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ASME
Section VIII – Division 2
Example Problem Manual

SETTING THE STANDARD

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ASME Section VIII - Division 2 Example Problem Manual

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FOREWORD

This document is the second edition of the ASME Section VIII – Division 2 example problem manual. The purpose of this second edition is to update the example problems to keep current with the changes incorporated into the 2013 edition of the ASME B&PV Code, Section VIII, Division 2. The example problems included in the first edition of the manual were based on the contents of the 2010 edition of the B&PV Code. In 2011, ASME transitioned to a two year publishing cycle for the B&PV Code without the release of addenda. The release of the 2011 addenda to the 2010 edition was the last addenda published by ASME and numerous changes to the Code were since adopted.

Known corrections to design equations and results have also been made in this second edition. Additionally, some formatting modifications were made to facilitate better use of the example manual, as applicable.

In 1998 the ASME Boiler and Pressure Vessel Standards Committee authorized a project to rewrite the ASME B&PV Code, Section VIII, Division 2. This decision was made shortly after the design margin on specified minimum tensile strength was lowered from 4.0 to 3.5 in Section I and Section VIII, Division 1. ASME saw the need to update Section VIII, Division 2 to incorporate the latest technologies and to be more competitive. In lieu of revising the existing standard, the decision was made to perform a clean sheet rewrite. By doing so it was felt that, not only could the standard be modernized with regard to the latest technical advances in pressure vessel construction, but it could be structured in a way to make it more user-friendly for both users and the committees that maintain it.

Much new ground was broken in the development of the new Section VIII, Division 2, including the process taken to write the new standard. Traditionally, development of new standards by ASME is carried out by volunteers who serve on the different committees responsible for any given standard. Depending upon the complexity of the standard, the development of the first drafts may take up to 15 years to complete based on past history. The prospect of taking 15 or more years to develop VIII-2 was unacceptable to ASME and the volunteer leadership. The decision was made to subcontract the development of the draft to the Pressure Vessel Research Council (PVRC) who in turn formed the Task Group on Continued Modernization of Codes to oversee the development of the new Section VIII, Division 2 Code. PVRC utilized professionals with both engineering and technical writing expertise to develop new technology and the initial drafts of the new Section VIII, Division 2.

A Steering Committee made up of ASME Subcommittee VIII members was formed to provide technical oversight and direction to the development team with the goal of facilitating the eventual balloting and approval process. ASME also retained a Project Manager to manage all the activities required to bring this new standard to publication.

The project began with the development of a detailed table of contents containing every paragraph heading that would appear in the new standard and identifying the source for the content that would be placed in this paragraph. In preparing such a detailed table of contents, the lead authors were able to quickly identify areas where major development effort was required to produce updated rules. A list of some of the new technology produced for VIII-2 rewrite includes:

- Adoption of a design margin on specified minimum tensile strength of 2.4,
- Toughness requirements,
- Design-by-rule for the creep range,
- Conical transition reinforcement requirements,
- Opening reinforcement rules,
- Local strain criteria for design-by-analysis using elastic-plastic analysis,
- Limit load and plastic collapse analysis for multiple loading conditions,
- Fatigue design for welded joints based on structural stress method, and
- Ultrasonic examination in lieu of radiographic examination.

Users of the Section VIII, Division 2 Code (manufacturers and owner/operators) were surveyed at the beginning of the project to identify enhancements that they felt the industry wanted and would lead to increased use of the standard. Since the initial focus of the Code was for the construction of pressure equipment for the chemical and petrochemical industry, the people responsible for specifying equipment for this sector were very much interested in seeing that common requirements that are routinely found in vessel specifications would become a requirement within this standard. This was accomplished by close participation of the petrochemical industry during the development of this standard. Some of the enhancements included:

- Alternatives provided for U.S. and Canadian Registered Professional Engineer certification of the User Design Specification and Manufacturers Design Report,
- Consolidation of weld joint details and design requirements,
- Introduction of a weld joint efficiency and the use of partial radiographic and ultrasonic examination,
- Introduction of the concept of a Maximum Allowable Working Pressure (MAWP) identical to VIII-1,
- Significant upgrade to the design-by-rule and design-by-analysis procedures,
- Extension of the time-independent range for low chrome alloys used in heavy wall vessels,
- Extension of fatigue rules to 900°F (400°C) for low-chrome alloys used in heavy wall vessels,
- Adoption of new examination requirements and simplification of presentation of the rules,
- User-friendly extensive use of equations, tables, and figures to define rules and procedures, and
- ISO format; logical paragraph numbering system and single column format,
- Many of these enhancements identified by users were included in the first release of Section VIII, Division 2 in 2007.

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PART 1

GENERAL REQUIREMENTS

PART CONTENTS

1.1 Introduction

ASME B&PV Code, Section VIII, Division 2 contains mandatory requirements, specific prohibitions, and non-mandatory guidance for the design, materials, fabrication, examination, inspection, testing, and certification of pressure vessels and their associated pressure relief devices. The 2007 edition of the code has been re-written and reorganized, and incorporates the latest technologies for pressure vessel design. Since this initial release the code has undergone further development in all of its Parts, including refinement of its Part 4 design-by-rule procedures and Part 5 design-by-analysis methods. These modifications are captured in this PTB document.

1.2 Scope

Example problems illustrating the use of the design-by-rule and design-by-analysis methods in ASME B&PV Code, Section VIII, Division 2 are provided in this document. Example problems are provided for all calculation procedures primarily in US Customary units, however select problems are shown using SI units.

1.3 Organization and Use

An introduction to the example problems is described in Part 2 of this document. The remaining Parts of this document contain the example problems. The Parts 3, 4, and 5 in this document coincide with the Parts 3, 4 and 5 in the ASME B&PV Code, Section VIII, Division 2. For example, example problems illustrating the design-by-rule calculations contained in Part 4 of Section VIII, Division 2 are provided in Part 4 of this document. All paragraph references are to the ASME B&PV Code, Section VIII, Division 2, 2013 Edition. [1].

The example problems in this manual follow the calculation procedures in ASME B&PV Code, Section VIII, Division 2. It is recommended that users of this manual obtain a copy of ASME PTB-1-2013 [2] that contains criteria and commentary on the use of the design rules.

It should be noted that VIII-2 permits the use of API 579-1/ASME FFS-1 [3] for some calculation procedures. When reviewing certain example problems in this manual, it is recommended that users obtain a copy of this standard.

1.4 References

1. ASME B&PV Code, Section VIII, Division 2, 2013, ASME, New York, New York, 2013.
2. Osage, D., *ASME Section VIII – Division 2 Criteria and Commentary*, PTB-1-2013, ASME, New York, New York, 2013.
3. API, API 579-1/ASME FFS-1 2007 *Fitness-For-Service*, American Petroleum Institute, Washington, D.C., 2007.

PART 2

EXAMPLE PROBLEM DESCRIPTIONS

PART CONTENTS

2.1 General

Example problems are provided for the following parts of the document;

- Part 3 – Materials Requirements
- Part 4 – Design By Rule Requirements
- Part 5 – Design By Analysis Requirements
- Part 6 – Fabrication Requirements
- Part 7 – Examination Requirements
- Part 8 – Pressure Testing Requirements

A summary of the example problems provided is contained in the Table of Contents.

2.2 Example Problem Format

In all of the example problems, with the exception of tubesheet design rules in paragraph 4.18, the code equations are shown with symbols and with substituted numerical values to fully illustrate the use of the code rules. Because of the complexity of the tubesheet rules, only the results for each step in the calculation producer is shown.

2.3 Calculation Precision

The calculation precision used in the example problems is intended for demonstration proposes only; an intended precision is not implied. In general, the calculation precision should be equivalent to that obtained by computer implementation, rounding of calculations should only be done on the final results.

PART 3

MATERIALS REQUIREMENTS

PART CONTENTS

3.1 Example E3.1 – Use of MDMT Exemptions Curves

Determine if Impact Testing is required for the proposed shell section, using only the rules of paragraph 3.11.2.3. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Nominal Thickness = $1.8125\ in$
- PWHT = Yes
- MDMT = $-20.0°F$
- Corrosion Allowance = $0.125\ in$

Per paragraph 3.11.2.3 for Carbon and Low Alloy Steel Except Bolting.

a) Since the vessel has been PWHT, Figure 3.8 (or Table 3.15) shall be used to establish impact testing exemptions based on the impact test exemption curve for the subject material specification, MDMT, and governing thickness of a welded part.

b) As noted in Figure 3.8, from the Material Assignment Table, a material specification of $SA-516,\ Grade\ 70,\ Norm.$ is designated a Curve D material.

c) The governing thickness t_g of a welded part is determined from the criteria of paragraph 3.11.2.3.b. For a butt joint in a cylindrical shell, t_g is equal to the nominal thickness of the thickest weld joint, see Figure 3.9 Sketch (a).

$$t_g = 1.8125\ in$$

d) If an MDMT and thickness combination for the subject material is on or above the applicable impact test exemption curve, then impact testing is not required for base metal. Requirements for weld metal and heat affected zones are provided in paragraph 3.11.8.

Interpreting the value of MDMT from Figure 3.8 is performed as follows. Enter the figure along the abscissa with a nominal governing thickness of $t_g = 1.8125\ in$ and project upward until an intersection with the Curve D material is achieved. Project this point left to the ordinate and interpret the MDMT. This results in an approximate value of $MDMT = -19.0°F$.

A more accurate value for MDMT can be achieved by using the tabular values found in Table 3.15. Linear interpolation between thicknesses for a $t_g = 1.8125\ in$ and a Curve D material results in the following value for MDMT.

$$y = \left(\frac{x - x_1}{x_2 - x_1} \right) (y_2 - y_1) + y_1$$

$$MDMT = \left(\frac{1.8125 - 1.75}{2.0 - 1.75} \right) \left[(-15.8) - (-20.2) \right] + (-20.2) = -19.1°F$$

Since the calculated MDMT of $-19.1°F$ is warmer than the required MDMT of $-20.0°F$, impact testing is required using only the rules in 3.11.2.3. However, impact testing may still be avoided by applying the rules of paragraph 3.11.2.5 or 3.11.2.8.

3.2 Example E3.2 – Use of MDMT Exemption Curves with Stress Reduction

Determine if impact testing is required for the proposed shell section in E3.1, using the rules of paragraph 3.11.2.5. The shell is cylindrical with all Category A joints made up of Type 1 butt welds which have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $356\ psi\ @\ 300°F$
- Inside Diameter = $150\ in$
- Nominal Thickness = $1.8125\ in$
- PWHT = Yes
- MDMT = $-20.0°F$
- Weld Joint Efficiency = 1.0
- Corrosion Allowance = $0.125\ in$
- Allowable Stress at Ambient Temperature = $22400\ psi$
- Allowable Stress at Design Temperature = $22400\ psi$
- Yield Strength at Ambient Temperature = $38000\ psi$

In accordance with paragraph 3.11.2.5, the procedure that is used to determine the exemption from impact testing based on design stress values is shown below.

a) STEP 1 – For the welded part under consideration, determine the nominal thickness of the part, t_n, and the required governing thickness t_g using paragraph 3.11.2.3.

$$t_n = t_g = 1.8125\ in$$

b) STEP 2 – Determine the applicable material toughness curve to be used. Since the vessel has been PWHT, Figure 3.8 (or Table 3.15) shall be used to establish impact testing exemptions based on the impact test exemption curve for the subject material specification, MDMT, and governing thickness of a welded part. As noted in Figure 3.8, from the Material Assignment Table, a material specification of $SA-516,\ Grade\ 70,\ Norm.$ is designated a Curve D material.

c) STEP 3 – Determine the MDMT from Figure 3.8 based on the applicable toughness curve and the governing thickness, t_g, From E3.1,

$$MDMT = -19.1°F$$

d) STEP 4 – Based on the design loading conditions at the MDMT, determine the stress ratio, R_{ts}, using the thickness basis, Equation (3.1).

$$R_{ts} = \frac{t_r E^*}{t_n - CA}$$

Where, t_r is the required thickness of the cylindrical shell at the specified MDMT of $-20.0°F$, using Equation 4.3.1 from Part 4.

$$t_r = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right] - 1\right)$$

$$where,$$

$$D = 150.0 + 2(Corrosion\ \ Allowance) = 150.0 + 2(0.125) = 150.25\ in$$

$$t_r = \frac{150.25}{2}\left(\exp\left[\frac{356}{22400(1.0)}\right] - 1\right) = 1.2035\ in$$

The variables E^*, t_n, and CA are defined as follows:

$$E^* = \max[E,\ 0.80] = \max[1.0, 0.8] = 1.0$$

$$t_n = 1.8125\ in$$

$$CA = 0.125\ in$$

Therefore,

$$R_{ts} = \frac{t_r E*}{t_n - CA} = \frac{1.2035(1.0)}{1.8125 - 0.125} = 0.7132$$

e) STEP 5 - Determine the final value of the MDMT and evaluate the results. Since the computed value of the ratio $R_{ts} > 0.24$ from STEP 4, the specified minimum yield strength, $S_y \le 50\ ksi$, and the shell was subject to PWHT, then the reduction in MDMT based on available thickness T_R is computed using Figure 3.13 (or Table 3.17).

Interpreting the value of T_R from Figure 3.13 is performed as follows. Enter the figure along the ordinate with a value of $R_{ts} = 0.7132$ project horizontally until an intersection with the $S_y \le 50\ ksi$ curve is achieved. Project this point downward to the abscissa and interpret T_R. This results in an approximate value of $T_R = -28.0°F$.

A more accurate value for T_R can be achieved by using the tabular values found in Table 3.17. Linear interpolation between ratios $R_{ts} = 0.7132$ and $S_y \le 50\ ksi$ results in the following value of T_R.

$$T_R = \left(\frac{0.7132-0.690}{0.734-0.690}\right)(25.8-31.1)+(31.1) = 28.3°F$$

The final computed value of the MDMT is determined using Equation (3.5).

$$MDMT = MDMT_{STEP3} - T_R = -19.1°F - 28.3°F = -47.4°F$$

Since the final value of MDMT is colder than the proposed MDMT, impact testing is not required.

3.3 Example E3.3 – Develop MDMT Using Fracture Mechanics (API 579-1/ASME FFS-1)

Using the fracture mechanics approach for determination of $MDMT$ given in paragraph 3.11.2.8, determine if an $MDMT$ of $-20°F$ is acceptable for the vessel shell shown below. Assume that the crack is located in the heat affected zone of a longitudinal double-v groove weld seam, oriented parallel to the weld seam.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $356\ psig\ @300°F$
- Inside Radius = $75.125\ in$
- Nominal Thickness = $1.6875\ in$
- PWHT = Yes
- MDMT = $-20.0°F$
- Longitudinal Weld Joint Efficiency = 1.0
- Ambient Yield Strength = $38000\ psi$
- Flaw Type per paragraph 3.11.2.8 = $Long,\ semi-ellip\ crack,$ $long\ weld,\ inside\ surface$
- Probability of Failure = 10^{-6}
- Coefficient of Variation = $0.10\ (Stresses\ well\ known)$
- Loads = $Pressure\ Only$

In accordance with paragraph 3.11.2.8, evaluate an assessment temperature of $-20°F$ using a fracture mechanics methodology. NOTE – for a typical fitness-for-service assessment involving crack-like flaws, the component under consideration shall be evaluated in its current state, where future corrosion allowance is not considered.

In accordance with paragraph 3.11.2.8.c, determine the values for a and $2c$. From the vessel data provided,

$$a = \frac{1}{4}t = \frac{1}{4}(1.6875) = 0.4219\ in$$

$$2c = 6a = 6(0.4219) = 2.5314\ in$$

Using API 579-1/ASME FFS-1, Part 9 Paragraph 9.4.3 Level 2 Assessment

a) STEP 1 – Evaluate operating conditions and determine the pressure, temperature and supplemental loading combinations to be evaluated.

$$P = 356\ psig @ 300°F$$

$$No\ Supplemental\ Loads$$

b) STEP 2 – Determine the stress distributions at the location of the flaw based on the applied loads in STEP 1 and classify the resulting stresses into the following stress categories: Primary Stress; Secondary Stress; and Residual Stress.

1) Primary Stress – The flaw is located away from all structural discontinuities. Therefore, the primary stress at the weld joint perpendicular to the crack face is a membrane hoop stress. From Annex C, Table C.1, the flaw geometry, component geometry, and loading condition correspond to KCSCLE2 and RCSCLE2, i.e. Cylinder - Surface Crack - Longitudinal Direction - Semi-Elliptical Shape - Internal Pressure. The stress intensity factor solution for KCSCLE2 is provided in Annex C, paragraph C.5.10. The reference stress solution for RCSCLE2 is provided in Annex D, paragraph D.5.10. The membrane and bending components of the primary stress for the calculation of the reference stress are given by Equations (D.47) and (D.48).

$$P_m = \frac{pR_i}{t} = \frac{356(75.125)}{1.6875} = 15848.5926\ psi$$

$$P_b = \frac{pR_o^2}{R_o^2 - R_i^2}\left[\frac{t}{R_i} - \frac{3}{2}\left(\frac{t}{R_i}\right)^2 + \frac{9}{5}\left(\frac{t}{R_i}\right)^3\right]$$

$$P_b = \frac{356(76.8125)^2}{(76.8125)^2 - (75.125)^2}\left[\frac{1.6875}{75.125} - \frac{3}{2}\left(\frac{1.6875}{75.125}\right)^2 + \frac{9}{5}\left(\frac{1.6875}{75.125}\right)^3\right]$$

$$P_b = 177.9865\ psi$$

2) Secondary Stress – Thermal gradients do not exist in the vessel at the location of the flaw, and the flaw is located away from all major structural discontinuities. Therefore, there are no secondary stresses.

3) Residual Stress – The flaw is located at a weld in a vessel that was subject to PWHT at the time of fabrication. From Annex E, paragraph E.3.2, to estimate the magnitude of the residual stress distribution at a weld joint, an estimate of the actual yield strength of the material must be made. The elevation of the effective yield strength above the specified minimum yield strength accounts for the typical elevation of actual properties above minimum requirements and is calculated per Equation (E.2)

$$\sigma_{ys}^r = \sigma_{ys} + 10\ ksi = 38.0 + 10.0 = 48.0\ ksi$$

For a longitudinal double-v weld seam subject to PWHT, the residual stress distribution perpendicular to the weld seam is determined in Annex E, paragraph E.4.4.1 per Equation (E.65). The value is considered to be a constant through-thickness distribution.

$$\sigma^r(x) = 0.2\sigma_{ys}^r = 0.2(48.0) = 9.6\ ksi$$

c) STEP 3 – Determine material properties; yield strength, tensile strength and fracture toughness,

K_{mat}, for the conditions being evaluated in STEP 1. The actual material properties for the plate are not available; therefore, the specified minimum yield strength and tensile strength are used.

$$\sigma_{ys} = 38.0\,ksi$$

$$\sigma_{uts} = 70.0\ ksi$$

The material fracture toughness is established using the MPC Charpy impact energy correlation found in Annex F, paragraph F.4.5.3, Equation (F.84).

$$K_{mat} = K_{IC} = \sigma_{ys}\left\{1.7 + \left(1.7 - \frac{27}{\sigma_{ys}}\right)\cdot \tanh\left[\frac{T-(T_o-75)}{C}\right]\right\}$$

$$K_{mat} = K_{IC} = 38.0\left\{1.7 + \left(1.7 - \frac{27}{38.0}\right)\cdot \tanh\left[\frac{(-20-(12-75))}{66}\right]\right\} = 86.1331\ ksi\sqrt{in}$$

Where,

$$T_o = 12°F$$

$T = -20°F$ (Assessment Temperature)

$$C = 66°F$$

σ_{ys} yield stress in ksi

d) STEP 4 – Determine crack-like flaw dimensions in accordance with paragraph 3.11.2.8.c.

$$a = 0.4219\ in$$

$$2c = 2.5314\ in$$

e) STEP 5 – Modify the primary stress, material fracture toughness, and flaw size using the Partial Safety Factors, PSF. If a given input value is known to be a conservative estimate (e.g. upper-bound stresses, lower-bound fracture toughness, or upper -bound flaw size), then an applicable PSF equal to 1.0 may be used in the assessment.

From Part 9, Table 9.3, modify the primary stress, material fracture toughness, and flaw size using the applicable PSF values. Based on an assumed depth of flaw, a coefficient of variation, COV_s, which is used to define the uncertainty in the primary stress distribution, and the probability of failure, p_f, the variable R_c used in Table 9.3 is determined as shown.

$$\left[\begin{array}{l} a = 0.4219\ in \\ COV_s = 0.10 \\ p_f = 10^{-6} \end{array}\right] \rightarrow R_c = 1.8\ \sqrt{in}$$

Calculation of the variable, R_{ky}, is also required to determine the values of PSF.

$$R_{ky} = \frac{K_{mat}^{mean}}{\sigma_{ys}} C_u = \left(\frac{86.1331}{38.0}\right) \cdot 1.0 = 2.2667\ \sqrt{in}$$

Where,

$$K_{mat}^{mean} = K_{IC} = 86.1331\ ksi\sqrt{in}$$

$$C_u = 1.0\ \left(if\ units\ of\ K_{mat}^{mean}\ are\ ksi\sqrt{in}\ and\ \sigma_{ys}\ are\ ksi\right)$$

Therefore, the values of PSF can now be determined from Table 9.3.

$$\left[\begin{array}{l} a = 0.4219\ in \\ COV_s = 0.10 \\ p_f = 10^{-6} \\ R_c = 1.8\ \sqrt{in} \\ R_{ky} = 2.2667\ \sqrt{in} \end{array}\right] \rightarrow \left[\begin{array}{l} PSF_s = 2.0 \\ PSF_k = 1.0 \\ PSF_a = 1.0 \end{array}\right]$$

1) Primary Membrane and Bending Stress - Modify the primary membrane and bending stress components determined in STEP 2.

$$P_m = P_m \cdot PSF_s = 15848.5926(2.0) = 31697.1852\ psi$$

$$P_b = P_b \cdot PSF_s = 177.9865(2.0) = 355.9730\ psi$$

2) Material Toughness - Modify the mean value of the material fracture toughness determined in STEP 3.

$$K_{mat} = \frac{K_{mat}}{PSF_k} = \frac{86.1331}{1.0} = 86.1331\ ksi\sqrt{in}$$

3) Flaw Size - Modify the flaw size determined in STEP 4.

$$a = a \cdot PSF_a = 0.4219(1.0) = 0.4219\ in$$

f) STEP 6 – Compute the reference stress for primary stresses, σ_{ref}^{P} based on the modified primary stress distribution and modified flaw size from STEP 5 and the reference stress solution in Annex D.

The reference stress solution for RCSCLE2 is provided in Annex D, paragraph D.5.10, Equation (D.74).

$$\sigma_{ref}^{P} = \frac{gP_b + \left[\left(gP_b\right)^2 + 9\left(M_s P_m \left(1-\alpha\right)^2\right)^2 \right]^{0.5}}{3\left(1-\alpha\right)^2}$$

$$\sigma_{ref}^{P} = \frac{0.9948\left(355.9730\right) + \left[\begin{matrix} \left(0.9948\left(355.9730\right)\right)^2 + \\ 9\left(1.0106\left(31697.1852\right)\left(1-0.1071\right)^2\right)^2 \end{matrix} \right]^{0.5}}{3\left(1-0.1071\right)^2}$$

$$\sigma_{ref}^{P} = 32181.5735 \ psi$$

Where, g is calculated using Annex D Equation (D.31) and α is calculated using Annex D Equation (D.75)

$$g = 1 - 20\left(\frac{a}{2c}\right)^{0.75} \alpha^3 = 1 - 20\left(\frac{0.4219}{2.5314}\right)^{0.75} \left(0.1071\right)^3 = 0.9948$$

$$\alpha = \left(\frac{\frac{a}{t}}{1 + \frac{t}{c}} \right) = \left(\frac{\left(\frac{0.4219}{1.6875}\right)}{1 + \left(\frac{1.6875}{1.2657}\right)} \right) = 0.1071$$

Where, M_s is the surface correction factor and is calculated per Annex D, paragraph D.2.3.3.b.1, Equation (D.18).

$$M_s = \frac{1}{1 - \frac{a}{t} + \frac{a}{t}\left(\frac{1}{M_t\left(\lambda_a\right)}\right)} = \frac{1}{1 - \frac{0.4219}{1.6875} + \frac{0.4219}{1.6875}\left(\frac{1}{1.0436}\right)} = 1.0106$$

In the above equation, $M_t\left(\lambda_a\right)$ is the surface correction factor for a cylindrical shell containing a longitudinal through-wall crack per Annex D, Equation (D.8), as a function of λ_a per Equation (D.19).

$$M_t(\lambda_a) = \left(\frac{1.02 + 0.4411\lambda_a^2 + 0.006124\lambda_a^4}{1.0 + 0.02642\lambda_a^2 + 1.533\left(10\right)^{-6} \lambda_a^4} \right)^{0.5}$$

$$M_t(\lambda_a) = \left(\frac{1.02 + 0.4411\left(0.4087\right)^2 + 0.006124\left(0.4087\right)^4}{1.0 + 0.02642\left(0.4087\right)^2 + 1.533\left(10\right)^{-6}\left(0.4087\right)^4} \right)^{0.5} = 1.0436$$

$$\lambda_a = \frac{1.818c}{\sqrt{R_i a}} = \frac{1.818\left(1.2657\right)}{\sqrt{75.125\left(0.4219\right)}} = 0.4087$$

g) STEP 7 – Compute the Load Ratio or the abscissa of the FAD using the reference stress for primary loads from STEP 6 and the yield strength from STEP 3.

$$L_r^P = \frac{\sigma_{ref}^P}{\sigma_{ys}} = \frac{32181.5735}{38000.0} = 0.8469$$

h) STEP 8 – Compute the stress intensity attributed to the primary loads K_I^P, using the modified primary stress distribution and modified flaw size from STEP 5, and the stress intensity factor solutions in Annex C.

The stress intensity factor solution for KCSCLE2 is provided in Annex C, paragraph C.5.11, Equation (C.192), with reference to paragraph C.5.10. For an equivalent membrane and bending stress distribution of a fourth order polynomial stress distribution, the higher order (non-linear) terms $G_2 \rightarrow G_4$ and $\sigma_2 \rightarrow \sigma_4$ can be eliminated from Equation (C.192). Therefore, the stress intensity factor solution from Equation (C.192) reduces to the following.

$$K_I^P = \left[G_o \left\{ \sigma_o + p_c \right\} + G_1 \sigma_1 \left(\frac{a}{t} \right) + G_2 \sigma_2 \left(\frac{a}{t} \right)^2 + G_3 \sigma_3 \left(\frac{a}{t} \right)^3 + G_4 \sigma_4 \left(\frac{a}{t} \right)^4 \right] \sqrt{\frac{\pi a}{Q}}$$

$$K_I^P = \left[G_o \left\{ \sigma_o + p_c \right\} + G_1 \sigma_1 \left(\frac{a}{t} \right) \right] \sqrt{\frac{\pi a}{Q}}$$

The value of K_I^P is calculated at the base of the flaw and at the surface of the flaw.

At the base of the flaw, $\varphi = 90°$:

$$K_I^P = \left[G_o \left\{ \sigma_o + p_c \right\} + G_1 \sigma_1 \left(\frac{a}{t} \right) \right] \sqrt{\frac{\pi a}{Q}}$$

$$K_I^P = \left[1.1512828 \left\{ 32053.1582 + 356 \right\} + 0.6975812 \left(-711.946 \right) \left(0.25 \right) \right] \sqrt{\frac{\pi \left(0.4219 \right)}{1.2389}}$$

$$K_I^P = 38.4648 \ ksi\sqrt{in}$$

At the surface of the flaw, $\varphi = 0°$:

$$K_I^P = \left[G_o \left\{ \sigma_o + p_c \right\} + G_1 \sigma_1 \left(\frac{a}{t} \right) \right] \sqrt{\frac{\pi a}{Q}}$$

$$K_I^P = \left[0.7418361 \left\{ 32053.1582 + 356 \right\} + 0.1159489 \left(-711.946 \right) \left(0.25 \right) \right] \sqrt{\frac{\pi \left(0.4219 \right)}{1.2389}}$$

$$K_I^P = 24.8464 \ ksi\sqrt{in}$$

Per Annex D, paragraph C.5.10.2, the influence coefficients G_0 and G_1 are determined using Equations (C.188) and (C.189), respectively.

At the base of the flaw, $\varphi = 90°$:

$$G_0 = A_{0,0} + A_{1,0}\beta + A_{2,0}\beta^2 + A_{3,0}\beta^3 + A_{4,0}\beta^4 + A_{5,0}\beta^5 + A_{6,0}\beta^6$$

$$G_0 = \begin{pmatrix} 0.7418361 + (\text{-}0.450705)(1) + 6.1501137(1)^2 + (\text{-}15.50397)(1)^3 \\ +19.978669(1)^4 + (\text{-}13.3919)(1)^5 + 3.627239(1)^6 \end{pmatrix} = 1.1512828$$

$$G_1 = A_{0,1} + A_{1,1}\beta + A_{2,1}\beta^2 + A_{3,1}\beta^3 + A_{4,1}\beta^4 + A_{5,1}\beta^5 + A_{6,1}\beta^6$$

$$G_1 = \begin{pmatrix} 0.1159489 + 0.1836592(1) + 2.2622005(1)^2 + (\text{-}4.094396)(1)^3 + \\ 4.6541091(1)^4 + (\text{-}3.530137)(1)^5 + 1.1061961(1)^6 \end{pmatrix} = 0.6975812$$

At the surface of the flaw, $\varphi = 0°$:

$$G_0 = A_{0,0} + A_{1,0}\beta + A_{2,0}\beta^2 + A_{3,0}\beta^3 + A_{4,0}\beta^4 + A_{5,0}\beta^5 + A_{6,0}\beta^6$$

$$G_0 = A_{0,0} = 0.7418361$$

$$G_1 = A_{0,1} + A_{1,1}\beta + A_{2,1}\beta^2 + A_{3,1}\beta^3 + A_{4,1}\beta^4 + A_{5,1}\beta^5 + A_{6,1}\beta^6$$

$$G_1 = A_{0,1} = 0.1159489$$

Where, β is given by Annex C, Equation (C.96) and the parameters $A_{i,j}$ are provided in Annex C, Table C.12 for an inside surface crack. For a surface crack with semi-elliptical shape, the elliptic angle φ is measured from the surface of the crack; see Annex C, Figure C.2.

At the base of the flaw, $\varphi = 90° = \dfrac{\pi}{2}\ rad$.

$$\beta = \frac{2\varphi}{\pi} = \frac{2\left(\dfrac{\pi}{2}\right)}{\pi} = 1$$

At the surface of the flaw, $\varphi = 0° = 0\ rad$.

$$\beta = \frac{2\varphi}{\pi} = \frac{2(0)}{\pi} = 0$$

The coefficients $A_{0,0} \to A_{6,0}$ and $A_{0,1} \to A_{6,1}$ are linearly interpolated from the values found in Annex C, Table C.12. The flaw ratios and parameters used in Table C.12 are as follows:

$$\left\{\begin{array}{l} \dfrac{t}{R_i} = \dfrac{1.6875}{75.125} = 0.0225 \\ \dfrac{a}{c} = \dfrac{0.4219}{1.2657} = 0.3333 \\ \dfrac{a}{t} = 0.25 \end{array}\right\} \Rightarrow \left\{\begin{array}{ll} A_{0,0} = 0.7418361 & A_{0,1} = 0.1159489 \\ A_{1,0} = -0.450705 & A_{1,1} = 0.1836592 \\ A_{2,0} = 6.1501137 & A_{2,1} = 2.2622005 \\ A_{3,0} = -15.50397 & A_{3,1} = -4.094396 \\ A_{4,0} = 19.978669 & A_{4,1} = 4.6541091 \\ A_{5,0} = -13.3919 & A_{5,1} = -3.530137 \\ A_{6,0} = 3.627239 & A_{6,1} = 1.1061961 \end{array}\right\}$$

The calculation procedure to determine coefficient $A_{0,0}$ for G_0 is shown for reference. The procedure requires 3 levels of interpolation within Table C.12.

1) Interpolate for $t/R_i = 0.0225$ between the values of 0.05 and 0.01667
2) Interpolate for $a/c = 0.333$ between the values of 0.25 and 0.50
3) Interpolate for $a/t = 0.25$ between the values of 0.2 and 0.4

The calculations begin with the interpolation for $a/t = 0.25$, which must be done for all conditions.

$t/R_i = 0.05,\ a/c = 0.5$		$t/R_i = 0.05,\ a/c = 0.25$		$t/R_i = 0.01667,\ a/c = 0.5$		$t/R_i = 0.01667,\ a/c = 0.25$	
$a/t = 0.2$	0.8998006	$a/t = 0.2$	0.6229529	$a/t = 0.2$	0.8987625	$a/t = 0.2$	0.6254105
$a/t = 0.4$	0.9875950	$a/t = 0.4$	0.7303090	$a/t = 0.4$	0.9959920	$a/t = 0.4$	0.7330250
$a/t = 0.25$	**0.9217492**	$a/t = 0.25$	**0.6497919**	$a/t = 0.25$	**0.9230699**	$a/t = 0.25$	**0.6523141**

The resulting values are used to interpolate for $a/c = 0.333$.

$t/R_i = 0.05$		$t/R_i = 0.01667$	
$a/c = 0.25$	0.6497919	$a/c = 0.25$	0.6523141
$a/c = 0.5$	0.9217492	$a/c = 0.5$	0.9230699
$a/c = 0.333$	**0.7400817**	$a/c = 0.333$	**0.7422050**

The final interpolation is for $t/R_i = 0.0225$.

$t/R_i = 0.01667$	0.7422050
$t/R_i = 0.05$	0.7400817
$t/R_i = 0.0225$	**0.7418361**

This results in a value of $A_0 = 0.7418361$ for G_0.

Per Annex D, paragraph C.5.10.2, the influence coefficients G_2, G_3, and G_4 are determined using paragraph C.14.3, Equations (C.274), (C.275), and (C.276), respectively. The value Q is calculated using Annex C, Equation (C.15) or (C.16). Crack and geometry dimensional limits are as follows:

$$0.0 \le a/t \le 0.8 \qquad a/t = 0.25 \qquad \textit{satisfied}$$

$$0.03125 \le a/c \le 2.0 \qquad a/c = 0.333 \qquad \textit{satisfied}$$

$$0 \le \varphi \le \pi \qquad \varphi = \pi/2 \qquad \textit{satisfied}$$

$$0.0 \le t/R_i \le 1.0 \qquad t/R_i = 0.0225 \qquad \textit{satisfied}$$

As noted above, the higher order (non-linear) terms $G_2 \rightarrow G_4$ are eliminated from Equation (C.192); therefore, are not required to be determined.

Where, Q is calculated per Equation (C.15), since $a/c \le 1.0$.

$$Q = 1.0 + 1.464\left(\frac{a}{c}\right)^{1.65} = 1.0 + 1.464\left(0.333\right)^{1.65} = 1.2389$$

The stress coefficients for a fourth-order polynomial distribution are described in Annex C, paragraph C.2.2.3, Equations (C.3), (C.4), and (C.5). For an equivalent membrane and bending stress distribution, the fourth order polynomial stress distributions reduce to the following for an inside surface crack.

$$\left\{\begin{array}{l} \sigma_m = \sigma_0 + \dfrac{\sigma_1}{2} \\ \sigma_b = -\dfrac{\sigma_1}{2} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \sigma_0 = \sigma_m - \dfrac{\sigma_1}{2} \\ \sigma_1 = -2\sigma_b \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \sigma_0 = \sigma_m + \sigma_b \\ \sigma_1 = -2\sigma_b \end{array}\right\}$$

Therefore, for a known membrane and bending stress, Equation (C.3) reduces to the following.

$$\sigma\left(\frac{x}{t}\right) = \left(\sigma_m + \sigma_b\right) - 2\sigma_b\left(\frac{x}{t}\right)$$

With $x/t = 1$,

$$\sigma_0 = \left(\sigma_m + \sigma_b\right) = \left(31697.1852 + 355.973\right) = 32053.1582 \ psi$$

$$\sigma_1 = -2\sigma_b\left(1.0\right) = -2\left(355.973\right) = -711.946 \ psi$$

i) STEP 9 – Compute the reference stress for secondary and residual stresses, σ_{ref}^{SR} based on the secondary and residual stress distributions from STEP 2, the modified flaw size from STEP 5, and the reference stress solutions in Annex D.

The reference stress solution for RCSCLE2 is provided in Annex D, paragraph D.5.10, Equation (D.74).

$$\sigma_{ref}^{SR} = \frac{gQ_b + \left[\left(gQ_b \right)^2 + 9\left(M_s \sigma_r \left(1-\alpha\right)^2 \right)^2 \right]^{0.5}}{3\left(1-\alpha\right)^2}$$

$$\sigma_{ref}^{SR} = \frac{(0.9948)(0.0) + \left[\begin{array}{l} \left(0.9948(0.0)\right)^2 + \\ 9\left[1.0106(9600.0)(1-0.1071)^2\right]^2 \end{array} \right]^{0.5}}{3\left(1-0.1071\right)^2} = 9701.76\ psi$$

Where, the following variables were determined in STEP 6.

$$g = 0.9948$$
$$\alpha = 0.1071$$
$$M_s = 1.0106$$

Additionally, the residual stress σ^r calculated in STEP 2 is substituted for P_m and the secondary bending stress Q_b is substituted for P_b.

$$\sigma^r = 9600.0\ psi$$
$$Q_b = 0.0$$

j) STEP 10 – Compute the stress intensity attributed to the secondary and residual stress, K_I^{SR}, using the secondary and residual stress distributions from STEP 2, the modified flaw size from STEP 5, and the stress intensity factor solutions in Annex C.

