the Charpy impact requirements from KM-234.2(a).

This problem assumes that the vessel does not contain lethal substances.

Vessel Data:

Liner

- Material = SA-705 Gr. XM-12 Condition H1100
- Design Temperature = 70°F
- Critical Stress Intensity Factor (K_{Ic}) = 104 ksi-in$^{0.5}$ (based on minimum fracture toughness and specification minimum yield strength – see methodology in problem E-KM-2.1.3)
- Wall Ratio (Y) = 1.50

Outer Body

- Material = SA-723 Gr. 2 Class 2
- Design Temperature = 70°F
- Critical Stress Intensity Factor (K_{Ic}) = 104 ksi-in$^{0.5}$ (problem E-KM-2.1.3)
- Wall Ratio (Y) = 2.083
- Design Pressure = 97,030 psi (problem E-KD-2.1.2)

The vessel can be assumed to be Leak-Before-Burst if the inner layer fails in fast fracture, the outer body can hold 120% of the design pressure without resulting in collapse.

The collapse pressure of a cylindrical shell can be determined by the Faupel burst equation of:

$$P_{burst} = \frac{2S_y}{\sqrt{3}}\left(2 - \frac{S_y}{S_U}\right)(\ln(Y) = 106{,}717\ psi$$

And 120% of the design pressure is 107,425 psi. Therefore, this cylinder is not considered to be Leak-Before-Burst.

5.3 Example Problem E-KD-3.1.3 – Fatigue Assessment of Welds – Elastic Analysis and Structural Stress

Evaluate an open ended vessel with the same dimensions as that given in Example E-KD-2.1.1 in accordance with the fatigue methodology provided in KD-340. For the purpose of this problem, the material of the vessel is SA-182 Grade F22. Design pressure and resulting stresses were calculated using the same methodology as in the previous problems with no autofrettage. Design requirements include only the pressure loading of an operating pressure of 24,500 psi for 10,000 cycles. Note that the vessel is in non-corrosive service with respect to environmental effects upon the fatigue behavior. Perform fatigue assessment for a theoretical radial-axial crack along the heat affected zone of a longitudinal seam weld in the vessel.

a) **STEP 1** – Determine a load history for the vessel.

   Per the User's Design Specification as described above, a full internal pressure cycle is the only loading event to be considered. The internal pressure is expected to cycle 10,000 times between 0 psi and the operating pressure of 24,500 psi.

b) **STEP 2** – Determine the individual stress-strain cycles.

   Since the full pressure cycle is the only event under consideration, the cyclic stress range is between the stress in the vessel at 0 psig and at 24,500 psi.

c) **STEP 3** – Determine the elastically calculated membrane and bending stress normal to the assumed hypothetical crack plane at the start and end points (${}^{m}t$ and ${}^{n}t$, respectively) for the cycle in Step 2. Using this data, calculate the membrane and bending stress ranges and the maximum, minimum and mean stress.

   The membrane stress is $\sigma_m = 24{,}671$ psi

   The bending stress is $\sigma_b = 11{,}279$ psi

Assume the end point of the cycle (${}^{n}t$) is in the shutdown condition where internal pressure is at 0 psig. Stress distributions were calculated for the radial and tangential (hoop) components due to the internal pressure on the vessel. Note, since this is an open-ended vessel, there is no axial stress component due to internal pressure. The longitudinal seam weld crack to be considered will be assumed to be radial and axial in orientation, meaning the stress component normal to the hypothetical crack plane is the hoop stress.

The equations for membrane, bending, maximum, minimum and mean stress are evaluated as follows using the through thickness hoop stress distribution from similar to that found in Example Problem E-KD-2.1.1, except at 24,500 psi (see Figure E-KD-3.1.3-1):

$$\Delta\sigma_m = m\sigma_m - n\sigma_m = 28.583 \cdot ksi \qquad \text{Eqn. KD-3.15}$$

$$\Delta\sigma_b = m\sigma_b - n\sigma_b = 12.25 \cdot ksi \qquad \text{Eqn. KD-3.16}$$

$$\sigma_{max} = \max\left[\left(m\sigma_m + m\sigma_b\right), \left(n\sigma_m + n\sigma_b\right)\right] = 40.833 \cdot ksi \qquad \text{Eqn. KD-3.17}$$

$$\sigma_{min} = \min\left[\left(m\sigma_m - m\sigma_b\right), \left(n\sigma_m - n\sigma_b\right)\right] = 0 \cdot ksi \qquad \text{Eqn. KD-3.18}$$

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = 20.416 \cdot ksi \qquad \text{Eqn. KD-3.19}$$

d) **STEP 4** – Determine the elastically calculated structural stress range using equation KD-3.20.

$$\Delta\sigma_e = \Delta\sigma_m + \Delta\sigma_b = 40.833 \text{ ksi} \qquad \text{Eqn. KD-3.20}$$

e) **STEP 5** – Determine the elastically calculated structural strain using equation KD-3.21 and the elastically calculated structural stress obtained in Step 4.

$$\Delta\varepsilon_e = \frac{\Delta\sigma_e}{E_{ya}} = 1.334 \times 10^{-3}$$

Eqn. KD-3.21

Where $E_{ya} = 30.6 \times 10^6$ psi (modulus of elasticity for 2¼Cr - 1 Mo material at 70°F) from ASME Section II, Part D.

The corresponding local nonlinear structural stress and strain ranges are determined by simultaneously solving Neuber's Rule (equation KD-3.22) and the material hysteresis loop stress-strain curve model (equation KD-3.23).

$$\Delta\sigma \cdot \Delta\varepsilon = \Delta\sigma_e \cdot \Delta\varepsilon_e \qquad \text{Eqn. KD-3.22}$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E_{ya}} + 2 \cdot \left(\frac{\Delta\sigma}{2 \cdot K_{css}}\right)^{\frac{1}{n_{css}}} \qquad \text{Eqn. KD-3.23}$$

Ds = 40.8 ksi and De = 0.133%

The values for the coefficients $K_{css} = 115.5$ and $n_{css} = 0.100$ are obtained from Table KD-360.1 at 70°F for 2¼Cr material.

Next, the nonlinear structural stress range is to be modified for low-cycle fatigue, as the transition between low and high cycle fatigue is not known. Equation KD-3.24 performs this modification.

f) **STEP 6** – Compute the equivalent structural stress range parameter.

$$\Delta\sigma = \left(\frac{E_{ya}}{1-\nu^2}\right)\cdot\Delta\varepsilon = 27.422$$ Eqn. KD-3.24

$$\Delta S_{ess} = \frac{\Delta\sigma}{t_{ess}^{\left(\frac{2-m_{ss}}{2\cdot m_{ss}}\right)}\cdot I^{\frac{1}{m_{ss}}}\cdot f_M} = 33.213\ \frac{ksi}{in^{\left(\frac{2-m_{ss}}{2\cdot m_{ss}}\right)}}$$ Eqn. KD-3.25

Where,

$$m_{ss} = 3.6$$ Eqn. KD-3.26

$$t_{ess} = 6\ in$$ Eqn. KD-3.29

$$I = \left(\frac{1.23 - 0.364\cdot R_b - 0.17\cdot R_b^{\,2}}{1.007 - 0.306\cdot R_b - 0.178\cdot R_b^{\,2}}\right)^{m_{ss}} = 2.104$$ Eqn. KD-3.30

$$R_b = \frac{|\Delta\sigma_b|}{|\Delta\sigma_m| + |\Delta\sigma_b|} = 0.3$$ Eqn. KD-3.31

$$f_M = 1.0$$ For R=0 (Eqn. KD-3.33)

$$R = \frac{\sigma_{min}}{\sigma_{max}} = 0$$ Eqn. KD-3.34

g) **STEP 7** – Determine the permissible number of cycles per the fatigue curves in KD-370 based on the equivalent structural stress range parameter as computed in Step 6.

The number of allowable design cycles, N, can be computed using equation KD-3.50 using constants provided in Table KD-370.1 for the lower 99% Prediction Interval (-3σ). These constants are to be used unless otherwise agreed upon by the Owner-User and the Manufacturer.

$$N = \frac{f_I}{f_E}\cdot\left(\frac{f_{MT}\cdot C}{\Delta S_{ess}}\right)^{\frac{1}{h}} = 11970$$ Eqn. KD-3.50

Where,

$f_I = 1$ No fatigue improvement performed

$f_E = 1$ Non-corrosive service

$E_T = 30.6\cdot 10^3\ ksi$ Elastic Modulus for Grade 22 at 70°F

$E_{ACS} = 29.4 \cdot 10^3 \ \text{ksi}$ — Elastic Modulus for Carbon Steel at 70°F

$f_{MT} = \dfrac{E_T}{E_{ACS}} = 1.041$ — Temp/Material adjustment for fatigue curves (Eqn. KD-3.55)

$C = 818.3$
$h = 0.31950$

Welded Joint Fatigue Curve coefficients for Lower 99% Prediction Interval per Table KD-370.1

h) **STEP 8 –** Determine the fatigue damage for the cycle history.

Per the User Design Specification above, 10,000 full pressure cycles are required (n = 10,000). Using equation KD-3.35 and the results of Step 7, the fatigue damage fraction can be calculated.

$$D_f = \frac{n}{N} = 0.835$$

Eqn. KD-3.35

i) **STEPS 9-11 –** Assessment of Steps 9-11 are not required for this vessel as the only stress range considered in this design is the 0 to 24,500 psig internal pressure operational cycle.