The stress intensity factor solution for KCSCLE2 is provided in Annex C, paragraph C.5.11, Equation (C.192), with reference to paragraph C.5.10. For an equivalent membrane and bending stress distribution of a fourth order polynomial stress distribution, the higher order (non-linear) terms $G_2 \rightarrow G_4$ and $\sigma_2 \rightarrow \sigma_4$ can be eliminated from Equation (C.192). Further, the residual stress is a constant value through-thickness; therefore, σ_1 can also be eliminated. This reduces the stress intensity factor solution from Equation (C.192) to the following.

$$K_I^{SR} = \left[G_o\left\{\sigma_r + p_c\right\} + G_1\sigma_1\left(\frac{a}{t}\right) + G_2\sigma_2\left(\frac{a}{t}\right)^2 + G_3\sigma_3\left(\frac{a}{t}\right)^3 + G_4\sigma_4\left(\frac{a}{t}\right)^4 \right]\sqrt{\frac{\pi a}{Q}}$$

$$K_I^{SR} = \left[G_o\left\{\sigma_r + p_c\right\} \right]\sqrt{\frac{\pi a}{Q}}$$

The value of K_I^{SR} is calculated at the base of the flaw and at the surface of the flaw.

At the base of the flaw, $\varphi = 90°$:

$$K_I^{SR} = \left[G_o\left\{\sigma_r + p_c\right\}\right]\sqrt{\frac{\pi a}{Q}}$$

$$K_I^{SR} = \left[1.1512828\left\{9600.0 + 0.0\right\}\right]\sqrt{\frac{\pi(0.4219)}{1.2389}} = 11.4318\ ksi\sqrt{in}$$

At the surface of the flaw, $\varphi = 0°$:

$$K_I^{SR} = \left[G_o\left\{\sigma_r + p_c\right\}\right]\sqrt{\frac{\pi a}{Q}}$$

$$K_I^{SR} = \left[0.7418361\left\{9600.0 + 0.0\right\}\right]\sqrt{\frac{\pi(0.4219)}{1.2389}} = 7.3662\ ksi\sqrt{in}$$

Where, the following variables were determined in STEP 8.

At the base of the flaw, $\varphi = 90° \rightarrow G_0 = 1.1512828$

At the surface of the flaw, $\varphi = 0° \rightarrow G_0 = 0.7418361$

Additionally, the residual stress σ^r calculated in STEP 2 is substituted for σ_0 and the residual stress does not act on the crack face pressure.

$$\sigma^r = 9600\ psi$$
$$p_c = 0.0$$

k) STEP 11 – Compute the plasticity interaction factor, Φ, using the following procedure.

1) STEP 11.1 - Since $K_I^{SR} \neq 0.0$ compute L_r^{SR}

$$L_r^{SR} = \frac{\sigma_{ref}^{SR}}{\sigma_{ys}} = \frac{9701.76}{38000.0} = 0.2553$$

2) STEP 11.2, Determine ψ and ϕ using Tables 9.4 and 9.6, and compute $\frac{\Phi}{\Phi_0}$ using Equation (9.19). The parameter L_r^P used to calculate ψ and ϕ shall be from STEP 7.

Using linear interpolation between the values of L_r^P and L_r^{SR} from Tables 9.4 and 9.6 produces the following.

$$\begin{Bmatrix} L_r^P = 0.8469 \\ L_r^{SR} = 0.2553 \end{Bmatrix} \Rightarrow \begin{Bmatrix} \psi = 0.0258 \\ \phi = 0.1351 \end{Bmatrix}$$

Therefore, per Equation (9.19),

$$\frac{\Phi}{\Phi_0} = 1 + \frac{\psi}{\phi} = 1 + \frac{0.0258}{0.1351} = 1.1910$$

3) STEP 11.3, Compute the plasticity interaction factor, Φ.

If $0 < L_r^{SR} \le 4.0$, then $\Phi_0 = 1.0$. Since $L_r^{SR} = 0.2553$,

$$\Phi = 1 + \frac{\psi}{\phi} = 1 + \frac{0.0258}{0.1351} = 1.1910$$

l) STEP 12 – Determine toughness ratio or ordinate of the FAD assessment point where K_I^P is the applied stress intensity due to the primary stress distribution from STEP 8, K_I^{SR} is the applied stress intensity due to the secondary and residual stress intensity from STEP 10, K_{mat} is the modified material toughness for STEP 5, and Φ is the plasticity correction factor from STEP 11.

At the base of the flaw, $\varphi = 90°$:

$$K_r = \left(\frac{K_I^P + \Phi K_I^{SR}}{K_{mat}} \right) = \left(\frac{38.4648 + (1.1910)(11.4318)}{86.1331} \right) = 0.6046$$

At the surface of the flaw, $\varphi = 0°$:

$$K_r = \left(\frac{K_I^P + \Phi K_I^{SR}}{K_{mat}} \right) = \left(\frac{24.8464 + (1.1910)(7.3662)}{86.1331} \right) = 0.3903$$

m) STEP 13 – Evaluate results; the FAD Assessment point for the current flaw size and operating conditions is defined as (K_r, L_r^P). Plot these points on the Failure Assessment Diagram (FAD), Fig. E3.3.1. The L_r cut-off for the material use in this assessment is 1.0

At the base of the flaw, $\varphi = 90° \rightarrow (K_r, L_r^P) = (0.6046,\ 0.8469)$:

At the surface of the flaw, $\varphi = 0° \rightarrow (K_r, L_r^P) = (0.3903,\ 0.8469)$:

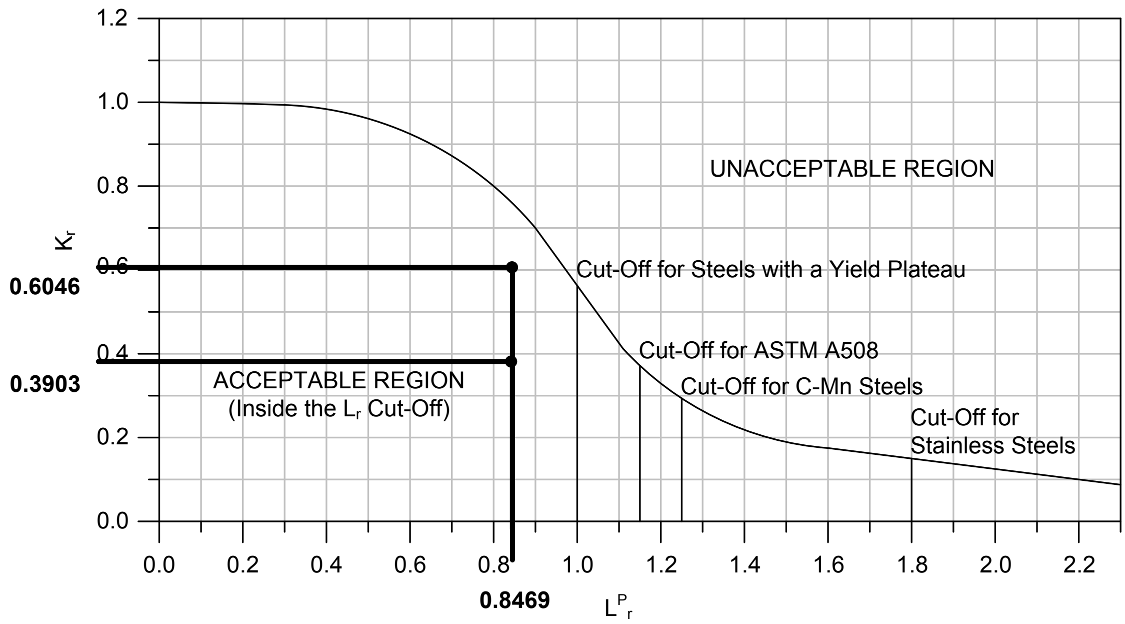


Figure E3.3.1 - The Failure assessment diagram (Fig. 9.20 of API 579-1)

Since both of the assessment points falls within the FAD envelop, the component containing this postulated flaw size is acceptable at the assessment temperature $-20°F$ per the Level 2 Assessment procedure.

PART 4

DESIGN BY RULE REQUIREMENTS

PART CONTENTS

4.1 General Requirements

4.1.1 Example E4.1.1 – Review of General Requirements for a Vessel Design

a) General Requirements

An engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 2 (VIII-2). The VIII-2 Code is being considered because the vessel in question is to be constructed of carbon steel with a specified corrosion allowance and a design pressure and temperature of 1650 psig at 200°F. As part of developing the design specification, the following items need to be evaluated.

b) Scope

1) The vessel may be designed using the design-by-rule procedures of Part 4, using the design-by-analysis rules of Part 5, or a combination of both Parts 4 and 5. Some limitations may apply to either design method.
2) The user of the vessel is responsible for defining all applicable loads and operating conditions that the vessel will be subject to. All loads and conditions must be specified on the User's Design Specification, see Part 2 paragraph 2.2.
3) A fatigue screening shall be applied to all vessel part designed in accordance with this Division to determine if a formal fatigue analysis is required, see Part 5 paragraph 5.5.2.

c) Minimum Thickness Requirements

Based on product form and process service, the parts of the vessel must meet the minimum thickness requirements presented in Part 4, paragraph 4.1.2.

d) Material Thickness Requirements

Fabrication tolerances must be considered in the design of the vessel components, based on forming, heat treatment and product form.

e) Corrosion Allowance in Design Equations

The equations used in a design-by-rule procedure of Part 4 or the thicknesses used in a design-by-analysis of Part 5 must be performed in a corroded condition. The term corrosion allowance is representative of loss of metal due to corrosion, erosion, mechanical abrasion, or other environmental effects. The corrosion allowance must be documented in the User's Design Specification.

f) Design Basis

1) The pressure used in the design of a vessel component together with the coincident design metal temperature must be specified. Where applicable, the pressure resulting from static head and other static or dynamic loads shall be included in addition to the specified design pressure. The typical loads that need to be considered in the design of a vessel are shown in Part 5, Table 5.1.
2) The specified design temperature shall not be less than the mean metal temperature expected coincidentally with the corresponding maximum pressure.

3) A minimum design metal temperature shall be determined and shall consider the coldest operating temperature, operational upsets, auto refrigeration, atmospheric temperature, and any other source of cooling.

4) All applicable loads and load case combinations shall be considered in the design to determine the minimum required wall thickness for a vessel part, see Part 4, Table 4.1.1.

g) Design Allowable Stress

Specifications for all materials of construction are provided in Part 3, Annex 3.A.

4.1.2 Example E4.1.2 – Required Wall Thickness of a Hemispherical Head

Determine the required thickness for a hemispherical head at the bottom of a vertical vessel considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

<u>Vessel Data:</u>

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $1650\ psig@300°F$
- Liquid Head = $60\ ft$
- Liquid Specific Gravity = 0.89
- Inside Diameter = $96.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0

The design pressure used to establish the wall thickness must be adjusted for the liquid head in accordance with Part 4, paragraph 4.1.5.2.a

$$Design\ \ Pressure = Specified\ \ Design\ \ Pressure + \gamma gh$$

$$Design\ \ Pressure = 1650 + \frac{0.89(62.4)(60)}{144} = 1673.140\ psig$$

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the bottom hemispherical head.

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right] - 1\right)$$

$$D = 96.0 + 2(Corrosion\ \ Allowance) = 96.0 + 2(0.125) = 96.25\ in$$

$$t = \frac{96.25}{2}\left(\exp\left[\frac{0.5(1673.14)}{22400(1.0)}\right] - 1\right) = 1.8313\ in$$

$$t = 1.8313 + Corrosion\ \ Allowance = 1.8313 + 0.125 = 1.9563\ in$$

The required thickness of the bottom head is $1.9563\ in$

4.1.3 Example E4.1.3 – Required Wall Thickness of a Hemispherical Head - Higher Strength Material

Determine the required thickness for a hemispherical head at the bottom of a vertical vessel in example E.4.1.2 considering the following design conditions. Note that a higher strength material is being used. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-537,\ Class\ 1$
- Design Conditions = $1650\ psig\ @300°F$
- Liquid Head = $60\ ft$
- Liquid Specific Gravity = 0.89
- Inside Diameter = $96.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $29000\ psi$
- Weld Joint Efficiency = 1.0

The design pressure used to establish the wall thickness must be adjusted for the liquid head in accordance with Part 4, paragraph 4.1.5.2.a

$$Design\ Pressure = Specified\ Design\ Pressure + \gamma gh$$

$$Design\ Pressure = 1650 + \frac{0.89(62.4)(60)}{144} = 1673.140\ psig$$

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the bottom hemispherical head.

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right] - 1\right)$$

$$D = 96.0 + 2(Corrosion\ Allowance) = 96.0 + 2(0.125) = 96.25\ in$$

$$t = \frac{96.25}{2}\left(\exp\left[\frac{0.5(1673.14)}{29000(1.0)}\right] - 1\right) = 1.4085\ in$$

$$t = 1.4085 + Corrosion\ Allowance = 1.4085 + 0.125 = 1.5335\ in$$

The required thickness of the bottom head constructed with a stronger material is $1.5335\ in$. This represents a savings in material costs of approximately 22%. Additional costs in welding and NDE are also expected. Similar cost savings can be achieved by using a stronger material for the cylinder shell. The design margins in Section VIII, Division 2 will typically result in a more efficient design when higher strength materials are used as shown in this example. For many fluid service environments, higher strength materials may be prone to cracking. However, if PWHT is specified for fluid service, as opposed to wall thickness requirements in accordance with Part 6, the use of higher strength materials may be justified and result in significant cost savings.

4.2 Welded Joints

4.2.1 Example E4.2.1 – Nondestructive Examination Requirement for Vessel Design

A significant change to the new VIII-2 is the introduction of rules for the examination of welded joints, which introduce Examination Groups. These Examination Groups permit partial radiography and also require surface examination as part of the nondestructive examination (NDE) requirements. Part 7, paragraph 7.4.2 and Table 7.1 of VIII-2 define the different Examination Groups assigned to welded joints based on the manufacturing complexity of the material group, the maximum thickness, the welding process, and the selected joint efficiency. Once the Examination Group is selected, Table 7.2 provides the required NDE, joint category designation, joint efficiency, and acceptable joint types.

An engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 2 (VIII-2). Based on the anticipated fabrication data provided, the engineer compares the Examination Groups to aid in the decision for NDE requirements for vessel design as follows.

<u>Vessel Data:</u>

- Material = $SA-516,\ Grade\ 70$
- Welding Processes = $SAW\ and\ SMAW$
- Weld Categories = $A,\ B,\ C,\ D$
- Weld Joint Types = $Type\ 1 \rightarrow Category\ A,\ B,\ and\ C$

  = $Type\ 7 \rightarrow Category\ D$

The definitions of Weld Categories and Weld Joint Types are provided in Part 4, Tables 4.2.1 and 4.2.2, respectively. Some acceptable weld joints for main shell weld seams, formed heads, flange attachments, and nozzles are provided in Tables 4.2.4, 4.2.5, 4.2.9, and 4.2.10, respectively.

Per Part 7, Table 7.1, a comparison of Examination Group 1b and Examination Group 3b for carbon steel material, $SA-516,\ Grade\ 70$ is as follows.

Parameter	Examination Group	
	1b	3b
Permitted Material	P-No. 1 Gr 1 and 2	P-No. 1 Gr 1 and 2
Maximum Thickness	Unlimited	50 mm (2 inches)
Welding Process	Unrestricted	Unrestricted
Design Basis	Part 4 or Part 5	Part 4

From the results of the comparison above, there are two parameters that will require a decision to be made by the engineer prior to assigning an Examination Group, maximum thickness of the vessel components and design basis. A preliminary check of the required wall thickness for the main cylinder and heads can be performed in accordance with the rules of Part 4. However, if there is one or several components that may require their design to be based on numerical analysis, i.e. finite element analysis per Part 5, only Examination Group 1b would be permitted.

Per Part 7, Table 7.2, a comparison of the required NDE for Examination Group 1b and Examination Group 3b for carbon steel material, $SA-516,\ Grade\ 70$ is as follows.

Examination Group			1b	3b
Joint Efficiency			1.0	0.85
Joint Category	Type of Weld	Type of NDE	Extent of NDE	
A	Type 1: Full Penetration Longitudinal Seam	RT or UT MT or PT	100% 10%	10% 10%
B	Type 1: Full Penetration Circumferential Seam	RT or UT MT or PT	100% 10%	10% 10%
C	Type 1: Full Penetration Flange/Nozzle Attachment	RT or UT MT or PT	100% 10%	10% 10%
D	Type 7: Full Penetration Corner Joint, Nozzle $d > 150\ mm\left(NPS\ 6\right) or$ $t > 16\ mm\left(0.625\ in\right)$	RT or UT MT or PT	100% 10%	10% 10%
	Type 7: Full Penetration Corner Joint, Nozzle $d \le 150\ mm\left(NPS\ 6\right) and$ $t \le 16\ mm\left(0.625\ in\right)$	MT or PT	10%	10%

A review of the above table indicates that more inspection is required for Examination Group 1b when compared to 3b. However, the increased costs for examination may be offset by the materials and fabrication savings. Consider the following comparison for a cylindrical shell.

<u>Vessel Data:</u>

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $725\ psig@300°F$
- Inside Diameter = $60.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$

For examination Group 1b, consider the requirements for a Category A Type 1 weld. The required wall thickness in accordance with Part 4, paragraph 4.3.3 is computed as shown below.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right)$$

$$D = 60.0 + 2(Corrosion\ \ Allowance) = 60.0 + 2(0.125) = 60.25\ in$$

$$t = \frac{60.25}{2}\left(\exp\left[\frac{725}{22400(1.0)}\right]-1\right) = 0.9910\ in$$

$$t = 0.9910 + Corrosion\ \ Allowance = 0.9910 + 0.125 = 1.1160\ in$$

Alternatively, for examination Group 3b, the required wall thickness for a Category A Type 1 weld in accordance with Part 4, paragraph 4.3.3 is computed as shown below.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right)$$

$$D = 60.0 + 2(Corrosion\ \ Allowance) = 60.0 + 2(0.125) = 60.25\ in$$

$$t = \frac{60.25}{2}\left(\exp\left[\frac{725}{22400(0.85)}\right]-1\right) = 1.1692\ in$$

$$t = 1.1692 + Corrosion\ \ Allowance = 1.1692 + 0.125 = 1.2942\ in$$

Examination Group 1b when compared to 3b results in an approximate 14% reduction in wall thickness. Cost savings for this reduction in wall thickness will include less material and less welding, and these reductions may offset the increased examination costs. It should also be noted that many refining and petrochemical companies invoke additional examination requirements in their associated corporate standards based on fluid service. Therefore, in some cases the increased examination requirements for Examination Group 1b may already be required based on fluid service.

4.2.2 Example E4.2.2 – Nozzle Detail and Weld Sizing

Determine the required fillet weld size and inside corner radius of a set-in type nozzle as shown in Table 4.2.10, Detail 4. The vessel and nozzle were designed such that their nominal thicknesses were established as follows.

Vessel Data:

- Cylinder Thickness = $0.625\ inches$
- Nozzle Diameter = $NPS\ 10$
- Nozzle Thickness = $Schedule\ XS \rightarrow 0.500\ inches$
- Corrosion Allowance = $0.125\ inches$

The minimum fillet weld throat dimension, t_c, is calculated as follows.

$$t_c \geq \min\left[0.7t_n, 6\,mm\ (0.25\ in)\right]$$
$$t_n = 0.500 - Corrosion\ \ Allowance = 0.5 - 0.125 = 0.375\ in$$
$$t_c \geq \min\left[0.70(0.375), 0.25\right]$$
$$t_c \geq 0.25\ in$$

The resulting fillet weld leg size is determined as, $\dfrac{t_c}{0.7} = 0.357\ in$. Therefore, a fillet weld leg size of $0.375\ in$ would be acceptable.

The minimum inside corner radius, r_1, is calculated as follows.

$$0.125t \leq r_1 \leq 0.5t$$
$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$
$$0.125(0.500) \leq r_1 \leq 0.5(0.500)$$
$$0.0625 \leq r_1 \leq 0.250\ in$$

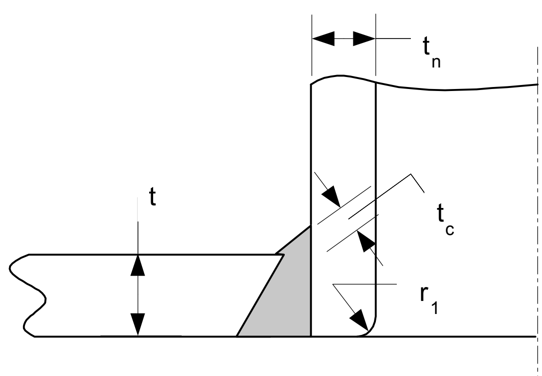


Figure E4.2.2 - Nozzle Detail

4.2.3 Example E4.2.3 – Nozzle Detail with Reinforcement Pad and Weld Sizing

Determine the required fillet weld sizes and inside corner radius of a set-in type nozzle with added reinforcement pad as shown in Table 4.2.11, Detail 2. The vessel and nozzle were designed such that their nominal thicknesses were established as follows.

Vessel Data:

- Cylinder Thickness = $0.625\ inches$
- Nozzle Diameter = $NPS\ 10$
- Nozzle Thickness = $Schedule\ XS \rightarrow 0.500\ inches$
- Reinforcement Pad Thickness = $0.625\ inches$
- Corrosion Allowance = $0.125\ inches$

The minimum fillet weld throat dimension, t_c, is calculated as follows.

$$t_c \geq \min\left[0.7t_n, 6\,mm\ (0.25\ in)\right]$$
$$t_n = 0.500 - Corrosion\ \ Allowance = 0.5 - 0.125 = 0.375\ in$$
$$t_c \geq \min\left[0.7(0.375), 0.25\right]$$
$$t_c \geq 0.25\ in$$

The resulting fillet weld leg size is determined as, $\dfrac{t_c}{0.7} = 0.357\ in$. Therefore, a fillet weld leg size of $0.375\ in$ would be acceptable.

The minimum fillet weld throat dimension, t_{f1}, is calculated as follows.

$$t_{f1} \geq \min\left[0.6t_e,\ 0.6t\right]$$
$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$
$$t_{f1} \geq \min\left[0.6(0.625), 0.6(0.500)\right]$$
$$t_{f1} \geq 0.300\ in$$

The resulting fillet weld leg size is determined as, $\dfrac{t_{f1}}{0.7} = 0.429\ in$. Therefore, a fillet weld leg size of $0.4375\ in$ would be acceptable.

The minimum inside corner radius, r_1, is calculated as follows.

$$0.125t \leq r_1 \leq 0.5t$$
$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.500\ in$$
$$0.125(0.500) \leq r_1 \leq 0.5(0.500)$$
$$0.0625 \leq r_1 \leq 0.250\ in$$

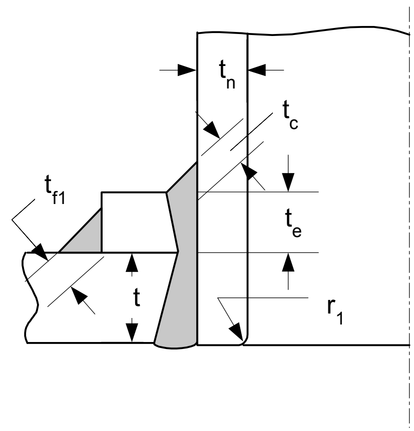


Figure E4.2.3 - Nozzle Detail

4.3 Internal Design Pressure

4.3.1 Example E4.3.1 – Cylindrical Shell

Determine the required thickness for a cylindrical shell considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $356\ psig@300°F$
- Inside Diameter = $90.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0

Adjust for corrosion allowance.

$$D = 90.0 + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

Evaluate per paragraph 4.3.3.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right] - 1\right)$$

$$t = \frac{90.25}{2}\left(\exp\left[\frac{356}{22400(1.0)}\right] - 1\right) = 0.7229\ in$$

$$t = 0.7229 + Corrosion\ \ Allowance = 0.7229 + 0.125 = 0.8479\ in$$

The required thickness is $0.8479\ in$.

Combined Loadings – cylindrical shells subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this example problem, the cylindrical shell is only subject to internal pressure.

4.3.2 Example E4.3.2 – Conical Shell

Determine the required thickness for a conical shell considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $356\ psig@300°F$
- Inside Diameter (Large End) = $150.0\ in$
- Inside Diameter (Small End) = $90.0\ in$
- Length of Conical Section = $78.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0

Adjust for corrosion allowance and determine the cone angle.

$$D_L = 150.0 + 2(Corrosion\ \ Allowance) = 150.0 + 2(0.125) = 150.25\ in$$

$$D_s = 90.0 + 2(Corrosion\ \ Allowance) = 150.0 + 2(0.125) = 90.25\ in$$

$$L_C = 78.0$$

$$\alpha = \arctan\left[\frac{0.5(D_L - D_S)}{L_C}\right] = \arctan\left[\frac{0.5(150.25 - 90.25)}{78.0}\right] = 21.0375\ deg$$

Evaluate per paragraph 4.3.4 using the large end diameter of the conical shell.

$$t = \frac{D}{2\cos[\alpha]}\left(\exp\left[\frac{P}{SE}\right] - 1\right)$$

$$t = \frac{150.25}{2\cos[21.0375]}\left(\exp\left[\frac{356}{22400(1.0)}\right] - 1\right) = 1.2894\ in$$

$$t = 1.2894 + Corrosion\ \ Allowance = 1.2894 + 0.125 = 1.4144\ in$$

The required thickness is $1.4144\ in$.

Combined Loadings – conical shells subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this example problem, the conical shell is only subject to internal pressure.

4.3.3 Example E4.3.3 – Spherical Shells

Determine the required thickness for a spherical shell considering the following design conditions. All Category A joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-542,\ Type\ D,\ Class\ 4a$
- Design Conditions = $2080\ psig@850°F$
- Inside Diameter = $149.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $28900\ psi$
- Weld Joint Efficiency = 1.0

Evaluate per paragraph 4.3.5.

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right]-1\right) = \frac{149.0}{2}\left(\exp\left[\frac{0.5(2080)}{28900(1.0)}\right]-1\right) = 2.7298\ in$$

The required thickness is $2.7298\ in$.

Combined Loadings – spherical shells and hemispherical heads subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this example problem, the spherical shell is only subject to internal pressure.

4.3.4 Example E4.3.4 – Torispherical Head

Determine the maximum allowable working pressure (MAWP) for the proposed seamless torispherical head. The Category B joint joining the head to the shell is a Type 1 butt weld and has been 100% radiographically examined.

Vessel Data:

- Material = $SA-387,\ Grade\ 11,\ Class\ 1$
- Design Temperature = $650°F$
- Inside Diameter = $72.0\ in$
- Crown Radius = $72.0\ in$
- Knuckle Radius = $4.375\ in$
- Thickness = $0.625\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $18000\ psi$
- Weld Joint Efficiency = 1.0
- Modulus of Elasticity at Design Temperature = $26.55E+06\ psi$
- Yield Strength at Design Temperature = $26900\ psi$

Evaluate per paragraph 4.3.6.

a) STEP 1 – Determine, D, assume values for L, r and t (known variables from above) and adjust for corrosion.

$$D = 72.0 + 2(Corrosion\ \ Allowance) = 72.0 + 2(0.125) = 72.25\ in$$
$$L = 72.0 + Corrosion\ \ Allowance = 72.0 + 0.125 = 72.125\ in$$
$$r = 4.375 + Corrosion\ \ Allowance = 4.375 + 0.125 = 4.5\ in$$
$$t = 0.625 - Corrosion\ \ Allowance = 0.625 - 0.125 = 0.5\ in$$

b) STEP 2 – Compute the head L/D, r/D, and L/t ratios and determine if the following equations are satisfied.

$$0.7 \le \left\{ \frac{L}{D} = \frac{72.125}{72.25} = 0.9983 \right\} \le 1.0 \qquad True$$

$$\left\{ \frac{r}{D} = \frac{4.5}{72.25} = 0.0623 \right\} \ge 0.06 \qquad True$$

$$20 \le \left\{ \frac{L}{t} = \frac{72.125}{0.5} = 144.25 \right\} \le 2000 \qquad True$$

c) STEP 3 – Calculate the geometric constants β_{th}, ϕ_{th}, R_{th}

$$\beta_{th} = \arccos\left[\frac{0.5D - r}{L - r}\right] = \arccos\left[\frac{0.5(72.25) - 4.5}{72.125 - 4.5}\right] = 1.0842\ rad$$

$$\phi_{th} = \frac{\sqrt{Lt}}{r} = \frac{\sqrt{72.125(0.5)}}{4.5} = 1.3345\ rad$$

Since $\phi_{th} \geq \beta_{th}$, calculate R_{th} as follows:

$$R_{th} = 0.5D = 0.5(72.25) = 36.125\ in$$

d) STEP 4 – Compute the coefficients C_1 and C_2

Since $\frac{r}{D} = 0.0623 \leq 0.08$, calculate C_1 and C_2 as follows:

$$C_1 = 9.31\left(\frac{r}{D}\right) - 0.086 = 9.31(0.0623) - 0.086 = 0.4940$$

$$C_2 = 1.25$$

e) STEP 5 – Calculate the value of internal pressure expected to produce elastic buckling of the knuckle, P_{eth}.

$$P_{eth} = \frac{C_1 E_T t^2}{C_2 R_{th}\left(\frac{R_{th}}{2} - r\right)} = \frac{(0.4940)(26.55E+06)(0.5)^2}{1.25(36.125)\left(\frac{36.125}{2} - 4.5\right)} = 5353.9445\ psi$$

f) STEP 6 – Calculate the value of internal pressure that will result in a maximum stress in the knuckle equal to the material yield strength, P_y.

$$P_y = \frac{C_3 t}{C_2 R_{th}\left(\frac{R_{th}}{2r} - 1\right)}$$

Since the allowable stress at design temperature is governed by time-independent properties, C_3 is the material yield strength at the design temperature, or $C_3 = S_y$.

$$P_y = \frac{26900(0.5)}{1.25(36.125)\left(\frac{36.125}{2(4.5)} - 1\right)} = 98.8274\ psi$$

g) STEP 7 – Calculate the value of internal pressure expected to result in a buckling failure of the knuckle, P_{ck}.

Calculate variable, G:

$$G = \frac{P_{eth}}{P_y} = \frac{5353.9445}{98.8274} = 54.1747$$

Since $G > 1.0$, calculate P_{ck} as follows:

$$P_{ck} = \left(\frac{0.77508G - 0.20354G^2 + 0.019274G^3}{1 + 0.19014G - 0.089534G^2 + 0.0093965G^3} \right) (P_y)$$

$$P_{ck} = \left(\frac{\begin{array}{l} 0.77508(54.1747) - 0.20354(54.1747)^2 + \\ 0.019274(54.1747)^3 \end{array}}{\begin{array}{l} 1 + 0.19014(54.1747) - 0.089534(54.1747)^2 + \\ 0.0093965(54.1747)^3 \end{array}} \right) (98.8274)$$

$$P_{ck} = 199.5671\ psi$$

h) STEP 8 – Calculate the allowable pressure based on a buckling failure of the knuckle, P_{ak}.

$$P_{ak} = \frac{P_{ck}}{1.5} = \frac{199.5671}{1.5} = 133.0447\ psi$$

i) STEP 9 – Calculate the allowable pressure based on rupture of the crown, P_{ac}.

$$P_{ac} = \frac{2SE}{\dfrac{L}{t} + 0.5} = \frac{2(18000)(1.0)}{\dfrac{72.125}{0.5} + 0.5} = 248.7047\ psi$$

j) STEP 10 – Calculate the maximum allowable internal pressure, P_a.

$$P_a = \min\left[P_{ak}, P_{ac}\right] = \min\left[133.0447, 248.7047\right] = 133.0\ psi$$

k) STEP 11 – If the allowable internal pressure computed from STEP 10 is greater than or equal to the design pressure, then the design is complete. If the allowable internal pressure computed from STEP 10 is less than the design pressure, then increase the head thickness and repeat steps 2 through 10.

The MAWP is $133.0\ psi$.

Combined Loadings – torispherical heads subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this calculation, the torispherical head shall be approximated as an equivalent spherical shell with a radius equal to L. In this example problem, the torisphercial head is only subject to internal pressure.

4.3.5 Example E4.3.5 – Elliptical Head

Determine the maximum allowable working pressure (MAWP) for the proposed seamless 2:1 elliptical head. The Category B joint joining the head to the shell is a Type 1 butt weld and has been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Temperature = $300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength at Design Temperature = $33600\ psi$

Evaluate per paragraph 4.3.7 and paragraph 4.3.6.

Calculate k, using the uncorroded inside diameter, D and depth of head, h.

$$k = \frac{D}{2h} = \frac{90.0}{2(22.5)} = 2.0$$

Verify variable k is within the established limits, permitting the use of the rules of paragraph 4.3.7.

$$1.7 \le \{k = 2\} \le 2.2 \qquad\qquad True$$

Determine the variables r and L using the uncorroded inside diameter, D.

$$r = D\left(\frac{0.5}{k} - 0.08\right) = 90.0\left(\frac{0.5}{2.0} - 0.08\right) = 15.3\ in$$

$$L = D(0.44k + 0.02) = 90.0(0.44(2.0) + 0.02) = 81.0\ in$$

Proceed with the design following the steps outlined in paragraph 4.3.6.

a) STEP 1 – Determine, D, assume values for $L,\ r$ and t (determined from paragraph 4.3.7) and adjust for corrosion.

$$D = 90.0 + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$L = 81.0 + \ Corrosion\ \ Allowance = 81.0 + 0.125 = 81.125\ in$$

$$r = 15.3 + \ Corrosion\ \ Allowance = 15.3 + 0.125 = 15.425\ in$$

$$t = 1.125 - \ Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

b) STEP 2 – Compute the head L/D, r/D, and L/t ratios and determine if the following equations are satisfied.

$$0.7 \le \left\{ \frac{L}{D} = \frac{81.125}{90.25} = 0.8989 \right\} \le 1.0 \qquad \textit{True}$$

$$\left\{ \frac{r}{D} = \frac{15.425}{90.25} = 0.1709 \right\} \ge 0.06 \qquad \textit{True}$$

$$20 \le \left\{ \frac{L}{t} = \frac{81.125}{1.000} = 81.125 \right\} \le 2000 \qquad \textit{True}$$

c) STEP 3 – Calculate the geometric constants β_{th}, ϕ_{th}, R_{th}

$$\beta_{th} = \arccos\left[\frac{0.5D-r}{L-r}\right] = \arccos\left[\frac{0.5(90.25)-15.425}{81.125-15.425}\right] = 1.1017\ rad$$

$$\phi_{th} = \frac{\sqrt{Lt}}{r} = \frac{\sqrt{81.125(1.0)}}{15.425} = 0.5839\ rad$$

Since $\phi_{th} < \beta_{th}$, calculate R_{th} as follows:

$$R_{th} = \frac{0.5D-r}{\cos\left[\beta_{th}-\phi_{th}\right]} + r = \frac{0.5(90.25)-15.425}{\cos\left[1.1017-0.5839\right]} + 15.425 = 49.6057\ in$$

d) STEP 4 – Compute the coefficients C_1 and C_2

Since $\frac{r}{D} = 0.1709 > 0.08$, calculate C_1 and C_2 as follows:

$$C_1 = 0.692\left(\frac{r}{D}\right) + 0.605 = 0.692(0.1709) + 0.605 = 0.7233$$

$$C_2 = 1.46 - 2.6\left(\frac{r}{D}\right) = 1.46 - 2.6(0.1709) = 1.0157$$

e) STEP 5 – Calculate the value of internal pressure expected to produce elastic buckling of the knuckle, P_{eth}.

$$P_{eth} = \frac{C_1 E_T t^2}{C_2 R_{th}\left(\frac{R_{th}}{2} - r\right)} = \frac{(0.7233)(28.3E+06)(1.0)^2}{1.0157(49.6057)\left(\frac{49.6057}{2} - 15.425\right)} = 43321.6096\ psi$$

f) STEP 6 – Calculate the value of internal pressure that will result in a maximum stress in the knuckle equal to the material yield strength, P_y.

$$P_y = \frac{C_3 t}{C_2 R_{th}\left(\dfrac{R_{th}}{2r}-1\right)}$$

Since the allowable stress at design temperature is governed by time-independent properties, C_3 is the material yield strength at the design temperature, or $C_3 = S_y$.

$$P_y = \frac{33600(1.0)}{1.0157(49.6057)\left(\dfrac{49.6057}{2(15.425)}-1\right)} = 1096.8927 \ psi$$

g) STEP 7 – Calculate the value of internal pressure expected to result in a buckling failure of the knuckle, P_{ck}.

Calculate variable, G:

$$G = \frac{P_{eth}}{P_y} = \frac{43321.6096}{1096.8927} = 39.4948$$

Since $G > 1.0$, calculate P_{ck} as follows:

$$P_{ck} = \left(\frac{0.77508G - 0.20354G^2 + 0.019274G^3}{1 + 0.19014G - 0.089534G^2 + 0.0093965G^3}\right)(P_y)$$

$$P_{ck} = \left(\frac{0.77508(39.4948) - 0.20354(39.4948)^2 + 0.019274(39.4948)^3}{1 + 0.19014(39.4948) - 0.089534(39.4948)^2 + 0.0093965(39.4948)^3}\right)(1096.8927)$$

$$P_{ck} = 2206.1634 \ psi$$

h) STEP 8 – Calculate the allowable pressure based on a buckling failure of the knuckle, P_{ak}.

$$P_{ak} = \frac{P_{ck}}{1.5} = \frac{2206.1634}{1.5} = 1470.8 \ psi$$

i) STEP 9 – Calculate the allowable pressure based on rupture of the crown, P_{ac}.

$$P_{ac} = \frac{2SE}{\dfrac{L}{t}+0.5} = \frac{2(22400)(1.0)}{\dfrac{81.125}{1.0}+0.5} = 548.9 \ psi$$

j) STEP 10 – Calculate the maximum allowable internal pressure, P_a.

$$P_a = \min\left[P_{ak}, P_{ac}\right] = \min\left[1470.8, 548.9\right] = 548.9 \ psi$$

k) STEP 11 – If the allowable internal pressure computed from STEP 10 is greater than or equal to the design pressure, then the design is complete. If the allowable internal pressure computed from STEP 10 is less than the design pressure, then increase the head thickness and repeat STEPs 2 through 10.

The MAWP is $548.9\ psi$.

Combined Loadings – ellipsoidal heads subject to internal pressure and other loadings shall satisfy the requirements of paragraph 4.3.10. In this calculation, the ellipsoidal head shall be approximated as an equivalent spherical shell with a radius equal to L. In this example problem, the ellipsoidal head is only subject to internal pressure.

4.3.6 Example E4.3.6 – Combined Loadings and Allowable Stresses

Determine the maximum tensile stress of the proposed cylindrical shell section considering the following design conditions and specified applied loadings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $356\ psig @ 300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0
- Axial Force = $-66152.5\ lbs$
- Net Section Bending Moment = $3.048E+06\ in-lbs$
- Torsional Moment = $0.0\ in-lbs$

Adjust variables for corrosion and determine outside dimensions.

$$D = 90.0 + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$D_o = 90.0 + 2(Uncorroded\ \ Thickness) = 90.0 + 2(1.125) = 92.25\ in$$

$$t = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

Evaluate per paragraph 4.3.10.

The loads transmitted to the cylindrical shell are given in the Table E4.3.6.2. Note that this table is given in terms of the load parameters and load combinations shown in Part 4, Table 4.1.1 and Table 4.1.2 (Table E4.3.6.1 of this example). As shown in Table E4.3.6.1, the acceptance criteria is that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition.

Determine applicability of the rules of paragraph 4.3.10 based on satisfaction of the following requirements.

The section of interest is at least $2.5\sqrt{Rt}$ away from any major structural discontinuity.

$$2.5\sqrt{Rt} = 2.5\sqrt{\left(\frac{90.25}{2}\right)(1.125)} = 16.7938\ in$$

Shear force is not applicable.

The shell R/t ratio is greater than 3.0, or:

$$\left\{ R/t = \frac{90.25/2}{1.0} = 45.125 \right\} > 3.0 \qquad \qquad True$$

Paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment as shown below. By inspection of the results shown in Table E4.3.6.3, Load Case 5 is determined to be the governing load case. The pressure and net section axial force, bending moment, and torsional moment at the location of interest for Load Case 5 are:

$$P = 320.4 \ psi$$
$$F_5 = -66152.5 \ lbs$$
$$M_5 = 1828800 \ in-lbs$$
$$M_{t5} = 0.0 \ in-lbs$$

a) STEP 1 – Calculate the membrane stress for the cylindrical shell. Note that the maximum bending stress occurs at $\theta = 0.0 \ deg$.

$$\sigma_{\theta m} = \frac{PD}{E(D_o - D)} = \frac{320.4(90.25)}{1.0(92.25-90.25)} = 14458.05 \ psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\frac{PD^2}{D_o^2 - D^2} + \frac{4F}{\pi\left(D_o^2 - D^2\right)} + \frac{32MD_o\cos[\theta]}{\pi\left(D_o^4 - D^4\right)} \right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(\begin{array}{l} \dfrac{320.4(90.25)^2}{(92.25)^2 - (90.25)^2} + \\ \dfrac{4(-66152.5)}{\pi\left[(92.25)^2 - (90.25)^2\right]} \pm \dfrac{32(1828800)(92.25)\cos[0.0]}{\pi\left[(92.25)^4 - (90.25)^4\right]} \end{array} \right)$$

$$\sigma_{sm} = \left\{ \begin{array}{l} 7149.8028 + (-230.7616) + 282.6779 = 7201.7191 \ psi \\ 7149.8028 + (-230.7616) - 282.6779 = 6636.3633 \ psi \end{array} \right\}$$

$$\tau = \frac{16M_tD_o}{\pi\left(D_o^4 - D^4\right)} = \frac{16(0.0)(92.25)}{\pi\left[(92.25)^4 - (90.25)^4\right]} = 0.0 \ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2} \right)$$

$$\sigma_1 = \left\{ \begin{array}{l} 0.5\left(14458.05 + 7201.7191 + \sqrt{(14458.05 - 7201.7191)^2 + 4(0.0)^2} \right) = 14458.05 \ psi \\ 0.5\left(14458.05 + 6636.3633 + \sqrt{(14458.05 - 6636.3633)^2 + 4(0.0)^2} \right) = 14458.05 \ psi \end{array} \right\}$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\tau^2} \right)$$

$$\sigma_2 = \begin{Bmatrix} 0.5\left(14458.05 + 7201.7191 - \sqrt{\left(14458.05 - 7201.7191\right)^2 + 4\left(0.0\right)^2} \right) \\ 0.5\left(14458.05 + 6636.3633 - \sqrt{\left(14458.05 - 6636.3633\right)^2 + 4\left(0.0\right)^2} \right) \end{Bmatrix}$$

$$\sigma_2 = \begin{Bmatrix} 7201.7191\ psi \\ 6636.3633\ psi \end{Bmatrix}$$

$$\sigma_3 = \sigma_r = -0.5P = -0.5\left(320.4\right) = -160.2\ psi$$

c) STEP 3 – At any point on the shell, the following limit shall be satisfied.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2 \right]^{0.5} \le SE$$

$$\sigma_e = \begin{Bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} \left(14458.05 - 7201.7191\right)^2 + \\ \left(7201.7191 - \left(-160.2\right)\right)^2 + \left(\left(-160.2\right) - 14458.05\right)^2 \end{bmatrix}^{0.5} = 12659.9\ psi \\ \frac{1}{\sqrt{2}}\begin{bmatrix} \left(14458.05 - 6636.3633\right)^2 + \\ \left(6636.3633 - \left(-160.2\right)\right)^2 + \left(\left(-160.2\right) - 14458.05\right)^2 \end{bmatrix}^{0.5} = 12670.1\ psi \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_e = 12659.9\ psi \\ \sigma_e = 12670.1\ psi \end{Bmatrix} \le \left\{ SE = 22400\ psi \right\}$$

Since the maximum tensile stress is less than the acceptance criteria, the shell section is adequately designed.

d) STEP 4 – For cylindrical and conical shells, if the meridional stress, σ_{sm} is compressive, then Equation (4.3.45) shall be satisfied where F_{xa} is evaluated using paragraph 4.4.12.2 with $\lambda = 0.15$.

Step 4 is not necessary in this example because the meridional stress, σ_{sm}, is tensile.

Table E4.3.6.1 - Design Loads and Load Combinations from VIII-2

Table 4.1.1 – Design Loads

Design Load Parameter	Description
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g. catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g. skirts, lugs, saddles, and legs), and appurtenances (e.g. platforms, ladders, etc.) • Weight of vessel contents under operating and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping • Transportation Loads (The static forces obtained as equivalent to the dynamic loads experienced during normal operation of a transport vessel – see paragraph 1.2.1.2.b)
L	• Appurtenance Live loading • Effects of fluid flow, steady state or transient • Loads resulting from wave action
E	Earthquake loads (see ASCE 7 for the specific definition of the earthquake load, as applicable)
W	Wind Loads (See 4.1.5.3.b)
S	Snow Loads
F	Loads due to Deflagration

Table 4.1.2 – Design Load Combinations

Design Load Combination (1)	General Primary Membrane Allowable Stress (2)
$P+P_s+D$	S
$P+P_s+D+L$	S
$P+P_s+D+S$	S
$0.9P+P_s+D+0.75L+0.75S$	S
$0.9P+P_s+D+(0.6W\ or\ 0.7E)$	S
$0.9P+P_S+D+0.75(0.6W\ or\ 0.7E)+0.75L+0.75S$	S
$0.6D+(0.6W\ or\ 0.7E)$ (3)	S
P_s+D+F	See Annex 4.D

Notes
1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) S is the allowable stress for the load case combination (see paragraph 4.1.5.3.c)
3) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7-10, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

Table E4.3.6.2 - Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	$P = 356.0$
P_s	Static head from liquid or bulk materials (e.g. catalyst)	$P_s = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -66152.5\ lbs$ $D_M = 0.0\ in-lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = 0.0\ lbs$ $L_M = 0.0\ in-lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 0.0\ in-lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 3.048E+06\ in-lbs$
S	Snow Loads	$S_F = 0.0\ lbs$ $S_M = 0.0\ in-lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in-lbs$

Based on these loads, the shell is required to be designed for the load case combinations shown in Table E4.3.6.3. Note that this table is given in terms of the load combinations shown in VIII-2, Table 4.1.2 (Table E4.3.6.1 of this example).

Table E4.3.6.3 - Load Case Combination at the Location of Interest

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P+P_s+D$	$P+P_s=356.0\ psi$ $F_1=-66152.5\ lbs$ $M_1=0.0\ in-lbs$	S
2	$P+P_s+D+L$	$P+P_s=356.0\ psi$ $F_2=-66152.5\ lbs$ $M_2=0.0\ in-lbs$	S
3	$P+P_s+D+S$	$P+P_s=356.0\ psi$ $F_3=-66152.5\ lbs$ $M_3=0.0\ in-lbs$	S
4	$0.9P+P_s+D+0.75L+0.75S$	$0.9P+P_s=320.4\ psi$ $F_4=-66152.5\ lbs$ $M_4=0.0\ in-lbs$	S
5	$0.9P+P_s+D+(0.6W\ or\ 0.7E)$	$0.9P+P_s=320.4\ psi$ $F_5=-66152.5\ lbs$ $M_5=1828800\ in-lbs$	S
6	$\left(\begin{array}{l}0.9P+P_S+D+0.75(0.6W\ or\ 0.7E)+\\0.75L+0.75S\end{array}\right)$	$0.9P+P_s=320.4\ psi$ $F_6=-66152.5\ lbs$ $M_6=1371600\ in-lbs$	S
7	$0.6D+(0.6W\ or\ 0.7E)$ Anchorage is included in the design. Therefore, consideration of this load combination is not required.	$F_7=-39691.5\ lbs$ $M_7=1828800\ in-lbs$	S
8	P_s+D+F	$P_s=0.0\ psi$ $F_8=-66152.5\ lbs$ $M_8=0.0\ in-lbs$	See Annex 4.D

4.3.7 Example E4.3.7 – Conical Transitions Without a Knuckle

Determine if the proposed large and small end cylinder-to-cone transitions are adequately designed considering the following design conditions and applied forces and moments. Evaluate the stresses in the cylinder and cone at both the large and small end junction.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $356\ psig@300°F$
- Inside Radius (Large End) = $75.0\ in$
- Thickness (Large End) = $1.8125\ in$
- Cylinder Length (Large End) = $60.0\ in$
- Inside Radius (Small End) = $45.0\ in$
- Thickness (Small End) = $1.125\ in$
- Cylinder Length (Small End) = $48.0\ in$
- Thickness (Conical Section) = $1.9375\ in$
- Length of Conical Section = $78.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle (See Figure E4.3.7) = $21.0375\ deg$
- Axial Force (Large End) = $-99167\ lbs$
- Net Section Bending Moment (Large End) = $5.406E+06\ in-lbs$
- Axial Force (Small End) = $-78104\ lbs$
- Net Section Bending Moment (Small End) = $4.301E+06\ in-lbs$

Adjust variables for corrosion.

$$R_L = 75.0 + Corrosion\ \ Allowance = 75.0 + 0.125 = 75.125\ in$$
$$R_S = 45.0 + Corrosion\ \ Allowance = 45.0 + 0.125 = 45.125\ in$$
$$t_L = 1.8125 - Corrosion\ \ Allowance = 1.8125 - 0.125 = 1.6875\ in$$
$$t_S = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$
$$t_C = 1.9375 - Corrosion\ \ Allowance = 1.9375 - 0.125 = 1.8125\ in$$
$$\alpha = 21.0375\ deg$$

Evaluate per paragraph 4.3.11.

Per paragraph 4.3.11.3, the length of the conical shell, measured parallel to the surface of the cone, shall be equal to or greater than the following:

$$L_C \geq 2.0\sqrt{\frac{R_L t_C}{\cos[\alpha]}} + 1.4\sqrt{\frac{R_S t_C}{\cos[\alpha]}}$$

$$2.0\sqrt{\frac{75.125(1.8125)}{\cos[21.0375]}} + 1.4\sqrt{\frac{45.125(1.8125)}{\cos[21.0375]}} = 37.2624\ in$$

$$L_C = 78.0 \geq 37.2624 \qquad True$$

Evaluate the Large End cylinder-to-cone junction per VIII-2, paragraph 4.3.11.4.

a) STEP 1 – Compute the large end cylinder thickness, t_L, using paragraph 4.3.3., (as specified in design conditions).

$$t_L = 1.6875\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the large end using paragraph 4.3.4., (as specified in design conditions).

$$\alpha = 21.0375\ deg$$

$$t_C = 1.8125\ in$$

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all of these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with Part 5. In the calculations, if $0\ deg < \alpha \leq 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \leq \left\{\frac{R_L}{t_L} = \frac{75.125}{1.6875} = 44.5185\right\} \leq 500 \qquad True$$

$$1 \leq \left\{\frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741\right\} \leq 2 \qquad True$$

$$\{\alpha = 21.0375\ deg\} \leq \{60\ deg\} \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force, F_L. Calculate the equivalent line load, X_L, using the specified net section axial force, F_L, and bending moment, M_L.

$$X_L = \frac{F_L}{2\pi R_L} \pm \frac{M_L}{\pi R_L^2} = \begin{Bmatrix} \dfrac{-99167}{2\pi(75.125)} + \dfrac{5406000}{\pi(75.125)^2} = 94.8111\ \dfrac{lbs}{in} \\ \dfrac{-99167}{2\pi(75.125)} - \dfrac{5406000}{\pi(75.125)^2} = -514.9886\ \dfrac{lbs}{in} \end{Bmatrix}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per Table 4.3.3 and Table 4.3.4, respectively. For calculated values of n other than those presented in Table 4.3.3 and Table 4.3.4, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741$$

$$H = \sqrt{\frac{R_L}{t_L}} = \sqrt{\frac{75.125}{1.6875}} = 6.6722$$

$$B = \tan\left[\alpha\right] = \tan\left[21.0375\right] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in Table 4.3.3 and Table 4.3.4 is required. The results of the interpolation are summarized with the following values for C_i.

Equation Coefficients C_i	VIII-2, Table 4.3.3		VIII-2, Table 4.3.4	
	Pressure Applied Junction Moment Resultant M_{sN}	Pressure Applied Junction Shear Force Resultant Q_N	Equivalent Line Load Junction Moment Resultant M_{sN}	Equivalent Line Load Junction Shear Force Resultant Q_N
1	-3.079751	-1.962370	-5.706141	-4.878520
2	3.662099	2.375540	0.004705	0.006808
3	0.788301	0.582454	0.474988	-0.018569
4	-0.226515	-0.107299	-0.213112	-0.197037
5	-0.080019	-0.103635	2.241065	2.033876
6	0.049314	0.151522	0.000025	-0.000085
7	0.026266	0.010704	0.002759	-0.000109
8	-0.015486	-0.018356	-0.001786	-0.004071
9	0.001773	0.006551	-0.214046	-0.208830
10	-0.007868	-0.021739	0.000065	-0.000781
11	---	---	-0.106223	0.004724

For the applied pressure case both M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\begin{bmatrix} C_1 + C_2\ln[H] + C_3\ln[B] + C_4\left(\ln[H]\right)^2 + C_5\left(\ln[B]\right)^2 + \\ C_6\ln[H]\ln[B] + C_7\left(\ln[H]\right)^3 + C_8\left(\ln[B]\right)^3 + \\ C_9\ln[H]\left(\ln[B]\right)^2 + C_{10}\left(\ln[H]\right)^2\ln[B] \end{bmatrix}$$

This results in

$$M_{sN} = -\exp\left[\begin{array}{l} -3.079751 + 3.662099 \cdot \ln[6.6722] + 0.788301 \cdot \ln[0.3846] + \\ (-0.226515)(\ln[6.6722])^2 + (-0.080019)(\ln[0.3846])^2 + \\ 0.049314 \cdot \ln[6.6722] \cdot \ln[0.3846] + \\ 0.026266(\ln[6.6722])^3 + (-0.015486)(\ln[0.3846])^3 + \\ 0.001773 \cdot \ln[6.6722] \cdot (\ln[0.3846])^2 + \\ (-0.007868)(\ln[6.6722])^2 \cdot \ln[0.3846] \end{array}\right] = -10.6148$$

$$Q_N = -\exp\left[\begin{array}{l} -1.962370 + 2.375540 \cdot \ln[6.6722] + 0.582454 \cdot \ln[0.3846] + \\ (-0.107299)(\ln[6.6722])^2 + (-0.103635)(\ln[0.3846])^2 + \\ 0.151522 \cdot \ln[6.6722] \cdot \ln[0.3846] + \\ 0.010704(\ln[6.6722])^3 + (-0.018356)(\ln[0.3846])^3 + \\ 0.006551 \cdot \ln[6.6722] \cdot (\ln[0.3846])^2 + \\ (-0.021739)(\ln[6.6722])^2 \cdot \ln[0.3846] \end{array}\right] = -4.0925$$

For the Equivalent Line Load case, M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\left[\frac{\left(\begin{array}{l} C_1 + C_3 \ln[H^2] + C_5 \ln[\alpha] + C_7 (\ln[H^2])^2 + \\ C_9 (\ln[\alpha])^2 + C_{11} \ln[H^2] \ln[\alpha] \end{array}\right)}{\left(\begin{array}{l} 1 + C_2 \ln[H^2] + C_4 \ln[\alpha] + C_6 (\ln[H^2])^2 + \\ C_8 (\ln[\alpha])^2 + C_{10} \ln[H^2] \ln[\alpha] \end{array}\right)}\right]$$

This results in

$$M_{sN} = -\exp\left[\frac{\begin{pmatrix} -5.706141 + 0.474988 \cdot \ln\left[6.6722^2\right] + \\ 2.241065 \cdot \ln\left[21.0375\right] + 0.002759\left(\ln\left[6.6722^2\right]\right)^2 + \\ (-0.214046)\left(\ln\left[21.0375\right]\right)^2 + \\ (-0.106223)\ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{pmatrix}}{\begin{pmatrix} 1 + 0.004705 \cdot \ln\left[6.6722^2\right] + (-0.213112)\ln\left[21.0375\right] + \\ 0.000025\left(\ln\left[6.6722^2\right]\right)^2 + (-0.001786)\left(\ln\left[21.0375\right]\right)^2 + \\ 0.000065 \cdot \ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{pmatrix}}\right] = -0.4912$$

$$Q_N = -\exp\left[\frac{\begin{pmatrix} -4.878520 + (-0.018569)\ln\left[6.6722^2\right] + \\ 2.033876 \cdot \ln\left[21.0375\right] + (-0.000109)\left(\ln\left[6.6722^2\right]\right)^2 + \\ (-0.208830)\left(\ln\left[21.0375\right]\right)^2 + \\ 0.004724 \cdot \ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{pmatrix}}{\begin{pmatrix} 1 + 0.006808 \cdot \ln\left[6.6722^2\right] + (-0.197037)\ln\left[21.0375\right] + \\ (-0.000085)\left(\ln\left[6.6722^2\right]\right)^2 + (-0.004071)\left(\ln\left[21.0375\right]\right)^2 + \\ (-0.000781)\ln\left[6.6722^2\right] \cdot \ln\left[21.0375\right] \end{pmatrix}}\right] = -0.1845$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\begin{array}{lll} \textit{Internal Pressure}: & M_{sN} = -10.6148, & Q_N = -4.0925 \\ \textit{Equivalent Line Load}: & M_{sN} = -0.4912, & Q_N = -0.1845 \end{array}$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in Table 4.3.1 for the Large End Junction.

Evaluate the Cylinder at the Large End:

Stress Resultant Calculations:

$$M_{sP} = Pt_L^2 M_{sN} = 356(1.6875)^2(-10.6148) = -10760.9194 \ \frac{in-lbs}{in}$$

$$M_{sX} = X_L t_L M_{sN} = \begin{Bmatrix} 94.8111(1.6875)(-0.4912) = -78.5889 \ \dfrac{in-lbs}{in} \\ -514.9886(1.6875)(-0.4912) = 426.8741 \ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$M_s = M_{sP} + M_{sX} = \begin{Bmatrix} -10760.9194 + (-78.5889) = -10839.5083 \ \dfrac{in-lbs}{in} \\ -10760.9194 + 426.8741 = -10334.0453 \ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$Q_P = Pt_L Q_N = 356(1.6875)(-4.0925) = -2458.5694 \ \frac{lbs}{in}$$

$$Q_X = X_L Q_N = \begin{Bmatrix} 94.8111(-0.1845) = -17.4926 \ \dfrac{lbs}{in} \\ -514.9886(-0.1845) = 95.0154 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$Q = Q_P + Q_X = \begin{Bmatrix} -2458.5694 + (-17.4926) = -2476.0620 \ \dfrac{lbs}{in} \\ -2458.5694 + 95.0154 = -2363.5540 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_L^2 t_L^2}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(75.125)^2(1.6875)^2}\right]^{0.25} = 0.1142 \ in^{-1}$$

$$N_s = \frac{PR_L}{2} + X_L = \begin{Bmatrix} \dfrac{356(75.125)}{2} + 94.8111 = 13467.0611 \ \dfrac{lbs}{in} \\ \dfrac{356(75.125)}{2} + (-514.9886) = 12857.2614 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_\theta = PR_L + 2\beta_{cy}R_L\left(-M_s\beta_{cy} + Q\right)$$

$$N_\theta = \begin{Bmatrix} 356(75.125) + 2(0.1142)(75.125)\left(-(-10839.5083)(0.1142) + (-2476.0620)\right) \\ 356(75.125) + 2(0.1142)(75.125)\left(-(-10334.0453)(0.1142) + (-2363.553)\right) \end{Bmatrix}$$

$$N_\theta = \begin{Bmatrix} 5498.9524 \ \dfrac{lbs}{in} \\ 6438.9685 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_s}{t_L} = \begin{Bmatrix} \dfrac{13467.0611}{1.6875} = 7980.4807 \ psi \\ \dfrac{12857.2614}{1.6875} = 7619.1179 \ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_s}{t_L^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(-10839.5083)}{(1.6875)^2(1.0)} = -22838.7994 \ psi \\ \dfrac{6(-10334.0453)}{(1.6875)^2(1.0)} = -21773.7909 \ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_L} = \begin{Bmatrix} \dfrac{5498.9524}{1.6875} = 3258.6385 \, psi \\ \dfrac{6438.9685}{1.6875} = 3815.6850 \ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_L^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(0.3)(-10839.5083)}{(1.6875)^2(1.0)} = -6851.6398 \ psi \\ \dfrac{6(0.3)(-10334.0453)}{(1.6875)^2(1.0)} = -6532.1373 \ psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\left\{\begin{array}{l}\sigma_{sm} = 7980.4807\ psi \\ \sigma_{sm} = 7619.1179\ psi\end{array}\right\} \leq \left\{1.5S = 1.5(22400) = 33600\ psi\right\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{sm} + \sigma_{sb} = 7980.4807 + (-22838.7994) = |-14858.3\ psi| \\ \sigma_{sm} - \sigma_{sb} = 7980.4807 - (-22838.7994) = 30819.3\ psi \\ \sigma_{sm} + \sigma_{sb} = 7619.1179 + (-21773.7909) = |-14154.7\ psi| \\ \sigma_{sm} - \sigma_{sb} = 7619.1179 - (-21773.7909) = 29392.9\ psi\end{array}\right\} \leq \left\{S_{PS} = 67200\ psi\right\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m} = 3258.6385\ psi \\ \sigma_{\theta m} = 3815.6850\ psi\end{array}\right\} \leq \left\{1.5S = 1.5(22400) = 33600\ psi\right\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m} + \sigma_{\theta b} = 3258.6385 + (-6851.6398) = |-3593.0\ psi| \\ \sigma_{\theta m} - \sigma_{\theta b} = 3258.6385 - (-6851.6398) = 10110.3\ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 3815.6850 + (-6532.1373) = |-2716.5\ psi| \\ \sigma_{\theta m} - \sigma_{\theta b} = 3815.6850 - (-6532.1373) = 10347.8\ psi\end{array}\right\} \leq \left\{S_{PS} = 67200\ psi\right\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cylinder at the cylinder-to-cone junction at the large end is adequately designed.

Evaluate the Cone at the Large End:

Stress Resultant Calculations - as determined above:

$$M_{csP} = M_{sP} = -10760.9194\ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \left\{\begin{array}{l}-78.5889\ \dfrac{in-lbs}{in} \\ 426.8741\ \dfrac{in-lbs}{in}\end{array}\right\}$$

$$M_{cs} = M_{csP} + M_{csX} = \left\{\begin{array}{l}-10760.9194 + (-78.5889) = -10839.5083\ \dfrac{in-lbs}{in} \\ -10760.9194 + 426.8741 = -10334.0453\ \dfrac{in-lbs}{in}\end{array}\right\}$$

$$Q_c = Q\cos[\alpha] + N_s \sin[\alpha]$$

$$Q_c = \left\{\begin{array}{l}\left((-2476.0620)\cos[21.0375] + (13467.0611)\sin[21.0375]\right) = 2523.3690\ \dfrac{lbs}{in} \\ \left((-2363.5540)\cos[21.0375] + (12857.2614)\sin[21.0375]\right) = 2409.4726\ \dfrac{lbs}{in}\end{array}\right\}$$

$$R_C = \frac{R_L}{\cos[\alpha]} = \frac{75.125}{\cos[21.0375]} = 80.4900\ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_C^2 t_C^2}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(80.4900)^2(1.8125)^2}\right]^{0.25} = 0.1064\ in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{Bmatrix} (13467.0611)\cos[21.0375] - (-2476.0620)\sin[21.0375] = 13458.2772\ \dfrac{lbs}{in} \\ (12857.2614)\cos[21.0375] - (-2363.5540)\sin[21.0375] = 12848.7353\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_{c\theta} = \frac{PR_L}{\cos[\alpha]} + 2\beta_{co}R_C\left(-M_{cs}\beta_{co} - Q_c\right)$$

$$N_{c\theta} = \begin{Bmatrix} \dfrac{356(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(-10839.5083)(0.1064) - 2523.3690\right) \\ \dfrac{356(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(-10334.0453)(0.1064) - 2409.4726\right) \end{Bmatrix}$$

$$N_{c\theta} = \begin{Bmatrix} 5187.9337\ \dfrac{lbs}{in} \\ 6217.6021\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{cpc} = 1.0$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{Bmatrix} \dfrac{13458.2772}{1.8125} = 7425.2564 \;\; psi \\ \dfrac{12848.7353}{1.8125} = 7088.9574 \;\; psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(-10839.5083)}{(1.8125)^2(1.0)} = -19797.2470 \;\; psi \\ \dfrac{6(-10334.0453)}{(1.8125)^2(1.0)} = -18874.0708 \;\; psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{Bmatrix} \dfrac{5187.9337}{1.8125} = 2862.3082 \;\; psi \\ \dfrac{6217.6021}{1.8125} = 3430.4012 \;\; psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(0.3)(-10839.5083)}{(1.8125)^2(1.0)} = -5939.1741 \;\; psi \\ \dfrac{6(0.3)(-10334.0453)}{(1.8125)^2(1.0)} = -5662.2213 \;\; psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\begin{Bmatrix} \sigma_{sm} = 7425.2564 \;\; psi \\ \sigma_{sm} = 7088.9574 \;\; psi \end{Bmatrix} \le \left\{1.5S = 1.5(22400) = 33600 \;\; psi\right\} \qquad True$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = 7425.2564 + (-19797.2470) = |-12371.9906 \;\; psi| \\ \sigma_{sm} - \sigma_{sb} = 7425.2564 - (-19797.2470) = 27222.5034 \;\; psi \\ \sigma_{sm} + \sigma_{sb} = 7088.9574 + (-18874.0708) = |-11785.1 \;\; psi| \\ \sigma_{sm} - \sigma_{sb} = 7088.9574 - (-18874.0708) = 25963.0 \;\; psi \end{Bmatrix} \le \left\{S_{PS} = 67200 \;\; psi\right\} True$$

$$\begin{Bmatrix} \sigma_{\theta m} = 2862.3082 \;\; psi \\ \sigma_{\theta m} = 3430.4012 \;\; psi \end{Bmatrix} \le \left\{1.5S = 1.5(22400) = 33600 \;\; psi\right\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = 2862.3082 + (-5939.1741) = |-3076.8659 \;\; psi| \\ \sigma_{\theta m} - \sigma_{\theta b} = 2862.3082 - (-5939.1741) = 8801.4823 \; psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 3430.4012 + (-5662.2213) = |-2231.8 \;\; psi| \\ \sigma_{\theta m} - \sigma_{\theta b} = 3430.4012 - (-5662.2213) = 9092.6 \; psi \end{Bmatrix} \le \left\{S_{PS} = 67200 \;\; psi\right\} \quad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cone at the cylinder-to-cone junction at the large end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore the design is complete.

<u>Evaluate the Small End cylinder-to-cone junction per paragraph 4.3.11.5.</u>

a) STEP 1 – Compute the small end cylinder thickness, t_S, using paragraph 4.3.3., (as specified in design conditions).

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the small end using paragraph 4.3.4., (as specified in design conditions

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all of these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with Part 5. In the calculations, if $0\ deg < \alpha \leq 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \leq \left(\frac{R_S}{t_S} = \frac{45.125}{1.0} = 45.125 \right) \leq 500 \qquad True$$

$$1 \leq \left(\frac{t_C}{t_S} = \frac{1.8125}{1.0} = 1.8125 \right) \leq 2 \qquad True$$

$$\{\alpha = 21.0375\ deg\} \leq \{60\ deg\} \qquad True$$

d) STEP 4 – Calculate the equivalent line load, X_S, given the net section axial force, F_S, and bending moment, M_S, applied at the conical transition.

$$X_S = \frac{F_S}{2\pi R_S} \pm \frac{M_S}{\pi R_S^2} = \left\{ \begin{array}{l} \dfrac{-78104.2}{2\pi(45.125)} + \dfrac{4301000}{\pi(45.125)^2} = 396.8622\ \dfrac{lbs}{in} \\ \dfrac{-78104.2}{2\pi(45.125)} - \dfrac{4301000}{\pi(45.125)^2} = -947.8060\ \dfrac{lbs}{in} \end{array} \right\}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per Table 4.3.5 and Table 4.3.6, respectively. For calculated values of n other than those presented in Table 4.3.5 and Table 4.3.6, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_S} = \frac{1.8125}{1.000} = 1.8125$$

$$H = \sqrt{\frac{R_S}{t_S}} = \sqrt{\frac{45.125}{1.000}} = 6.7175$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in Table 4.3.5 and Table 4.3.6 is required. The results of the interpolation is summarized with the following values for C_i.

Equation Coefficients C_i	VIII-2, Table 4.3.5		VIII-2, Table 4.3.6	
	Pressure Applied Junction Moment Resultant M_{sN}	Pressure Applied Junction Shear Force Resultant Q_N	Equivalent Line Load Junction Moment Resultant M_{sN}	Equivalent Line Load Junction Shear Force Resultant Q_N
1	-15.144683	0.569891	0.006792	-0.408044
2	3.036812	-0.000027	0.000290	0.021200
3	6.460714	0.008431	-0.000928	-0.325518
4	-0.155909	0.002690	0.121611	-0.003988
5	-1.462862	-0.002884	0.010581	-0.111262
6	-0.369444	0.000000	-0.000011	0.002204
7	0.007742	-0.000005	-0.000008	0.000255
8	0.143191	-0.000117	0.005957	-0.014431
9	0.040944	-0.000087	0.001310	0.000820
10	0.007178	0.000001	0.000186	0.000106
11	---	-0.003778	0.194433	---

For the applied pressure case M_{sN} is calculated using the following equation

$$M_{sN} = \exp\begin{bmatrix} C_1 + C_2 \ln\left[H^2\right] + C_3 \ln\left[\alpha\right] + C_4\left(\ln\left[H^2\right]\right)^2 + C_5\left(\ln\left[\alpha\right]\right)^2 + \\ C_6 \ln\left[H^2\right]\ln\left[\alpha\right] + C_7\left(\ln\left[H^2\right]\right)^3 + C_8\left(\ln\left[\alpha\right]\right)^3 + \\ C_9 \ln\left[H^2\right]\left(\ln\left[\alpha\right]\right)^2 + C_{10}\left(\ln\left[H^2\right]\right)^2 \ln\left[\alpha\right] \end{bmatrix}$$

This results in

$$M_{sN} = \exp\left[\begin{array}{l} -15.144683 + 3.036812 \cdot \ln\left[6.7175^2\right] + 6.460714 \cdot \ln\left[21.0375\right] + \\ (-0.155909)\left(\ln\left[6.7175^2\right]\right)^2 + (-1.462862)\left(\ln\left[21.0375\right]\right)^2 + \\ (-0.369444)\ln\left[6.7175^2\right] \cdot \ln\left[21.0375\right] + \\ 0.007742\left(\ln\left[6.7175^2\right]\right)^3 + 0.143191\left(\ln\left[21.0375\right]\right)^3 + \\ 0.040944 \cdot \ln\left[6.7175^2\right] \cdot \left(\ln\left[21.0375\right]\right)^2 + \\ 0.007178\left(\ln\left[6.7175^2\right]\right)^2 \cdot \ln\left[21.0375\right] \end{array}\right] = 9.2135$$

For the applied pressure case Q_N is calculated using the following equation

$$Q_N = \left(\frac{C_1 + C_3 H^2 + C_5 \alpha + C_7 H^4 + C_9 \alpha^2 + C_{11} H^2 \alpha}{1 + C_2 H^2 + C_4 \alpha + C_6 H^4 + C_8 \alpha^2 + C_{10} H^2 \alpha}\right)$$

This results in

$$Q_N = \left(\frac{\left(\begin{array}{l} 0.569891 + 0.008431(6.7175)^2 + (-0.002884)(21.0375) + \\ (-0.000005)(6.7175)^4 + (-0.000087)(21.0375)^2 + \\ (-0.003778)(6.7175)^2(21.0375) \end{array}\right)}{\left(\begin{array}{l} 1 + (-0.000027)(6.7175)^2 + 0.002690(21.0375) + \\ 0.000000(6.7175)^4 + (-0.000117)(21.0375)^2 + \\ 0.000001(6.7175)^2(21.0375) \end{array}\right)}\right) = -2.7333$$

For the Equivalent Line Load case, M_{sN} is calculated using the following equation

$$M_{sN} = \left(\frac{C_1 + C_3 H + C_5 B + C_7 H^2 + C_9 B^2 + C_{11} HB}{1 + C_2 H + C_4 B + C_6 H^2 + C_8 B^2 + C_{10} HB}\right)$$

This results in

$$M_{sN}=\left(\frac{\begin{pmatrix}0.006792+(-0.000928)(6.7175)+0.010581(0.3846)+\\(-0.000008)(6.7175)^2+0.001310(0.3846)^2+\\0.194433(6.7175)(0.3846)\end{pmatrix}}{\begin{pmatrix}1+0.000290(6.7175)+0.121611(0.3846)+\\(-0.000011)(6.7175)^2+0.005957(0.3846)^2+\\0.000186(6.7175)(0.3846)\end{pmatrix}}\right)=0.4828$$

For the Equivalent Line Load case, Q_N is calculated using the following equation

$$Q_N=\begin{pmatrix}C_1+C_2\ln[H]+C_3\ln[B]+C_4(\ln[H])^2+C_5(\ln[B])^2+C_6\ln[H]\ln[B]+\\C_7(\ln[H])^3+C_8(\ln[B])^3+C_9\ln[H](\ln[B])^2+C_{10}(\ln[H])^2\ln[B]\end{pmatrix}$$

This results in

$$Q_N=\begin{pmatrix}-0.408044+0.021200\cdot\ln[6.7175]+(-0.325518)\ln[0.3846]+\\(-0.003988)(\ln[6.7175])^2+(-0.111262)(\ln[0.3846])^2+\\0.002204\cdot\ln[6.7175]\cdot\ln[0.3846]+0.000255\cdot(\ln[6.7175])^3+\\(-0.014431)(\ln[0.3846])^3+0.000820\cdot\ln[6.7175]\cdot(\ln[0.3846])^2+\\0.000106(\ln[6.7175])^2\cdot\ln[0.3846]\end{pmatrix}=-0.1613$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\begin{array}{lll}\textit{Internal Pressure}: & M_{sN}=9.2135, & Q_N=-2.7333\\ \textit{Equivalent Line Load}: & M_{sN}=0.4828, & Q_N=-0.1613\end{array}$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in Table 4.3.2 for the Small End Junction.