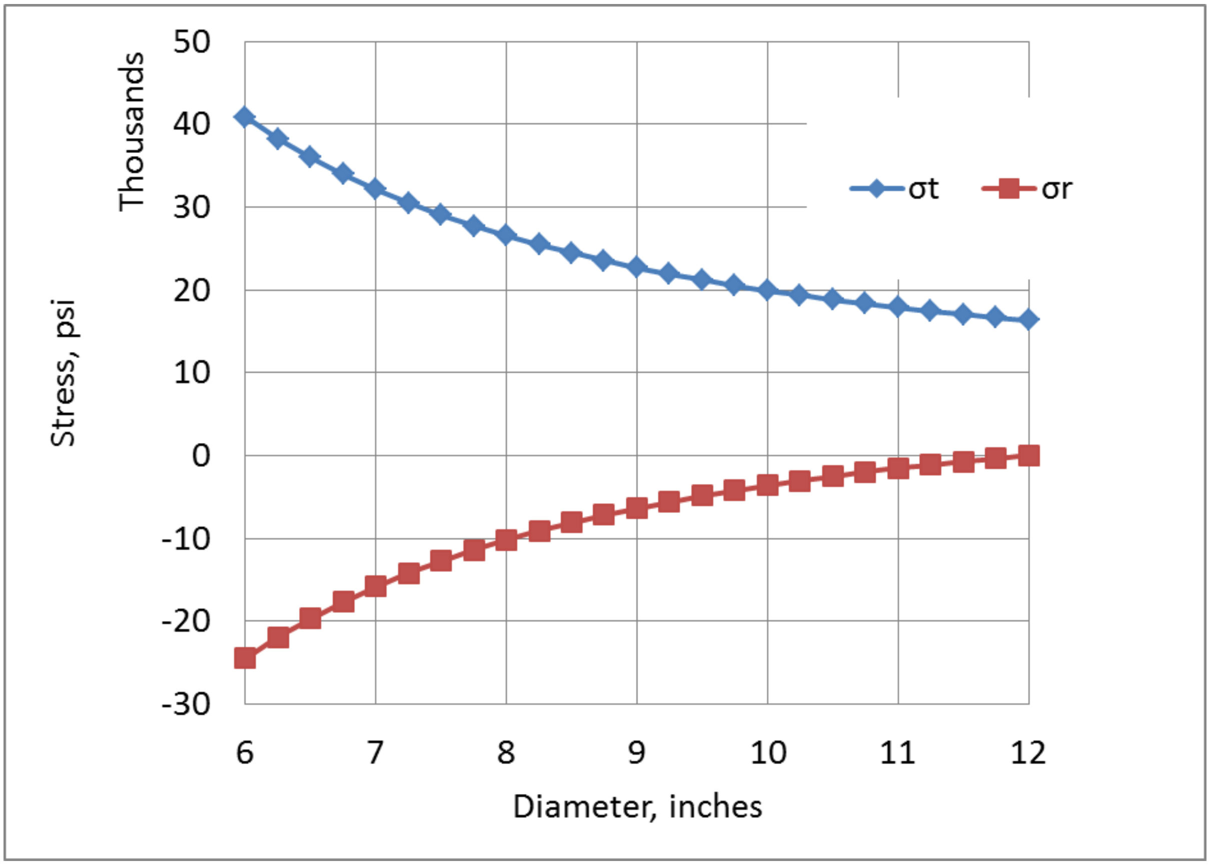

Figure 33 – E-KD-3.1.3-1 – Stress Distribution in Monoblock Open End Shell from E-KD-2.1.1 Evaluated at 24,500 psi

5.4 Example Problem E-KD-3.1.4 – Non-Welded Vessel using Design Fatigue Curves

Evaluate an open ended monowall vessel described in Example E-KD-2.1.1 in accordance with the fatigue methodology provided in KD-340. Design pressure and residual stresses were calculated using the methodology found in problems E-KD-2.1.1 and E-KD-3.1.3. Design requirements include only the cyclic pressure loading from zero to an operating pressure of 24,500 psi for 3,500 cycles, as listed in its User's Design Specification. Note that the vessel is in non-corrosive service with respect to environmental effects upon the fatigue behavior. The objective of this problem is to perform fatigue of the vessel bore and determine the cumulative effect of the number of design cycles on the vessel results in a number of design cycles in excess of this number.

It should be noted that based on past operational experience, this vessel is considered Leak-Before-Break in accordance with KD-141(d). The User's Design report includes documentation for vessels of similar size that all resulted in Leak-Before-Break Mode of Failure.

The surface roughness of the bore of the vessel is noted as 120 R_a on the drawing. The modulus of elasticity used in the analysis of the vessel is 28.5 x 10^6 psi from Table TM-1 from Section II Part D.

a) **STEP 1** – Determine a load history for the vessel and the associated stresses at each of the load cycles specified.

Per the User's Design Specification as described above, a full internal pressure cycle is the only loading event to be considered. The internal pressure is expected to cycle 2,000 times between 0 psig and the operating pressure of 24,500 psig. E-KD-3.1.4-1 is a compilation of these principal stresses.

Table 5 – E-KD-3.1.4-1 – Principal Stresses in Cylinder

	Operating Pressure	Zero Pressure
First Principal Stress (s_1)	40,833 psi	0 psi
Second Principal Stress (s_2)	0 psi	0 psi
Third Principal Stress (s_3)	-24,500 psi	0 psi

b) **STEP 2** – Determine the Stress Intensities(S_{ij}) – The second step is to determine the operational stress intensities for the complete operating cycle. In this point, there are only two points to be evaluated, at operating pressure and at zero pressure. Table E-KD-3.1.4-2 has the results of these calculations at the top of the table listed as S_{ij} for each of the differences (1-2, 2-3, and 3-1).

c) **STEP 3** – Determine the Alternating Stress Intensities ($S_{alt\ i,j}$) – The next step is to evaluate the alternating stress intensities by the absolute value of the difference of maximum and minimum stress intensities throughout the complete operational cycle. These are listed in Table E-KD-3.1.4-2 as $S_{alt\ i,j}$.

d) **STEP 4** – Determine the Associated Mean Stress ($s_{n\ i,j}$)– The next step is to determine the associated mean stress for each of the directions of stress. These are listed in Table E-KD-3.1.4-2 as $s_{n\ i,j}$.

e) **STEP 5** – Determine the Stress Normal to the Plane of Maximum Shear ($s_{n\ i,j}$) – The next step is to determine the associated mean normal stress. It is noted here that the cylinder is a non-welded monowall construction and not made of austenitic stainless steel. The values calculated are shown in Table E-KD-3.1.4-2 as $s_{n\ i,j}$.for both the operating and zero pressure case

f) **STEP 6** – Determine the Associated Mean Stress ($s_{nm\ i,j}$) – The next step is to determine the mean normal stress, $s_{nm\ i,j}$. This is determined by taking the average of the stress normal to the plane of maximum shear calculated in Step 5.

g) **STEP 7** – Determine the Appropriate Fatigue Curve for Use and Surface Roughness Factor (K_r) – Figure KD-320.3 is the curve to be used for pressure equipment made of this material per KD-322(c). The influence of the surface roughness of this cylinder is taken into account by inclusion of the surface roughness factor, K_r. which is found using Figure KD-320.5(b). The equation for this is:

$$K_r = \max\left(1, \frac{1}{-0.16998\log(R_a) + 1.2166}\right) = 1.163$$

Further, it should also be noted that the modulus of elasticity for Figure KD-320.3 is 29 x 10^6 psi and that the allowable amplitude of the alternating stress component ($S`_a$) when s_{nm} equals 0 and $N = 10^6$ cycles is 42,800 psi.

h) **STEP 8** – Determine the Equivalent Alternating Stress Intensity ($S_{eq\ i,j}$) – This cylinder being evaluated is for non-welded construction. Paragraph KD-312.4 states that for 17-4 or 15-5 stainless steel, the value of b shall be either 0.2 or 0.5 depending on the Associated Mean Stress

($\sigma_{nm\,i,j}$) determined in Step 6. These are listed in Table E-KD-3.1.4-2 along with a calculation for the denominator of equation KD-3.11 from paragraph KD-312.4. The limit of this factor is 0.9, but the table shows that this is below 0.9 for all cases considered. The value of Equivalent Alternating Stress for each of the directions considered are shown in the table. The fatigue life will be evaluated based on the maximum value calculated of 64,279 psi.

i) **STEP 9** – Determine the Alternating Stress for Use in Evaluation of Design Life (S_a) – The alternating stress that will be used in conjunction with the curve is then found using equation KD-3.12:

$$S_a = K_r S_{eq} \frac{E(curve)}{E(analysis)} = 76{,}039\ psi$$

Table 6 – E-KD-3.1.4-2 – Calculated Stress Intensities and other Values for Fatigue

	1 - 2	2 – 3	3 – 1
Stress Intensities ($S_{ij} = s_i - s_j$)			
Operating Pressure (psi)	40,833 psi	0	-24,500 psi
Zero Pressure (psi)	0 psi	0 psi	0 psi
Alternating Stress Intensities, $S_{alt\,i,j} = \lvert 0.5(S_{ij\,max} - S_{ij\,min})\rvert$			
	20,417 psi	12,250 psi	32,666 psi
Stress Normal to the Plane of Maximum Shear, $\sigma_{n\,i,j} = 0.5\,(\sigma_i + \sigma_j)$			
Operating Pressure (psi)	20,417 psi	-12,250 psi	8166 psi
Zero Pressure (psi)	0 psi	0 psi	0 psi
Associated Mean Normal Stresses $\sigma_{nm\,i,j} = 0.5\,(\sigma_{n\,i,j\,max} + \sigma_{n\,i,j\,min})$			
	10,208 psi	-6,125 psi	4083 psi
β			
	0.5	0.2	0.5
Factor ($1 - \beta\sigma_{nm\,i,j}/S`_a$)			
	0.881	1.029	0.952
Equivalent Alternating Stress Intensity $S_{eq\,i,j} = S_{alt\,i,j}\frac{1}{1-\beta\sigma_{nm\,i,j}/S`_a}$			
	23,181 psi	11,909 psi	34,303 psi

j) **STEP 10** – Determine the Design Life of the Vessel (N_f) – The design life of the vessel is then determined by using Figure KD-320.3. There are three acceptable methods for determining the number of cycles including interpolation from tabular values listed in Table KD-320.1 for Figure KD-320.3, use of the equations below Table KD-320.1 for Figure KD-320.2 or graphically from Figure KD-320.3. The method of interpolation was used here. Table E-KD-3.1.4-3 contains the values from Table KD-320.1 used for interpolation.

The equation for interpolation of the fatigue curve from the notes to Table KD-320.1 is re-written here as:

$$N_f = N_{fi}\left(\frac{N_{fj}}{N_{fi}}\right)^{\frac{\log\left(\frac{S_{ai}}{S_a}\right)}{\log\left(\frac{S_{ai}}{S_{aj}}\right)}} = 2{,}013{,}735\ cycles$$

On that basis, the cylinder meets the requirements specified for a design life of 3500 cycles.

Table 7 – E-KD-3.1.4-3 – Values for Interpolation from Table KD-320.1 for Figure KD-320.3

	i	**j**
S_a	42,800 psi	40,600 psi
N_f	1,000,000	2,000,000

PART 6

Example Problems Life Assessment Using Fracture Mechanics

6 EXAMPLE PROBLEMS LIFE ASSESSMENT USING FRACTURE MECHANICS

6.1 Example Problem E-KD-4.1.1 – Determine the Design Life of a Vessel from E-KD-2.1.1

Determine the design life of the vessel wall found in E-KD-2.1.1 using the fracture mechanics design approach of KD-4. The vessel wall under consideration is an open end vessel where the end load is not supported by the vessel. The failure mode to be analyzed is a semi-elliptical surface connected flaw in the ID of the wall in the axial-radial plane.

The cylinder being analyzed is from E-KD-2.1.1.

- Outside Diameter (D_O) = 12.0 in
- Design Pressure (P_d) = 45,000 psi
    - (Maximum Design Pressure per E-KD-2.1.1 – 56,079 psi)
- Operating Pressure = 40,000 psi (Assumed to be approximately 90% of P_d)
- Operational Temperature = 70°F
- Vessel is to be operated cycling between the operational pressure and the 0 pressure state. No autofrettage is to be considered.
- The number of design cycles is to be 10,000 cycles per the User's Design Specification.
- Assumed initial crack size = 0.0625 in (a) x 0.188 in (2c)
- Assumed Crack Aspect Ratio (2c/a) = 3:1 per KD-410(b)
- Material fracture toughness = 100 ksi-in$^{.05}$

Note that it was determined that this failure mode is not able to be considered "Leak-Before-Burst" (see E-KD-3.1.1).

The initial crack size is based on the size flaw specified in the User's Design Specification for the vessel and demonstrated to the Authorized Inspector (see KD-411(a)).

Residual stresses due to autofrettage are not considered as part of this analysis.

STEP 1 – Evaluate the stresses at the extremes of the operational conditions

The stresses due to operating pressure were evaluated based on the equations of KD-250. Figure E-KD-4.1.1-1 shows a plot of these stress distributions for the vessel wall. There are no residual stresses considered in the evaluation of design life of the vessel. Pressure was added to the stress distribution in the evaluation of the crack growth.

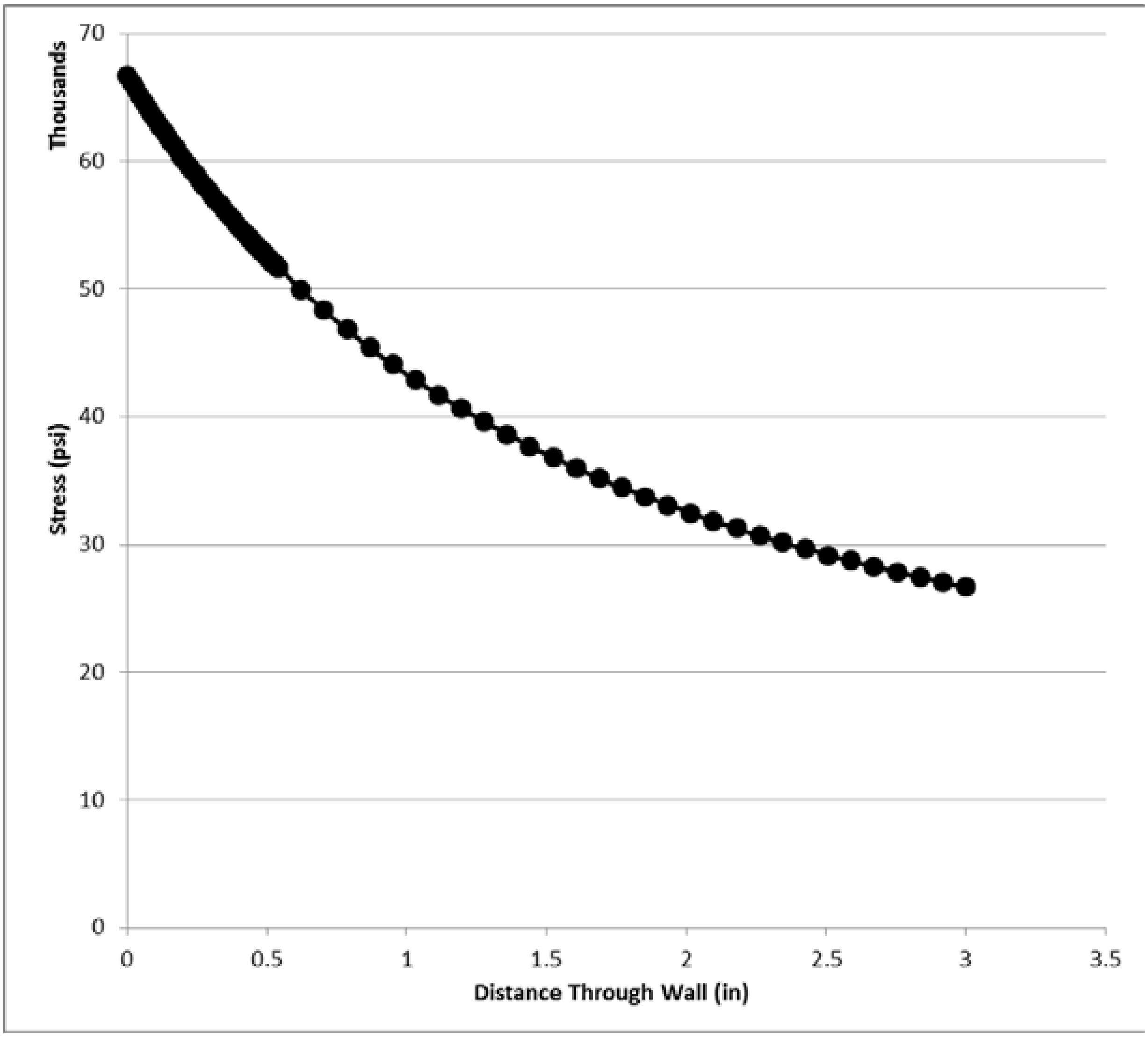

Figure 34 – E-KD-4.1.1-1 – Stress through the Vessel Wall due to Operating Pressure (40 ksi)

<u>**STEP 2** – Calculate the critical crack growth for the crack following the method of KD-430</u>

The crack growth should be calculated by means of the equations in KD-430. Specifically, the equation for crack growth at the deepest point in a two dimensional crack is shown in equation 1 as:

$$\frac{da}{dN} = C\,[f(R_K)]\,(\Delta K)^m$$

And the growth along the free surface is

$$\frac{dl}{dN} = 2C\,[f(R_K)]\,(\Delta K)^m$$

Where $l = 2c$, and

$$\Delta K = K_{I\,max}^{*} - K_{I\,min}^{*}$$

And

$$R_K = \frac{K^*_{I\,min} + K_{I\,res}}{K^*_{I\,max} + K_{I\,res}}$$

The function $f(R_K)$ is based on the function from Table D-500 of Appendix D of VIII-3 for martensitic precipitation hardened steels, where $C_3 = 3.48$ and $C_2 = 1.5$ and

$$R_K \geq 0.67 \qquad f(R_K) = 30.53\,R_K - 17.0$$

$$0 < R_K \leq 0.67 \qquad f(R_K) = 1.0 + C_3\,R_K$$

$$R_K < 0 \qquad f(R_K) = \left[\frac{C_2}{C_2 - R_K}\right]^m$$

The stress intensity at the crack tips due to the various loading conditions is calculated separately. The stress intensity factor for the pressurized state will follow the methodology found in problem E-KD-3.1.1 for a semi-elliptical ID surface connected crack in the longitudinal direction. This methodology will not be repeated here.

In this instance the $K^*_{I\,min}$ and $K^*_{I\,max}$ are the stress intensity at the zero pressure and operating pressure states, respectively. It should be noted that the crack is a surface connected flaw which results in pressure acting on the crack faces. This is included in the calculation of stress intensity by adding the pressure to the stresses shown in Figure E-KD-4.1.1. The $K_{I\,res}$ is the stress intensity at due to any residual stresses that may be present including due to shrink fitting of the component, autofrettage, or yielding during normal hydrostatic testing operations. In the failure mode that is being evaluated, there are no residual stresses in the component and the zero pressure state will respectively result in $K_{I\,res}$ and $K^*_{I\,min}$ equaling zero.

Figure E-KD-4.1.1-2 shows a plot of the stress intensity factors calculated for the case in question here. Note that the surface stress is assumed to be constant along the surface of the crack and the crack is assumed to be remote from any discontinuities that may affect it throughout the life of the crack.

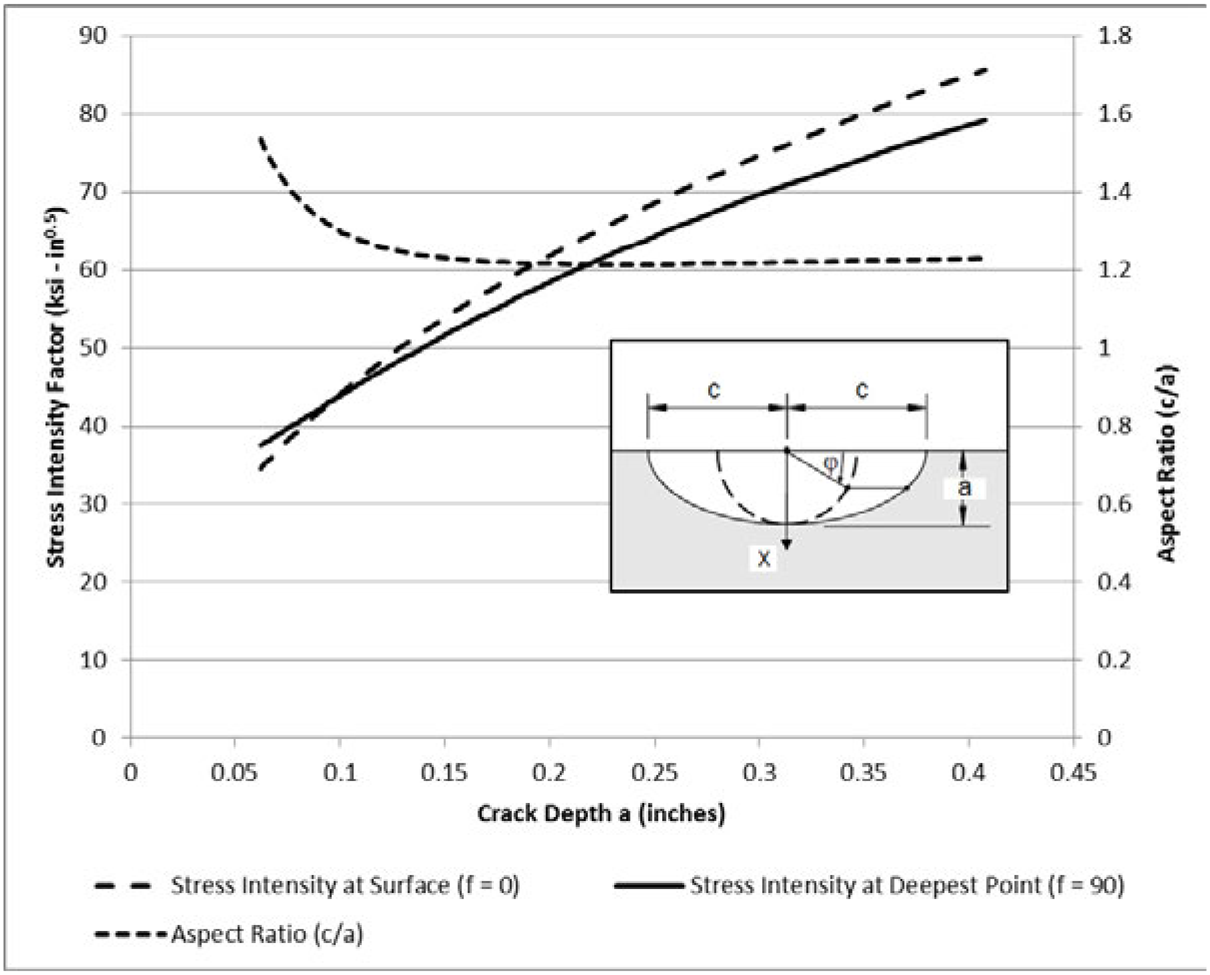

Figure 35 – E-KD-4.1.1-2 – Stress Intensity Factor for the Crack and Aspect Ratio vs. Crack Depth

The crack growth is required to be a two dimensional crack growth per KD-430. Therefore, the aspect ratio of the crack changes as the crack grows. The stress intensities shown in the figure are based on the numerical integration of the two crack growth equations from KD-430. The stress intensities shown correspond to the stress intensity of the crack at the surface corresponds to the stress of the crack at the deepest point based on the aspect ratio of the crack for that dimension.