Evaluate the Cylinder at the Small End:

Stress Resultant Calculations:

$$M_{sP} = Pt_S^2 M_{sN} = 356(1.000)^2(9.2135) = 3280.0060 \ \frac{in-lbs}{in}$$

$$M_{sX} = X_S t_S M_{sN} = \begin{cases} 396.8622(1.0000)(0.4828) = 191.6051 \ \dfrac{in-lbs}{in} \\ -947.8060(1.0000)(0.4828) = -457.6007 \ \dfrac{in-lbs}{in} \end{cases}$$

$$M_s = M_{sP} + M_{sX} = \begin{cases} 3280.0060 + (191.6051) = 3471.6111 \ \dfrac{in-lbs}{in} \\ 3280.0060 + (-457.6007) = 2822.4053 \ \dfrac{in-lbs}{in} \end{cases}$$

$$Q_P = Pt_S Q_N = 356(1.0000)(-2.7333) = -973.0548 \ \frac{lbs}{in}$$

$$Q_X = X_S Q_N = \begin{cases} 396.8622(-0.1613) = -64.0139 \ \dfrac{lbs}{in} \\ -947.8060(-0.1613) = 152.8811 \ \dfrac{lbs}{in} \end{cases}$$

$$Q = Q_P + Q_X = \begin{cases} -973.0548 + (-64.0139) = -1037.0687 \dfrac{lbs}{in} \\ -973.0548 + 152.8811 = -820.1737 \dfrac{lbs}{in} \end{cases}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_S^2 t_S^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(45.1250)^2(1.000)^2}\right]^{0.25} = 0.1914 \ in^{-1}$$

$$N_s = \frac{PR_S}{2} + X_S = \begin{cases} \dfrac{356(45.125)}{2} + 396.8622 = 8429.1122 \ \dfrac{lbs}{in} \\ \dfrac{356(45.125)}{2} + (-947.8060) = 7084.4440 \ \dfrac{lbs}{in} \end{cases}$$

$$N_\theta = PR_S + 2\beta_{cy}R_S\left(-M_s\beta_{cy} - Q\right)$$

$$N_\theta = \begin{Bmatrix} 356(45.125)+2(0.1914)(45.125)\left(-(3471.6111)(0.1914)-(-1037.0687)\right) \\ 356(45.125)+2(0.1914)(45.125)\left(-(2822.4053)(0.1914)-(-820.1737)\right) \end{Bmatrix}$$

$$N_\theta = \begin{Bmatrix} 22500.7769 \ \dfrac{lbs}{in} \\ 20900.5790 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{pc} = 1.0$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_s}{t_S} = \begin{Bmatrix} \dfrac{8429.1122}{1.0000} = 8429.1122 \ psi \\ \dfrac{7084.4440}{1.0000} = 7084.4440 \ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(3471.6111)}{(1.0000)^2(1.0)} = 20829.6666 \ psi \\ \dfrac{6(2822.4053)}{(1.0000)^2(1.0)} = 16934.4318 \ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_S} = \begin{Bmatrix} \dfrac{22500.7769}{1.0000} = 22500.7769 \ psi \\ \dfrac{20900.5790}{1.0000} = 20900.5790 \ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_S^2 K_{pc}} = \begin{Bmatrix} \dfrac{6(0.3)(3471.6111)}{(1.0000)^2(1.0)} = 6248.8999 \ psi \\ \dfrac{6(0.3)(2822.4053)}{(1.0000)^2(1.0)} = 5080.3295 \ psi \end{Bmatrix}$$

Check Acceptance Criteria:

$$\left\{\begin{array}{l}\sigma_{sm}=8429.1122\ psi\\ \sigma_{sm}=7084.4440\ psi\end{array}\right\}\le\left\{1.5S=1.5\left(22400\right)=33600\ psi\right\}\qquad True$$

$$\left\{\begin{array}{l}\sigma_{sm}+\sigma_{sb}=8429.1122+\left(20829.6666\right)=29258.8\ psi\\ \sigma_{sm}-\sigma_{sb}=8429.1122-\left(20829.6666\right)=\left|-12400.6\ psi\right|\\ \sigma_{sm}+\sigma_{sb}=7084.4440+\left(16934.4318\right)=24018.9\ psi\\ \sigma_{sm}-\sigma_{sb}=7084.4440-\left(16934.4318\right)=\left|-9850.0\ psi\right|\end{array}\right\}\le\left\{S_{PS}=67200\ psi\right\}\qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m}=22500.7769\ psi\\ \sigma_{\theta m}=20900.5790\ psi\end{array}\right\}\le\left\{1.5S=1.5\left(22400\right)=33600\ psi\right\}\qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m}+\sigma_{\theta b}=22500.7769+\left(6248.8999\right)=28749.7\ psi\\ \sigma_{\theta m}-\sigma_{\theta b}=22500.7769-\left(6248.8999\right)=16251.9\ psi\\ \sigma_{\theta m}+\sigma_{\theta b}=20900.5790+\left(5080.3295\right)=25981.0\ psi\\ \sigma_{\theta m}-\sigma_{\theta b}=20900.5790-\left(5080.3295\right)=15820.2\ psi\end{array}\right\}\le\left\{S_{PS}=67200\ psi\right\}\qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cylinder at the cylinder-to-cone junction at the small end is adequately designed.

Evaluate the Cone at the Small End:

Stress Resultant Calculations:

$$M_{csP}=M_{sP}=3280.0060\ \frac{in-lbs}{in}$$

$$M_{csX}=M_{sX}=\left\{\begin{array}{l}191.6051\ \dfrac{in-lbs}{in}\\ -457.6007\ \dfrac{in-lbs}{in}\end{array}\right\}$$

$$M_{cs}=M_{csP}+M_{csX}=\left\{\begin{array}{l}3280.0060+191.6051=3471.6111\ \dfrac{in-lbs}{in}\\ 3280.0060+\left(-457.6007\right)=2822.4053\ \dfrac{in-lbs}{in}\end{array}\right\}$$

$$Q_c=Q\cos\left[\alpha\right]+N_s\sin\left[\alpha\right]$$

$$Q_c=\left\{\begin{array}{l}-1037.0687\cos\left[21.0375\right]+8429.1122\sin\left[21.0375\right]=2057.9298\ \dfrac{lbs}{in}\\ -820.1737\cos\left[21.0375\right]+7084.4440\sin\left[21.0375\right]=1777.6603\ \dfrac{lbs}{in}\end{array}\right\}$$

$$R_c = \frac{R_c}{\cos[\alpha]} = \frac{45.1250}{\cos[21.0375]} = 48.3476\ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_c^{\,2} t_c^{\,2}}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(48.3476)^2(1.8125)^2}\right]^{0.25} = 0.1373\ in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{Bmatrix} \left(8429.1122\cos[21.0375] - (-1037.0687)\sin[21.0375]\right) = 8239.5612\ \dfrac{lbs}{in} \\ \left(7084.4440\cos[21.0375] - (-820.1737)\sin[21.0375]\right) = 6906.6602\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_{c\theta} = \frac{PR_s}{\cos[\alpha]} + 2\beta_{co} R_c\left(-M_{cs}\beta_{co} + Q_c\right)$$

$$N_{c\theta} = \begin{Bmatrix} \dfrac{356(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(3471.6111)(0.1373) + 2057.9298\right) \\ \dfrac{356(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(2822.4053)(0.1373) + 1777.6603\right) \end{Bmatrix}$$

$$N_{c\theta} = \begin{Bmatrix} 38205.1749\ \dfrac{lbs}{in} \\ 35667.6380\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{cpc} = 1.0$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{cases} \dfrac{8239.5612}{1.8125} = 4545.9648 \ \ psi \\ \dfrac{6906.6602}{1.8125} = 3810.5711 \ \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{cases} \dfrac{6(3471.6111)}{(1.8125)^2(1.0)} = 6340.5406 \ \ psi \\ \dfrac{6(2822.4053)}{(1.8125)^2(1.0)} = 5154.8330 \ \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{cases} \dfrac{38205.1749}{1.8125} = 21078.7172 \ \ psi \\ \dfrac{35667.6380}{1.8125} = 19678.6968 \ \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{cases} \dfrac{6(0.3)(3471.6111)}{(1.8125)^2(1.0)} = 1902.1622 \ \ psi \\ \dfrac{6(0.3)(2822.4053)}{(1.8125)^2(1.0)} = 1546.4499 \ \ psi \end{cases}$$

Check Acceptable Criteria:

$$\begin{Bmatrix} \sigma_{sm} = 4545.9648 \ \ psi \\ \sigma_{sm} = 3810.5711 \ \ psi \end{Bmatrix} \leq \left\{ 1.5S = 1.5(22400) = 33600 \ \ psi \right\} \qquad True$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = 4545.9648 + (6340.5406) = 10886.5 \ \ psi \\ \sigma_{sm} - \sigma_{sb} = 4545.9648 - (6340.5406) = \left| -1794.6 \ \ psi \right| \\ \sigma_{sm} + \sigma_{sb} = 3810.5711 + (5154.8330) = 8965.4 \ \ psi \\ \sigma_{sm} - \sigma_{sb} = 3810.5711 - (5154.8330) = \left| -1344.3 \ \ psi \right| \end{Bmatrix} \leq \left\{ S_{PS} = 67200 \ \ psi \right\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = 21078.7172 \ \ psi \\ \sigma_{\theta m} = 19678.6968 \ \ psi \end{Bmatrix} \leq \left\{ 1.5S = 1.5(22400) = 33600 \ \ psi \right\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = 21078.7172 + (1902.1622) = 22980.9 \ \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 21078.7172 - (1902.1622) = 19176.6 \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 19678.6968 + (1546.4499) = 21225.1 \ \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 19678.6968 - (1546.4499) = 18132.2 \ psi \end{Bmatrix} \leq \left\{ S_{PS} = 67200 \ \ psi \right\} \qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ are both tensile, the condition of local buckling need not be considered. Therefore, the cone at the cylinder-to-cone junction at the small end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore the design is complete.

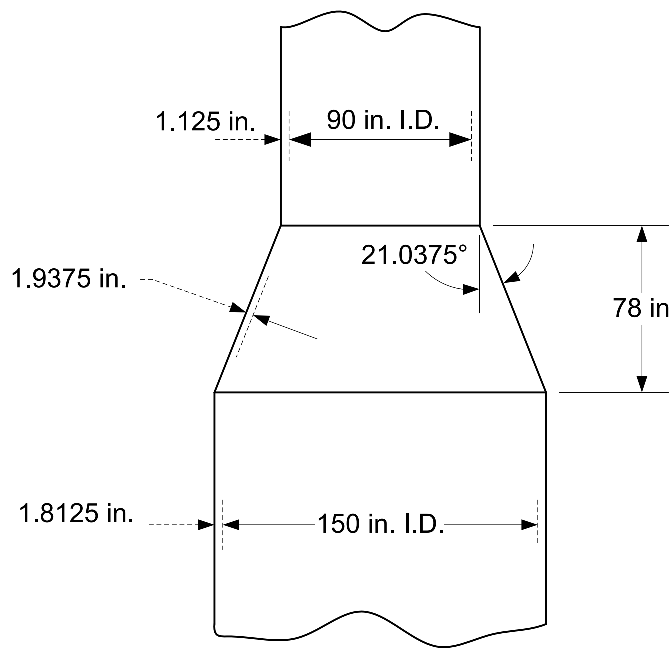


Figure E4.3.7 - Conical Transition

4.3.8 Example E4.3.8 – Conical Transitions with a Knuckle

Determine if the proposed design for the large end of a cylinder-to-cone junction with a knuckle is adequately designed considering the following design conditions and applied forces and moments. See Figure 4.3.8 for details

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $280\ psig@300°F$
- Inside Diameter (Large End) = $120.0\ in$
- Inside Radius (Large End) = $60.0\ in$
- Knuckle Radius = $10.0\ in$
- Large End Thickness = $1.0\ in$
- Cone Thickness = $1.0\ in$
- Knuckle Thickness = $1.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $22400\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle = $30.0\ deg$
- Axial Force (Large End) = $-10000\ lbs$
- Net Section Bending Moment (Large End) = $2.0E+06\ in-lbs$

Evaluate per paragraph 4.3.12.

a) STEP 1 – Compute the large end cylinder thickness, t_L, using paragraph 4.3.3.

$$t_L = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{120.0}{2}\left(\exp\left[\frac{280.0}{22400.0(1.0)}\right]-1\right) = 0.7547\ in$$

As specified in the design conditions,

$$t_L = 1.0\ in$$

Since the required thickness is less than the design thickness, the cylinder is adequately designed for internal pressure.

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the large end using paragraph 4.3.4.

$$\alpha = 30.0\ deg$$

$$t_C = \frac{D_i}{2\cos[\alpha]}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{117.3205}{2\cos[30.0]}\left(\exp\left[\frac{280.0}{22400.0(1.0)}\right]-1\right) = 0.8520\ in$$

Where, the value of D_i is substituted for D and is calculated as follows,

$$D_i = D - 2r\left(1-\cos\left[\alpha\right]\right) = 120.0 - 2\left(10.0\right)\left(1-\cos\left[30.0\right]\right) = 117.3205\ in$$

As specified in the design conditions,

$$t_C = 1.0\ in$$

Since the required thickness is less than the design thickness, the cone is adequately designed for internal pressure.

c) STEP 3 – Proportion the transition geometry by assuming a value for the knuckle radius, r_k, and knuckle thickness, t_k, such that the following equations are satisfied. If all of these equations cannot be satisfied, the cylinder-to-cone junction shall be designed in accordance with Part 5.

$$\left\{t_k = 1.0\ in\right\} \geq \left\{t_L = 1.0\ in\right\} \qquad True$$

$$\left\{r_k = 10.0\ in\right\} > \left\{3t_k = 3.0\ in\right\} \qquad True$$

$$\left\{\frac{r_k}{R_L} = \frac{10.0}{60.0} = 0.1667\right\} > \left\{0.03\right\} \qquad True$$

$$\left\{\alpha = 30\ deg\right\} \leq \left\{60\ deg\right\} \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition at the location of the knuckle. The thrust load due to pressure shall not be included as part of the axial force, F_L.

$$F_L = 10000\ lbs$$

$$M_L = 2.0E+06\ in-lbs$$

e) STEP 5 – Compute the stresses in the cylinder, knuckle and cone at the junction using the equations in Table 4.3.7.

Determine if the knuckle is considered to be compact or non-compact.

$$\alpha r_k < 2K_m\left(\left\{R_k\left(\alpha^{-1}\tan\left[\alpha\right]\right)^{0.5} + r_k\right\}t_k\right)^{0.5}$$

$$\left\{0.5236\left(10.0\right)\right\} = \left\{2\left(0.7\right)\left(\left\{50.0\left(\left(0.5236\right)^{-1}\tan\left[0.5236\right]\right)^{0.5} + 10\right\}1\right)^{0.5}\right\}$$

$$\left\{5.2360\ in\right\} < \left\{11.0683\ in\right\} \qquad True$$

Where,

$$K_m = 0.7$$

$$\alpha = \frac{30.0}{180}\pi = 0.5236\ rad$$

$$R_k = R_L - r_k = 60.0 - 10.0 = 50.0\ in$$

Therefore, analyze the knuckle junction as a compact knuckle.

Stress Calculations: Determine the hoop and axial membrane stresses at the knuckle:

$$\sigma_{\theta m} = \frac{PK_m\left(R_L\sqrt{R_L t_L} + L_k\sqrt{L_k t_C}\right) + \alpha\left(PL_{1k}r_k - 0.5P_e L_{1k}^2\right)}{K_m\left(t_L\sqrt{R_L t_L} + t_C\sqrt{L_k t_C}\right)\alpha t_k r_k}$$

$$\sigma_{sm} = \frac{P_e L_{1k}}{2t_k}$$

Where,

$$L_{1k} = R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5} + r_k = 50.0\left(\left(0.5236\right)^{-1}\tan[0.5236]\right)^{0.5} + 10.0 = 62.5038\ in$$

$$L_k = \frac{R_k}{\cos[\alpha]} + r_k = \frac{50.0}{\cos[0.5236]} + 10.0 = 67.7351\ in$$

$$P_e = P + \frac{F_L}{\pi L_{1k}^2\cos^2\left[\frac{\alpha}{2}\right]} \pm \frac{2M_L}{\pi L_{1k}^3\cos^3\left[\frac{\alpha}{2}\right]}$$

$$P_e = \begin{Bmatrix} 280 + \dfrac{-10000.0}{\pi\left(62.5038\right)^2\cos^2\left[\dfrac{0.5236}{2}\right]} + \dfrac{2\left(2.0E+06\right)}{\pi\left(62.5038\right)^3\cos^3\left[\dfrac{0.5236}{2}\right]} \\ 280 + \dfrac{-10000.0}{\pi\left(62.5038\right)^2\cos^2\left[\dfrac{0.5236}{2}\right]} - \dfrac{2\left(2.0E+06\right)}{\pi\left(62.5038\right)^3\cos^3\left[\dfrac{0.5236}{2}\right]} \end{Bmatrix}$$

$$P_e = \begin{Bmatrix} 284.9125\ psi \\ 273.3410\ psi \end{Bmatrix}$$

Therefore,

$$\sigma_{\theta m}=\begin{cases}\dfrac{\begin{pmatrix}280(0.7)\left(60.0\sqrt{60.0(1.0)}+67.7351\sqrt{67.7351(1.0)}\right)+\\0.5236\left(280(62.5038)(10.0)-0.5(284.9125)(62.5038)^{2}\right)\end{pmatrix}}{0.7\left(1.0\sqrt{60.0(1.0)}+1.0\sqrt{67.7351(1.0)}\right)+0.5236(1.0)(10.0)}=35.8767\ psi\\[2ex]\dfrac{\begin{pmatrix}280(0.7)\left(60.0\sqrt{60.0(1.0)}+67.7351\sqrt{67.7351(1.0)}\right)+\\0.5236\left(280(62.5038)(10.0)-0.5(273.3410)(62.5038)^{2}\right)\end{pmatrix}}{0.7\left(1.0\sqrt{60.0(1.0)}+1.0\sqrt{67.7351(1.0)}\right)+0.5236(1.0)(10.0)}=756.6825\ psi\end{cases}$$

And,

$$\sigma_{sm}=\begin{cases}\dfrac{284.9125(62.5038)}{2(1.0)}=8904.0570\ psi\\[2ex]\dfrac{273.3410(62.5038)}{2(1.0)}=8542.4256\ psi\end{cases}$$

Check Acceptable Criteria:

$$\begin{Bmatrix}\sigma_{\theta m}=35.9\ psi\\ \sigma_{\theta m}=756.7\ psi\end{Bmatrix}\le\{S=22400\ psi\}\qquad True$$

$$\begin{Bmatrix}\sigma_{sm}=8904.1\ psi\\ \sigma_{sm}=8542.4\ psi\end{Bmatrix}\le\{S=22400\ psi\}\qquad True$$

Since the longitudinal membrane stress, σ_{sm} and the circumferential membranes stress, $\sigma_{\theta m}$ in the knuckle are both tensile, the condition of local buckling need not be considered. Therefore, the knuckle at the cylinder-to-cone junction at the large end is adequately designed.

f) STEP 6 – The stress acceptance criterion in STEP 5 is satisfied for the knuckle. Therefore, the design is complete.

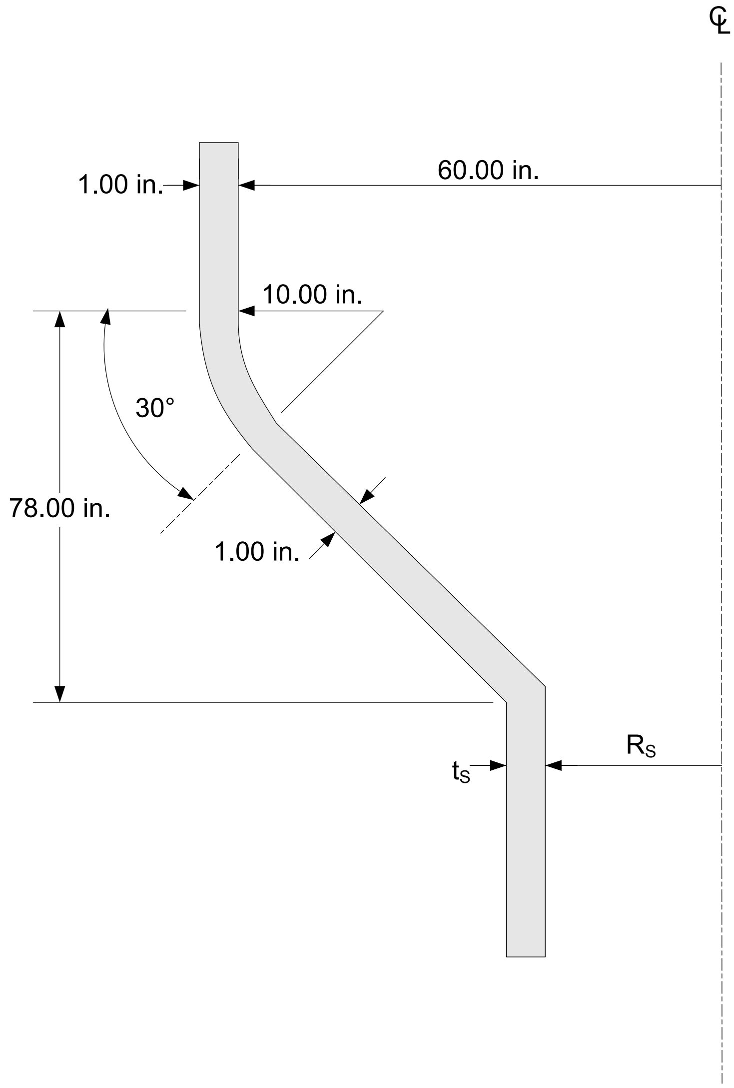


Figure E4.3.8 - Knuckle Detail

4.4 Shells Under External Pressure and Allowable Compressive Stresses

4.4.1 Example E4.4.1 – Cylindrical Shell

Determine the maximum allowable external pressure (MAEP) for a cylindrical shell considering the following design conditions.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Temperature = $300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Unsupported Length = $636.0\ in$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$

Evaluate per paragraph 4.4.5.

a) STEP 1 – Assume an initial thickness, t, and unsupported length, L.

$$t = t - Corrosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$
$$L = 636.0\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{1.6C_h E_y t}{D_o} = \frac{1.6(0.0092)(28.3\text{E+}06)(1.0)}{92.25} = 4515.7290\ psi$$

Where,

$$D_o = D + 2(Uncorroded\ Thickness) = 90.0 + 2(1.125) = 92.25\ in$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{\left(\frac{92.25}{2}\right)1.0}} = 93.6459$$

$$2\left(\frac{D_o}{t}\right)^{0.94} = 2\left(\frac{92.25}{1.0}\right)^{0.94} = 140.6366$$

Since $13 < M_x < 2\left(\frac{D_o}{t}\right)^{0.94}$, calculate C_h as follows:

$$C_h = 1.12M_x^{-1.058} = 1.12(93.6459)^{-1.058} = 0.0092$$

c) STEP 3 – Calculate the predicted buckling stress, F_{ic}.

$$\frac{F_{he}}{S_y} = \frac{4515.7290}{33600.0} = 0.1344$$

Since $\frac{F_{he}}{S_y} \leq 0.552$, calculate F_{ic} as follows:

$$F_{ic} = F_{he} = 4515.7290 \ psi$$

d) STEP 4 – Calculate the value of design factor, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0 \ psi$$

Since $F_{ic} \leq 0.55S_y$, calculate FS as follows:

$$FS = 2.0$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{D_o}\right) = 2(2257.8645)\left(\frac{1.0}{92.25}\right) = 48.9 \ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{4515.7290}{2.0} = 2257.8645 \ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness or reduce the unsupported length of the shell (i.e. by the addition of a stiffening rings) and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The allowable external pressure is $P_a = 48.9 \ psi$

Combined Loadings – cylindrical shells subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the cylindrical shell is only subject to external pressure.

4.4.2 Example E4.4.2 – Conical Shell

Determine the maximum allowable external pressure (MAEP) for a conical shell considering the following design conditions.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Temperature = $300°F$
- Inside Diameter (Large End) = $150.0\ in$
- Thickness (Large End) = $1.8125\ in$
- Inside Diameter (Small End) = $90.0\ in$
- Thickness (Small End) = $1.125\ in$
- Thickness (Conical Section) = $1.9375\ in$
- Axial Cone Length = $78.0\ in$
- One-Half Apex Angle = $21.0375\ deg$
- Corrosion Allowance = $0.125\ in$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$

Evaluate per paragraph 4.4.6. and 4.4.5.

The required thickness of a conical shell subjected to external pressure loading shall be determined using the equations for a cylinder by making the following substitutions:

a) The value of t_c is substituted for t in the equations in paragraph 4.4.5.

$$t_c = t = 1.9375 - Corrosion\ Allowance = 1.9375 - 0.125 = 1.8125\ in$$

b) For offset cones, the cone angle, α, shall satisfy the requirements of paragraph 4.3.4.

The conical shell in this example problem is not of the offset type. Therefore, no additional requirements are necessary.

c) The value of $0.5(D_L + D_S)/\cos[\alpha]$ is substituted for D_o in the equations in paragraph 4.4.5, (concentric cone design with common center line per Figure 4.4.7 Sketch (a)).

$$D_o = \frac{0.5(D_L + D_s)}{\cos[\alpha]} = \frac{0.5\left[\left(150.0 + 2(1.8125)\right) + \left(90.0 + 2(1.125)\right)\right]}{\cos[21.0375]} = 131.7170\ in$$

d) The value of $L_{ce}/\cos[\alpha]$ is substituted for L in the equations in paragraph 4.4.5 where L_{ce} is determined as shown below. For Sketches (a) and (e) in Figure 4.4.7:

$$L_{ce} = L_c$$

$$L = \frac{L_{ce}}{\cos[\alpha]} = \frac{78.0}{\cos[21.0375]} = 83.5703\ in$$

e) Note that the half-apex angle of a conical transition can be computed knowing the shell geometry with the following equations. These equations were developed with the assumption that the conical transition contains a cone section, knuckle, or flare. If the transition does not contain a knuckle or flare, the radii of these components should be set to zero when computing the half-apex angle (see Figure 4.4.7).

$$If\ (R_L - r_k) \geq (R_S + r_f):$$

$$\alpha = \beta + \phi = 0.3672 - 0 = 0.3672\ rad = 21.0375\ deg$$

$$\beta = \arctan\left[\frac{(R_L - r_k) - (R_S + r_f)}{L_c}\right] = \arctan\left[\frac{(75.0-0)-(45.0+0)}{78.0}\right] = 0.3672\ rad$$

$$\phi = \arcsin\left[\frac{(r_f + r_k)\cos[\beta]}{L_c}\right] = \arcsin\left[\frac{(0.0+0.0)\cos[0.3672]}{78.0}\right] = 0.0\ rad$$

Proceed with the design following the steps outlined in paragraph 4.4.5.

a) STEP 1 – Assume an initial thickness, t, and unsupported length, L (see Figures 4.4.1 and 4.4.2).

$$t = 1.8125\ in$$

$$L = 83.5703\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = \frac{1.6C_h E_y t}{D_o} = \frac{1.6(0.1301)(28.3E+06)(1.8125)}{131.7170} = 81062.4824\ psi$$

Where,

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{83.5703}{\sqrt{\left(\frac{131.7170}{2.0}\right)1.8125}} = 7.6490$$

$$2\left(\frac{D_o}{t}\right)^{0.94} = 2\left(\frac{131.7170}{1.8125}\right)^{0.94} = 112.3859$$

Since $1.5 < M_x < 13$, calculate C_h as follows:

$$C_h = \frac{0.92}{M_x - 0.579} = \frac{0.92}{7.6490 - 0.579} = 0.1301$$

c) STEP 3 – Calculate the predicted buckling stress, F_{ic}.

$$\frac{F_{he}}{S_y} = \frac{81062.4824}{33600.0} = 2.4126$$

Since $0.552 < \left(\frac{F_{he}}{S_y}\right) < 2.439$, calculate F_{ic} as follows:

$$F_{ic} = 0.7S_y\left(\frac{F_{he}}{S_y}\right)^{0.4} = 0.7(33600.0)\left(\frac{81062.4824}{33600.0}\right)^{0.4} = 33452.5760\ psi$$

d) STEP 4 – Calculate the value of design factor, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0\ psi$$

Since $0.55S_y < F_{ic} < S_y$, calculate FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{33452.5760}{33600.0}\right) = 1.6693$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{D_o}\right) = 2(20039.8826)\left(\frac{1.8125}{131.7170}\right) = 551.5\ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{33452.5760}{1.6693} = 20039.8826\ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness or reduce the unsupported length of the shell (i.e. by the addition of a stiffening rings) and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 551.5\ psi$

Combined Loadings – conical shells subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the conical shell is only subject to external pressure.

4.4.3 Example E4.4.3 – Spherical Shell and Hemispherical Head

Determine the maximum allowable external pressure (MAEP) for a hemispherical head considering the following design conditions.

Vessel Data:

- Material = $SA-542,\ Type\ D,\ Class\ 4a$
- Design Temperature = $350°F$
- Inside Diameter = $149.0\ in$
- Thickness = $2.8125\ in$
- Corrosion Allowance = $0.0\ in$
- Modulus of Elasticity at Design Temperature = $29.1E+06\ psi$
- Yield Strength = $58000\ psi$

Evaluate per paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the spherical shell.

$$t = 2.8125\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075E_y\left(\frac{t}{R_o}\right) = 0.075\left(29.1E+06\right)\left(\frac{2.8125}{\frac{149.0}{2}+2.8125}\right) = 79395.7154\ psi$$

c) STEP 3 – Calculate the predicted buckling stress, F_{ic}.

$$\frac{F_{he}}{S_y} = \frac{79395.7154}{58000.0} = 1.3689$$

Since $0.55 < \left(\frac{F_{he}}{S_y}\right) < 1.6$, calculate F_{ic} as follows:

$$F_{ic} = 0.18F_{he} + 0.45S_y = 0.18\left(79395.7154\right) + 0.45\left(58000.0\right) = 40391.2288\ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55\left(58000.0\right) = 31900.0\ psi$$

Since $0.55S_y < F_{ic} < S_y$, calculate the FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{40391.2288}{58000.0}\right) = 1.8910$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(21359.7191)\left(\frac{2.8125}{\frac{149.0}{2}+2.8125}\right) = 1554.1\ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{40391.2288}{1.8910} = 21359.7191\ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 1554.1\ psi$

Combined Loadings – spherical shells and hemispherical heads subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the hemispherical head is only subject to external pressure.

4.4.4 Example E4.4.4 – Torispherical Head

Determine the maximum allowable external pressure (MAEP) for a torispherical head considering the following design conditions.

<u>Vessel Data:</u>

- Material = $SA-387,\ Grade\ 11,\ Class\ 1$
- Design Temperature = $650°F$
- Inside Diameter = $72.0\ in$
- Crown Radius = $72.0\ in$
- Knuckle Radius = $4.375\ in$
- Thickness = $0.625\ in$
- Corrosion Allowance = $0.125\ in$
- Modulus of Elasticity at Design Temperature = $26.55E+06\ psi$
- Yield Strength at Design Temperature = $26900\ psi$

Evaluate per paragraph 4.4.8 and 4.4.7.

The required thickness of a torispherical head subjected to external pressure loading shall be determined using the equations for a spherical shell in paragraph 4.4.7 by substituting the outside crown radius for R_o.

$$R_o = 72.0 + 0.625 = 72.625\ in$$

Restrictions on Torispherical Head Geometry – the restriction of paragraph 4.3.6 shall apply. See paragraph 4.3.6.1.b, STEP 2 of E4.3.4.

Torispherical heads With Different Dome and Knuckle Thickness – heads with this configuration shall be designed in accordance with Part 5. In this example problem, the dome and knuckle thickness are the same.

Proceed with the design following the steps outlined in paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the torispherical head.

$$t = 0.625 - Corrosion\ Allowance = 0.625 - 0.125 = 0.500\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075 E_y \left(\frac{t}{R_o}\right) = 0.075\left(26.55E+06\right)\left(\frac{0.500}{72.625}\right) = 13709.1222\ psi$$

c) STEP 3 – Calculate the predicted buckling stress, F_{ic}.

$$\frac{F_{he}}{S_y} = \frac{13709.1222}{26900.0} = 0.5096$$

Since $\frac{F_{he}}{S_y} \le 0.55$, calculate F_{ic} as follows:

$$F_{ic} = F_{he} = 13709.1222\ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per paragraph 4.4.2.

$$0.55 S_y = 0.55\left(26900.0\right) = 14795.0\ psi$$

Since $F_{ic} \le 0.55 S_y$, calculate the FS as follows:

$$FS = 2.0$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2\left(6854.5611\right)\left(\frac{0.500}{72.625}\right) = 94.4\ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{13709.1222}{2.0} = 6854.5611\ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 94.4\ psi$

Combined Loadings – torispherical heads subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the torispherical head is only subject to external pressure.

4.4.5 Example E4.4.5 – Elliptical Head

Determine the maximum allowable external pressure (MAEP) for a 2:1 elliptical head considering the following design conditions.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Temperature = $300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$

Evaluate per paragraph 4.4.9 and 4.4.7.

The required thickness of an elliptical head subjected to external pressure loading shall be determined using the equations for a spherical shell in paragraph 4.4.7 by substituting $K_o D_o$ for R_o where K_o is given by the following equation.

$$K_o = 0.25346 + 0.13995\left(\frac{D_o}{2h_o}\right) + 0.12238\left(\frac{D_o}{2h_o}\right)^2 - 0.015297\left(\frac{D_o}{2h_o}\right)^3$$

$$K_o = \left(\begin{array}{l} 0.25346 + 0.13995\left(\dfrac{92.25}{2(23.0625)}\right) + 0.12238\left(\dfrac{92.25}{2(23.0625)}\right)^2 - \\ 0.015297\left(\dfrac{92.25}{2(23.0625)}\right)^3 \end{array}\right) = 0.9005$$

$$D_o = 90.0 + 2(1.125) = 92.25$$

$$h_o = \left(\frac{D_o}{4}\right) = \frac{92.25}{4} = 23.0625\ in$$

Therefore,

$$R_o = K_o D_o = 0.9005(92.25) = 83.0711\ in$$

Proceed with the design following the steps outlined in paragraph 4.4.7.

a) STEP 1 – Assume an initial thickness, t for the elliptical head.

$$t = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$

b) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075E_y\left(\frac{t}{R_o}\right) = 0.075(28.3E+06)\left(\frac{1.0}{83.0711}\right) = 25550.4020\ psi$$

c) STEP 3 – Calculate the predicted buckling stress, F_{ic}.

$$\frac{F_{he}}{S_y} = \frac{25550.4020}{33600} = 0.7604$$

Since $0.55 < \left(\frac{F_{he}}{S_y}\right) \le 1.6$, calculate F_{ic} as follows:

$$F_{ic} = 0.18F_{he} + 0.45S_y$$
$$F_{ic} = 0.18(25550.4020) + 0.45(33600.0) = 19719.0724\ psi$$

d) STEP 4 – Calculate the value of design margin, FS, per paragraph 4.4.2.

$$0.55S_y = 0.55(33600.0) = 18480.0\ psi$$

Since $0.55S_y < F_{ic} < S_y$, calculate the FS as follows:

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{19719.0724}{33600.0}\right) = 1.9721$$

e) STEP 5 – Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(9999.0226)\left(\frac{1.0}{83.0711}\right) = 240.7\ psi$$

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{19719.0724}{1.9721} = 9999.0226\ psi$$

f) STEP 6 – If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2. Repeat this process until the allowable external pressure is equal to or greater than the design external pressure.

The maximum allowable external pressure $P_a = 240.7\ psi$

Combined Loadings – ellipsoidal heads subject to external pressure and other loadings shall satisfy the requirements of paragraph 4.4.12. In this example problem, the ellipsoidal head is only subject to external pressure.

4.4.6 Example E4.4.6 – Combined Loadings and Allowable Compressive Stresses

Determine the allowable compressive stresses of the proposed cylindrical shell section considering the following design conditions and specified applied loadings.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $-14.7\ psig\ @\ 300°F$
- Inside Diameter = $90.0\ in$
- Thickness = $1.125\ in$
- Corrosion Allowance = $0.125\ in$
- Unsupported Length = $636.0\ in$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength = $33600\ psi$
- Axial Force = $-66152.5\ lbs$
- Net Section Bending Moment = $3.048E+06\ in-lbs$
- Shear Force = $11257.6\ lbs$

Adjust variables for corrosion and determine outside dimensions.

$$D_o = 90.0 + 2(Uncorroded\ Thickness) = 90.0 + 2(1.125) = 92.25\ in$$

$$R_o = 0.5D_o = 0.5(92.25) = 46.125\ in$$

$$D_i = 90.0 + 2(Corrosion\ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$t = 1.125 - Corrosion\ Allowance = 1.125 - 0.125 = 1.0\ in$$

Evaluate per paragraph 4.4.12.2

The loads transmitted to the cylindrical shell are given in the Table E4.4.6.2. Note that this table is given in terms of the load parameters shown in Part 4, Table 4.1.1 (Table E4.4.6.1 of this example). As shown in Table E4.4.6.1, the acceptance criteria is that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition

Paragraph 4.4.12.2, the following procedure shall be used to determine the allowable compressive stresses for cylindrical shells that are based on loading conditions. By inspection of the results shown in Table E4.4.6.3, Load Case 5 is determined to be the governing load case. The pressure and net section axial force, bending moment, and radial shear force at the location of interest for Load Case 5 are:

$$0.9P + P_s = -14.7\ psi \qquad (Conservatively)$$

$$F_5 = -66152.5\ lbs$$

$$M_5 = 1828800\ in-lbs$$

$$V_5 = 6754.6\ lbs$$

Common parameters used in each of the loading conditions are given in paragraph 4.4.12.2.k.

Per paragraph 4.4.12.2.k:

$$A = \frac{\pi\left(D_o^{\ 2} - D_i^2\right)}{4} = \frac{\pi\left(92.25^2 - 90.25^2\right)}{4} = 286.6703\ in^2$$

$$S = \frac{\pi\left(D_o^{\ 4} - D_i^4\right)}{32D_o} = \frac{\pi\left(92.25^4 - 90.25^4\right)}{32(92.25)} = 6469.5531\ in^3$$

$$f_h = \frac{PD_o}{2t} = \frac{14.7(92.25)}{2(1.0)} = 678.0375\ psi$$

$$f_b = \frac{M}{S} = \frac{1828800}{6469.5531} = 282.6779\ psi$$

$$f_a = \frac{F}{A} = \frac{66152.5}{286.6703} = 230.7616\ psi$$

$$f_q = \frac{P\pi D_i^2}{4A} = \frac{14.7(\pi)(90.25)^2}{4(286.6703)} = 328.0341\ psi$$

$$f_v = \frac{V\sin[\phi]}{A} = \frac{6754.6\cdot\sin[90.0]}{286.6703} = 23.5623\ psi$$

Note: ϕ is defined as the angle measured around the circumference from the direction of the applied shear force to the point under consideration. For this example problem, $\phi = 90^\circ$ to maximize the shear force.

$$r_g = 0.25\sqrt{D_o^{\ 2} + D_i^2} = 0.25\sqrt{\left(92.25^2 + 90.25^2\right)} = 32.2637\ in$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{(46.125)1.0}} = 93.6459$$

The value of the slenderness factor for column buckling, λ_c is calculated in paragraph 4.4.12.2.b.

Per paragraph 4.4.12.2:

a) External Pressure Acting Alone – the allowable hoop compressive membrane stress of a cylinder subject to external pressure acting alone, F_{ha}, is computed using the equations in paragraph 4.4.5.

From Example E4.4.1,

$$F_{ha} = 2257.8645\ psi$$

b) Axial Compressive Stress Acting Alone – the allowable axial compressive membrane stress of a cylinder subject to an axial compressive load acting alone, F_{xa}, is computed using the following equations:

The value of the slenderness factor for column buckling, λ_c is dependent on the calculated value of F_{xa}, defined as the allowable compressive membrane stress of a cylinder due to an axial compressive load, with $\lambda_c \le 0.15$. The value of λ_c determines the procedure to be used in obtaining the allowable axial compressive stress, either due to local buckling, $\lambda_c \le 0.15$, or column buckling, $\lambda_c > 0.15$. Therefore, an initial calculation is required to determine the value of F_{xa} with an assumed value of $\lambda_c \le 0.15$. The actual value of λ_c is then calculated and the procedure to obtain the allowable axial compressive stress is determined.

The design factor FS used in paragraph 4.4.12.2.b is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in paragraph 4.4.2. An initial calculation is required to determine the value of F_{xa} by setting $FS = 1.0$, with $F_{ic} = F_{xa}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.b.

1) For $\lambda_c \le 0.15$, (Local Buckling)

$$F_{xa} = \min\left[F_{xa1},\ F_{xa2}\right]$$

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

Since $\frac{D_o}{t} \le 135$, calculate F_{xa1} as follows with an initial value of $FS = 1.0$.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.0} = 33600\ psi$$

The value of F_{xa2} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{xa2} = \frac{F_{xe}}{FS}$$

$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since $\frac{D_o}{t} \le 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \frac{D_o}{t}\right)},\ 0.9\right]$$

Since $M_x \ge 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389+\dfrac{92.25}{1.0}},\ 0.9\right] = 0.8499$$

Therefore,

$$F_{xe} = \frac{0.8499(28.3E+06)(1.0)}{92.25} = 260728.1301\ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{260728.1301}{1.0} = 260728.1301\ psi$$

$$F_{xa} = \min[33600,\ 260728] = 33600\ psi$$

With a value of $F_{ic} = F_{xa} = 33600$, in accordance with paragraph 4.4.2, it is determined the value of $FS = 1.667$ since $\{F_{ic} = 33600\} = \{S_y = 33600\}$. Using this computed value of $FS = 1.667$ in paragraph 4.4.12.2.b, F_{xa} is calculated as follows.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.667} = 20155.9688\ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{260728.1301}{1.667} = 156405.5969\ psi$$

$$F_{xa} = \min[20155.9688,\ 156405.5969] = 20155.9688\ psi$$

With F_{xa} calculated, determine the value of λ_c from paragraph 4.4.12.2.k. For a cylinder with end conditions with one end free and the other end fixed, $K_u = 2.1$.

$$\lambda_c = \frac{K_u L_u}{\pi r_g}\left(\frac{F_{xa}\cdot FS}{E_y}\right)^{0.5} = \frac{2.1(636.0)}{\pi(32.2637)}\left(\frac{20155.9688(1.667)}{28.3E+06}\right)^{0.5} = 0.4540$$

Since $\lambda_c > 0.15$, the allowable axial compressive membrane stress of the cylinder is due to Column Buckling, per paragraph 4.4.12.2.b.2.

2) For $\lambda_c > 0.15$ and $\dfrac{K_u L_u}{r_g} < 200$ (Column Buckling)

$$\left\{\frac{K_u L_u}{r_g} = \frac{2.1(636.0)}{32.2637} = 41.3964\right\} < \{200\} \qquad True$$

Since $0.15 < \lambda_c < 1.147$, calculate F_{ca} as follows:

$$F_{ca} = F_{xa}\left[1-0.74(\lambda_c - 0.15)\right]^{0.3}$$

$$F_{ca} = 20155.9688\left[1-0.74(0.4540-0.15)\right]^{0.3} = 18672.4331\ psi$$

c) Compressive Bending Stress – the allowable axial compressive membrane stress of a cylindrical shell subject to a bending moment acting across the full circular cross section, F_{ba}, is computed using the following equations:

Similar to the procedure used in paragraph 4.4.12.2.b, the design factor FS used in paragraph 4.4.12.2.c is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in paragraph 4.4.2. An initial calculation is required to determine the value of F_{ba} by setting $FS = 1.0$, with $F_{ic} = F_{ba}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.c.

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

$$\gamma = \frac{S_y D_o}{E_y t} = \frac{33600(92.25)}{(28.3E+06)(1.0)} = 0.1095$$

Since $\dfrac{D_o}{t} \le 100$ and $\gamma < 0.11$, calculate F_{ba} as follows with an initial value of $FS = 1.0$:

$$F_{ba} = \frac{S_y(1.4 - 2.9\gamma)}{FS} = \frac{33600(1.4 - 2.9(0.1095))}{1.0} = 36370.32 \ psi$$

With a value of $F_{ic} = F_{ba} = 36370.32$, in accordance with paragraph 4.4.2, it is determined the value of $FS = 1.667$ since $\{F_{ic} = 36370.32\} \ge \{S_y = 33600\}$. Using this computed value of $FS = 1.667$ in paragraph 4.4.12.2.c, F_{ba} is calculated as follows.

$$F_{ba} = \frac{S_y(1.4 - 2.9\gamma)}{FS} = \frac{33600(1.4 - 2.9(0.1095))}{1.667} = 21817.8284 \ psi$$

d) Shear Stress – the allowable shear stress of a cylindrical shell, F_{va}, is computed using the following equations:

Similar to the procedure used in paragraph 4.4.12.2.b, the design factor FS used in paragraph 4.4.12.2.d is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in paragraph 4.4.2. An initial calculation is required to determine the value of F_{va} by setting $FS = 1.0$, with $F_{ic} = F_{va}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.d.

The value of F_{va} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{va} = \frac{\eta_v F_{ve}}{FS}$$

$$F_{ve} = \alpha_v C_v E_y \left(\frac{t}{D_o} \right)$$

For a value of $M_x = 93.6459$,

$$4.347\left(\frac{D_o}{t}\right) = 4.347\left(\frac{95.25}{1.0}\right) = 401.0108$$

Since $26 < M_x < 4.347\left(\frac{D_o}{t}\right)$, calculate C_v as follows:

$$C_v = \frac{1.492}{M_x^{0.5}} = \frac{1.492}{(93.6459)^{0.5}} = 0.1542$$

Since $\left(\frac{D_o}{t}\right) \le 500$, calculate α_v as follows:

$$\alpha_v = 0.8$$

It follows then,

$$F_{ve} = 0.8(0.1542)(28.3E+06)\left(\frac{1.0}{92.25}\right) = 37843.7724 \ psi$$

$$\frac{F_{ve}}{S_y} = \frac{37843.7724}{33600} = 1.1263$$

Since $0.48 < \left(\frac{F_{ve}}{S_y}\right) < 1.7$, calculate η_v as follows:

$$\eta_v = 0.43\left(\frac{S_y}{F_{ve}}\right) + 0.1 = 0.43\left(\frac{33600}{37843.7724}\right) + 0.1 = 0.4818$$

Therefore,

$$F_{va} = \frac{0.4818(37843.7724)}{1.0} = 18233.1295 \ psi$$

With a value of $F_{ic} = F_{va} = 18233.1295$, in accordance with VIII-2, paragraph 4.4.2, it is determined the value of $FS = 2.0$ since $\{F_{ic} = 18233.1295\} \le \{0.55S_y = 18480\}$. Using this computed value of $FS = 2.0$ in paragraph 4.4.12.2.d, F_{va} is calculated as follows.

$$F_{va} = \frac{0.4818(37843.7724)}{2.0} = 9116.5648 \, psi$$

e) Axial Compressive Stress and Hoop Compression – the allowable compressive stress for the combination of uniform axial compression and hoop compression, F_{xha}, is computed using the following equations:

1) For $\lambda_c \le 0.15$, F_{xha} is computed using the following equation with F_{ha} and F_{xa} evaluated using the equations in paragraphs 4.4.12.2.a and 4.4.12.2.b.1, respectively.

Although, $0.15 < \lambda_c \le 1.2$, the procedure in VIII-2, paragraph 4.4.12.2.e.1 to calculate F_{xha} is required per paragraph 4.4.12.2.e.2 with the modifications noted, (see next step in procedure).

$$F_{xha} = \left[\left(\frac{1}{F_{xa}^2}\right) - \left(\frac{C_1}{C_2 F_{xa} F_{ha}}\right) + \left(\frac{1}{C_2^2 F_{ha}^2}\right)\right]^{-0.5}$$

$$F_{xha} = \left[\begin{array}{l} \left(\dfrac{1}{(20155.9688)^2}\right) - \\ \left(\dfrac{0.1344}{0.8241(20155.9688)(2257.8645)}\right) + \\ \left(\dfrac{1}{(0.8241)^2 (2257.8645)^2}\right) \end{array}\right]^{-0.5} = 1864.3312\ psi$$

Where,

$$C_1 = \frac{(F_{xa} \cdot FS + F_{ha} \cdot FS)}{S_y} - 1.0 = \frac{20155.9688(1.667) + 2257.8645(2.0)}{33600} - 1.0$$

$$C_1 = 0.1344$$

$$C_2 = \frac{f_x}{f_h} = \frac{558.7957}{678.0375} = 0.8241$$

$$f_x = f_a + f_q = 230.7616 + 328.0341 = 558.7957\ psi$$

2) For $0.15 < \lambda_c \le 1.2$, F_{xha}, is computed from the following equation with $F_{ah1} = F_{xha}$ evaluated using the equations in paragraph 4.4.12.2.e.1, and F_{ah2} using the following procedure. The value of F_{ca} used in the calculation for F_{ah2} is evaluated using the equation in VIII-2, paragraph 4.4.12.2.b.2 with $F_{xa} = F_{xha}$ as determined in VIII-2, paragraph 4.4.12.2.e.1. As noted, the load on the end of a cylinder due to external pressure does not contribute to column buckling and therefore F_{ah1} is compared with f_a rather than f_x. The stress due to the pressure load does, however, lower the effective yield stress and the quantity in $(1 - f_q / S_y)$ accounts for this reduction.

$$F_{xha} = \min[F_{ah1}, F_{ah2}] = \min[1864.3312, 1710.2496] = 1710.2496\ psi$$

$$F_{ah1} = F_{xha} = 1864.3312\ psi$$

$$F_{ah2} = F_{ca}\left(1 - \frac{f_q}{S_y}\right) = 1727.1112\left(1 - \frac{328.0341}{33600}\right) = 1710.2496 \ psi$$

Where,

$$F_{ca} = F_{xha}\left[1 - 0.74\left(\lambda_c - 0.15\right)\right]^{0.3}$$

$$F_{ca} = 1864.3312\left[1 - 0.74\left(0.4540 - 0.15\right)\right]^{0.3} = 1727.1112 \ psi$$

3) For $\lambda_c \leq 0.15$, the allowable hoop compressive membrane stress, F_{hxa}, is given by the following equation.

$$F_{hxa} = \frac{F_{xha}}{C_2}$$

Note: this step is not required since $\lambda_c > 0.15$.

f) Compressive Bending Stress and Hoop Compression – the allowable compressive stress for the combination of axial compression due to a bending moment and hoop compression, F_{bha}, is computed using the following equations.

1) An iterative solution procedure is utilized to solve these equations for C_3 with F_{ha} and F_{ba} evaluated using the equations in paragraphs 4.4.12.2.a and 4.4.12.2.c, respectively.

$$F_{bha} = C_3 C_4 F_{ba} = 0.9968\left(0.0431\right)\left(21817.8284\right) = 937.3393 \ psi$$

Where,

$$C_4 = \left(\frac{f_b}{f_h}\right)\left(\frac{F_{ha}}{F_{ba}}\right) = \left(\frac{282.6779}{678.0375}\right)\left(\frac{2257.8645}{21817.8284}\right) = 0.0431$$

$$C_3^2\left(C_4^2 + 0.6C_4\right) + C_3^{2n} - 1 = 0$$

$$n = 5 - \frac{4F_{ha} \cdot FS}{S_y} = 5 - \frac{4\left(2257.8645\right)\left(2.0\right)}{33600} = 4.4624$$

1st attempt at solving for C_3, using an interval halving approach, with an initial guess at C_3 as follows:

$$C_3 = \frac{Upper \ Bound + Lower \ Bound}{2} = \frac{1.0 + 0.0}{2} = 0.5$$

The following results are obtained:

$$0.5^2\left(\left(0.0431\right)^2 + 0.6\left(0.0431\right)\right) + 0.5^{2(4.4624)} - 1 = -0.9910$$

2nd attempt at solving for C_3, with an second guess of C_3 as follows:

$$C_3 = \frac{1.0 + 0.5}{2} = 0.75$$

The following results are obtained:

$$0.75^2\left(\left(0.0431\right)^2+0.6\left(0.0431\right)\right)+0.75^{2(4.4624)}-1=-0.9077$$

Successive iterations are performed at solving for C_3 until the following value is obtained.

$$C_3=0.9968$$

The following results are obtained which satisfy the equation within a tolerance of ± 0.001:

$$0.9968^2\left(\left(0.0431\right)^2+0.6\left(0.0431\right)\right)+0.9968^{2(4.4624)}-1=-0.0007$$

2) The allowable hoop compressive membrane stress, F_{hba}, is given by the following equation.

$$F_{hba}=F_{bha}\left(\frac{f_h}{f_b}\right)=937.3393\left(\frac{678.0375}{282.6779}\right)=2248.3229\ psi$$

g) Shear Stress and Hoop Compression – the allowable compressive stress for the combination of shear, F_{vha}, and hoop compression is computed using the following equations.

Note: This load combination is only applicable for shear stress and hoop compression, in the absence of axial compressive stress and compressive bending stress. It is shown in this example problem for informational purposes only. The effect of shear is accounted for in the interaction equations of paragraphs 4.4.12.2.h and 4.4.12.2.i through the variable K_s.

1) The allowable shear stress is given by the following equation with F_{ha} and F_{va} evaluated using the equations in paragraphs 4.4.12.2.a and 4.4.12.2.d, respectively.

$$F_{vha}=\left[\left(\frac{F_{va}^2}{2C_5F_{ha}}\right)^2+F_{va}^2\right]^{0.5}-\frac{F_{va}^2}{2C_5F_{ha}}$$

$$F_{vha}=\left[\left(\frac{(9116.5648)^2}{2(0.0348)(2257.8645)}\right)^2+(9116.5648)^2\right]^{0.5}-\left[\frac{(9116.5648)^2}{2(0.0348)(2257.8645)}\right]$$

$$F_{vha}=78.5678\,psi$$

Where,

$$C_5=\frac{f_v}{f_h}=\frac{23.5623}{678.0375}=0.0348$$

2) The allowable hoop compressive membrane stress, F_{hva}, is given by the following equation.

$$F_{hva}=\frac{F_{vha}}{C_5}=\frac{78.5678}{0.0348}=2257.6954\ psi$$

h) Axial Compressive Stress, Compressive Bending Stress, Shear Stress, and Hoop Compression – the allowable compressive stress for the combination of uniform axial compression, axial compression due to a bending moment, and shear in the presence of hoop compression is computed using the following interaction equations.

1) The shear coefficient is determined using the following equation with F_{va} from paragraph 4.4.12.2.d.

$$K_s = 1.0 - \left(\frac{f_v}{F_{va}}\right)^2 = 1.0 - \left(\frac{23.5623}{9116.5648}\right)^2 = 0.9999$$

2) For $\lambda_c \le 0.15$ the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{xha} and F_{bha} evaluated using the equations in paragraphs 4.4.12.2.e.1 and 4.4.12.2.f.1, respectively.

$$\left(\frac{f_a}{K_s F_{xha}}\right)^{1.7} + \left(\frac{f_b}{K_s F_{bha}}\right) \le 1.0$$

Note: this step is not required since $\lambda_c > 0.15$.

3) For $0.15 < \lambda_c \le 1.2$ the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{xha} and F_{bha} evaluated using the equations in paragraphs 4.4.12.2.e.2 and 4.4.12.2.f.1, respectively.

$$\frac{f_a}{K_s F_{xha}} = \frac{230.7616}{0.9999(1710.2496)} = 0.1349$$

Since $\dfrac{f_a}{K_s F_{xha}} < 0.2$, the following equation shall be used:

$$\left(\frac{f_a}{2K_s F_{xha}}\right) + \left(\frac{\Delta f_b}{K_s F_{bha}}\right) \le 1.0$$

$$\left\{\left(\frac{230.7616}{2(0.9999)(1710.2496)}\right) + \left(\frac{1.0024(282.6779)}{0.9999(937.3393)}\right) = 0.3698\right\} \le \{1.0\} \qquad True$$

Where,

$$\Delta = \frac{C_m}{1 - \left(\dfrac{f_a \cdot FS}{F_e}\right)} = \frac{1.0}{1 - \left(\dfrac{230.7616(1.667)}{162990.2785}\right)} = 1.0024$$

$$F_e = \frac{\pi^2 E_y}{\left(\dfrac{K_u L_u}{r_g}\right)^2} = \frac{\pi^2 (28.3E+06)}{\left(\dfrac{2.1(636.0)}{32.2637}\right)^2} = 162990.2785\ psi$$

i) Axial Compressive Stress, Compressive Bending Stress, and Shear Stress – the allowable compressive stress for the combination of uniform axial compression, axial compression due to a bending moment, and shear in the absence of hoop compression is computed using the following interaction equations.

1) The shear coefficient is determined using the equation in paragraph 4.4.12.2.h.1 with F_{va} from paragraph 4.4.12.2.d.

$$K_s = 1.0 - \left(\frac{f_v}{F_{va}}\right)^2 = 1.0 - \left(\frac{23.5623}{9116.5648}\right)^2 = 0.9999$$

2) For $\lambda_c \le 0.15$ the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{xa} and F_{ba} evaluated using the equations in paragraphs 4.4.12.2.b.1 and 4.4.12.2.c, respectively.

$$\left(\frac{f_a}{K_s F_{xa}}\right)^{1.7} + \left(\frac{f_b}{K_s F_{ba}}\right) \le 1.0$$

Note: this step is not required since $\lambda_c > 0.15$.

3) For $0.15 < \lambda_c \le 1.2$ the acceptability of a member subject to compressive axial and bending stresses, f_a and f_b, respectively, is determined using the following interaction equation with F_{ca} and F_{ba} evaluated using the equations in paragraphs 4.4.12.2.b.2 and 4.4.12.2.c, respectively. The coefficient Δ is evaluated using the equations in paragraph 4.4.12.2.h.3.

$$\frac{f_a}{K_s F_{ca}} = \frac{230.7616}{0.9999(18672.4331)} = 0.0124$$

Since $\dfrac{f_a}{K_s F_{ca}} < 0.2$, the following equation shall be used:

$$\left(\frac{f_a}{2K_s F_{ca}}\right) + \left(\frac{\Delta f_b}{K_s F_{ba}}\right) \le 1.0$$

$$\left\{\left(\frac{230.7616}{2(0.9999)(18672.4331)}\right) + \left(\frac{1.0024(282.6779)}{0.9999(21817.8284)}\right) = 0.0192\right\} \le \{1.0\} \quad True$$

From paragraph 4.4.12.2.h.3:

$$\Delta = 1.0024$$

$$F_e = 162990.2785 \ psi$$

j) The maximum deviation , e, may exceed the value e_x given in paragraph 4.4.4.2 if the maximum axial stress is less than F_{xa} for shells designed for axial compression only, or less than F_{xha} for shells designed for combinations of axial compression and external pressure. The change in buckling stress, F'_{xe}, is given by Equation (4.4.114). The reduced allowable buckling stress, $F_{xa(reduced)}$, is determined using Equation (4.4.115) where e is the new maximum deviation, F_{xa} is determined using Equation 4.4.61, and FS_{xa} is the value of the stress reduction factor used to determine F_{xa}.

$$F'_{xe} = \left(0.944 - \left|0.286 \log\left[\frac{0.0005e}{e_x}\right]\right|\right)\left(\frac{E_y t}{R}\right)$$

$$F'_{xe} = \left(0.944 - \left|0.286 \log\left[\frac{0.0005(0.2501)}{(0.0913)}\right]\right|\right)\left(\frac{(28.3E+06)(1.0)}{46.125}\right) = 76737.5098 \ psi$$

$$F_{xa(reduced)} = \frac{F_{xa} \cdot FS_{xa} - F'_{xe}}{FS_{xa}}$$

$$F_{xa(reduced)} = \frac{20155.9688(1.667) - (76737.5098)}{1.667} = -25877.3304 \ psi$$

From paragraph 4.4.4.1, assuming the measurements are taken using the outside radius:

$$e = \min[e_c, 2t] = \min\left[0.2501, 2(1.0)\right] = 0.2501 \ in$$

$$e_c = 0.0165t\left(\frac{L_{ec}}{\sqrt{Rt}} + 3.25\right)^{1.069} = 0.0165(1.0)\left(\frac{64.3134}{\sqrt{46.125(1.0)}} + 3.25\right)^{1.069} = 0.2501 \ in$$

$$L_{ec} = 2R\sin\left[\frac{\pi}{2n}\right] = 2(46.125)\sin\left(\frac{\pi}{2(2.0362)}\right) = 64.3134 \ in$$

$$n = \xi\left(\sqrt{\frac{R}{t}} \cdot \left(\frac{R}{L}\right)\right)^{\psi} = (2.80)\left(\sqrt{\frac{46.125}{1.0}}\left(\frac{46.125}{636.0}\right)\right)^{0.4498} = 2.0362$$

$$\xi = \min\left[2.28\left(\frac{R}{t}\right)^{0.54}, 2.80\right] = \min\left[2.28\left(\frac{46.125}{1.0}\right)^{0.54}, 2.80\right]$$

$$\xi = \min\left[18.05, 2.80\right]$$

$$\xi = 2.80$$

$$\psi = \min\left[0.38\left(\frac{R}{t}\right)^{0.044}, 0.485\right] = \min\left[0.38\left(\frac{46.125}{1.0}\right)^{0.044}, 0.485\right]$$

$$\psi = \min\left[0.4498, 0.485\right]$$

$$\psi = 0.4498$$

From paragraph 4.4.4.2:

$$e_x = 0.002R_m = 0.002(45.625) = 0.0913\ in$$

$$R_m = \frac{(D_o + D_i)}{4} = \frac{(92.25 + 90.25)}{4} = 45.625\ in$$

A summary of the allowable compressive stresses are as follows:

Paragraph 4.4.12.2.a, External Pressure Acting Alone

$$F_{ha} = 2257.8645\ psi$$

Paragraph 4.4.12.2.b, Axial Compressive Stress Acting Alone

$$F_{xa} = 20155.9688\ psi$$

$$F_{ca} = 18672.4331\ psi$$

Paragraph 4.4.12.2.c, Compressive Bending Stress

$$F_{ba} = 21817.8284\ psi$$

Paragraph 4.4.12.2.d, Shear Stress

$$F_{va} = 9116.5648\ psi$$

Paragraph 4.4.12.2.e, Axial Compressive Stress and Hoop Compression

$$F_{xha} = 1710.2496\ psi$$

Paragraph 4.4.12.2.f, Compressive Bending Stress and Hoop Compression

$$F_{bha} = 937.3393\ psi$$

$$F_{hba} = 2248.3229\ psi$$

Paragraph 4.4.12.2.g, Shear Stress and Hoop Compression

$$F_{vha} = 78.5678\ psi$$

$$F_{hva} = 2257.6954\ psi$$

Table E4.4.6.1 - Design Loads and Load Combinations from VIII-2

Table 4.1.1 – Design Loads

Design Load Parameter	Description
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g. catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g. skirts, lugs, saddles, and legs), and appurtenances (e.g. platforms, ladders, etc.) • Weight of vessel contents under operating and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping • Transportation Loads (The static forces obtained as equivalent to the dynamic loads experienced during normal operation of a transport vessel – see paragraph 1.2.1.2.b)
L	• Appurtenance Live loading • Effects of fluid flow, steady state or transient • Loads resulting from wave action
E	Earthquake loads (see ASCE 7 for the specific definition of the earthquake load, as applicable)
W	Wind Loads (See 4.1.5.3.b)
S	Snow Loads
F	Loads due to Deflagration

Table 4.1.2 – Design Load Combinations

Design Load Combination (1)	General Primary Membrane Allowable Stress (2)
$P+P_s+D$	S
$P+P_s+D+L$	S
$P+P_s+D+S$	S
$0.9P+P_s+D+0.75L+0.75S$	S
$0.9P+P_s+D+\left(0.6W \; or \; 0.7E\right)$	S
$0.9P+P_S+D+0.75\left(0.6W \; or \; 0.7E\right)+0.75L+0.75S$	S
$0.6D+\left(0.6W \; or \; 0.7E\right)$ (3)	S
P_s+D+F	See Annex 4.D

Notes
1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) S is the allowable stress for the load case combination (see paragraph 4.1.5.3.c)
3) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7-10, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

Table E4.4.6.2 - Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	$P = -14.7$
P_s	Static head from liquid or bulk materials (e.g. catalyst)	$P_s = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -66152.5\ lbs$ $D_M = 0.0\ in-lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = 0.0\ lbs$ $L_M = 0.0\ in-lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 0.0\ in-lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 3.048E+06\ in-lbs$ $W_V = 11257.6\ lbs$
S	Snow Loads	$S_F = 0.0\ lbs$ $S_M = 0.0\ in-lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in-lbs$

Based on these loads, the shell is required to be designed for the load case combinations shown in Table E4.4.6.3. Note that this table is given in terms of the load combinations shown in VIII-2, Table 4.1.2 (Table E4.4.6.1 of this example).

Table E4.4.6.3 - Load Case Combination at the Location of Interest

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P+P_s+D$	$P+P_s=-14.7\ psi$ $F_1=-66152.5\ lbs$ $M_1=0.0\ in-lbs$	S
2	$P+P_s+D+L$	$P+P_s=-14.7\ psi$ $F_2=-66152.5\ lbs$ $M_2=0.0\ in-lbs$	S
3	$P+P_s+D+S$	$P+P_s=-14.7\ psi$ $F_3=-66152.5\ lbs$ $M_3=0.0\ in-lbs$	S
4	$0.9P+P_s+D+0.75L+0.75S$	$0.9P+P_s=-13.2\ psi$ $F_4=-66152.5\ lbs$ $M_4=0.0\ in-lbs$	S
5	$0.9P+P_s+D+(0.6W\ or\ 0.7E)$	$0.9P+P_s=-13.4\ psi$ $F_5=-66152.5\ lbs$ $M_5=1828800\ in-lbs$ $V_5=6754.6\ lbs$	S
6	$\left(\begin{array}{l}0.9P+P_S+D+0.75(0.6W\ or\ 0.7E)+\\0.75L+0.75S\end{array}\right)$	$0.9P+P_s=320.4\ psi$ $F_6=-66152.5\ lbs$ $M_6=1371600\ in-lbs$ $V_6=5065.9\ lbs$	S
7	$0.6D+(0.6W\ or\ 0.7E)$ Anchorage is included in the design. Therefore, consideration of this load combination is not required.	$F_7=-39691.5\ lbs$ $M_7=1828800\ in-lbs$ $V_7=6754.6\ lbs$	S
8	P_s+D+F	$P_s=0.0\ psi$ $F_8=-66152.5\ lbs$ $M_8=0.0\ in-lbs$	See Annex 4.D

4.4.7 Example E4.4.7 – Conical Transitions without a Knuckle

Determine if the proposed large and small end cylinder-to-cone transitions are adequately designed considering the following design conditions and applied forces and moments.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $-14.7\ psig @ 300°F$
- Inside Radius (Large End) = $75.0\ in$
- Thickness (Large End) = $1.8125\ in$
- Inside Radius (Small End) = $45.0\ in$
- Thickness (Small End) = $1.125\ in$
- Thickness (Conical Section) = $1.9375\ in$
- Length of Conical Section = $78.0\ in$
- Unsupported Length of Large Cylinder = $732.0\ in$
- Unsupported Length of Small Cylinder = $636.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Yield Strength = $33600\ psi$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle (See E4.3.2) = $21.0375\ deg$
- Axial Force (Large End) = $-99167\ lbs$
- Net Section Bending Moment (Large End) = $5.406E+06\ in-lbs$
- Axial Force (Small End) = $-78104\ lbs$
- Net Section Bending Moment (Small End) = $4.301E+06\ in-lbs$

Adjust variables for corrosion.

$$R_L = 75.0 + Corrosion\ \ Allowance = 75.0 + 0.125 = 75.125\ in$$
$$R_S = 45.0 + Corrosion\ \ Allowance = 45.0 + 0.125 = 45.125\ in$$
$$t_L = 1.8125 - Corrosion\ \ Allowance = 1.8125 - 0.125 = 1.6875\ in$$
$$t_S = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$
$$t_C = 1.9375 - Corrosion\ \ Allowance = 1.9375 - 0.125 = 1.8125\ in$$

Evaluate per paragraphs 4.4.13 and 4.3.11.

The design rules in paragraph 4.3.11 shall be satisfied. In these calculations, a negative value of pressure shall be used in all applicable equations.

Proceed with the design following the steps outlined in paragraph 4.3.11.3.

The length of the conical shell, measured parallel to the surface of the cone, shall be equal to or greater than the following:

$$L_C \geq 2.0\sqrt{\frac{R_L t_C}{\cos[\alpha]}} + 1.4\sqrt{\frac{R_S t_C}{\cos[\alpha]}}$$

$$2.0\sqrt{\frac{75.125(1.8125)}{\cos[21.0375]}} + 1.4\sqrt{\frac{45.125(1.8125)}{\cos[21.0375]}} = 37.2624\ in$$

$$L_C = 78.0 \geq 37.2624 \qquad True$$

Evaluate the Large End cylinder-to-cone junction per paragraph 4.3.11.4.

a) STEP 1 – Compute the large end cylinder thickness, t_L, using paragraph 4.3.3., (as specified in design conditions).

$$t_L = 1.6875\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the large end using paragraph 4.3.4., (as specified in design conditions).

$$\alpha = 21.0375\ deg$$

$$t_C = 1.8125\ in$$

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all of these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with Part 5. In the calculations, if $0\ deg < \alpha \leq 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \leq \left\{\frac{R_L}{t_L} = \frac{75.125}{1.6875} = 44.5185\right\} \leq 500 \qquad True$$

$$1 \leq \left\{\frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741\right\} \leq 2 \qquad True$$

$$\{\alpha = 21.0375\ deg\} \leq 60\ deg \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force, F_L. Calculate the equivalent line load, X_L, using the specified net section axial force, F_L, and bending moment, M_L.

$$X_L = \frac{F_L}{2\pi R_L} \pm \frac{M_L}{\pi R_L^2} = \left\{\begin{array}{l} \dfrac{-99167}{2\pi(75.125)} + \dfrac{5.406E+06}{\pi(75.125)^2} = 94.8111\ \dfrac{lbs}{in} \\ \dfrac{-99167}{2\pi(75.125)} - \dfrac{5.406E+06}{\pi(75.125)^2} = -514.9886\ \dfrac{lbs}{in} \end{array}\right\}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per Table 4.3.3 and Table 4.3.4, respectively. For calculated values of n other than those presented in Table 4.3.3 and Table 4.3.4, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_L} = \frac{1.8125}{1.6875} = 1.0741$$

$$H = \sqrt{\frac{R_L}{t_L}} = \sqrt{\frac{75.125}{1.6875}} = 6.6722$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in Table 4.3.3 and Table 4.3.4 is required. The results of the interpolation are summarized with the following values for C_i (see paragraph 4.3.11.4, STEP 5 of E4.3.7).

For the applied pressure case both M_{sN} and Q_N are calculated using the following equation

$$M_{sN}, Q_N = -\exp\left[\begin{array}{l} C_1 + C_2\ln[H] + C_3\ln[B] + C_4\left(\ln[H]\right)^2 + C_5\left(\ln[B]\right)^2 + \\ C_6\ln[H]\ln[B] + C_7\left(\ln[H]\right)^3 + C_8\left(\ln[B]\right)^3 + \\ C_9\ln[H]\left(\ln[B]\right)^2 + C_{10}\left(\ln[H]\right)^2\ln[B] \end{array}\right]$$

This results in the following (see paragraph 4.3.11.4, STEP 5 of E4.3.7):

$$M_{sN} = -10.6148$$

$$Q_N = -4.0925$$

For the Equivalent Line Load case, M_{sN} and Q_N are calculated using the following equation.

$$M_{sN}, Q_N = -\exp\left[\frac{\left(\begin{array}{l} C_1 + C_3\ln\left[H^2\right] + C_5\ln[\alpha] + C_7\left(\ln\left[H^2\right]\right)^2 + \\ C_9\left(\ln[\alpha]\right)^2 + C_{11}\ln\left[H^2\right]\ln[\alpha] \end{array}\right)}{\left(\begin{array}{l} 1 + C_2\ln\left[H^2\right] + C_4\ln[\alpha] + C_6\left(\ln\left[H^2\right]\right)^2 + \\ C_8\left(\ln[\alpha]\right)^2 + C_{10}\ln\left[H^2\right]\ln[\alpha] \end{array}\right)}\right]$$

This results in the following (see paragraph 4.3.11.4, STEP 5 of E4.3.7):

$$M_{sN} = -0.4912$$

$$Q_N = -0.1845$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$Internal\ Pressure: \qquad M_{sN} = -10.6148, \qquad Q_N = -4.0925$$
$$Equivalent\ Line\ Load: \qquad M_{sN} = -0.4912, \qquad Q_N = -0.1845$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in Table 4.3.1 for the Large End Junction.

Evaluate the Cylinder at the Large End:

Stress Resultant Calculations:

$$M_{sP} = Pt_L^2 M_{sN} = -14.7(1.6875)^2(-10.6148) = 444.3413\ \frac{in-lb}{in}$$

$$M_{sX} = X_L t_L M_{sN} = \begin{Bmatrix} 94.8111(1.6875)(-0.4912) = -78.5889\ \frac{in-lbs}{in} \\ -514.9886(1.6875)(-0.4912) = 426.8741\ \frac{in-lbs}{in} \end{Bmatrix}$$

$$M_s = M_{sP} + M_{sX} = \begin{Bmatrix} 444.3413 + (-78.5889) = 365.7524\ \frac{in-lbs}{in} \\ 444.3413 + 426.8741 = 871.2154\ \frac{in-lbs}{in} \end{Bmatrix}$$

$$Q_P = Pt_L Q_N = -14.7(1.6875)(-4.0925) = 101.5196\ \frac{lb}{in}$$

$$Q_X = X_L Q_N = \begin{Bmatrix} 94.8111(-0.1845) = -17.4926\ \frac{lbs}{in} \\ -514.9886(-0.1845) = 95.0154\ \frac{lbs}{in} \end{Bmatrix}$$

$$Q = Q_P + Q_X = \begin{Bmatrix} 101.5196 + (-17.4926) = 84.0270\ \frac{lbs}{in} \\ 101.5196 + 95.0154 = 196.5350\ \frac{lbs}{in} \end{Bmatrix}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_L^2 t_L^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(75.125)^2(1.6875)^2}\right]^{0.25} = 0.1142\ in^{-1}$$

$$N_s = \frac{PR_L}{2} + X_L = \begin{Bmatrix} \frac{-14.7(75.125)}{2} + 94.8111 = -457.3577\ \frac{lbs}{in} \\ \frac{-14.7(75.125)}{2} + (-514.9886) = -1067.1574\ \frac{lbs}{in} \end{Bmatrix}$$

$$N_\theta = PR_L + 2\beta_{cy} R_L \left(-M_s \beta_{cy} + Q\right)$$

$$N_\theta = \begin{cases} -14.7(75.125) + 2(0.1142)(75.125)\left(-(365.7524)(0.1142) + 84.0270\right) \\ -14.7(75.125) + 2(0.1142)(75.125)\left(-(871.2154)(0.1142) + 196.5350\right) \end{cases}$$

$$N_\theta = \begin{cases} -379.2502 \ \dfrac{lbs}{in} \\ 560.7660 \ \dfrac{lbs}{in} \end{cases}$$

$$K_{pc} = 1.0$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_s}{t_L} = \begin{cases} \dfrac{-457.3577}{1.6875} = -271.0268 \ psi \\ \dfrac{-1067.1574}{1.6875} = -632.3895 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_s}{t_L^2 K_{pc}} = \begin{cases} \dfrac{6(365.7524)}{(1.6875)^2(1.0)} = 770.6388 \ psi \\ \dfrac{6(871.2154)}{(1.6875)^2(1.0)} = 1835.6472 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_L} = \begin{cases} \dfrac{-379.2502}{1.6875} = -224.7409 \ psi \\ \dfrac{560.7660}{1.6875} = 332.3058 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_L^2 K_{pc}} = \begin{cases} \dfrac{6(0.3)(365.7524)}{(1.6875)^2(1.0)} = 231.1916 \ psi \\ \dfrac{6(0.3)(871.2154)}{(1.6875)^2(1.0)} = 550.6942 \ psi \end{cases}$$

Check Acceptance Criteria:

$$\left\{\begin{matrix} \sigma_{sm} = -271.0268\ psi \\ \sigma_{sm} = -632.3895\ psi \end{matrix}\right\} \le \{1.5S,\ not\ applicable\ due\ to\ compressive\ stress\}$$

$$\left\{\begin{matrix} \sigma_{sm} + \sigma_{sb} = -271.0268 + 770.6388 = 499.6\ psi \\ \sigma_{sm} - \sigma_{sb} = -271.0268 - 770.6388 = |-1041.7|\ psi \\ \sigma_{sm} + \sigma_{sb} = -632.3895 + 1835.6472 = 1203.3\ psi \\ \sigma_{sm} - \sigma_{sb} = -632.3895 - 1835.6472 = |-2468.0|\ psi \end{matrix}\right\} \le \{S_{PS} = 67200\ psi\} \qquad True$$

$$\left\{\begin{matrix} \sigma_{\theta m} = -224.7409 \\ \sigma_{\theta m} = 332.3058 \end{matrix}\right\} \le \left\{\begin{matrix} 1.5S,\ not\ applicable\ due\ to\ compressive\ stress \\ 1.5S = 1.5(22400) = 33600\ psi \end{matrix}\right\} \qquad True$$

$$\left\{\begin{matrix} \sigma_{\theta m} + \sigma_{\theta b} = -224.7409 + 231.1916 = 6.5\ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -224.7409 - 231.1916 = |-455.9|\ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 332.3058 + 550.6942 = 883.0\ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 332.3058 - 550.6942 = |-218.4|\ psi \end{matrix}\right\} \le \{S_{PS} = 67200\ psi\} \qquad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limit is satisfied.

$$\sigma_{\theta m} \le F_{ha}$$
$$\sigma_{sm} \le F_{xa}$$

F_{ha} is evaluated using paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he} = \frac{0.4E_y t}{D_o}$$

In accordance with paragraph 4.4.5.1, the value of F_{ha} is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t = 1.6875\ in$$
$$L \rightarrow Not\ required,\ as\ the\ equation\ for\ F_{he}\ is\ independent\ of\ L$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}

$$F_{he} = \frac{0.4E_y t}{D_o} = \frac{0.4(28.3E+06)(1.6875)}{153.625} = 124344.9959\ psi$$

3) STEP 3 – Calculate the predicted buckling stress, F_{ic}

$$\frac{F_{he}}{S_y} = \frac{124344.9959}{33600} = 3.7007$$

Since $\dfrac{F_{he}}{S_y} \geq 2.439$, calculate F_{ic} as follows:

$$F_{ic} = S_y = 33600 \ psi$$

4) STEP 4 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $F_{ic} = S_y = 33600 \ psi$, calculate FS as follows:

$$FS = 1.667$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ to the allowable hoop compressive membrane stress, F_{ha} per following criteria.

$$\left\{\sigma_{\theta m} = 224.7 \ psi\right\} \leq \left\{F_{ha} = 20156.0 \ psi\right\} \qquad True$$

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

F_{xa} is evaluated using paragraph 4.4.12.2.b with $\lambda = 0.15$. The design factor FS used in paragraph 4.4.12.2.b is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in paragraph 4.4.2. An initial calculation is required to determine the value of F_{xa} by setting $FS = 1.0$, with $F_{ic} = F_{xa}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.b.