The number of cycles at the final crack size evaluated is 4,600 cycles.

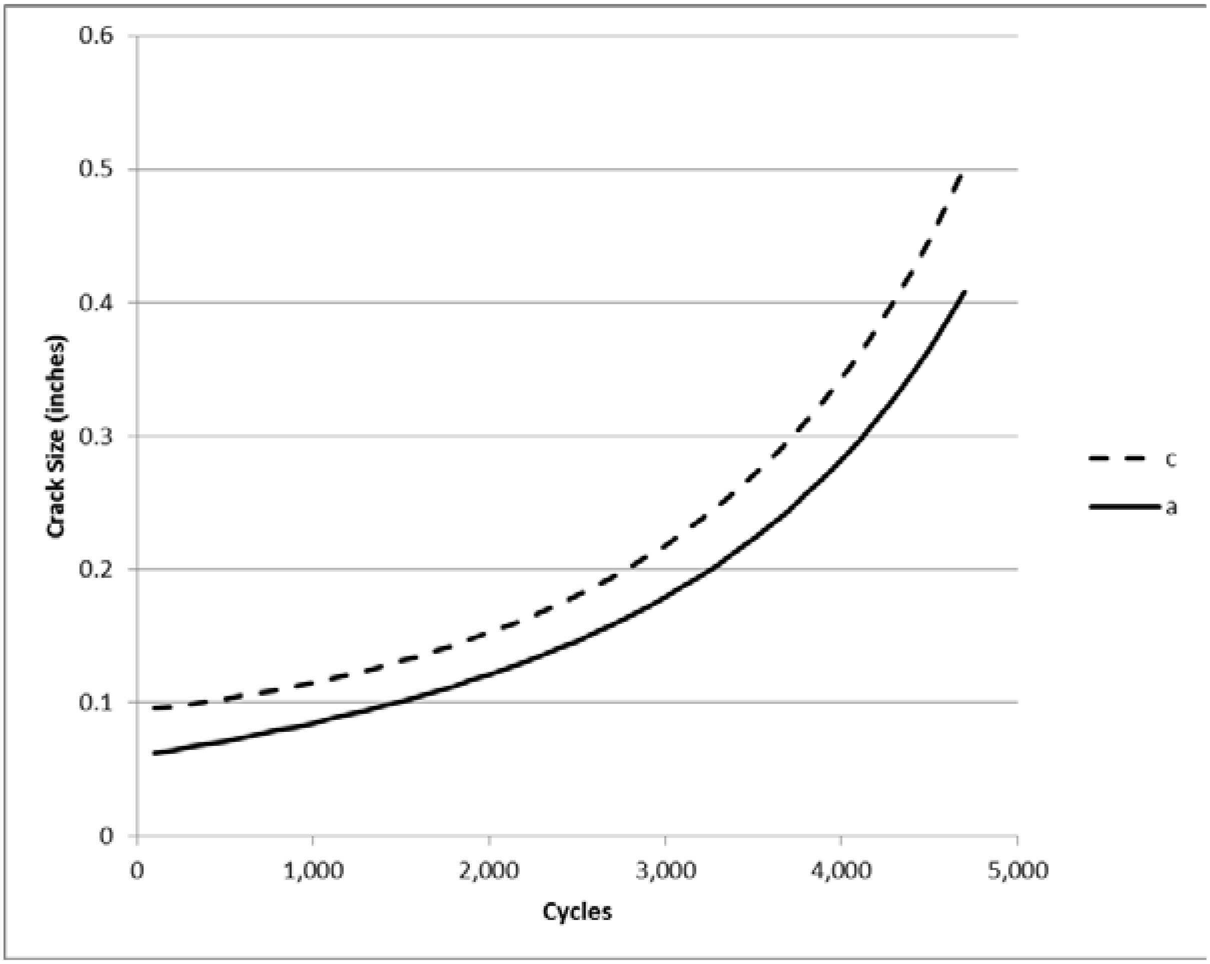

Figure 36 – E-KD-4.1.1-3 – Crack Size vs. Number of Cycles

STEP 3 – Determine the Critical Crack size in accordance with the Failure Assessment Diagram of API 579-1 / ASME FFS-1

The critical crack depth for the failure mode being evaluated is determined using the failure assessment diagram from API 579-1 / ASME FFS-1. This is performed in accordance with paragraph 9.4.3.

In this methodology, the cracking is plotted K_r vs. L^P_r on the FAD as shown in Figure E-KD-4.1.1-4, where:

$$L_r^P = \frac{\sigma_{ref}^P}{\sigma_{ys}}$$

σ_{ref}^P is the reference stress for the crack in question from Annex D - D.5.10 and s_{ys} is the yield strength of the material.

And

$$K_r = \frac{K_I^P + \Phi K_I^{SR}}{K_{mat}}$$

Where:

K_I^P is the applied stress intensity due to the primary stress distribution, K_I^{SR} is the applied stress intensity due to the secondary and residual stress distributions, K_{mat} is the material toughness and Φ is the plasticity correction factor. The complete details of this are found in Part 9 of API 579-1 / ASME FFS-1.

Figure E-KD-4.1.1-4 specifically shows the bounding curve for the FAD including the "acceptable region" and the unacceptable regions.

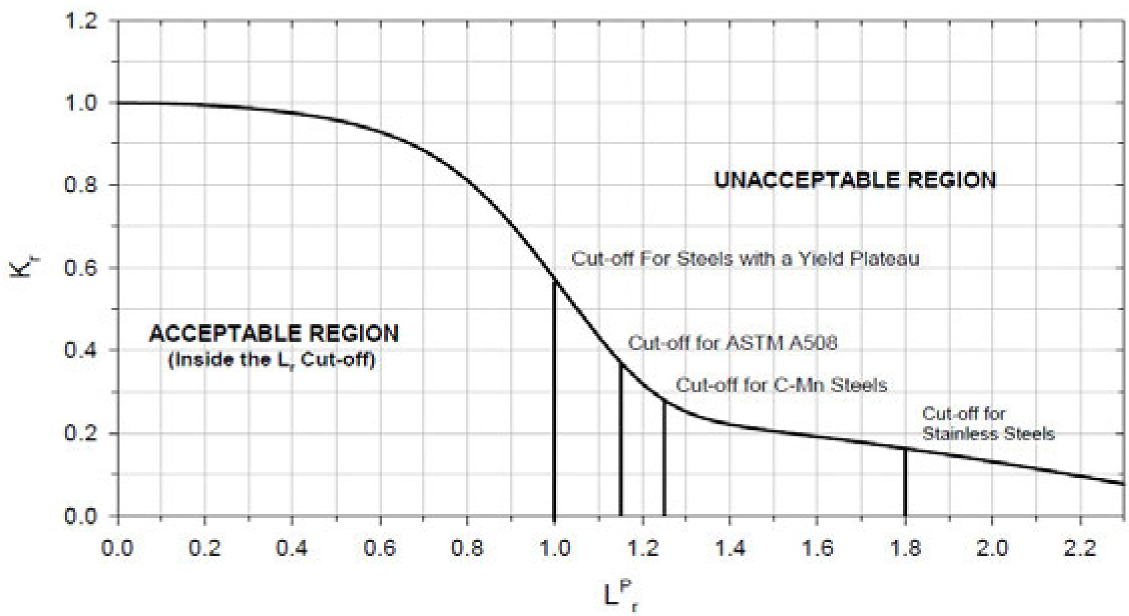

Figure 37 – E-KD-4.1.1-4 – Example of a Failure Assessment Diagram (from API 579-1 / ASME FFS-1 Fig 9.20)

The bounding curve is defined as:

$$K_r = (1 - 0.14\,(L_r^P)^2)\,(0.3 + 0.7\exp[-0.65\,(L_r^P)^6])$$

The bounding curve is stopped at a value of L_r^P equal to 1.109 for this problem based on the SA-705 XM-12 H1100 steel used.

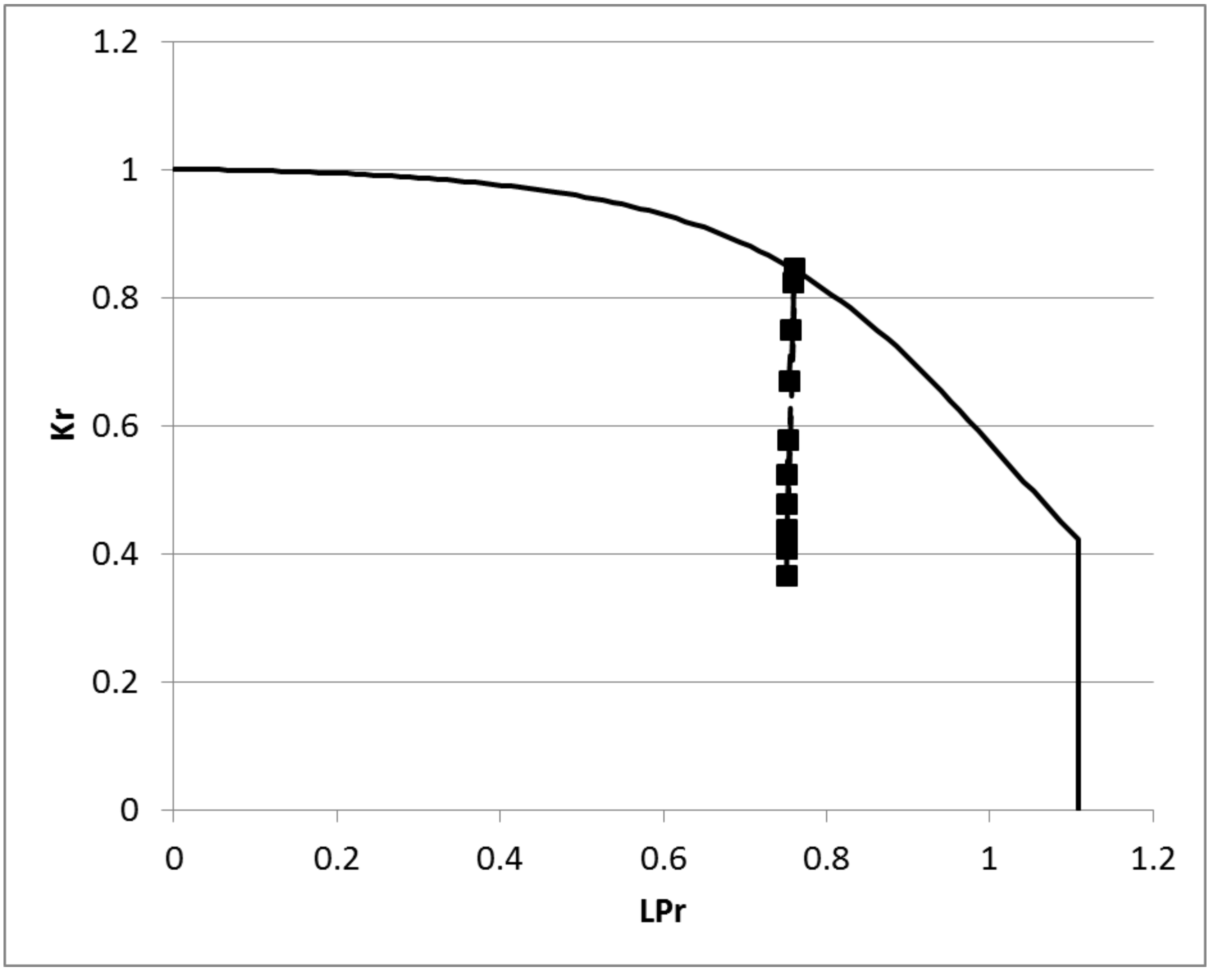

Figure 38 – E-KD-4.1.1-5 – Failure Assessment Diagram for E-KD-4.1.1

Figure E-KD-4.1.1-5 shows a plot of the failure assessment diagram for problem E-KD-4.1.1. The initial crack size of 0.188 long x 0.0625 inch deep resulted in the point at the lower end of the assessment curve. The crack was then grown as described in Step 2, analyzed and plotted at those sizes. The result is a crack who intersects the FAD bounding curve at a size of 1.0032 inch long (2c) x 0.408 deep (a). This size crack has a value of K_r equal to 0.845 and L_r^P equal to 0.761.