For $\lambda_c = 0.15$, (Local Buckling)

$$F_{xa} = \min\left[F_{xa1},\, F_{xa2}\right]$$

$$\frac{D_o}{t} = \frac{153.625}{1.6875} = 91.0370$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{732.0}{\sqrt{76.8125(1.6875)}} = 64.2944$$

Since $\dfrac{D_o}{t} \leq 135$, calculate F_{xa1} as follows with an initial value of $FS = 1.0$.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.0} = 33600 \ psi$$

The value of F_{xa2} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{xa2} = \frac{F_{xe}}{FS}$$

$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since $\frac{D_o}{t} \leq 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \frac{D_o}{t}\right)},\ 0.9\right]$$

Since $M_x \geq 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389 + \frac{153.625}{1.6875}},\ 0.9\right] = 0.8520$$

Therefore,

$$F_{xe} = \frac{0.8520(28.3E+06)(1.6875)}{153.625} = 264854.8413\ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{264854.8413}{1.0} = 264854.8413\ psi$$

$$F_{xa} = \min[33600,\ 264855] = 33600\ psi$$

With a value of $F_{ic} = F_{xa} = 33600$, in accordance with paragraph 4.4.2, it is determined the value of $FS = 1.667$ since $\{F_{ic} = 33600\} = \{S_y = 33600\}$. Using this computed value of $FS = 1.667$ in paragraph 4.4.12.2.b, F_{xa} is calculated as follows.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.667} = 20155.9688\ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{264854.8413}{1.667} = 158881.1286\ psi$$

$$F_{xa} = \min[20155.9688,\ 158881.1286] = 20155.9688\ psi$$

Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria.

$$\left\{\sigma_{sm} = 632.4 \ psi\right\} \le \left\{F_{xa} = 20156.0 \ psi\right\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

The cylinder at the cylinder-to-cone junction at the large end is adequately designed.

<u>Evaluate the Cone at the Large End:</u>

Stress Resultant Calculations, as determined above.

$$M_{csP} = M_{sP} = 444.3413 \ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \left\{ \begin{array}{l} -78.5889 \ \dfrac{in-lbs}{in} \\ 426.8741 \ \dfrac{in-lbs}{in} \end{array} \right\}$$

$$M_{cs} = M_{csP} + M_{csX} = \left\{ \begin{array}{l} 444.3413 + (-78.5889) = 365.7524 \ \dfrac{in-lbs}{in} \\ 444.3413 + 426.8741 = 871.2154 \ \dfrac{in-lbs}{in} \end{array} \right\}$$

$$Q_c = Q\cos[\alpha] + N_s \sin[\alpha]$$

$$Q_c = \left\{ \begin{array}{l} 84.0270\left(\cos[21.0375]\right) + (-457.3577)\sin[21.0375] = -85.7555 \ \dfrac{lbs}{in} \\ 196.5350\left(\cos[21.0375]\right) + (-1067.1574)\sin[21.0375] = -199.6519 \ \dfrac{lbs}{in} \end{array} \right\}$$

$$R_C = \frac{R_L}{\cos[\alpha]} = \frac{75.125}{\cos[21.0375]} = 80.4900 \ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_C^2 t_C^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(80.4900)^2 (1.8125)^2}\right]^{0.25} = 0.1064 \ in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \left\{ \begin{array}{l} -457.3577\left(\cos[21.0375]\right) - 84.0270\sin[21.0375] = -457.0368 \ \dfrac{lbs}{in} \\ -1067.1574\left(\cos[21.0375]\right) - 196.5350\sin[21.0375] = -1066.5786 \ \dfrac{lbs}{in} \end{array} \right\}$$

$$N_{c\theta} = \frac{PR_L}{\cos[\alpha]} + 2\beta_{co} R_C \left(-M_{cs}\beta_{co} - Q_c\right)$$

$$N_{c\theta} = \begin{cases} \dfrac{-14.7(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(365.7524)(0.1064) - (-85.7555)\right) \\ \dfrac{-14.7(75.125)}{\cos[21.0375]} + 2(0.1064)(80.4900)\left(-(871.2154)(0.1064) - (-199.6519)\right) \end{cases}$$

$$N_{c\theta} = \begin{cases} -380.9244 \ \dfrac{lbs}{in} \\ 648.7441 \ \dfrac{lbs}{in} \end{cases}$$

$$K_{cpc} = 1.0$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses.

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{cases} \dfrac{-457.0368}{1.8125} = -252.1582 \ psi \\ \dfrac{-1066.5786}{1.8125} = -588.4572 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{cases} \dfrac{6(365.7524)}{(1.8125)^2(1.0)} = 668.0091 \ psi \\ \dfrac{6(871.2154)}{(1.8125)^2(1.0)} = 1591.1853 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{cases} \dfrac{-380.9244}{1.8125} = -210.1652 \ psi \\ \dfrac{648.7441}{1.8125} = 357.9278 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{cases} \dfrac{6(0.3)(365.7524)}{(1.8125)^2(1.0)} = 200.4027 \ psi \\ \dfrac{6(0.3)(871.2154)}{(1.8125)^2(1.0)} = 477.3556 \ psi \end{cases}$$

Check Acceptance Criteria:

$$\begin{Bmatrix} \sigma_{sm} = -252.1582 \ psi \\ \sigma_{sm} = -588.4572 \ psi \end{Bmatrix} \leq \{1.5S, \ not \ applicable \ due \ to \ compressive \ stress\}$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = -252.1582 + 668.0091 = 415.6 \ psi \\ \sigma_{sm} - \sigma_{sb} = -252.1582 - 668.0091 = |-920.2| \ psi \\ \sigma_{sm} + \sigma_{sb} = -588.4572 + 1591.1853 = 1002.7 \ psi \\ \sigma_{sm} - \sigma_{sb} = -588.4572 - 1591.1853 = |-2179.6| \ psi \end{Bmatrix} \leq \{S_{PS} = 67200 \ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = -210.1652 \\ \sigma_{\theta m} = 357.9278 \end{Bmatrix} \leq \begin{Bmatrix} 1.5S, \ not \ applicable \ due \ to \ compressive \ stress \\ 1.5S = 1.5(22400) = 33600 \ psi \end{Bmatrix} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = -210.1652 + 200.4027 = |-9.7| \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -210.1652 - 200.4027 = -410.6 \ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = 357.9278 + 477.3556 = 835.3 \ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = 357.9278 - 477.3556 = |-119.4| \ psi \end{Bmatrix} \leq \{S_{PS} = 60000 \ psi\} \qquad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

Using the procedure shown above for the cylindrical shell and substituting the cone thickness, t_c for the cylinder thickness, t, the allowable compressive hoop membrane and axial membrane stresses, F_{ha} and F_{xa}, respectively, are calculated as follows.

$$F_{ha} = 20156.0 \ psi$$
$$F_{xa} = 20156.0 \ psi$$

Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ and axial compressive membrane stress, σ_{sm}, to the allowable hoop compressive membrane stress, F_{ha} and axial compressive membrane stress, F_{xa} per following criteria.

$$\{\sigma_{\theta m} = 210.2 \ psi\} \leq \{F_{ha} = 20156.0 \ psi\} \qquad True$$
$$\{\sigma_{sm} = 588.5 \ psi\} \leq \{F_{xa} = 20156.0 \ psi\} \qquad True$$

Therefore, local buckling due to hoop and axial compressive membrane stress is not a concern.

The cone at the cylinder-to-cone junction at the large end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore, the design is complete.

Evaluate the Small End cylinder-to-cone junction per paragraph 4.3.11.5.

a) STEP 1 – Compute the small end cylinder thickness, t_S, using paragraph 4.3.3., (as specified in design conditions)

$$t_S = 1.0\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the small end using paragraph 4.3.4., (as specified in design conditions)

$$\alpha = 21.0375\ deg$$

$$t_C = 1.8125\ in$$

c) STEP 3 – Proportion the cone geometry such that the following equations are satisfied. If all of these equations are not satisfied, then the cylinder-to-cone junction shall be designed in accordance with Part 5. In the calculations, if $0\ deg < \alpha \le 10\ deg$, then use $\alpha = 10\ deg$.

$$20 \le \left\{ \frac{R_S}{t_S} = \frac{45.125}{1.0} = 45.125 \right\} \le 500 \qquad True$$

$$1 \le \left(\frac{t_C}{t_S} = \frac{1.8125}{1.0} = 1.8125 \right) \le 2 \qquad True$$

$$\{\alpha = 21.0375\ deg\} \le 60\ deg \qquad True$$

d) STEP 4 – Determine the net section axial force, F_S, and bending moment, M_S, applied to the conical transition (as specified in design conditions). The thrust load due to pressure shall not be included as part of the axial force, F_S. Calculate the equivalent line load, X_S, using the specified net section axial force, F_S, and bending moment, M_S.

$$X_S = \frac{F_S}{2\pi R_S} \pm \frac{M_S}{\pi R_S^2} = \begin{Bmatrix} \dfrac{-78104}{2\pi(45.125)} + \dfrac{4.301E+06}{\pi(45.125)^2} = 396.8629\ \dfrac{lbs}{in} \\ \dfrac{-78104}{2\pi(45.125)} - \dfrac{4.301E+06}{\pi(45.125)^2} = -947.8053\ \dfrac{lbs}{in} \end{Bmatrix}$$

e) STEP 5 – Compute the junction transition design parameters (the normalized resultant moment, M_{sN}, and shear force, Q_N) for the internal pressure and equivalent line load per Table 4.3.5 and Table 4.3.6, respectively. For calculated values of n other than those presented in Table 4.3.5 and Table 4.3.6, linear interpolation of the equation coefficients, C_i, is permitted.

$$n = \frac{t_C}{t_S} = \frac{1.8125}{1.0} = 1.8125$$

$$H = \sqrt{\frac{R_S}{t_S}} = \sqrt{\frac{45.125}{1.0}} = 6.7175$$

$$B = \tan[\alpha] = \tan[21.0375] = 0.3846$$

Linear interpolation of the equation coefficients, C_i in Table 4.3.5 and Table 4.3.6 is required. The results of the interpolation are summarized with the following values for C_i (see paragraph 4.3.11.5, STEP 5 of E4.3.7)

For the applied pressure case M_{sN} is calculated using the following equation

$$M_{sN} = \exp\left[\begin{array}{l} C_1 + C_2 \ln\left[H^2\right] + C_3 \ln\left[\alpha\right] + C_4\left(\ln\left[H^2\right]\right)^2 + C_5\left(\ln\left[\alpha\right]\right)^2 + \\ C_6 \ln\left[H^2\right]\ln\left[\alpha\right] + C_7\left(\ln\left[H^2\right]\right)^3 + C_8\left(\ln\left[\alpha\right]\right)^3 + \\ C_9 \ln\left[H^2\right]\left(\ln\left[\alpha\right]\right)^2 + C_{10}\left(\ln\left[H^2\right]\right)^2 \ln\left[\alpha\right] \end{array}\right]$$

This results in the following (see paragraph 4.3.11.5, STEP 5 of E4.3.7)

$$M_{sN} = 9.2135$$

For the applied pressure case Q_N is calculated using the following equation

$$Q_N = \left(\frac{C_1 + C_3 H^2 + C_5 \alpha + C_7 H^4 + C_9 \alpha^2 + C_{11} H^2 \alpha}{1 + C_2 H^2 + C_4 \alpha + C_6 H^4 + C_8 \alpha^2 + C_{10} H^2 \alpha}\right)$$

This results in the following (see paragraph 4.3.11.5, STEP 5 of E4.3.7)

$$Q_N = -2.7333$$

For the Equivalent Line Load case, M_{sN} is calculated using the following equation

$$M_{sN} = \left(\frac{C_1 + C_3 H + C_5 B + C_7 H^2 + C_9 B^2 + C_{11} HB}{1 + C_2 H + C_4 B + C_6 H^2 + C_8 B^2 + C_{10} HB}\right)$$

This results in the following (see paragraph 4.3.11.5, STEP 5 of E4.3.7)

$$M_{sN} = 0.4828$$

For the Equivalent Line Load case, Q_N is calculated using the following equation

$$Q_N = \left(\begin{array}{l} C_1 + C_2 \ln\left[H\right] + C_3 \ln\left[B\right] + C_4\left(\ln\left[H\right]\right)^2 + C_5\left(\ln\left[B\right]\right)^2 + C_6 \ln\left[H\right]\ln\left[B\right] + \\ C_7\left(\ln\left[H\right]\right)^3 + C_8\left(\ln\left[B\right]\right)^3 + C_9 \ln\left[H\right]\left(\ln\left[B\right]\right)^2 + C_{10}\left(\ln\left[H\right]\right)^2 \ln\left[B\right] \end{array}\right)$$

This results in the following (see paragraph 4.3.11.5, STEP 5 of E4.3.7)

$$Q_N = -0.1613$$

Summarizing, the normalized resultant moment M_{sN}, and shear force Q_N for the internal pressure and equivalent line load are as follows:

$$\textit{Internal Pressure}: \qquad M_{sN} = 9.2135, \qquad Q_N = -2.7333$$
$$\textit{Equivalent Line Load}: \qquad M_{sN} = 0.4828, \qquad Q_N = -0.1613$$

f) STEP 6 – Compute the stresses in the cylinder and cone at the junction using the equations in Table 4.3.2. for the Small End Junction

Evaluate the Cylinder at the Small End.

Stress Resultant Calculations.

$$M_{sP} = Pt_S^2 M_{sN} = -14.7(1.0)^2(9.2135) = -135.4385\ \frac{in-lbs}{in}$$

$$M_{sX} = X_S t_S M_{sN} = \begin{Bmatrix} 396.8629(1.0)(0.4828) = 191.6054\ \frac{in-lbs}{in} \\ -947.8053(1.0)(0.4828) = -457.6004\ \frac{in-lbs}{in} \end{Bmatrix}$$

$$M_s = M_{sP} + M_{sX} = \begin{Bmatrix} -135.4385 + (191.6054) = 56.1669\ \frac{in-lbs}{in} \\ -135.4385 + (-457.6004) = -593.0389\ \frac{in-lbs}{in} \end{Bmatrix}$$

$$Q_P = Pt_S Q_N = -14.7(1.0)(-2.7333) = 40.1795\ \frac{lbs}{in}$$

$$Q_X = X_S Q_N = \begin{Bmatrix} 396.8629(-0.1613) = -64.0140\ \frac{lbs}{in} \\ -947.8053(-0.1613) = 152.8810\ \frac{lbs}{in} \end{Bmatrix}$$

$$Q = Q_P + Q_X = \begin{Bmatrix} 40.1795 + (-64.0140) = -23.8345\ \frac{lbs}{in} \\ 40.1795 + 152.8810 = 193.0605\ \frac{lbs}{in} \end{Bmatrix}$$

$$\beta_{cy} = \left[\frac{3(1-\nu^2)}{R_S^2 t_S^2}\right]^{0.25} = \left[\frac{3(1-(0.3)^2)}{(45.1250)^2(1.000)^2}\right]^{0.25} = 0.1914\ in^{-1}$$

$$N_s = \frac{PR_S}{2} + X_S = \begin{Bmatrix} \dfrac{-14.7(45.125)}{2} + 396.8629 = 65.1942\ \dfrac{lbs}{in} \\ \dfrac{-14.7(45.125)}{2} + (-947.8053) = -1279.4741\ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_\theta = PR_S + 2\beta_{cy} R_S \left(-M_s \beta_{cy} - Q\right)$$

$$N_\theta = \begin{cases} -14.7(45.125) + 2(0.1914)(45.125)\left(-(56.1669)(0.1914) - (-23.8345)\right) \\ -14.7(45.125) + 2(0.1914)(45.125)\left(-(-593.0389)(0.1914) - 193.0605\right) \end{cases}$$

$$N_\theta = \begin{cases} -437.3238 \ \dfrac{lbs}{in} \\ -2037.5216 \ \dfrac{lbs}{in} \end{cases}$$

$$K_{pc} = 1.0$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_s}{t_S} = \begin{cases} \dfrac{65.1942}{1.0} = 65.1942 \ psi \\ \dfrac{-1279.4741}{1.0} = -1279.4741 \ psi \end{cases}$$

$$\sigma_{sb} = \frac{6M_s}{t_S^2 K_{pc}} = \begin{cases} \dfrac{6(56.1669)}{(1.0)^2(1.0)} = 337.0014 \ psi \\ \dfrac{6(-593.0389)}{(1.0)^2(1.0)} = -3558.2334 \ psi \end{cases}$$

$$\sigma_{\theta m} = \frac{N_\theta}{t_S} = \begin{cases} \dfrac{-437.3238}{1.0} = -437.3238 \ psi \\ \dfrac{-2037.5216}{1.0} = -2037.5216 \ psi \end{cases}$$

$$\sigma_{\theta b} = \frac{6\nu M_s}{t_S^2 K_{pc}} = \begin{cases} \dfrac{6(0.3)(56.1669)}{(1.0)^2(1.0)} = 101.1004 \ psi \\ \dfrac{6(0.3)(-593.0389)}{(1.0)^2(1.0)} = -1067.4700 \ psi \end{cases}$$

Check Acceptance Criteria:

$$\begin{Bmatrix} \sigma_{sm} = 65.1942\ psi \\ \sigma_{sm} = -1279.4741\ psi \end{Bmatrix} \leq \begin{Bmatrix} 1.5S = 1.5(22400) = 33600\ psi \\ 1.5S,\ not\ applicable\ due\ to\ compressive\ stress \end{Bmatrix} \qquad True$$

$$\begin{Bmatrix} \sigma_{sm} + \sigma_{sb} = 65.1942 + 337.0014 = 402.2\ psi \\ \sigma_{sm} - \sigma_{sb} = 65.1942 - 337.0014 = |-271.8|\ psi \\ \sigma_{sm} + \sigma_{sb} = -1279.4741 + (-3558.2334) = |-4837.7|\ psi \\ \sigma_{sm} - \sigma_{sb} = -1279.4741 - (-3558.2334) = 2278.8\ psi \end{Bmatrix} \leq \{S_{PS} = 67200\ psi\} \qquad True$$

$$\begin{Bmatrix} \sigma_{\theta m} = -437.3238 \\ \sigma_{\theta m} = -2037.5216 \end{Bmatrix} \leq \{1.5S,\ not\ applicable\ due\ to\ compressive\ stress\}$$

$$\begin{Bmatrix} \sigma_{\theta m} + \sigma_{\theta b} = -437.3238 + 101.1004 = |-336.2|\ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -437.3238 - 101.1004 = |-538.4|\ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = -2037.5216 + (-1067.4700) = |-3105.0|\ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -2037.5216 - (-1067.4700) = |-970.1|\ psi \end{Bmatrix} \leq \{S_{PS} = 67200\ psi\} \qquad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limit is satisfied.

$$\sigma_{\theta m} \leq F_{ha}$$
$$\sigma_{sm} \leq F_{xa}$$

F_{ha} is evaluated using paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he} = \frac{0.4E_y t}{D_o}$$

In accordance with paragraph 4.4.5.1, the value of F_{ha} calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t = 1.0\ in$$
$$L \rightarrow Not\ required,\ as\ the\ equation\ for\ F_{he}\ is\ independent\ of\ L$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}

$$F_{he} = \frac{0.4E_y t}{D_o} = \frac{0.4(28.3E+06)(1.0)}{92.25} = 122710.0271\ psi$$

3) STEP 3 – Calculate the predicted buckling stress, F_{ic}

$$\frac{F_{he}}{S_y} = \frac{122710.0271}{33600} = 3.6521$$

Since $\frac{F_{he}}{S_y} \geq 2.439$, calculate F_{ic} as follows:

$$F_{ic} = S_y = 33600 \ psi$$

4) STEP 4 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $F_{ic} = S_y = 33600 \ psi$, calculate FS as follows:

$$FS = 1.667$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ to the allowable hoop compressive membrane stress, F_{ha} per following criteria.

$$\left\{\sigma_{\theta m} = 2037.5 \ psi\right\} \leq \left\{F_{ha} = 20156.0 \ psi\right\} \qquad True$$

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

F_{xa} is evaluated using paragraph 4.4.12.2.b with $\lambda = 0.15$. The design factor FS used in paragraph 4.4.12.2.b is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in paragraph 4.4.2. An initial calculation is required to determine the value of F_{xa} by setting $FS = 1.0$, with $F_{ic} = F_{xa}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.b.

For $\lambda_c = 0.15$, (Local Buckling)

$$F_{xa} = \min\left[F_{xa1}, F_{xa2}\right]$$

$$\frac{D_o}{t} = \frac{92.25}{1.0} = 92.25$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{636.0}{\sqrt{46.125(1.0)}} = 93.6459$$

Since $\frac{D_o}{t} \leq 135$, calculate F_{xa1} as follows with an initial value of $FS = 1.0$.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.0} = 33600 \ psi$$

The value of F_{xa2} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{xa2} = \frac{F_{xe}}{FS}$$

$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since $\frac{D_o}{t} \le 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \frac{D_o}{t}\right)},\ 0.9\right]$$

Since $M_x \ge 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389 + \frac{92.25}{1.0}},\ 0.9\right] = 0.8499$$

Therefore,

$$F_{xe} = \frac{0.8520(28.3E+06)(1.0)}{92.25} = 260728.1301 \ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{260728.1301}{1.0} = 260728.1301 \ psi$$

$$F_{xa} = \min[33600,\ 260728] = 33600 \ psi$$

With a value of $F_{ic} = F_{xa} = 33600$, in accordance with paragraph 4.4.2, it is determined the value of $FS = 1.667$ since $\{F_{ic} = 33600\} = \{S_y = 33600\}$. Using this computed value of $FS = 1.667$ in paragraph 4.4.12.2.b, F_{xa} is calculated as follows.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$
$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{260728.1301}{1.6670} = 156405.5969 \ psi$$
$$F_{xa} = \min\left[20155.9688,\ 156405.5969\right] = 20155.9688 \ psi$$

Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria

$$\left\{\sigma_{sm} = 1279.5 \ psi\right\} \leq \left\{F_{xa} = 20156.0 \ psi\right\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

The cylinder at the cylinder-to-cone junction at the small end is adequately designed.

Evaluate the Cone at the Small End.

Stress Resultant Calculations as determined above.

$$M_{csP} = M_{sP} = -135.4385 \ \frac{in-lbs}{in}$$

$$M_{csX} = M_{sX} = \begin{Bmatrix} 191.6054 \ \dfrac{in-lbs}{in} \\ -457.6004 \ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$M_{cs} = M_{csP} + M_{csX} = \begin{Bmatrix} -135.4385 + 191.6054 = 56.1669 \ \dfrac{in-lbs}{in} \\ -135.4385 + (-457.6004) = -593.0389 \ \dfrac{in-lbs}{in} \end{Bmatrix}$$

$$Q_c = Q\cos[\alpha] + N_s \sin[\alpha]$$

$$Q_c = \begin{Bmatrix} (-23.8345)\cos[21.0375] + 65.1942\sin[21.0375] = 1.1575 \ \dfrac{lbs}{in} \\ 193.0605\cos[21.0375] + (-1279.4741)\sin[21.0375] = -279.1120 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$R_c = \frac{R_c}{\cos[\alpha]} = \frac{45.1250}{\cos[21.0375]} = 48.3476 \ in$$

$$\beta_{co} = \left[\frac{3(1-\nu^2)}{R_c^{\,2} t_c^{\,2}}\right]^{0.25} = \left[\frac{3\left(1-(0.3)^2\right)}{(48.3476)^2(1.8125)^2}\right]^{0.25} = 0.1373 \ in^{-1}$$

$$N_{cs} = N_s \cos[\alpha] - Q\sin[\alpha]$$

$$N_{cs} = \begin{Bmatrix} 65.1942\cos[21.0375] - (-23.8345)\sin[21.0375] = 69.4048 \ \dfrac{lbs}{in} \\ (-1279.4741)\cos[21.0375] - 193.0605\sin[21.0375] = -1263.4963 \ \dfrac{lbs}{in} \end{Bmatrix}$$

$$N_{c\theta} = \frac{PR_s}{\cos[\alpha]} + 2\beta_{co}R_c\left(-M_{cs}\beta_{co} + Q_c\right)$$

$$N_{c\theta} = \begin{Bmatrix} \dfrac{-14.7(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(56.1669)(0.1373) + 1.1575\right) \\ \dfrac{-14.7(45.125)}{\cos[21.0375]} + 2(0.1373)(48.3476)\left(-(-593.0389)(0.1373) + (-279.1120)\right) \end{Bmatrix}$$

$$N_{c\theta} = \begin{Bmatrix} -797.7248 \dfrac{lbs}{in} \\ -3335.2619 \dfrac{lbs}{in} \end{Bmatrix}$$

$$K_{cpc} = 1.0$$

Stress Calculations: Determine the meridional and circumferential membrane and bending stresses:

$$\sigma_{sm} = \frac{N_{cs}}{t_C} = \begin{Bmatrix} \dfrac{69.4048}{1.8125} = 38.2923 \ psi \\ \dfrac{-1263.4963}{1.8125} = -697.1014 \ psi \end{Bmatrix}$$

$$\sigma_{sb} = \frac{6M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(56.1669)}{(1.8125)^2(1.0)} = 102.5831 \ psi \\ \dfrac{6(-593.0389)}{(1.8125)^2(1.0)} = -1083.1246 \ psi \end{Bmatrix}$$

$$\sigma_{\theta m} = \frac{N_{c\theta}}{t_C} = \begin{Bmatrix} \dfrac{-797.7248}{1.8125} = -440.1240 \ psi \\ \dfrac{-3335.2619}{1.8125} = -1840.1445 \ psi \end{Bmatrix}$$

$$\sigma_{\theta b} = \frac{6\nu M_{cs}}{t_C^2 K_{cpc}} = \begin{Bmatrix} \dfrac{6(0.3)(56.1669)}{(1.8125)^2(1.0)} = 30.7749 \ psi \\ \dfrac{6(0.3)(-593.0389)}{(1.8125)^2(1.0)} = -324.9374 \ psi \end{Bmatrix}$$

Check Acceptable Criteria:

$$\left\{\begin{array}{l}\sigma_{sm} = 38.2923\ psi \\ \sigma_{sm} = -697.1014\ psi\end{array}\right\} \le \left\{\begin{array}{l}1.5S = 1.5(22400) = 33600\ psi \\ 1.5S,\ not\ applicable\ due\ to\ compressive\ stress\end{array}\right\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{sm} + \sigma_{sb} = 38.2923 + 102.5831 = 140.9\ psi \\ \sigma_{sm} - \sigma_{sb} = 38.2923 - 102.5831 = |-64.3|\ psi \\ \sigma_{sm} + \sigma_{sb} = -697.1014 + (-1083.1246) = |-1780.2|\ psi \\ \sigma_{sm} - \sigma_{sb} = -697.1014 - (-1083.1246) = 386.0\ psi\end{array}\right\} \le \{S_{PS} = 67200\ psi\} \qquad True$$

$$\left\{\begin{array}{l}\sigma_{\theta m} = -440.1240 \\ \sigma_{\theta m} = -1840.1445\end{array}\right\} \le \{1.5S,\ not\ applicable\ due\ to\ compressive\ stress\}$$

$$\left\{\begin{array}{l}\sigma_{\theta m} + \sigma_{\theta b} = -440.1240 + 30.7749 = |-409.3|\ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -440.1240 - 30.7749 = |-470.9|\ psi \\ \sigma_{\theta m} + \sigma_{\theta b} = -1840.1445 + (-324.9374) = |-2164.9|\ psi \\ \sigma_{\theta m} - \sigma_{\theta b} = -1840.1445 - (-324.9374) = |-1515.1|\ psi\end{array}\right\} \le \{S_{PS} = 67200\ psi\} \qquad True$$

Since the hoop membrane stress, $\sigma_{\theta m}$ and the axial membrane stress, σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

Using the procedure shown above for the cylindrical shell and substituting the cone thickness, t_c for the cylinder thickness, t, the allowable compressive hoop membrane and axial membrane stresses, F_{ha} and F_{xa}, respectively, are calculated as follows.

$$F_{ha} = 20156.0\ psi$$
$$F_{xa} = 20156.0\ psi$$

Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ and axial compressive membrane stress, σ_{sm}, to the allowable hoop compressive membrane stress, F_{ha} and axial compressive membrane stress, F_{xa} per following criteria.

$$\{\sigma_{\theta m} = 1840.1\ psi\} \le \{F_{ha} = 20156.0\ psi\} \qquad True$$
$$\{\sigma_{sm} = 697.1\ psi\} \le \{F_{xa} = 20156.0\ psi\} \qquad True$$

Therefore, local buckling due to hoop and axial compressive membrane stress is not a concern.

The cone at the cylinder-to-cone junction at the small end is adequately designed.

g) STEP 7 – The stress acceptance criterion in STEP 6 is satisfied for both the cylinder and cone. Therefore, the design is complete.

4.4.8 Example E4.4.8 – Conical Transitions with a Knuckle

Determine if the proposed design for the large end of a cylinder-to-cone junction with a knuckle is adequately designed considering the following design conditions and applied forces and moments.

Vessel Data:

- Material = $SA-516,\ Grade\ 70,\ Norm.$
- Design Conditions = $-14.7\ psig@300°F$
- Inside Diameter (Large End) = $120.0\ in$
- Large End Thickness = $1.0\ in$
- Inside Diameter (Small End) = $60.0\ in$
- Small End Thickness = $1.0\ in$
- Knuckle Radius = $10.0\ in$
- Cone Thickness = $1.0\ in$
- Knuckle Thickness = $1.0\ in$
- Length of Conical Section = $73.0\ in$
- Unsupported Length of Large Cylinder = $240.0\ in$
- Unsupported Length of Small Cylinder = $360.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $22400\ psi$
- Yield Strength = $33600\ psi$
- Modulus of Elasticity = $28.3E+06\ psi$
- Weld Joint Efficiency = 1.0
- One-Half Apex Angle = $30.0\ deg$
- Axial Force (Large End) = $-10000\ lbs$
- Net Section Bending Moment (Large End) = $2.0E+06\ in-lbs$

Evaluate per paragraphs 4.4.14 and 4.3.12.

The design rules in paragraph 4.3.12 shall be satisfied. In these calculations, a negative value of pressure shall be used in all applicable equations.

a) STEP 1 – Compute the large end cylinder thickness, t_L, using paragraph 4.4.5, (as specified in design conditions)

$$t_L = 1.0\ in$$

b) STEP 2 – Determine the cone half-apex angle, α, and compute the cone thickness, t_C, at the large end using paragraph 4.4.5, (as specified in design conditions).

$$\alpha = 30\ deg$$

$$t_C = 1.0\ in$$

c) STEP 3 – Proportion the transition geometry by assuming a value for the knuckle radius, r_k, and knuckle thickness, t_k, such that the following equations are satisfied. If all of these equations cannot be satisfied, the cylinder-to-cone junction shall be designed in accordance with Part 5.

$$\{t_k = 1.0\ in\} \geq \{t_L = 1.0\ in\} \qquad True$$

$$\{r_k = 10.0\ in\} > \{3t_k = 3.0\ in\} \qquad True$$

$$\left\{\frac{r_k}{R_L} = \frac{10.0}{60.0} = 0.1667\right\} > \{0.03\} \qquad True$$

$$\{\alpha = 30\ deg\} \leq \{60\ deg\} \qquad True$$

d) STEP 4 – Determine the net section axial force, F_L, and bending moment, M_L, applied to the conical transition at the location of the knuckle. The thrust load due to pressure shall not be included as part of the axial force, F_L.

$$F_L = -10000\ lbs$$
$$M_L = 2.0E+06\ in-lbs$$

e) STEP 5 – Compute the stresses in the knuckle at the junction using the equations in Table 4.3.7.

Determine if the knuckle is considered to be compact or non-compact.

$$\alpha r_k < 2K_m\left(\left\{R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5} + r_k\right\}t_k\right)^{0.5}$$

$$\{0.5236(10.0)\} < \left\{2(0.7)\left(\left\{50.0\left((0.5236)^{-1}\cdot\tan[0.5236]\right)^{0.5} + 10\right\}1\right)^{0.5}\right\}$$

$$\{5.2360\ in\} < \{11.0683\ in\} \qquad True$$

Where,

$$K_m = 0.7$$
$$\alpha = \frac{30.0}{180}\pi = 0.5236\ rad$$
$$R_k = R_L - r_k = 60.0 - 10.0 = 50.0\ in$$

Therefore, analyze the knuckle junction as a compact knuckle.

Stress Calculations: Determine the circumferential and longitudinal membrane and bending stresses:

$$\sigma_{\theta m} = \frac{PK_m\left(R_L\sqrt{R_L t_L} + L_k\sqrt{L_k t_C}\right) + \alpha\left(PL_{1k}r_k - 0.5P_e L_{1k}^2\right)}{K_m\left(t_L\sqrt{R_L t_L} + t_C\sqrt{L_k t_C}\right)\alpha t_k r_k}$$

$$\sigma_{sm} = \frac{P_e L_{1k}}{2t_k}$$

Where,

$$L_{1k} = R_k\left(\alpha^{-1}\tan[\alpha]\right)^{0.5} + r_k = 50.0\left((0.5236)^{-1}\tan[0.5236]\right)^{0.5} + 10.0 = 62.5038\ in$$

$$L_k = \frac{R_k}{\cos[\alpha]} + r_k = \frac{50.0}{\cos[0.5236]} + 10.0 = 67.7351\ in$$

$$P_e = P + \frac{F_L}{\pi L_{1k}^2 \cos^2\left[\frac{\alpha}{2}\right]} \pm \frac{2M_L}{\pi L_{1k}^3 \cos^3\left[\frac{\alpha}{2}\right]}$$

$$P_e = \begin{cases} -14.7 + \dfrac{-10000.0}{\pi(62.5038)^2\cdot\cos^2\left[\dfrac{0.5236}{2}\right]} + \dfrac{2(2.0E+06)}{\pi(62.5038)^3\cdot\cos^3\left[\dfrac{0.5236}{2}\right]} \\ -14.7 + \dfrac{-10000.0}{\pi(62.5038)^2\cdot\cos^2\left[\dfrac{0.5236}{2}\right]} - \dfrac{2(2.0E+06)}{\pi(62.5038)^3\cdot\cos^3\left[\dfrac{0.5236}{2}\right]} \end{cases}$$

$$P_e = \begin{cases} -9.7875\ psi \\ -21.3590\ psi \end{cases}$$

Therefore,

$$\sigma_{\theta m} = \begin{cases} \dfrac{\left(\begin{array}{l} (-14.7)(0.7)\left(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}\right) + \\ 0.5236\left((-14.7)(62.5038)(10.0) - 0.5(-9.7875)(62.5038)^2\right) \end{array}\right)}{0.7\left(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}\right) + 0.5236(1.0)(10.0)} = -323.9558\ psi \\ \dfrac{\left(\begin{array}{l} (-14.7)(0.7)\left(60.0\sqrt{60.0(1.0)} + 67.7351\sqrt{67.7351(1.0)}\right) + \\ 0.5236\left((-14.7)(62.5038)(10.0) - 0.5(-21.3590)(62.5038)^2\right) \end{array}\right)}{0.7\left(1.0\sqrt{60.0(1.0)} + 1.0\sqrt{67.7351(1.0)}\right) + 0.5236(1.0)(10.0)} = 396.8501\ psi \end{cases}$$

And,

$$\sigma_{sm} = \begin{cases} \dfrac{-9.7875(62.5038)}{2(1.0)} = -305.8780\ psi \\ \dfrac{-21.3590(62.5038)}{2(1.0)} = -667.5093\ psi \end{cases}$$

Check Acceptable Criteria:

$$\begin{Bmatrix} \sigma_{\theta m} = -324.0\ psi \\ \sigma_{\theta m} = 396.9\ psi \end{Bmatrix} \le \begin{Bmatrix} S,\ not\ applicable\ due\ to\ compressive\ stress \\ S = 22400\ psi \qquad\qquad True \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{sm} = -305.9\ psi \\ \sigma_{sm} = -667.5\ psi \end{Bmatrix} \le \{S,\ not\ applicable\ due\ to\ compressive\ stress\}$$

Since the hoop membrane stress $\sigma_{\theta m}$ and axial membrane stress σ_{sm} are compressive, the condition of local buckling shall be considered. Local buckling is not a concern if the following limits are satisfied.

$$\sigma_{\theta m} \le F_{ha}$$

$$\sigma_{sm} \le F_{xa}$$

F_{ha} is evaluated using paragraph 4.4.5.1, but substituting F_{he} with the following equation.

$$F_{he} = \frac{0.4E_y t}{D_o}$$

In accordance with paragraph 4.4.5.1, the value of F_{ha} is calculated as follows.

1) STEP 1 – Assume an initial thickness, t and unsupported length, L.

$$t = 1.0\ in$$

$$L \rightarrow Not\ required,\ as\ the\ equation\ for\ F_{he}\ is\ independent\ of\ L$$

2) STEP 2 – Calculate the predicted elastic buckling stress, F_{he}

$$F_{he} = \frac{0.4E_y t}{D_o} = \frac{0.4(28.3E+06)(1.0)}{122.0} = 92786.8853\ psi$$

3) STEP 3 – Calculate the predicted buckling stress, F_{ic}

$$\frac{F_{he}}{S_y} = \frac{92786.8853}{33600} = 2.7615$$

Since $\frac{F_{he}}{S_y} \ge 2.439$, calculate F_{ic} as follows:

$$F_{ic} = S_y = 33600\ psi$$

4) STEP 4 – Calculate the value of design factor, FS per paragraph 4.4.2.

Since $F_{ic} = S_y = 33600\ psi$, calculate FS as follows:

$$FS = 1.667$$

5) STEP 5 – Calculate the allowable hoop compressive membrane stress as follows:

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{33600}{1.667} = 20155.9688 \ psi$$

6) STEP 6 – Compare the calculated hoop compressive membrane stress, $\sigma_{\theta m}$ to the allowable hoop compressive membrane stress, F_{ha} per following criteria.

$$\{\sigma_{\theta m} = -324.0 \ psi\} \le \{F_{ha} = 20156.0 \ psi\} \qquad True$$

Therefore, local buckling due to hoop compressive membrane stress is not a concern.

F_{xa} is evaluated using paragraph 4.4.12.2.b with $\lambda = 0.15$. The design factor FS used in paragraph 4.4.12.2.b is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in VIII-2, paragraph 4.4.2. An initial calculation is required to determine the value of F_{xa} by setting $FS = 1.0$, with $F_{ic} = F_{xa}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.b.For $\lambda_c = 0.15$, (Local Buckling)

$$F_{xa} = \min\left[F_{xa1},\, F_{xa2}\right]$$

$$\frac{D_o}{t} = \frac{122.0}{1.0} = 122.0$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{240.0}{\sqrt{61.0(1.0)}} = 30.7289$$

Since $\dfrac{D_o}{t} \le 135$, calculate F_{xa1} as follows with an initial value of $FS = 1.0$.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.0} = 33600 \ psi$$

The value of F_{xa2} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{xa2} = \frac{F_{xe}}{FS}$$

$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since $\dfrac{D_o}{t} \le 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389+\dfrac{D_o}{t}\right)},\ 0.9\right]$$

Since $M_x \geq 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389+\dfrac{122.0}{1.0}},\ 0.9\right] = 0.8004$$

Therefore,

$$F_{xe} = \frac{0.8004(28.3E+06)(1.0)}{122.0} = 185666.5574\ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{185666.5574}{1.0} = 185666.5574\ psi$$

$$F_{xa} = \min[33600, 185666.5574] = 33600\ psi$$

With a value of $F_{ic} = F_{xa} = 33600$, in accordance with VIII-2, paragraph 4.4.2, it is determined the value of $FS = 1.667$ since $\{F_{ic} = 33600\} = \{S_y = 33600\}$. Using this computed value of $FS = 1.667$ in paragraph 4.4.12.2.b, F_{xa} is calculated as follows.

$$F_{xa1} = \frac{S_y}{FS} = \frac{33600}{1.667} = 20155.9688\ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{185666.5574}{1.6670} = 111377.6589\ psi$$

$$F_{xa} = \min[20155.9688, 111377.6589] = 20155.9688\ psi$$

Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per the following criteria:

$$\left\{\begin{array}{l}\sigma_{sm} = 305.9\ psi \\ \sigma_{sm} = 667.5\ psi\end{array}\right\} \leq \{F_{xa} = 20156.0\ psi\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

The cylinder-to-cone junction with a knuckle at the large end is adequately designed.

f) STEP 6 – The stress acceptance criterion in STEP 6 is satisfied. Therefore, the design is complete.

4.5 Shells Openings in Shells and Heads

4.5.1 Example E4.5.1 – Radial Nozzle in Cylindrical Shell and Weld Strength Analysis

Design an integral nozzle and perform a weld strength analysis. The parameters used in this design procedure are shown in Figure E4.5.1.

Vessel and Nozzle Data:

- Design Conditions = $356\ psig@300°F$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0
- Shell Material = $SA-516,\ Grade\ 70,\ Norm.$
- Shell Allowable Stress = $22400\ psi$
- Nozzle Material = $SA-105$
- Nozzle Allowable Stress = $21200\ psi$
- Shell Inside Diameter = $150.0\ in$
- Shell Thickness = $1.8125\ in$
- Nozzle Outside Diameter = $19.0\ in$
- Nozzle Hub Outside Diameter = $25.5\ in$
- Nozzle Hub Height = $7.1875\ in$
- Nozzle Thickness = $4.75\ in$
- External Nozzle Projection = $14.1875\ in$
- Internal Nozzle Projection = $0.0 in$

The nozzle is inserted through the shell, i.e. set–in type nozzle, see Figure 4.5.13.

Establish the corroded dimensions.

Shell:

$$D_i = 150.0 + 2\left(Corrosion\ Allowance\right) = 150.0 + 2(0.125) = 150.25\ in$$

$$t = 1.8125 - Corrosion\ Allowance = 1.8125 - 0.125 = 1.6875\ in$$

Nozzle:

$$t_n = 4.75 - Corrosion\ Allowance = 4.75 - 0.125 = 4.625\ in$$

$$R_n = \frac{D - 2(t_n)}{2} = \frac{25.5 - 2(4.625)}{2} = 8.125\ in$$

The procedure, per paragraph 4.5.5, to design a radial nozzle in a cylindrical shell subject to pressure loading is shown below.

> Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in paragraph 4.5.13 would need to be checked.

a) STEP 1 – Determine the effective radius of the shell as follows

$$R_{eff} = 0.5D_i = 0.5(150.25) = 75.125\ in$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall.

For set–in, integrally reinforced nozzles:

$$L_R = \min\left[\sqrt{R_{eff}t},\ 2R_n\right]$$

$$L_R = \min\left[\sqrt{(75.125)(1.6875)},\ 2(8.125)\right] = \min[11.2594, 16.25] = 11.2594\ in$$

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface.

For set–in nozzles:

$$L_{H1} = \min\left[1.5t,\ t_e\right] + \sqrt{R_n t_n} = \min\left[1.5(1.6875),\ 0.0\right] + \sqrt{8.125(4.625)} = 6.1301\ in$$

$$L_{H2} = L_{pr1} = 14.1875 in$$

$$L_{H3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5\ in$$

$$L_H = \min\left[L_{H1},\ L_{H2},\ L_{H3}\right] + t = \min[6.1301,\ 15.875,\ 13.5] + 1.6875 = 7.8176\ in$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable

$$L_{I1} = \sqrt{R_n t_n} = \sqrt{8.125(4.625)} = 6.1301\ in$$

$$L_{I2} = L_{pr2} = 0.0$$

$$L_{I3} = 8(t + t_e) = 8(1.6875 + 0.0) = 13.5\ in$$

$$L_I = \min\left[L_{I1},\ L_{I2},\ L_{I3}\right] = \min[6.1301,\ 0.0,\ 13.5] = 0.0$$

e) STEP 5 – Determine the total available area near the nozzle opening, see Figure 4.5.2. Do not include any area that falls outside of the limits defined by L_H, L_R, and L_I. For variable thickness nozzles, see Figure 4.5.13 for metal area definitions of A_2.

For set–in nozzles:

$$A_T = A_1 + f_{rn}(A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp}A_5$$

$$A_1 = (tL_R)\cdot\max\left[\left(\frac{\lambda}{5}\right)^{0.85},\ 1.0\right] = 1.6875(11.2594)\cdot\max\left[\left(\frac{1.3037}{5}\right)^{0.85},\ 1.0\right]$$

$$A_1 = 19.0002\ in^2$$

$$\lambda = \min\left[\left\{\frac{(2R_n + t_n)}{\sqrt{(D_i + t_{eff})t_{eff}}}\right\}, 12.0\right] = \min\left[\frac{2(8.125)+4.625}{\sqrt{150.25+1.6875(1.6875)}}, 12.0\right] = 1.3037$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 1.6875 + \left(\frac{0.0(0.0)}{11.2594}\right) = 1.6875\ in$$

$$f_{rn} = \min\left[\frac{S_n}{S}, 1\right] = \frac{21200}{22400} = 0.9464$$

$$f_{rp} = \min\left[\frac{S_p}{S}, 1\right] = 0.0$$

Since $\{L_H = 7.8176\ in\} \le \{L_{x3} = L_{pr3} + t = 7.1875 + 1.6875 = 8.875\ in\}$, calculate A_2 as follows, see Figure 4.5.13:

$$A_2 = t_n L_H = 4.625(7.8176) = 36.1564\ in^2$$

$$A_3 = t_n L_I = 0.0$$

$$A_{41} = 0.5L_{41}^2 = 0.5(0.375)^2 = 0.0703\ in^2$$

$$A_{42} = 0.5L_{42}^2 = 0.0$$

$$A_{43} = 0.5L_{43}^2 = 0.0$$

$$A_5 = \min\left[A_{5a}, A_{5b}\right]$$

$$A_{5a} = Wt_e = 0.0$$

$$A_{5b} = L_R t_e = 0.0$$

$$A_5 = 0.0$$

$$A_T = 19.0002 + 0.9464(36.1564 + 0.0) + 0.0703 + 0.0 + 0.0 + 0.0 = 53.2889\ in^2$$

f) STEP 6 – Determine the applicable forces

For set–in nozzles:

$$f_N = PR_{xn} L_H = 356(10.2644)(7.8176) = 28566.4985\ lbs$$

$$R_{xn} = \frac{t_n}{\ln\left[\frac{R_n + t_n}{R_n}\right]} = \frac{4.625}{\ln\left[\frac{8.125 + 4.625}{8.125}\right]} = 10.2644\ in$$

$$f_S = PR_{xs}(L_R + t_n) = 356(75.9656)(11.2594 + 4.625) = 429573.7997\ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\dfrac{R_{eff}+t_{eff}}{R_{eff}}\right]} = \frac{1.6875}{\ln\left[\dfrac{75.125+1.6875}{75.125}\right]} = 75.9656\ in$$

$$f_Y = PR_{xs}R_{nc} = 356(75.9656)(8.125) = 219730.4980\ lbs$$

Note, for radial nozzles, $R_{nc} = R_n$.

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection.

$$\sigma_{avg} = \frac{(f_N + f_S + f_Y)}{A_T} = \frac{28566.4985 + 429573.7997 + 219730.4980}{53.2889} = 12720.6753\ psi$$

$$\sigma_{circ} = \frac{PR_{xs}}{t_{eff}} = \frac{356(75.9656)}{1.6875} = 16025.9281\ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection.

$$P_L = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\},\ \sigma_{circ}\right]$$

$$P_L = \max\left[\left\{2(12720.6753) - 16025.9281\right\},\ 16025.9281\right] = 16025.9281\ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy Equation 4.5.56. If the nozzle is subjected to internal pressure, then the allowable stress, S_{allow}, is given by Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by Equation 4.5.58 where F_{ha} is evaluated in paragraph 4.4 for the shell geometry being evaluated (e.g. cylinder, spherical shell, or formed head). The allowable stress shall be the minimum of the shell or nozzle material evaluated at the design temperature.

$$\left\{P_L = 16025.9281\ psi\right\} \le \left\{S_{allow} = 1.5SE = 1.5(21200)(1.0) = 31800\ psi\right\}$$

j) STEP 10 – Determine the maximum allowable working pressure at the nozzle intersection.

$$P_{\max 1} = \frac{S_{allow}}{\dfrac{2A_p}{A_T} - \dfrac{R_{xs}}{t_{eff}}} = \frac{1.5(21200)(1.0)}{\left(\dfrac{2(1904.1315)}{53.2889}\right) - \left(\dfrac{75.9656}{1.6875}\right)} = 1202.3676\ psi$$

$$A_p = \frac{f_N + f_S + f_Y}{P} = \frac{28566.4985 + 429573.7997 + 219730.4980}{356.0} = 1904.1315\ in^2$$

$$P_{\max 2} = S\left(\frac{t}{R_{xs}}\right) = 22400\left(\frac{1.6875}{75.9656}\right) = 497.5936\ psi$$

$$P_{\max} = \min\left[P_{\max 1},\ P_{\max 2}\right] = \min\left[1202.3676,\ 497.5936\right] = 497.5936\ psi$$

The nozzle is acceptable because $P_{max} = 497.6\ psi$ is greater than the specified design pressure of $356\ psig$.

Weld Strength Analysis

The procedure to evaluate attachment welds of nozzles in a cylindrical, conical, or spherical shell or formed head subject to pressure loading per paragraph 4.5.14.2 is shown below.

a) STEP 1 – Determine the discontinuity force factor,

For set-in nozzles:

$$k_y = \frac{R_{nc} + t_n}{R_{nc}} = \frac{8.125 + 4.625}{8.125} = 1.5692$$

Note, for radial nozzles, $R_{nc} = R_n$.

b) STEP 2 – Calculate weld length resisting continuity force,

Weld length of nozzle to shell weld, for radial nozzles:

$$L_\tau = \frac{\pi}{2}\left(R_n + t_n\right) = \frac{\pi}{2}\left(8.125 + 4.625\right) = 20.0277\ in$$

Weld length of pad to shell weld, for radial nozzles:

$$L_{\tau p} = \frac{\pi}{2}\left(R_n + t_n + W\right) \qquad Not\ \ Applicable$$

c) STEP 3 – Compute the weld throat dimensions, as applicable

$$L_{41T} = 0.7071 L_{41} = 0.7071\left(0.375\right) = 0.2652\ in$$
$$L_{42T} = 0.0$$
$$L_{43T} = 0.0$$

d) STEP 4 – Determine if the weld sizes are acceptable. If the nozzle is integrally reinforced, and the computed shear stress in the weld given by Equation (4.5.182) satisfies Equation (4.5.183), then the design is complete. If the shear stress in the weld does not satisfy Equation (4.5.183), increase the weld size and return to Step 3.

$$\tau = \frac{f_{welds}}{L_\tau\left(0.49 L_{41T} + 0.6 t_{w1} + 0.49 L_{43T}\right)}$$

$$\tau = \frac{45450.9764}{20.0277\left(0.49\left(0.2652\right) + 0.6\left(1.6875\right) + 0.49\left(0.0\right)\right)} = 1986.4411\ psi$$

Where,

$$f_{welds} = \min\left[f_Y k_y,\ 1.5S_n\left(A_2 + A_3\right),\ \frac{\pi}{4} P R_n^2 k_y^2 \right]$$

$$f_{welds} = \min\left[\begin{array}{l} \left\{219730.4980(1.5692) = 344801.0975\right\} \\ \left\{1.5(21200)(36.1564 + 0) = 1149773.52\right\} \\ \left\{\frac{\pi}{4}(356)(8.125)^2(1.5692)^2 = 45450.9764\right\} \end{array} \right] = 45450.9764\ lbs$$

$$\left\{\tau = 1986.4\ \ psi\right\} \le \left\{S = 22400\ \ psi\right\}$$

The weld strength is acceptable.

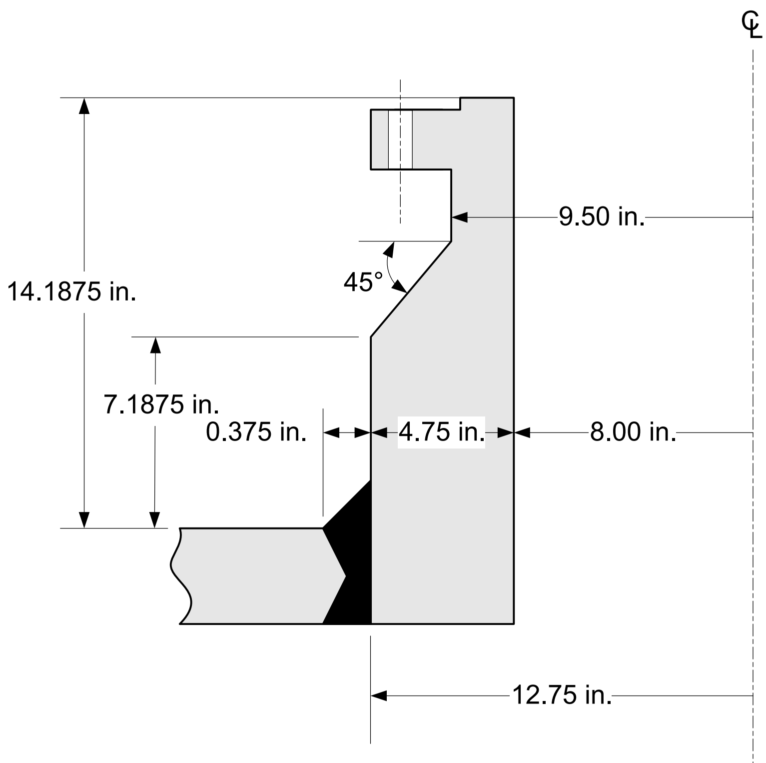


Figure E4.5.1 - Nozzle Detail

4.5.2 Example E4.5.2 – Hillside Nozzle in Cylindrical Shell and Weld Strength Analysis

Design an integral hillside nozzle in a cylindrical shell and perform a weld strength analysis. The parameters used in this design procedure are shown in Figure E4.5.2.

Vessel and Nozzle Data:

- Design Conditions = $356\ psig@300°F$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0
- Shell Material = $SA-516,\ Grade\ 70,\ Norm.$
- Shell Allowable Stress = $22400\ psi$
- Nozzle Material = $SA-105$
- Nozzle Allowable Stress = $21200\ psi$
- Shell Inside Diameter = $150.0\ in$
- Shell Thickness = $1.8125\ in$
- Nozzle Outside Diameter = $11.56\ in$
- Nozzle Thickness = $1.97\,in$
- External Nozzle Projection = $19.0610\ in$
- Internal Nozzle Projection = $0.0\,in$
- Distance from cylinder/nozzle centerlines = $34.875\ in$

The hillside nozzle is inserted through the shell, i.e. set–in type nozzle, see Figure 4.5.4.

Establish the corroded dimensions.

Shell:

$$D_i = 150.0 + 2\left(Corrosion\ Allowance\right) = 150.0 + 2(0.125) = 150.25\ in$$

$$t = 1.8125 - Corrosion\ Allowance = 1.8125 - 0.125 = 1.6875\ in$$

Nozzle:

$$t_n = 1.97 - Corrosion\ Allowance = 1.97 - 0.125 = 1.845\ in$$

$$R_n = \frac{D - 2\left(t_n\right)}{2} = \frac{11.56 - 2\left(1.845\right)}{2} = 3.935\ in$$

For a hillside nozzle in a cylindrical shell (see Figure 4.5.4), the design procedure in paragraph 4.5.5 shall be used with the following substitutions from paragraph 4.5.6.

$$R_{nc} = \max\left[\left(\frac{R_{ncl}}{2}\right),\ R_n\right]$$

Where,

$$R_{ncl} = R_{eff}\left(\theta_1 - \theta_2\right)$$

$$\theta_1 = \cos^{-1}\left[\frac{D_X}{R_{eff}}\right] = \cos^{-1}\left[\frac{34.875}{75.125}\right] = 62.3398\ deg = 1.0880\ rad$$

$$\theta_2 = \cos^{-1}\left[\frac{D_X + R_n}{R_{eff}}\right] = \cos^{-1}\left[\frac{34.875+3.935}{75.125}\right] = 58.8952\ deg = 1.0279\ rad$$

$$R_{ncl} = 75.125\left(1.0880 - 1.0279\right) = 4.5150\ in$$

$$R_{nc} = \max\left[\left(\frac{4.5150}{2}\right),\ 3.935\right] = 3.935\ in$$

The procedure in paragraph 4.5.5 is shown below.

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in paragraph 4.5.13 would need to be checked.

a) STEP 1 – Determine the effective radius of the shell as follows:

$$R_{eff} = 0.5D_i = 0.5\left(150.25\right) = 75.125\ in$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall:

For set–in, integrally reinforced nozzles:

$$L_R = \min\left[\sqrt{R_{eff}t},\ 2R_n\right] = \min\left[\sqrt{75.125(1.6875)},\ 2(3.935)\right] = 7.8700\ in$$

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface.

For set–in nozzles:

$$L_{H1} = \min\left[1.5t,\ t_e\right] + \sqrt{R_n t_n} = \min\left[1.5(1.6875),\ 0.0\right] + \sqrt{3.935(1.845)} = 2.6945\ in$$

$$L_{H2} = L_{pr1} = 19.0610\ in$$

$$L_{H3} = 8\left(t + t_e\right) = 8\left(1.6875 + 0.0\right) = 13.5\ in$$

$$L_H = \min\left[L_{H1},\ L_{H2},\ L_{H3}\right] + t = \min\left[2.6945,\ 19.0610,\ 13.5\right] + 1.6875 = 4.3820\ in$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable:

$$L_{I1} = \sqrt{R_n t_n} = \sqrt{3.935(1.845)} = 2.6945$$

$$L_{I2} = L_{pr2} = 0.0$$

$$L_{I3} = 8\left(t + t_e\right) = 8\left(1.6875 + 0.0\right) = 13.5\ in$$

$$L_I = \min\left[L_{I1},\ L_{I2},\ L_{I3}\right] = 0.0$$

e) STEP 5 – Determine the total available area near the nozzle opening, see Figures 4.5.1. Do not include any area that falls outside of the limits defined by L_H, L_R, and L_I.

For set–in nozzles:

$$A_T = A_1 + f_{rn}(A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp}A_5$$

$$A_1 = \left(tL_R\right)\cdot \max\left[\left(\frac{\lambda}{5}\right)^{0.85},\ 1.0\right] = 1.6875\left(7.8700\right)\cdot \max\left[\left(\frac{0.6067}{5}\right)^{0.85},\ 1.0\right] = 13.2806$$

$$\lambda = \min\left[\left\{\frac{\left(2R_n + t_n\right)}{\sqrt{\left(D_i + t_{eff}\right)t_{eff}}}\right\},\ 12.0\right] = \min\left[\left\{\frac{2\left(3.935\right) + 1.845}{\sqrt{\left(150.25 + 1.6875\right)\left(1.6875\right)}}\right\},\ 12.0\right] = 0.6067$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 1.6875 + \left(\frac{0.0\left(0.0\right)}{7.87}\right) = 1.6875\ in$$

$$f_{rn} = \frac{S_n}{S} = \frac{21200}{22400} = 0.9464$$

$$f_{rp} = \frac{S_p}{S} = 0.0$$

Since $\left\{t_n = 1.845\ in\right\} = \left\{t_{n2} = 1.845\ in\right\}$, calculate A_2 as follows:

$$A_2 = t_n L_H = 1.845\left(4.3820\right) = 8.0848\ in^2$$

$$A_3 = t_n L_I = 1.845\left(0.0\right) = 0.0$$

$$A_{41} = 0.5L_{41}^2 = 0.5\left(0.375\right) = 0.0703\ in^2$$

$$A_{42} = 0.5L_{42}^2 = 0.0$$

$$A_{43} = 0.5L_{43}^2 = 0.0$$

$$A_5 = \min\left[A_{5a},\ A_{5b}\right]$$

$$A_{5a} = Wt_e = 0.0$$

$$A_{5b} = L_R t_e = 0.0$$

$$A_5 = 0.0$$

$$A_T = 13.2806 + 0.9464\left(8.0848 + 0.0\right) + 0.0 + 0.0703 + 0.0 + 0.0 = 21.0024\ in^2$$

f) STEP 6 – Determine the applicable forces:

For set–in nozzles:

$$f_N = PR_{xn}L_H = 356(4.7985)(4.3820) = 7485.6216\ lbs$$

$$R_{xn} = \frac{t_n}{\ln\left[\dfrac{R_n + t_n}{R_n}\right]} = \frac{1.845}{\ln\left[\dfrac{3.935 + 1.845}{3.935}\right]} = 4.7985\ in$$

$$f_S = PR_{xs}(L_R + t_n) = 356(75.9656)(7.87 + 1.845) = 262730.0662\ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[\dfrac{R_{eff} + t_{eff}}{R_{eff}}\right]} = \frac{1.6875}{\ln\left[\dfrac{75.125 + 1.6875}{75.125}\right]} = 75.9656\ in$$

$$f_Y = PR_{xs}R_{nc} = 356(75.9656)(3.935) = 106417.1704\ lbs$$

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection:

$$\sigma_{avg} = \frac{(f_N + f_S + f_Y)}{A_T} = \frac{(7485.6216 + 262730.0662 + 106417.1704)}{21.0024} = 17932.8485\ psi$$

$$\sigma_{circ} = \frac{PR_{xs}}{t_{eff}} = \frac{356(75.9656)}{1.6875} = 16025.9281\ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection:

$$P_L = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\},\ \sigma_{circ}\right]$$

$$P_L = \max\left[\left\{2(17932.8485) - 16025.9281\right\},\ 16025.9281\right] = 19839.7689\ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy Equation 4.5.56. If the nozzle is subjected to internal pressure, then the allowable stress, S_{allow}, is given by Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by Equation 4.5.58 where F_{ha} is evaluated in paragraph 4.4 for the shell geometry being evaluated (e.g. cylinder, spherical shell, or formed head). The allowable stress shall be the minimum of the shell or nozzle material evaluated at the design temperature.

$$\left\{P_L = 19839.7689\ psi\right\} \le \left\{S_{allow} = 1.5SE = 1.5(21200)(1.0) = 31800\ psi\right\}$$

j) STEP 10 – Determine the maximum allowable working pressure of the nozzle:

$$P_{\max 1} = \frac{S_{allow}}{\dfrac{2A_p}{A_T} - \dfrac{R_{xs}}{t_{eff}}} = \frac{1.5(21200)(1.0)}{\dfrac{2(1057.9575)}{21.0024} - \dfrac{75.9656}{1.6875}} = 570.6114\ psi$$

$$A_p = \frac{(f_N + f_S + f_Y)}{A_T} = \frac{(7485.6216 + 262730.0662 + 106417.1704)}{356} = 1057.9575\ in^2$$

$$P_{\max 2} = S\left(\frac{t}{R_{xs}}\right) = 22400\left(\frac{1.6875}{75.9656}\right) = 497.5936\ psi$$

$$P_{\max} = \min\left[P_{\max 1}, P_{\max 2}\right] = \min\left[570.6114,\ 497.5936\right] = 497.5936\ psi$$

The nozzle is acceptable because $P_{max} = 497.6\ psi$ is greater than the specified design pressure of $356\ psig$.

Weld Strength Analysis.

The procedure to evaluate attachment welds of nozzles in a cylindrical, conical, or spherical shell or formed head subject to pressure loading per paragraph 4.5.14 is shown below.

a) STEP 1 – Determine the discontinuity force factor,

For set–in Nozzles:

$$k_y = \frac{R_{nc} + t_n}{R_{nc}} = \frac{3.935 + 1.845}{3.935} = 1.4689$$

b) STEP 2 – Calculate weld length resisting continuity force,

Weld length of nozzle to shell weld, for non–radial nozzles:

$$L_\tau = \frac{\pi}{2}\sqrt{\frac{\left(R_{nc} + t_n\right)^2 + \left(R_n + t_n\right)^2}{2}} = \frac{\pi}{2}\sqrt{\frac{\left(3.935 + 1.845\right)^2 + \left(3.935 + 1.845\right)^2}{2}} = 9.0792\ in$$

Weld length of pad to shell weld, for non–radial nozzles:

$$L_{\tau p} = \frac{\pi}{2}\sqrt{\frac{\left(R_{nc} + t_n + W\right)^2 + \left(R_n + t_n + W\right)^2}{2}} \qquad Not\ Applicable$$

c) STEP 3 – Compute the weld throat dimensions, as applicable

$$L_{41T} = 0.7071 L_{41} = 0.7071\left(0.375\right) = 0.2652\ in$$

$$L_{42T} = 0.0$$

$$L_{43T} = 0.0$$

d) STEP 4 – Determine if the weld sizes are acceptable. If the nozzle is integrally reinforced, and the computed shear stress in the weld given by Equation (4.5.182) satisfies Equation (4.5.183), then the design is complete. If the shear stress in the weld does not satisfy Equation (4.5.183), increase the weld size and return to Step 3.

$$\tau = \frac{f_{welds}}{L_\tau\left(0.49 L_{41T} + 0.6 t_{w1} + 0.49 L_{43T}\right)}$$

$$\tau = \frac{9341.4397}{9.0792\left(0.49\left(0.2652\right) + 0.6\left(1.6875\right) + 0.49\left(0.0\right)\right)} = 900.5955\ psi$$

Where,

$$f_{welds} = \min\left[f_Y k_y,\ 1.5S_n\left(A_2 + A_3\right),\ \frac{\pi}{4}PR_n^2k_y^2 \right]$$

$$f_{welds} = \min\left[\begin{array}{l} \{106417.1704(1.4689) = 156316.1816\} \\ \{1.5(21200)(8.0848+0) = 257096.64\} \\ \left\{\frac{\pi}{4}(356)(3.935)^2(1.4689)^2 = 9341.4397\right\} \end{array} \right] = 9341.4397\ lbs$$

$$\{\tau = 900.6\ \ psi\} \le \{S = 22400\ \ psi\}$$

The weld strength is acceptable.

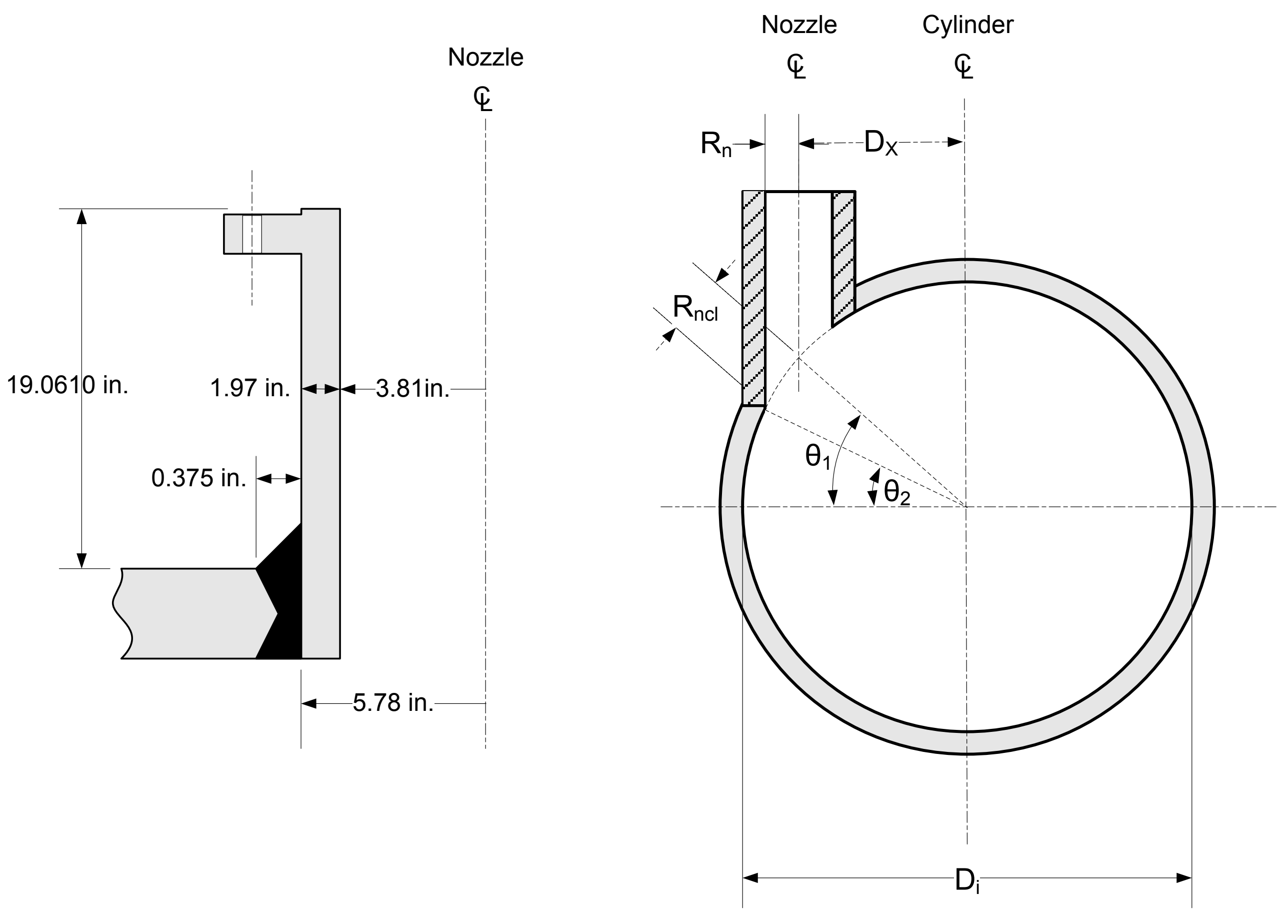


Figure E4.5.2 - Nozzle Detail

4.5.3 Example E4.5.3 – Radial Nozzle in Elliptical Head and Weld Strength Analysis

Design an integral radial nozzle centered in a 2:1 elliptical head and perform a weld strength analysis. The parameters used in this design procedure are shown in Figure E4.5.3.

Vessel and Nozzle Data:

- Design Conditions = $356\ psig@300°F$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0
- Head Material = $SA-516,\ Grade\ 70,\ Norm.$
- Head Allowable Stress = $22400\ psi$
- Nozzle Material = $SA-105$
- Nozzle Allowable Stress = $21200\ psi$
- Head Inside Diameter = $90.0\ in$
- Height of the 2:1 Elliptical Head = $22.625\ in$
- Head Thickness = $1.0\ in$
- Nozzle Outside Diameter = $15.94\ in$
- Nozzle Thickness = $2.28\ in$
- External Nozzle Projection = $13.5\ in$
- Internal Nozzle Projection = $0.0 in$
- Distance from head/nozzle centerlines = $0.0\ in$

The nozzle is inserted centrally through the head, i.e. set–in type nozzle, see Figure 4.5.9.

Establish the corroded dimensions.

Head:

$$D_i = 90.0 + 2(Corrosion\ Allowance) = 90.0 + 2(0.125) = 90.25\ in$$

$$R = \frac{D_i}{2} = \frac{90.25}{2} = 45.125\ in$$

$$t = 1.0 - Corrosion\ Allowance = 1.0 - 0.125 = 0.875\ in$$

Nozzle:

$$t_n = 2.28 - Corrosion\ Allowance = 2.28 - 0.125 = 2.155\ in$$

$$R_n = \frac{D - 2(t_n)}{2} = \frac{15.94 - 2(2.155)}{2} = 5.815\ in$$

The procedure, per paragraph 4.5.10, to design a radial nozzle in an elliptical head subject to pressure loading is shown below.

Note: This is an analysis of a single nozzle; therefore, the spacing criterion is automatically satisfied. If there were multiple nozzles in the shell, the spacing requirements for nozzles in paragraph 4.5.13 would need to be checked.

a) STEP 1 – Determine the effective radius of the shell or formed head as follows.

For ellipsoidal heads:

$$R_{eff}=\frac{0.9D_i}{6}\left[2+\left(\frac{D_i}{2h}\right)^2\right]=\frac{0.9(90.25)}{6}\left[2+\left(\frac{90.25}{2(22.625)}\right)^2\right]=80.9262\ in$$

b) STEP 2 – Calculate the limit of reinforcement along the vessel wall.

For set–in, integrally reinforced nozzles in ellipsoidal heads,

$$L_R=\min\left[\sqrt{R_{eff}t},\ 2R_n\right]=\min\left[\sqrt{80.9262(0.875)},\ 2(5.8150)\right]=8.4149\ in$$

c) STEP 3 – Calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface, see Figures 4.5.9 and 4.5.10.

For set–in nozzles,

$$L_H=\min\left[t+t_e+F_p\sqrt{R_nt_n},\ L_{pr1}+t\right]$$

For ellipsoidal heads,

$$X_o=\min\left[D_R+(R_n+t_n)\cdot\cos[\theta],\ \frac{D_i}{2}\right]$$

$$X_o=\min\left[0.0+(5.8150+2.1550)\cdot\cos[0.0],\ \frac{90.25}{2}\right]=7.9700\ in$$

$$\theta=\arctan\left[\left(\frac{h}{R}\right)\cdot\left(\frac{D_R}{\sqrt{R^2-D_R^2}}\right)\right]=\arctan\left[\left(\frac{22.625}{45.125}\right)\cdot\left(\frac{0.0}{\sqrt{45.125^2-0.0^2}}\right)\right]=0.0\ rad$$

Since $\{X_o=7.9700\ in\}\le\{0.35D_i=0.35(90.25)=31.5875\ in\}$, calculate F_p as follows:

$$F_p=\left\{C_n=\min\left[\left(\frac{t+t_e}{t_n}\right)^{0.35},1.0\right]=\min\left[\left(\frac{0.875+0.0}{2.1550}\right)^{0.35},1.0\right]=0.7295\right\}$$

$$L_H=\min\left[0.875+0.0+(0.7295)\sqrt{5.8150(2.1550)},\ 13.5+0.875\right]=3.4574\ in$$

d) STEP 4 – Calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface, if applicable.

$$L_{pr2}=0.0$$

$$L_I=\min\left[F_p\sqrt{R_nt_n},L_{pr2}\right]=0.0$$

e) STEP 5 – Determine the total available area near the nozzle opening, see Figures 4.5.1, where f_{rn} and f_{rp} are given by Equations (4.5.21) and (4.5.22) respectively. Do not include any area that falls outside of the limits defined by L_H, L_R, and L_I.