<u>**STEP 4** – Determine the Allowable Final Crack Depth and the Number of Design Cycles (N_P) for the Cylinder</u>

The allowable final crack depth is determined in accordance with KD-412. Note, the total number of cycles at the critical flaw size as predicted by the FAD is 4600 cycles.

The allowable final crack depth per KD-412.1 is the minimum of

- 25% of the section thickness considered = 0.75 in
- 25% of the critical crack depth = 25% x 0.386 in deep = 0.0965 in

KD-412 states for this case that the number of "design" cycles (N_P) is the minimum of

- ½ of the cycles to reach the critical crack depth = ½ (4,600 cycles) = 2300 cycles
- the number of cycles to reach the "allowable final crack depth" (0.0965 in) = 1,300 cycles

Therefore, the number of "design" cycles (N_P) in accordance with KD-4 is 1,300 cycles.

PART 7

Example Problems on Residual Stresses using Autofrettage

7 EXAMPLE PROBLEMS ON RESIDUAL STRESSES USING AUTOFRETTAGE

7.1 Example Problem E-KD-5.1.1 – Determine Residual Stresses in Autofrettaged Cylinder Wall with known Autofrettage Pressure

Determine the residual stress in the vessel wall found in E-KD-2.1.1 for an autofrettaged cylinder with an autofrettage pressure of 65,000 psi using the methodology in KD-521. Assume this is an open end vessel where the end load is not supported by the vessel.

The cylinder being analyzed is from E-KD-2.1.1.

- Outside Diameter (D_O) = 12.0 in
- Design Pressure (P_d) = 50,581 psi (Calculated in E-KD-2.1.1)
- Autofrettage Pressure (P_A) = 65,000 psi

It should be noted that for an actual vessel using this calculation, the actual measured yield strength for the material (KD-502) of the vessel should be used, in lieu of the specification minimum. The specification minimum is used in this example.

STEP 1 – Calculate the elastic-plastic interface for the given autofrettage pressure using KD-521.3

The elastic-plastic interface (D_p) is calculated iteratively using the following equation:

$$P_A = 1.15 \cdot S_y \cdot \left[\ln\left(\frac{D_P}{D_I}\right) + \frac{\left(D_O^2 - D_P^2\right)}{2 \cdot D_O^2} \right]$$

Therefore, the elastic-plastic interface diameter is:

$$D_p = 7.080 \text{ in}$$

Note that the maximum overstrain ratio which KD-521.3 is appropriate is 0.4. The overstrain ratio here is:

$$(D_p - D_I) / (D_O - D_I) = 0.18$$

STEP 2 – Determine the Linear Elastic Stress at the ID at Autofrettage Pressure

The theoretical linear elastic stress at the ID of the vessel can be found using the equations of KD-250:

$$\sigma_t = P_A \cdot \frac{D_O^2 + D_I^2}{D_O^2 - D_I^2} \qquad \sigma_t = 108333 \cdot psi$$

$$\sigma_r = -P_A \qquad \sigma_r = -65000 \cdot psi$$

$$\sigma_l = 0 \cdot psi$$

Note the longitudinal stress (σ_l) is zero for a vessel that does not support the pressure end load as in this case.

STEP 3 – Verify of the Average permanent tangential strain at ID

Paragraph KD-510 limits the permanent tangential strain at the bore surface resulting from the autofrettage operation to not exceed 2%. The equations of KD-521.2 can be used for this purpose by solving the left side of the equation for ε_p:

$$FAC = (1 - 2\cdot\nu)\cdot\left[\ln\left[\left(\frac{D_I}{D_P}\right)^2\right] - 1\right] + (2 - \nu)\cdot\left(\frac{D_P}{D_I}\right)^2 + (1 - \nu)\cdot\left(\frac{D_P}{D_O}\right)^2$$

$$\varepsilon_p = \frac{1.15\cdot S_y}{2\cdot E}\cdot\left[FAC - \frac{\left[\ln\left[\left(\frac{D_P}{D_I}\right)^2\right] + \frac{\left(D_O^2 - D_P^2\right)}{D_O^2}\right]\cdot\left[1 - \nu + (1 + \nu)\cdot\left(\frac{D_O}{D_I}\right)^2\right]}{\left(\frac{D_O}{D_I}\right)^2 - 1}\right]$$

$\varepsilon_p = 0.0338\%$ which is less than 2% so this is acceptable per KD-510.

STEP 4 – Calculate the Theoretical Residual Stresses Between the Bore and the Elastic-Plastic Interface

The theoretical residual stress distribution for the cylinder without the effect of reverse yielding should be determined using the equations of KD-522.1.

$$\sigma_{tRA}(D) = S_y\cdot\left[\frac{D_P^2 + D_O^2}{2\cdot D_O^2} + \ln\left(\frac{D}{D_P}\right) - \left(\frac{D_I^2}{D_O^2 - D_I^2}\right)\cdot\left(\frac{D_O^2 - D_P^2}{2\cdot D_O^2} + \ln\left(\frac{D_P}{D_I}\right)\right)\cdot\left(1 + \frac{D_O^2}{D^2}\right)\right]$$

$$\sigma_{rRA}(D) = S_y\cdot\left[\frac{D_P^2 - D_O^2}{2\cdot D_O^2} + \ln\left(\frac{D}{D_P}\right) - \left(\frac{D_I^2}{D_O^2 - D_I^2}\right)\cdot\left(\frac{D_O^2 - D_P^2}{2\cdot D_O^2} + \ln\left(\frac{D_P}{D_I}\right)\right)\cdot\left(1 - \frac{D_O^2}{D^2}\right)\right]$$

The residual stress without reverse yielding is $\sigma_{tRA}(D_I) = -35.724$ ksi and $\sigma_{rRA}(D_I) = 0$ ksi

STEP 5 – Correction for Bauschinger Factor for Reverse Yielding

The residual stresses are then corrected for the effect of a reduced yield strength termed as the "Bauschinger Effect Factor". This is done in accordance with the methods of KD-522.2.

a) The first step is to determine the diameter where the residual hoop stress minus the radial stress is equal to zero. This is done using the equations in KD-522.1 as shown in Step 4 iteratively. This is defined as D_Z., which for this problem equals 6.869 in.

b) Define the hoop stress at the ID without reverse yielding as σ_{AD}. Note: $\sigma_{AD} = -35.724$ psi.

c) Determine the overstrain ratio as:

$$M = (D_P - D_I)/(D_O - D_I) = 0.18$$

d) Determine the corrected value of tangential stress at the ID based on the equations in KD-522.2(b)

$$\sigma_{CD1} = \sigma_{AD}\cdot\left(1.0388 - 0.1651Y + 0.6307 - 1.8871M + 1.9837M^2 - 0.7296M^3\right) = -37853\cdot psi$$

$$\sigma_{CD2} = \sigma_{AD}\cdot\left(-0.5484 + 1.8141Y - 0.6502Y^2 + 0.0791Y^3\right) = -39718\cdot psi$$

$$\sigma_{CD} = \max\left(\sigma_{CD1}, \sigma_{CD2}\right) \qquad \sigma_{CD} = -37853\cdot psi$$

Note that σ_{CD}/S_Y = -0.33, so no further correction is needed to limit σ_{CD}/S_Y to -0.7.

e) The residual stresses are then calculated in a piecewise continuous fashion.

1) From $D_I < D < D_Z$

$$\sigma_{tR}(D) = \sigma_{CD}\cdot\left[\frac{D_z\cdot\left(\ln\left(\frac{D}{D_I}\right)+1\right)+D_I-2\cdot D}{D_z - D_I}\right] \qquad \sigma_{rR}(D) = \sigma_{CD}\cdot\left(\frac{D_z\cdot\ln\left(\frac{D}{D_I}\right)+D_I-D}{D_z - D_I}\right)$$

And for this case $\sigma_{tR}(D_I)$ = -37,853 psi & $\sigma_{rR}(D_I)$ = 0 psi and $\sigma_{tR}(D_I) = \sigma_{rR}(D_Z)$ = -2617 psi.

2) A correction factor is then applied for the stresses for $D>D_Z$: $F_b = \sigma_{rR}(D_Z) / \sigma_{rRA}(D_Z) = 1.134$

Where $\sigma_{rRA}(D_Z)$ = -2307 psi when calculated using the equations from Step 4 and KD-522.1.

3) The stresses for $D_Z<D<D_P$ are then calculated by multiplying stresses from KD-522.1 by the correction factor F_b.

4) The stresses for $D_P<D<D_O$ are then calculated using the equations from KD-523:

$$\sigma_{tRB}(D) = S_y\cdot F_b\cdot\left(1+\frac{D_O^2}{D^2}\right)\cdot\left[\frac{D_P^2}{2\cdot D_O^2}+\frac{D_I^2}{D_O^2-D_I^2}\cdot\left(\frac{D_P^2-D_O^2}{2\cdot D_O^2}-\ln\left(\frac{D_P}{D_I}\right)\right)\right]$$

$$\sigma_{rRB}(D) = S_y\cdot F_b\cdot\left(1-\frac{D_O^2}{D^2}\right)\cdot\left[\frac{D_P^2}{2\cdot D_O^2}+\frac{D_I^2}{D_O^2-D_I^2}\cdot\left(\frac{D_P^2-D_O^2}{2\cdot D_O^2}-\ln\left(\frac{D_P}{D_I}\right)\right)\right]$$

Where $\sigma_{tRB}(D_P)$ = 5,169 psi, $\sigma_{rRB}(D_P)$ = -2,500 psi which is at the elastic / plastic interface and where $\sigma_{tRB}(D_O)$ = 2670 psi, $\sigma_{rRB}(D_O)$ = 0 psi which is at the outer diameter.

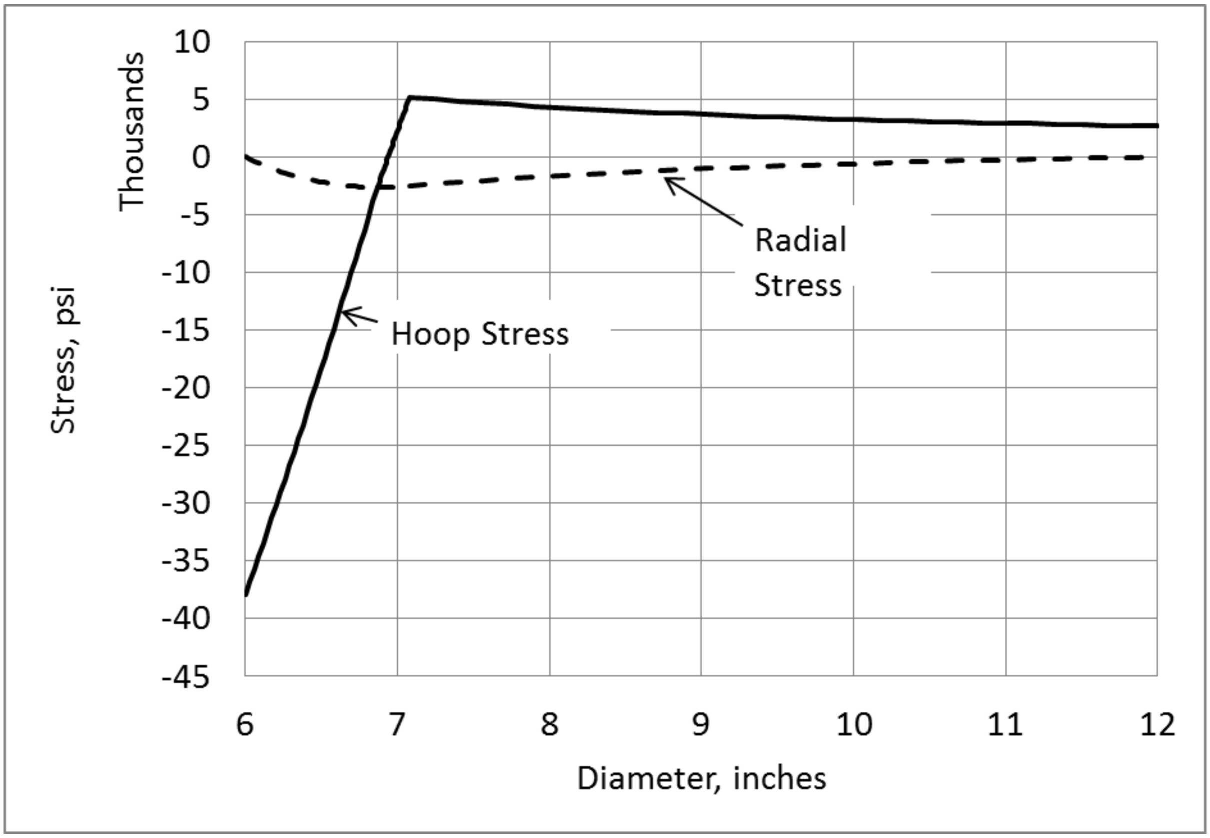

Figure 39 – E-KD-5.1.2-1 – Stress Distribution In Vessel Wall

7.2 Example Problem E-KD-5.1.2 – Determine the Autofrettage Pressure in a Cylinder Wall with known Residual ID Tangential Strain

Determine the autofrettage pressure (P_A) in the vessel wall found in E-KD-2.1.1 when the residual tangential ID strain is known using the methodology in KD-521. Assume this is an open end vessel where the end load is not supported by the vessel.

The cylinder being analyzed is from E-KD-2.1.1.

- Outside Diameter (D_O) = 12.0 in
- Design Pressure (P_d) = 50,581 psi (Calculated in E-KD-2.1.1)
- Residual ID tangential strain ($\in_P$) = 0.0338%

It should be noted that for an actual vessel using this calculation, the actual measured yield strength for the material (KD-502) of the vessel should be used. The minimum yield strength of 115 ksi from E-KD-2.1.1 is used here.

STEP 1 – Calculate the elastic-plastic interface (D_p) for the given autofrettage pressure using KD-521.2

The elastic-plastic interface (D_p) is calculated iteratively using the following equation:

$$\varepsilon_p = \frac{1.15 \cdot S_y}{2 \cdot E} \cdot \left[(1 - 2 \cdot \nu) \cdot \left[\ln\left[\left(\frac{D_I}{D_P} \right)^2 \right] - 1 \right] + (2 - \nu) \cdot \left(\frac{D_P}{D_I} \right)^2 + (1 - \nu) \cdot \left(\frac{D_P}{D_O} \right)^2 - \frac{\left[\ln\left[\left(\frac{D_P}{D_I} \right)^2 \right] + \frac{\left(D_O^2 - D_P^2 \right)}{D_O^2} \right] \cdot \left[1 - \nu + (1 + \nu) \cdot \left(\frac{D_O}{D_I} \right)^2 \right]}{\left(\frac{D_O}{D_I} \right)^2 - 1} \right]$$

The elastic plastic interface can then be found to be 7.080 inches

STEP 2 – Calculate the autofrettage pressure (P_A) using KD-521.3

The autofrettage pressure can then be determined using the equation in KD-521.3.

$$P_A = 1.15 \cdot S_y \cdot \left[\ln\left(\frac{D_P}{D_I} \right) + \frac{\left(D_O^2 - D_P^2 \right)}{2 \cdot D_O^2} \right]$$

Therefore, the autofrettage pressure is:

$$P_A = 65{,}000 \text{ psi}$$

PART 8

Example Problems in Closures and Connections

8 EXAMPLE PROBLEMS IN CLOSURES AND CONNECTIONS

8.1 Example Problem E-KD-6.1.1 – Evaluation of a Connection in a 60 ksi Pressure Vessel at 100°F

Determine the suitability of an industry standard 9/16 tubing connection [6][7] for use in an ASME Section VIII Division 3 pressure vessel made of SA-723 Grade 2 Class 2 material rated at 60 ksi at 100F. The vessel will be designed with sufficient wall thickness to accommodate an opening of this size using the elastic-plastic finite element methods of KD-230.

Note, that it is typical to consider the boundary of a pressure vessel to end at the first connection. However, it is noted that in KD-6, the rules regarding the geometry of the connection machined into the vessel are mandatory for all vessels.

Also, it is assumed that this is a connection on the exterior surface of a pressure vessel such as the center connection on a head. This assumptions means that the dimension of the material surrounding the connection (in the direction perpendicular to the centerline of the connection as shown in Figure E-KD-6.1.1-1) is large compared with the dimensions of the connection. Therefore, it is assumed that the radial displacement is negligible.

It is stated that the connection does not experience cyclic loading in service in the User's Design Specification for the pressure vessel. Further, the UDS states that there are no externally imposed loads on this connection.

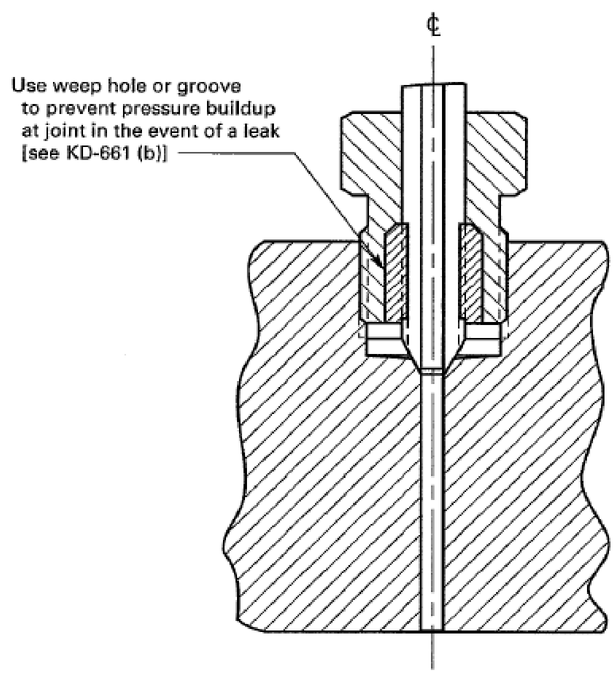

Figure 40 – E-KD-6.1.1-1 – Typical High Pressure Connection (from Appendix H of ASME Section VIII, Division 3)

Material

Vessel Material

SA-723 Grade 2 Class 2 material rated at 100F

S_y = 120,000 psi at 100F Yield Strength of the material

S_u = 135,000 psi at 100F Tensile Strength of the material

Vessel and Opening Dimension and Loading Data

Ds = 0.296 in Opening Seal Diameter

b = 0.005 in Assumed Seal Width

m = 6.5 Seal Factor

$$W = 0.785D_s^2P + (2b \times 3.14D_s mP)$$

W = 7,755 lbf Endload on connection using ASME Section VIII Division 1[5] Appendix 2 Section 2-5 for connections with metal seat. Note that this is not a cyclic load as the connection operates at a pressure much lower than the pre-torque on the connection applies.

Thread is 1 1/8-14UNS Class 2B thread machined into the body

p = 1/14 in Thread pitch

D_{major} = 1.113 in Min material condition of Major Diameter of Male Thread

D_{minor} = 1.064 in Min material condition of Minor Diameter of Female Thread

D_{pitch} = 1.071 in Pitch Diameter of Connection Thread

Droot = 1.0384 in Nominal Root Diameter of Male Thread

L = 0.438 in Minimum Engaged Gland Thread Length per KD-626 (this does not include incomplete or partial threads)

STEP 1 – Determine the Average Thread Shear Stress (KD-623)

The average thread shear stress is limited to 30,000 psi (0.25S_y). This is calculated by:

$$\tau = \frac{W}{\pi D_{pitch}\frac{L}{2}} = 10{,}525\ psi$$

STEP 2 –Determination of the Average Thread Bearing Stress (KD-623)

The average thread bearing stress due to the maximum design load is limited to 90,000 psi (0.75 S_y). This is calculated by:

$$\sigma = \frac{W}{A_{bearing}} = 7671\ psi$$

Where

$$A_{bearing} = \pi D_{pitch}\left(D_{major} - D_{minor}\right)\frac{L}{p} = 1.011\ in^2$$

STEP 3 – Determine the Length of Engagement Required (KD-626)

The minimum thread engagement length is the minimum based on the drawing tolerances and without credit for the first and last partial thread in the engaged length. KD-626(a) states that connections with imposed loads must comply with the length of engagement for bolts in KD-626(b). The UDS states that there are no externally imposed loads on these connections, so therefore, this requirement does not apply.

Note: The connections listed here are "industry standard" but typically machined to manufacturer's published standards such as listed in the references for this manual.

8.2 Example Problem E-KD-6.1.2 – Alternative Evaluation of Stresses in Threaded End Closures

In lieu of performing a numerical simulation, such as a finite element analysis, of a closure to determine the stresses for a fatigue or fracture mechanics analysis, KD-630 provides guidance on the evaluation of these stresses. This problem is to evaluate the stresses at the first thread in the pressure vessel evaluated in example problem E-AE-2.2.1 and E-KD-2.3.1.

It is noted that a vent hole will be incorporated into the closure for use in the event of seal failure, as required by KD-661. This will either be as a small weep hole through the side of the vessel or by venting the nut by grooving the face and possibly drilling an intersecting hole axially through the nut.

Vessel Dimension and Loading Data (see E-AE-2.2.1 and E-KD-2.3.1 for complete details)

Design Pressure (P_D) = 11,000 psi

Outside Diameter of the Vessel (D_O) = 12 in

Inside Diameter of the Vessel (D_I) = 10 in

Pitch Diameter of the Threads (D_p) = 10.443 in

Root Diameter of the Threads (D_{root}) = 10.769 in

Thread Pitch (P_T) = 0.5 in

Total number of threads (n) = 10

F_s = 863,938 lbf (from E-AE-2.2.1)

STEP 1 – Evaluate the Longitudinal Bending Stress at the First Thread (KD-631.1)

The primary longitudinal bending stress in the vessel at the first thread is found using:

$$\sigma_{LB} = 3.0\frac{F_s}{A_{Longitudinal}} = 117{,}737\ psi$$

Where

$$A_{Longintudinal} = \frac{\pi}{4}\left(D_O^2 - D_{root}^2\right) = 22.014\ in^2$$

This stress is conservatively assumed to be present throughout the entire thickness.

STEP 2 – Evaluate the Circumferential Stresses in the Wall Thickness (KD-631.2)

The circumferential stresses at the first thread are assumed to be those calculated for a vessel with an ID equal to that of the thread root diameter and the OD the same as the vessel. The stresses are then calculated using the equations in KD-250 for the circumferential stress. Using this method the resultant radial stress is 102,027 psi at the internal surface of the thread root. Figure E-KD-6.1.2-1 shows a plot of the circumferential stress as a function of the radius using this method.

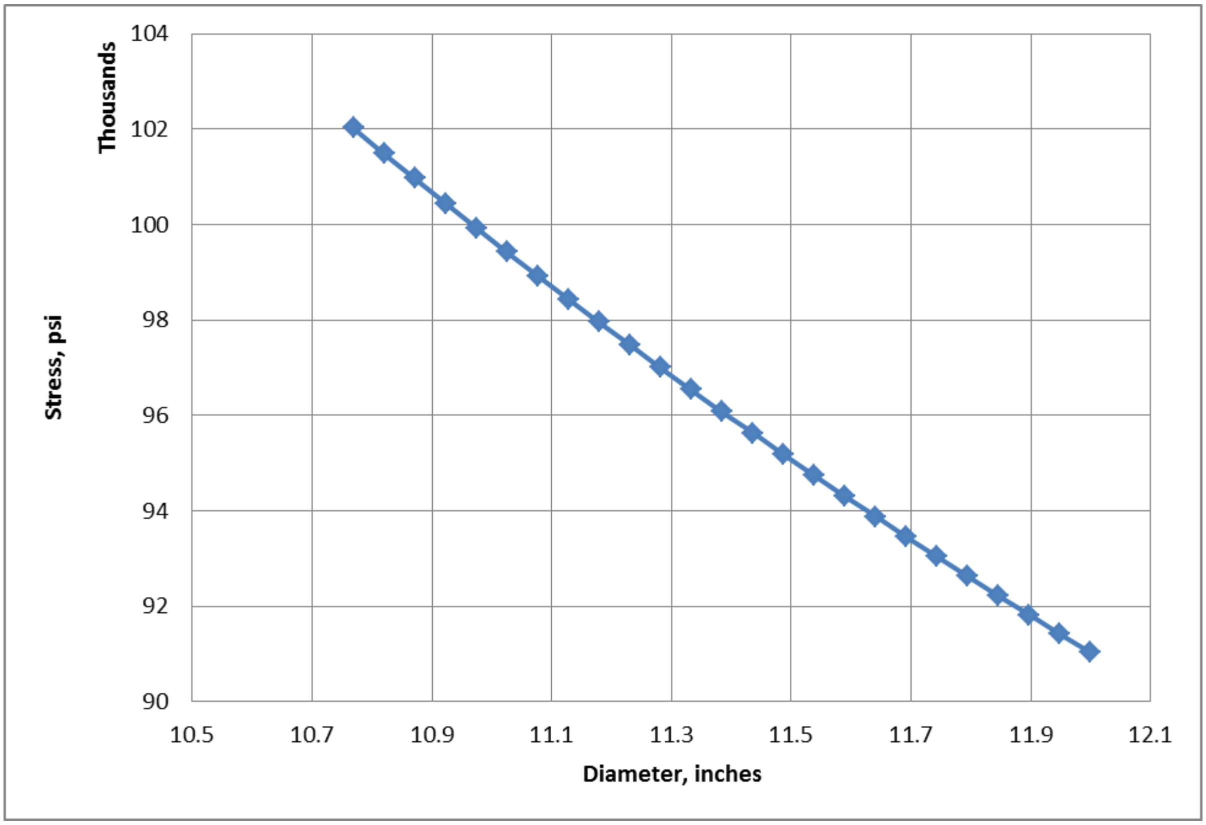

Figure 41 – E-KD-6.1.2-1 – Circumferential Stress at First Thread in Vessel Closure using KD-631.2

PART 9

Example Problems on Residual Stresses in Multiwall Vessels

9 EXAMPLE PROBLEMS ON RESIDUAL STRESSES IN MULTIWALL VESSELS

9.1 Example Problem E-KD-8.1.1 – Dual Wall Cylindrical Vessel Stress Distribution

Evaluate the hoop and radial residual stress distributions in the liner and outer wall of the dual wall cylindrical vessel found in Example Problem E-KD-2.1.2 in accordance with KD-8 stress given the following data and that from the original problem. The vessel does not have any additional residual stresses such as from autofrettage. The area of the wall being analyzed is remote from any discontinuities in the vessel shell.

- Liner Material = SA-705 Gr. XM-12 Condition H1100
    - Elastic Modulus = 28,300 ksi @ 100°F per Table TM-1 of Section II, Part D
    - Poisson's Ratio = 0.31 per Table PRD of Section II, Part D
- Body Material = SA-723 Gr. 2 Class 2
    - Elastic Modulus = 27,600 ksi @ 100°F per Table TM-1 of Section II, Part D
    - Poisson's Ratio = 0.30 per Table PRD of Section II, Part D
- Overall Diameter Ratio (Y) = 3.125
- Liner Wall Ratio (Y_i) = 1.5
- Outer Body Ratio (Y_o) = 2.083
- Diametral Interference (□) 0.050 in
- Interface Diameter (D_{if}) = 24 in (from problem E-KD-2.1.2)

<u>**STEP 1** – Calculate the Interference Pressure between the cylinders using KD-811.1</u>

The interface pressure (P_{if}) is calculated using the following equation:

$$P_{if} = \frac{\delta}{D_{if} A}$$

Where:

$$A = \frac{1}{E_i}\left(\frac{D_I^2 + D_{if}^2}{D_{if}^2 - D_I^2} - \nu_i\right) + \frac{1}{E_O}\left(\frac{D_{if}^2 + D_O^2}{D_O^2 - D_{if}^2} + \nu_O\right)$$

Therefore, the interface pressure is:

$$P_{if} = 13{,}915 \text{ psi}$$

The residual stresses at any point in the inner layer ($D_i < D < D_{if}$) are calculated from equations (1) and (2) of KD-811.2.

$$\sigma_{trl} = \frac{-P_{if} \cdot Y_i^2}{Y_i^2 - 1} \cdot \left(1 + \frac{D_i^2}{D^2}\right)$$

$$\sigma_{rrl} = \frac{-P_{if} \cdot Y_i^2}{Y_i^2 - 1} \cdot \left(1 - \frac{D_i^2}{D^2}\right)$$

And for the outer layer, the residual stresses for the outer layer ($D_{if} \le D \le D_o$)are calculated using equations (3) and (4)

$$\sigma_{trb} = \frac{P_{if}}{Y_o^2 - 1} \cdot \left(1 + \frac{D_o^2}{D^2}\right)$$

$$\sigma_{rrb} = \frac{P_{if}}{Y_o^2 - 1} \cdot \left(1 - \frac{D_o^2}{D^2}\right)$$

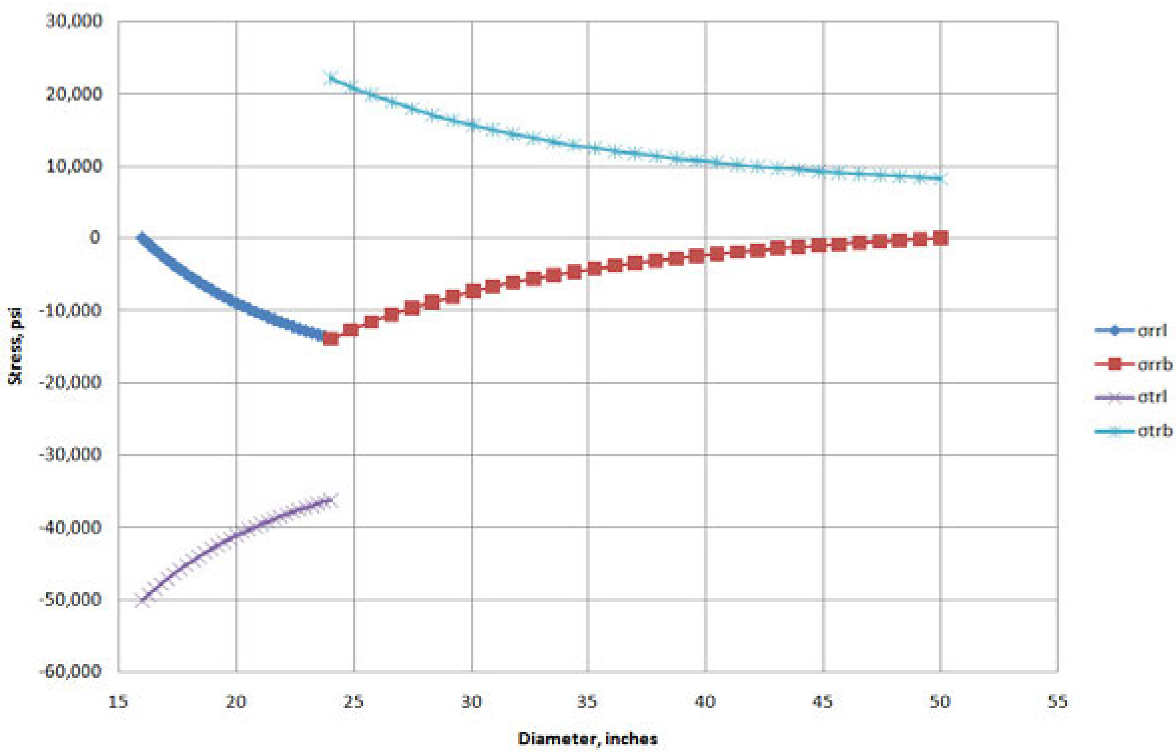

Figure 42 – E-KD-8.1.1-1 – Stress Distribution In Dual Wall Vessel Liner and Body

PART 10
Example Problems in Determination of Hydrostatic Test Pressure

10 EXAMPLE PROBLEMS IN DETERMINATION OF HYDROSTATIC TEST PRESSURE

10.1 Example Problem E-KT-3.1.1 – Determination of Hydrostatic Test Pressure in Cylindrical Vessel

Determine the hydrostatic test pressure for a monobloc cylindrical vessel from problem E-KD-2.1.1 in accordance with the requirements of Article KT-3. Perform calculations for both open and closed-end vessels.

<u>Vessel Data:</u>

- Material = SA-705 Gr. XM-12 Condition H1100
- Design Temperature = 70°F
- Inside Diameter = 6.0 in
- Outside Diameter = 12.0 in
- Diameter Ratio (Y) = 2.0 [KD-250]
- Yield Strength = 115,000 psi @ 100°F per Table Y-1 of Section II, Part D
- Tensile Strength = 140,000 psi @ 100°F
- Test Temperature = 70°F

STEP 1 – Evaluate the lower limit on hydrostatic test pressure per KT-311

From example problem E-KD-2.1.1, the design pressure computed for both an open-ended and a closed-ended cylinder are:

$P_d = 50{,}581$ psi   Open Ended

$P_d = 53{,}141$ psi   Closed Ended

Assuming the test temperature is 70°F, the following hydrostatic test pressures are determined:

$$P_t = 1.25 P_d \left[\frac{\left(S_y\right)_{T_1}}{\left(S_y\right)_{T_2}} \right]$$

Where: $\left(S_y\right)_{T_1}$ = Yield strength at test temperature

$\left(S_y\right)_{T_2}$ = Yield strength at design temperature

The minimum test pressures for the vessels are:

$P_t = 63{,}226$ psi   Open Ended

$P_t = 73{,}648$ psi   Closed Ended

<u>**STEP 2** – Evaluate the upper limit on hydrostatic test pressure per KT-312</u>

The upper limit on an open ended cylindrical shell for the shell in question ($Y = 2.00$) per KT-312.1 is:

$$P_T = 3.232 \cdot S_y \cdot \left(Y^{0.268} - 1\right)$$

The upper limit of hydrostatic test pressure for a closed ended cylindrical shell is $P_T = S_y \ln(Y)$

Therefore, on that basis the upper limit of hydrostatic test pressure is:

$P_t = 75{,}874$ psi   Open Ended

$P_t = 79{,}712$ psi   Closed Ended

It should also be noted that the test pressure in KT-312.2 may be exceeded per KT-312.3, provided that the Designer evaluates the suitability and integrity of the vessel and documents that evaluation in the Manufacturer's Design Report.

PART 11

Example Problems Using the Methods of Appendix E

11 EXAMPLE PROBLEMS USING THE METHODS OF APPENDIX E

11.1 Example Problem E-AE-2.1.1 – Blind End Dimensions and Corner Stresses in a Vessel without Detailed Stress Analysis – Thick Wall Pressure Vessel

Determine the dimensions of the bottom of the vessel including the maximum opening size in example problem E-KD-2.1.1 and the stresses in the corner radius using the methods of Appendix E for design without detailed analysis.

Vessel Dimension and Loading Data

The vessel is to be the same as that found in E-KD-2.1.1.

$t_w = 3$ in	Wall Thickness
$Y = 2.0$	Diameter Ratio (D_o / D_i)
$D_i = 6.0$ in	Inside Diameter
$P_d = 53{,}141$ psi	Design Pressure from E-KD-2.1.1

Bottom central opening – One standard ¼ inch high pressure connection with ∅ 0.093 port and ∅ 9/16 -18 UNF thread.

STEP 1 – Determine the minimum inside corner radius (R_c)

This is to be a minimum of 25% of the design wall thickness. In this case:

$$R_c = 0.25 \text{ x } 3.00 \text{ inch} = 0.75 \text{ in}$$

STEP 2 – Determine the minimum thickness of the blind end (t_b)

For diameter ratios (Y) between 1.25 to 2.25 and where R_c follows the rules of Step 1 the minimum bottom thickness is:

$$t_b = t_w\,(-1.0667Y^3 + 6.80Y^2 - 15.433Y + 13.45) = 3.751 \text{ in}$$

In this case, $t_w = 3$ in and $Y = 2.0$.

STEP 3 – Angle of Bottom

This is valid for bottom angle $a \leq 10°$.

STEP 4 – Determine maximum diameter of bottom opening (D_{OP})

Maximum size of a centrally located bottom opening is 15% of the inner diameter.

$$D_{OP} = 0.15 * 6.0 \text{ in} = 0.900 \text{ in}$$

Therefore, the ¼ inch standard opening in the center of the bottom with the ∅ 9/16 -18 UNF thread and ∅ 0.093 port is acceptable.

STEP 5 – Determination of the corner radius principal stresses in the bottom radius (σ_1, s_2, σ_3)

The principal stresses in the vessel bottom can be found using the equations in E-110(b):

$$\sigma_1 = Pd\cdot\left(0.6320 - 1.5160\cdot Y + 4.5731\cdot\frac{Y}{\ln(Y)} - 19.1428\cdot\frac{1}{\sqrt{Y}} + 31.0567\cdot\frac{\ln(Y)}{Y^2}\right) = 140340\,\text{psi}$$

$$\sigma_2 = Pd\cdot\left(-0.5718 + 0.1141\cdot Y + 1.0208\cdot\frac{1}{\ln(Y)} + 1.6096\cdot\frac{\ln(Y)}{Y^2} - 1.3667\cdot\frac{1}{Y^2}\right) = 56667\,\text{psi}$$

$$\sigma_3 = -Pd = -53141\,\text{psi}$$

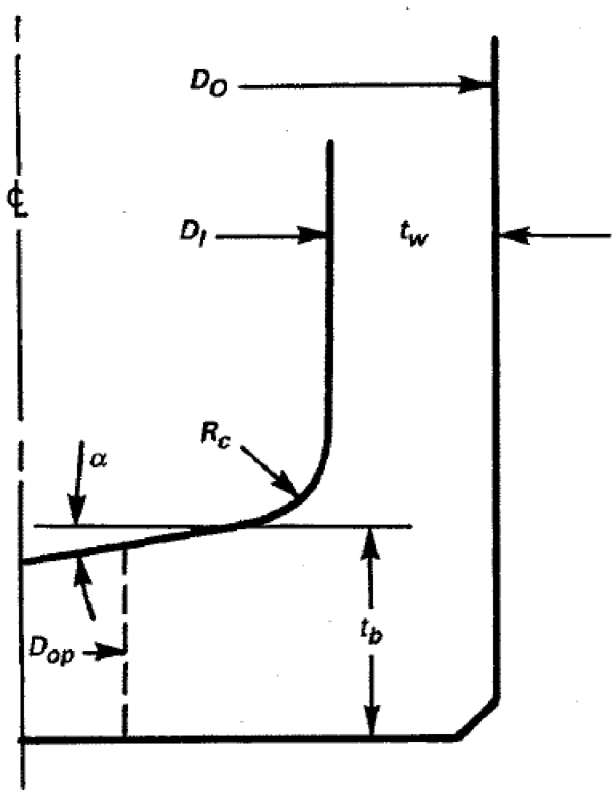

Figure 43 – E-AE-2.2.2-1 – Dimensions of Blind End of a Thick Walled Pressure Vessel (from Figure E-110)

11.2 Example Problem E-AE-2.1.2 – Blind End Dimensions and Corner Stresses in a Vessel without Detailed Stress Analysis – Thin Wall Pressure Vessel

Determine the dimensions of the bottom of the vessel including the maximum opening size in example problem E-KD-2.3.1 and the stress intensity in the corner radius using the methods of Appendix E for design without detailed analysis.

Vessel Dimension and Loading Data

The vessel is to be the same as that found in E-KD-2.3.1.

t_w = 1 in — Wall Thickness

Y = 1.2 — Diameter Ratio (D_o / D_i)

D_i = 10.0 in — Inside Diameter

R_c = 2 in — Inside Corner Radius

Pd = 15,315 psi — Design Pressure

STEP 1 – Determine the minimum thickness of the vessel bottom (t_b)

For diameter ratios (Y) less than 1.25, the minimum bottom thickness is:

$$t_b = Di \cdot \left(1.5 \cdot C \cdot \frac{P_d}{S_y}\right)^{0.5} = 2.939\,\text{in}$$

Where the bottom factor (C) is used in calculating the bottom thickness based on the dimensions of the vessel. In this case, it is less than three times the end thickness (R_c = 2 in) Therefore, C = 0.44 per E-120(c).

STEP 2 – Determination of the stress intensity in the bottom radius (σ_1, S_2, σ_3)

The principal stresses in the vessel bottom can be found using the equations in E-110(b):

S = 1.8 C (Di / t_b)^2 Pd = 140,400 psi

11.3 Example Problem E-AE-2.2.1 – Thread Load Distribution

Determine the loads applied on the body threads for the example problem E-KD-2.3.1. For the configuration given in example problem E-KD-2.3.1 assuming the load on the last thread is unity, the load on the individual threads is determined as shown below.

Vessel Dimension and Loading Data

P_D = 11,000 psi	Design Pressure
D_O = 12 in	Outside Diameter of the Vessel
D_I = 10 in	Inside Diameter of the Vessel
D_p = 10.443 in	Pitch Diameter of the Threads
P_T = 0.5 in	Thread Pitch
n = 10	Total number of threads which is less than 20 but greater than 4 per E-200.
A_B = 27.45 in^2	Cross-sectional area of the vessel normal to the vessel axis through the internal threads
A_C = 85.65 in^2	Cross-sectional area of the vessel normal to the vessel axis through the external threads
C_M = 0.024054 in^{-1}	Combined flexibility factor of the body and closure
C_T = 0.19152 in^{-1}	Flexibility factor of the threads

Thread Helix Angle = atan($P_T / \pi D_p$) = 0.873977° which is less than the 2°

The total load acting at the seal connection between the closure and the body is equal to $F_S = P_D \times \pi/4 \times D_I^{\,2} = 863{,}938$ lbf

Table 8 – E-AE-2.2.1-1 – Thread Load Distribution

Thread	F_i [Note 1]	F_{sum}	F_i % [Note 2]	C_m/C_t x F_{sum}	F_i %× F_S (lbf) [Note 3]
9	1.000	1.000	3.02	0.126	26087
8	1.126	2.126	3.40	0.267	29363
7	1.393	3.158	4.20	0.442	36327
6	1.834	5.353	5.54	0.672	47854
5	2.507	7.860	7.57	0.987	65391
4	3.494	11.354	10.55	1.426	91141
3	4.92	16.274	14.85	2.044	128338
2	6.963	23.238	21.03	2.919	181653
1	9.882	33.121	29.84	4.160	257784

Notes:

1) $F_T = 33.118$ (obtained by adding nine F_i values)
2) F_i % = Percentage total seal load (F_S) carried by individual threads
3) F_i %× F_S = End load carried by individual thread

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