For set–in nozzles:

$$A_T = A_1 + f_{rn}(A_2 + A_3) + A_{41} + A_{42} + A_{43} + f_{rp}A_5$$

$$A_1 = tL_R = 0.875(8.4149) = 7.3630\ in^2$$

Since $\{t_n = 2.1550\ in\} = \{t_{n2} = 2.1550\ in\}$, calculate A_2 as follows:

$$A_2 = t_n L_H = 2.1550(3.4574) = 7.4507\ in^2$$

$$A_3 = t_n L_I = 0.0$$

$$A_{41} = 0.5L_{41}^2 = 0.5(0.375)^2 = 0.0703\ in^2$$

$$A_{42} = 0.5L_{42}^2 = 0.0$$

$$A_{43} = 0.5L_{43}^2 = 0.0$$

$$t_e = 0.0\ in$$

$$A_{5a} = Wt_e = 0.0$$

$$A_{5b} = (L_R - t_n)t_e = 0.0$$

$$A_5 = \min[A_{5a}, A_{5b}] = 0.0$$

$$f_{rn} = \frac{S_n}{S} = \frac{21200}{22400} = 0.9464$$

$$f_{rp} = \frac{S_p}{S} = 0.0$$

$$A_T = 7.363 + 0.9464(7.4507 + 0.0) + 0.0703 + 0.0 + 0.0 + 0.0(0.0) = 14.4846\ in^2$$

f) STEP 6 – Determine the applicable forces.

For set–in nozzles,

$$f_N = PR_{xn}L_H = 356(6.8360)(3.4572) = 8413.4972\ lbs$$

$$R_{xn} = \frac{t_n}{\ln\left[1 + \dfrac{t_n}{R_n}\right]} = \frac{2.1550}{\ln\left[1 + \dfrac{2.1550}{5.8150}\right]} = 6.8360\ in$$

$$f_S = \frac{PR_{xs}(L_R + t_n)}{2} = \frac{356(81.3629)(8.4149 + 2.1550)}{2} = 153079.5936\ lbs$$

$$R_{xs} = \frac{t_{eff}}{\ln\left[1 + \dfrac{t_{eff}}{R_{eff}}\right]} = \frac{0.875}{\ln\left[1 + \dfrac{0.875}{80.9262}\right]} = 81.3629\ in$$

$$t_{eff} = t + \left(\frac{A_5 f_{rp}}{L_R}\right) = 0.875 + \left(\frac{0.0}{8.4149}\right) = 0.875\ in$$

$$f_Y = \frac{PR_{xs}R_{nc}}{2} = \frac{356(81.3629)(5.8150)}{2} = 84216.2969\ lbs$$

g) STEP 7 – Determine the average local primary membrane stress and the general primary membrane stress at the nozzle intersection.

$$\sigma_{avg} = \frac{(f_N + f_S + f_Y)}{A_T} = \frac{8413.4972 + 153079.5936 + 84216.2969}{14.4846} = 16963.4914\ psi$$

$$\sigma_{circ} = \frac{PR_{xs}}{2t_{eff}} = \frac{356(81.3629)}{2(0.875)} = 16551.5385\ psi$$

h) STEP 8 – Determine the maximum local primary membrane stress at the nozzle intersection.

$$P_L = \max\left[\left\{2\sigma_{avg} - \sigma_{circ}\right\},\ \sigma_{circ}\right]$$

$$P_L = \max\left[\left\{2(16963.4914) - 16551.5385\right\},\ 16551.5385\right] = 17375.4443\ psi$$

i) STEP 9 – The calculated maximum local primary membrane stress should satisfy Equation 4.5.146. If the nozzle is subjected to internal pressure, then the allowable stress, S_{allow}, is given by Equation 4.5.57. If the nozzle is subjected to external pressure, then the allowable stress is given by Equation 4.5.58.

$$\left\{P_L = 17375.4443\right\} \le \left\{S_{allow} = 1.5SE = 1.5(21200)(1.0) = 31800\ psi\right\}$$

j) STEP 10 – Determine the maximum allowable working pressure of the nozzle.

$$P_{\max 1} = \frac{S_{allow}}{\left(\frac{2A_p}{A_T}\right) - \left(\frac{R_{xs}}{2t_{eff}}\right)} = \frac{1.5(21200)(1.0)}{\left(\frac{2(690.1949)}{14.4846}\right) - \left(\frac{81.3629}{2(0.875)}\right)} = 651.5402\ psi$$

$$A_p = \frac{(f_N + f_S + f_Y)}{P}$$

$$A_p = \frac{8413.4972 + 153079.5936 + 84216.2969}{356} = 690.1949\ in^2$$

$$P_{\max 2} = 2S\left(\frac{t}{R_{xs}}\right) = 2(22400)\left(\frac{0.875}{81.3629}\right) = 481.7921\ psi$$

$$P_{\max} = \min\left[P_{\max 1},\ P_{\max 2}\right] = \min\left[651.5402,\ 481.7921\right] = 481.7921\ psi$$

The nozzle is acceptable because $P_{max} = 481.7921\ psi$ is greater than the specified design pressure of $356\ psig$.

Weld Strength Analysis

The procedure to evaluate attachment welds of nozzles in a cylindrical, conical, or spherical shell or formed head subject to pressure loading per paragraph 4.5.14.2 is shown below.

a) STEP 1 – Determine the discontinuity force factor,

For set–in Nozzles:

$$k_y = \frac{R_{nc} + t_n}{R_{nc}} = \frac{5.8150 + 2.155}{5.8150} = 1.3706$$

Note, for radial nozzles, $R_{nc} = R_n$.

b) STEP 2 – Calculate weld length resisting continuity force,

Weld length of nozzle to shell weld, for radial nozzles:

$$L_\tau = \frac{\pi}{2}\left(R_n + t_n\right) = \frac{\pi}{2}\left(5.8150 + 2.155\right) = 12.5192\ in$$

Weld length of pad to shell weld, for radial nozzles:

$$L_{\tau p} = \frac{\pi}{2}\left(R_n + t_n + W\right) \qquad Not\ Applicable$$

c) STEP 3 – Compute the weld throat dimensions, as applicable.

$$L_{41T} = 0.7071 L_{41} = 0.7071\left(0.375\right) = 0.2652\ in$$

$$L_{42T} = 0.7071 L_{42} = 0.7071\left(0.0\right) = 0.0$$

$$L_{43T} = 0.7071 L_{43} = 0.7071\left(0.0\right) = 0.0$$

e) STEP 4 – Determine if the weld sizes are acceptable. If the nozzle is integrally reinforced, and the computed shear stress in the weld given by Equation (4.5.182) satisfies Equation (4.5.183), then the design is complete. If the shear stress in the weld does not satisfy Equation (4.5.183), increase the weld size and return to Step 3. For nozzles on heads, A_2 and A_3 are to be calculated using $F_p = 1.0$, when computing f_{welds} using Equation (4.5.184).

From STEP 3 of paragraph 4.5.10, re–calculate the limit of reinforcement along the nozzle wall projecting outside the vessel surface using $F_p = 1.0$.

$$L_H = \min\left[t + t_e + F_p\sqrt{R_n t_n},\ L_{pr1} + t\right]$$

$$L_H = \min\left[0.875 + 0.0 + \left(1.0\right)\sqrt{5.8150\left(2.1550\right)},\ 13.5 + 0.875\right] = 4.4150\ in$$

Re–calculate the values of A_2 using the new value of L_H.

$$A_2 = t_n L_H = 2.1550\left(4.4150\right) = 9.5143\ in^2$$

From STEP 4 of paragraph 4.5.10, re–calculate the limit of reinforcement along the nozzle wall projecting inside the vessel surface using $F_p = 1.0$. Since the nozzle does not have an internal projection, the value of $L_I = 0.0$; therefore, $A_3 = 0.0\ in^2$.

$$\tau = \frac{f_{welds}}{L_\tau\left(0.49L_{41T} + 0.6t_{w1} + 0.49L_{43T}\right)}$$

$$\tau = \frac{17760.7284}{12.5192\left(0.49(0.2652) + 0.6(0.875) + 0.49(0.0)\right)} = 2166.0944\ psi$$

Where,

$$f_{welds} = \min\left[f_Y k_y,\ 1.5S_n\left(A_2 + A_3\right),\ \frac{\pi}{4}PR_n^2k_y^2\right]$$

$$f_{welds} = \min\left[\begin{array}{l}\{219730.4980(1.3706) = 301162.6206\}, \\ \{1.5(21200)(9.5143+0) = 302554.74\}, \\ \left\{\frac{\pi}{4}(356)(5.8150)^2(1.3706)^2 = 17760.7284\right\}\end{array}\right] = 17760.7284\ lbs$$

$$\{\tau = 2166.1\ psi\} \le \{S = 22400\ psi\}$$

The weld strength is acceptable.

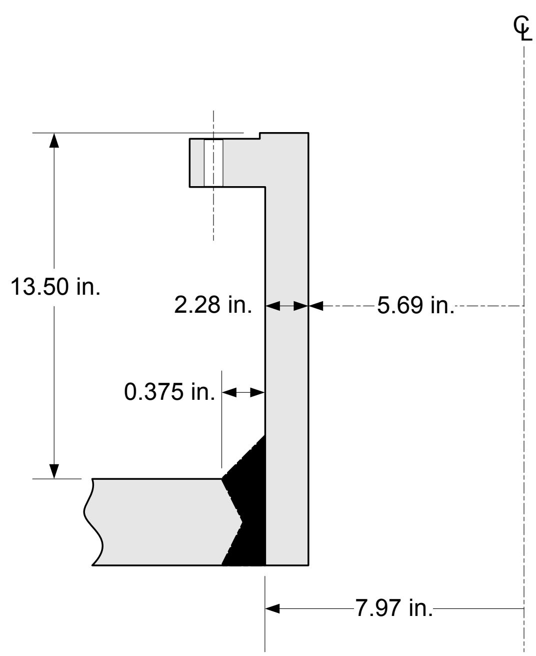


Figure E4.5.3 - Nozzle Details

4.6 Flat Heads

4.6.1 Example E4.6.1 – Flat Un-stayed Circular Heads

Determine the required thickness for a heat exchanger blind flange.

Blind Flange Data:

- Material = $SA-105$
- Design Conditions = $135\ psig\ @\ 650°F$
- Flange Bolt-Up Temperature = $100°F$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $17800\ psi$
- Allowable Stress at Flange Bolt-Up Temp. = $24000\ psi$
- Weld Joint Efficiency = 1.0
- Mating flange information and gasket details are provided in Example Problem E4.16.1.

Using the procedure in paragraph 4.6.2.3, calculate the minimum required thickness for the heat exchanger blind flange.

The minimum required thickness of a flat unstayed non-circular head, cover, or blind flange that is attached with bolting that results in an edge moment (see Table 4.6.1, Detail 7) shall be calculated by the equations shown below. The operating and gasket seating bolt loads, W_o and W_g, and the moment arm of this load, h_G, in these equations shall be computed based on the flange geometry and gasket material as described in paragraph 4.16.

a) STEP 1 – Calculate the gasket moment arm, h_G, and the diameter of the gasket load reaction d in accordance with paragraph 4.16, as demonstrated in Example Problem E4.16.1.

Flange Design Procedure, STEP 6: $h_G = 0.875\ in$

Design Bolt Loads, STEP 3: $d = G = 29.5\ in$

b) STEP 2 – Calculate the operating and gasket seating bolt loads, W_o and W_g, in accordance with paragraph 4.16, as demonstrated in Example Problem E4.16.1.

Design Bolt Loads, STEP 4: $W_o = 111329.5\ lbs$

Design Bolt Loads, STEP 5: $W_g = 237626.3\ lbs$

c) STEP 3 – Identify the appropriate attachment factor, C, from Table 4.6.1.

Per Detail 7: $C = 0.3$

d) STEP 4 - The required thickness of the blind flange is the maximum of the thickness required for the operating and gasket seating conditions.

$$t = \max\left[t_o, t_g\right]$$

1) The required thickness in the operating condition is in accordance with Equation (4.6.3):

$$t_o = d\sqrt{\left(\frac{CP}{S_{ho}E}\right) + \left(\frac{1.9W_o h_G}{S_{ho}Ed^3}\right)} + CA$$

$$t_o = (29.5)\sqrt{\left(\frac{0.3(135)}{17800(1.0)}\right) + \left(\frac{1.9(111329.5)(0.875)}{17800(1.0)(29.5)^3}\right)} + 0.125 = 1.6523\ in$$

2) The required thickness in the gasket seating condition is in accordance with Equation (4.6.4):

$$t_g = d\sqrt{\frac{1.9W_g h_G}{S_{hg}Ed^3}} + CA$$

$$t_g = (29.5)\sqrt{\frac{1.9(237626.3)(0.875)}{24000(1.0)(29.5)^3}} + 0.125 = 0.8720\ in$$

$$t = \max\left[1.6523, 0.8720\right] = 1.6523\ in$$

The required thickness is $1.6523\ in$.

4.6.2 Example E4.6.2 – Flat Un-stayed Non-Circular Heads Attached by Welding

Determine the required thickness for an air-cooled heat exchanger end plate. The end plate is welded to the air-cooled heat exchanger box with full penetration Category C, Type 7 corner joints.

End Plate Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $400\ psig\ @\ 500°F$
- Short Span Length = $7.125\ in$
- Long Span Length = $9.25\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20600\ psi$
- Weld Joint Efficiency = 1.0

Using the procedure in paragraph 4.6.3.1, calculate the minimum required thickness for the end plate.

The minimum required thickness of a flat unstayed non-circular head or cover that is not attached with bolting that results in an edge moment shall be calculated by the following equations.

a) STEP 1 – Determine the short and long span dimensions of the non-circular plate, d and D, respectively (in the corroded state) as demonstrated in Example Problem E4.12.1.

$$d = 7.125 + 2(0.125) = 7.375\ in$$

$$D = 9.250 + 2(0.125) = 9.500\ in$$

Note, the variables d and D used in paragraph 4.6.3 are denoted as H and h, respectively in paragraph 4.12.

b) STEP 2 – Calculate the Z factor in accordance with Equation (4.6.6).

$$Z = \min\left[2.5, \left(3.4 - \left(\frac{2.4d}{D}\right)\right)\right] = \min\left[2.5, \left(3.4 - \left(\frac{2.4(7.375)}{9.5}\right)\right)\right] = 1.5368\ in$$

c) STEP 3 - The appropriate attachment factor, C, is taken from paragraph 4.12.2.6. For end closures of non-circular vessels constructed of flat plate, the design rules of paragraph 4.6 shall be used except that 0.20 shall be used for the value of C in all of the calculations.

$$C = 0.2$$

d) STEP 4 - Calculate the required thickness, t using Equation (4.6.5).

$$t = d\sqrt{\frac{ZCP}{S_{ho}E}} + CA = 7.375\sqrt{\frac{1.5368(0.2)(400)}{20600(1.0)}} + 0.125 = 0.6947\ in$$

The required thickness is $0.6947\ in$

4.7 Spherically Dished Bolted Covers

4.7.1 Example E4.7.1 – Thickness Calculation for a Type D Head

Determine if the proposed Type D spherically dished bolted cover is adequately designed, considering the following design conditions. The spherically dished head is seamless. See Figure E4.7.1 for details.

Tubeside Data:

- Design Conditions = $213\ psig\ @\ 400°F$
- Corrosion Allowance (CAT) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Shellside Data:

- Design Conditions = $305\ psig\ @\ 250°F$
- Corrosion Allowance (CAS) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Flange Data:

- Material = $SA-105$
- Allowable Stress at Ambient Temperature = $24000\ psi$
- Allowable Stress at Tubeside Design Temperature = $20500\ psi$
- Allowable Stress at Shellside Design Temperature = $21600\ psi$

Head Data:

- Material = $SA-515,\ Grade\ 60$
- Allowable Stress At Ambient Temperature = $21300\ psi$
- Allowable Stress at Tubeside Design Temperature = $18200\ psi$
- Allowable Stress at Shellside Design Temperature = $19200\ psi$
- Yield Stress at Shellside Design Temperature = $28800\ psi$
- Modulus of Elasticity at Shellside Design Temp. = $28.55E+06\ psi$

Bolt Data:

- Material = $SA-193,\ Grade\ B7$
- Diameter = $0.75\ in$
- Cross-Sectional Root Area = $0.302\ in^2$
- Number of Bolts = 20
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Allowable Stress at Tubeside Design Temperature = $25000\ psi$
- Allowable Stress at Shellside Design Temperature = $25000\ psi$

Gasket Data

• Material	=	Solid Flat Metal (Iron/Soft Steel)
• Gasket Factor	=	5.5
• Gasket Seating Factor	=	$18000\ psi$
• Inside Diameter	=	$16.1875\ in$
• Outside Diameter	=	$17.0625\ in$

Per paragraph 4.7.1.3, calculations shall be performed using dimensions in the corroded condition and the uncorroded condition, and the more severe case shall control. This example only evaluates the spherically dished bolted cover in the corroded condition.

Per paragraph 4.7.5.1, the thickness of the head for a Type D Head Configuration (see Figure 4.7.4) shall be determined by the following equations.

a) Internal pressure (pressure on the concave side) – the head thickness shall be determined using Equation (4.7.2).

$$t = \left(\frac{5PL}{6S}\right) = \frac{5(213)(16.125)}{6(18200)} = 0.1573\ in$$

Where,

$$L = 16.0 + CAT = 16.0 + 0.125 = 16.125\ in$$

This thickness is increased for the corrosion allowance on both the shell and tube side.

$$t = t + CAS + CAT = 0.1573 + 0.125 + 0.125 = 0.4073\ in$$

b) External pressure (pressure on the convex side) – the head thickness shall be determined in accordance with the ruled in paragraph 4.4.

Per paragraph 4.4.7.1, the required thickness of a spherical shell or hemispherical head subjected to external pressure loading shall be determined using the following procedure

1) STEP 1 - Assume an initial thickness, t, for the spherical shell.

The specified head thickness shall consider corrosion from tubeside and shellside, resulting in the following.

$$t = t - CAS - CAT$$

$$t = 0.875 - 0.125 - 0.125 = 0.625\ in$$

2) STEP 2 - Calculate the predicted elastic buckling stress, F_{he}.

$$F_{he} = 0.075 E_y \left(\frac{t}{R_o}\right) = 0.075(28.55E+06)\left(\frac{0.625}{16.75}\right) = 79897.3881\ psi$$

Where,

$$R_o = L + t = 16.125 + 0.625 = 16.75\ in$$

3) STEP 3 - Calculate the predicted buckling stress, F_{ic}.

$$\frac{F_{he}}{S_y} = \frac{79897.3881}{28800} = 2.7742$$

Since $1.6 < \frac{F_{he}}{S_y} < 6.25$, use Equation (4.4.55), to calculate F_{ic}

$$F_{ic} = \frac{1.31S_y}{\left(1.15 + \frac{S_y}{F_{he}}\right)} = \frac{1.31(28800)}{\left(1.15 + \frac{28800}{79897.3881}\right)} = 24977.7825\ psi$$

4) STEP 4 - Calculate the value of the design margin, FS per paragraph 4.4.2.

$$0.55S_y = 0.55(28800) = 15840\ psi$$

Since $0.55S_y < F_{ic} < S_y$, use Equation (4.4.2) to calculate FS.

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{24977.7825}{28800}\right) = 1.7643$$

5) STEP 5 - Calculate the allowable external pressure, P_a.

$$P_a = 2F_{ha}\left(\frac{t}{R_o}\right) = 2(14157.3329)\left(\frac{0.625}{16.75}\right) = 1056.5\ psi$$

Where,

$$F_{ha} = \frac{F_{ic}}{FS} = \frac{24977.7825}{1.7643} = 14157.3329\ psi$$

6) STEP 6 - If the allowable external pressure, P_a, is less than the design external pressure, increase the shell thickness and go to STEP 2.

Since $\{P_a = 1056.5\ psi\} > \{P = 305\ psi\}$, the specified head thickness is acceptable for external pressure.

The flange thickness of the head for a Type D Head Configuration is determined per paragraph 4.7.5.2. To compute the flange thickness calculations, the flange operating and gasket seating moments are determined using the flange design procedure from paragraphs 4.16.6 and 4.16.7.

Paragraph 4.16.6: Design Bolt Loads. The procedure to determine the bolt loads for the operating and gasket seating conditions is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint.

$$Tubeside\ Conditions: P = 213\ psig\ at\ 400°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 4.16.1.

$$m = 5.5$$
$$y = 18000\ psi$$

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5(GOD - GID) = 0.5(17.0625 - 16.1875) = 0.4375\ in$$

From Table 4.16.3, Facing Sketch Detail 2, Column I,

$$b_o = \frac{w+N}{4} = \frac{(0.125+0.4375)}{4} = 0.1406\ in$$

Where,

$$w = raised\ nubbin\ width = 0.125\ in$$

For $b_o \le 0.25\ in$,

$$b = b_o = 0.1406\ in$$
$$G = mean\ diameter\ of\ the\ gasket\ contact\ face$$
$$G = 0.5(17.0625 + 16.1875) = 16.625\ in$$

d) STEP 4 – Determine the design bolt load for the operating condition.

$$W_o = \frac{\pi}{4}G^2P + 2b\pi GmP \qquad for\ non-self-energized\ gaskets$$
$$W_o = \frac{\pi}{4}(16.625)^2(213) + 2(0.1406)\pi(16.625)(5.5)(213) = 63442.9\ lbs$$

e) STEP 5 – Determine the design bolt load for the gasket seating condition.

$$W_g = \left(\frac{A_m + A_b}{2}\right)S_{bg} = \left(\frac{5.2872+6.04}{2}\right)25000 = 141590.0\ lbs$$

Where,

$$A_b = (Number\ of\ bolts)(Root\ area\ of\ one\ bolt) = (20)(0.302) = 6.04\ in^2$$

$$A_m = \max\left[\left(\frac{W_o + F_A + \frac{4M_E}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right] = \max\left[\left(\frac{63442.9+0+0}{25000}\right), \left(\frac{132181.1}{25000}\right)\right]$$

$$A_m = \max[2.5377,\ 5.2872] = 5.2872\ in^2$$

Note, $F_A = 0$ and $M_E = 0$ since there are no externally applied net-section axial forces or bending moments.

And,

$$W_{gs} = \pi b G\left(C_{us} y\right) \qquad for\ non-self-energized\ gaskets$$
$$W_{gs} = \pi\left(0.1406\right)\left(16.625\right)\left(1.0\left(18000\right)\right) = 132181.1\ lbs$$

Paragraph 4.16.7: Flange Design Procedure. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint and the external net-section axial force, F_A, and bending moment, M_E.

$$Tubeside\ Conditions: P = 213\ psig\ at\ 400°F$$
$$Shellside\ Conditions: P = 305\ psig\ at\ 250°F$$

$$F_A = 0$$

$$M_E = 0$$

b) STEP 2 – Determine the design bolt loads for operating condition W_o, and the gasket seating condition W_g, and the corresponding actual bolt load area A_b, from paragraph 4.16.6.

$$W_o = 63442.9\ lbs$$

$$W_g = 141590.0\ lbs$$

$$A_b = 6.04\ in^2$$

c) STEP 3 – Determine an initial flange geometry (see Figure E4.7.1), in addition to the information required to determine the bolt load, the following geometric parameters are required.

1) Flange bore

$$B = \left[16.25 + 2\left(CAT\right)\right] = \left[16.25 + 2\left(0.125\right)\right] = 16.50\ in$$

2) Bolt circle diameter

$$C = 18.125\ in$$

3) Outside diameter of the flange

$$A = \left[19.625 - 2\left(CAS\right)\right] = \left[19.625 - 2\left(0.125\right)\right] = 19.375\ in$$

4) Flange thickness, (see Figure E4.7.1)

$$T = 2.375 - 2\left(CAS\right) = 2.375 - 2\left(0.125\right) = 2.125\ in$$

5) Thickness of the hub at the large end

$$Not\ Applicable$$

6) Thickness of the hub at the small end

$$Not\ Applicable$$

7) Hub length

$$Not\ Applicable$$

d) STEP 4 – Determine the flange stress factors using the equations in Table 4.16.4 and 4.16.5.

$$Not\ \ Applicable$$

e) STEP 5 – Determine the flange forces.

Tubeside Conditions:

$$H_D = \frac{\pi}{4}B^2P = \frac{\pi}{4}(16.5)^2(213) = 45544.7\ lbs$$

$$H = \frac{\pi}{4}G^2P = \frac{\pi}{4}(16.625)^2(213) = 46237.3\ lbs$$

$$H_T = H - H_D = 46237.3 - 45544.7 = 692.6\ lbs$$

$$H_G = W_o - H = 63442.9 - 46237.3 = 17205.6\ lbs$$

Shellside Conditions:

$$H_D = \frac{\pi}{4}B^2P = \frac{\pi}{4}(16.5)^2(305) = 65216.5\ lbs$$

$$H = \frac{\pi}{4}G^2P = \frac{\pi}{4}(16.625)^2(305) = 66208.4\ lbs$$

$$H_T = H - H_D = 66208.4 - 65216.5 = 991.9\ lbs$$

$$H_G = Not\ \ Applicable$$

f) STEP 6 – Determine the flange moment for the operating condition using Equation (4.16.14) or Equation (4.16.15), as applicable. When specified by the user or his designated agent, the maximum bolt spacing (B_{smax}) and the bolt spacing correction factor (B_{sc}) shall be applied in calculating the flange moment for internal pressure using the equations in Table 4.16.11. The flange moment M_o for the operating condition and flange moment M_g for the gasket seating condition without correction for bolt spacing $B_{sc} = 1$ is used for the calculation of the rigidity index in Step 10. In these equations, h_D is determined from Equation (4.7.21), as referenced in paragraph 4.7.5.2, and h_T and h_G are determined from Table 4.16.6. Since $F_A = 0$ and $M_E = 0$, the flange cross-section bending moment of inertia, I, and polar moment of inertia, I_p, need not be calculated; and the flange design moment calculation for net-section bending moment and axial force supplemental loads, $M_{oe} = 0$. Additionally, $F_s = 1.0$ for non-split rings.

For internal pressure (Tubeside Conditions):

$$M_o = abs\left[\left(\left(H_D h_D + H_T h_T + H_G h_G\right)B_{sc} + M_{oe}\right)F_s\right]$$

$$M_o = abs\left[\left(\left(45544.7(0.8125) + 692.6(0.7813) + 17205.6(0.75)\right)1.0 + 0\right)1.0\right]$$

$$M_o = 50450.4\ in-lbs$$

For external pressure (Shellside Conditions):

$$M_o = abs\left[\left(\left(H_D h_D + H_T h_T + H_G h_G\right)B_{sc} + M_{oe}\right)F_s\right]$$

$$M_o = abs\left[\left(\left(65216.5(0.8125-0.75)+991.9(0.7813-0.75)\right)1.0+0\right)1.0\right]$$

$$M_o = 4107.1\ in-lbs$$

From Equation (4.7.21),

$$h_D = \frac{C-B}{2} = \frac{18.125-16.50}{2} = 0.8125\ in$$

From Table 4.16.6, for loose type flanges,

$$h_G = \frac{C-G}{2} = \frac{18.125-16.625}{2} = 0.75\ in$$

$$h_T = \frac{h_D + h_T}{2} = \frac{0.8125+0.75}{2} = 0.7813\ in$$

g) STEP 7 – Determine the flange moment for the gasket seating condition using Equation (4.16.17) or Equation (4.16.18), as applicable.

For internal pressure (Tubeside Conditions):

$$M_g = \frac{W_g\left(C-G\right)B_{sc}F_s}{2} = \frac{(141590.0)(18.125-16.625)(1.0)(1.0)}{2} = 106192.5\ in-lbs$$

For external pressure (Shellside Conditions):

$$M_g = W_g h_G F_s = (141590.0)(0.75)(1.0) = 106192.5\ in-lbs$$

Per paragraph 4.7.5.2, the flange thickness of the head for a Type D Head Configuration shall be determined by the following equations. When determining the flange design moment for the design condition, M_o, an additional moment term, M_r, computed using Equation (4.7.22) shall be added to M_o as defined in paragraph 4.16. The term M_{oe} in the equation for M_o as defined in paragraph 4.16 shall be set to zero in this calculation. Note that this term may be positive or negative depending on the location of the head–to–flange ring intersection with relation to the flange ring centroid. Since the head–to–flange ring intersection is above the flange centroid, the sign of the M_r value is negative.

$$T = \max\left[T_g, T_o\right] = \max\left[T_g, \max\left[T_{o(tubeside)}, T_{o(shellside)}\right]\right]$$

Where,

$$T_g = \sqrt{\frac{M_g}{S_{fg}B}\left(\frac{A+B}{A-B}\right)} + CAS + CAS$$

$$T_o = Q + \sqrt{Q^2 + \frac{M_o}{S_{fo} B}\left(\frac{A+B}{A-B}\right)} + CAS + CAS$$

And,

$$Q = \frac{|P| B \sqrt{4L^2 - B^2}}{8 S_{fo} (A-B)}$$

Gasket Seating Conditions:

$$T_g = \sqrt{\left(\frac{106192.5}{(24000)(16.5)}\right) \cdot \left(\frac{(19.375+16.5)}{(19.375-16.5)}\right)} + 0.125 + 0.125 = 2.0793\ in$$

Tubeside Conditions:

$$T_o = 0.2065 + \sqrt{(0.2065)^2 + \left(\frac{30838.6}{(20500)(16.5)}\right) \cdot \left(\frac{(19.375+16.5)}{(19.375-16.5)}\right)} + 0.125 + 0.125$$

$$T_o = 1.5429\ in$$

Where,

$$Q = \frac{|P| B \sqrt{4L^2 - B^2}}{8 S_{fo} (A-B)} = \frac{|213|(16.5)\sqrt{\left(4(16.125)^2 - (16.5)^2\right)}}{8(20500)(19.375-16.5)} = 0.2065$$

$$M_o = M_o \pm M_r F_s = 50450.4 - 19611.8(1.0) = 30838.6\ in-lbs$$

$$M_r = H_r h_r = 78447.1(0.25) = 19611.8\ in-lbs$$

$$H_r = \left(0.785 B^2 P \cot[\beta]\right) = \left[(0.785)(16.5)^2 (213)\cot[30.1259]\right] = 78447.1\ lbs$$

$$\beta = \arcsin\left[\frac{B}{2L+t}\right] = \arcsin\left[\frac{(16.5)}{2(16.125)+(0.625)}\right] = \begin{Bmatrix} 0.5258\ rad \\ 30.1259\ deg \end{Bmatrix}$$

$$h_r = \frac{T}{2} - (X - CAS) = \frac{2.125}{2} - (0.9375 - 0.125) = 0.25\ in$$

Shellside Conditions:

$$T_o = 0.2807 + \sqrt{(0.2807)^2 + \left(\frac{23975.5}{(21600)(16.5)}\right) \cdot \left(\frac{(19.375+16.5)}{(19.375-16.5)}\right)} + 0.125 + 0.125$$

$$T_o = 1.2389\ in$$

Where,

$$Q = \frac{|P| B\sqrt{4L^2 - B^2}}{8S_{fo}(A-B)} = \frac{|305|(16.5)\sqrt{\left(4(16.125)^2 - (16.5)^2\right)}}{8(21600)(19.375-16.5)} = 0.2807$$

$$M_o = M_o \pm M_r F_s = 4107.1 - 28082.6(1.0) = -23975.5\ in-lbs$$

$$M_r = H_r h_r = 112330.3(0.25) = 28082.6\ in-lbs$$

$$H_r = \left(0.785B^2 P\cot[\beta]\right) = \left[(0.785)(16.5)^2(305)\cot[30.1259]\right] = 112330.3\ lbs$$

Therefore,

$$T = \max\left[T_g, T_o\right] = \max\left[T_g, \max\left[T_{o(tubeside)}, T_{o(shellside)}\right]\right]$$

$$T = \max\left[2.0793,\ \max[1.5429,\ 1.2389]\right]$$

$$T = 2.0793\ in$$

Since the specified head thickness, $\{t = 0.875\ in\} > \{t_{req} = 0.4073\ in\}$, and the specified flange thickness, $\{T = 2.375\ in\} > \{T_{req} = 2.0793\ in\}$ are shown to be greater than the required thickness for both internal pressure (tubeside conditions) and external pressure (shellside conditions), the proposed Type D spherically dished bolted cover is adequately designed.

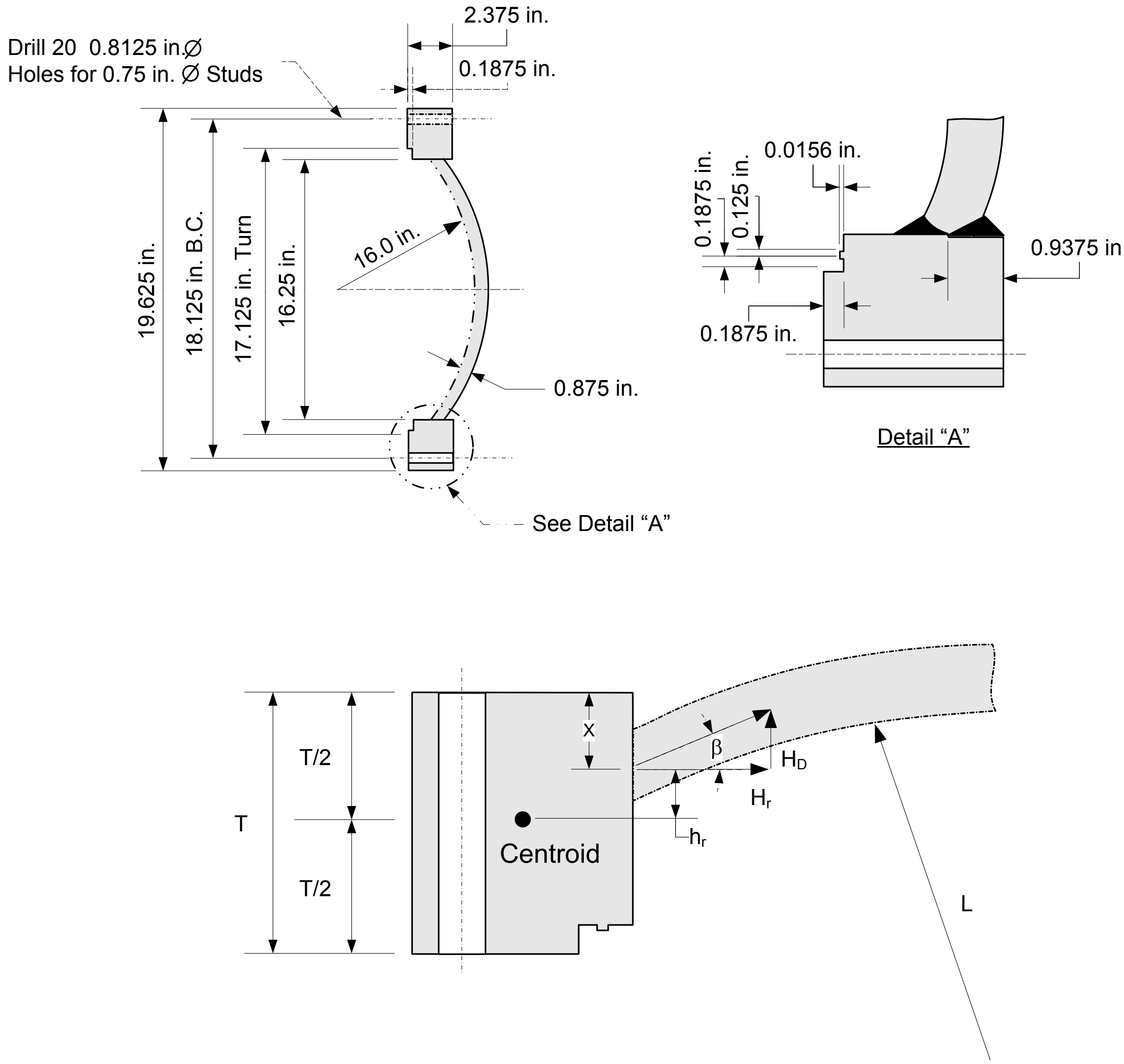


Figure E4.7.1 - Spherically Dished Bolted Cover

4.7.2 Example E4.7.2 – Thickness Calculation for a Type D Head Using the Alternative Rule in Paragraph 4.7.5.3

Determine if the proposed Type D spherically dished bolted cover is adequately designed, considering the following design conditions. The spherically dished head is seamless. Evaluate using the alternative procedure in paragraph 4.7.5.3.

Tubeside Data:

- Design Conditions = $213\ psig\ @\ 400°F$
- Corrosion Allowance (CAT) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Shellside Data:

- Design Conditions = $305\ psig\ @\ 250°F$
- Corrosion Allowance (CAS) = $0.125\ in$
- Weld Joint Efficiency = 1.0

Flange Data:

- Material = $SA-105$
- Allowable Stress at Ambient Temperature = $24000\ psi$
- Allowable Stress at Tubeside Design Temperature = $20500\ psi$
- Allowable Stress at Shellside Design Temperature = $21600\ psi$

Head Data:

- Material = $SA-515,\ Grade\ 60$
- Allowable Stress At Ambient Temperature = $21300\ psi$
- Allowable Stress at Tubeside Design Temperature = $18200\ psi$
- Allowable Stress at Shellside Design Temperature = $19200\ psi$
- Yield Stress at Shellside Design Temperature = $28800\ psi$
- Modulus of Elasticity at Shellside Design Temp. = $28.55E+06\ psi$

Bolt Data:

- Material = $SA-193,\ Grade\ B7$
- Diameter = $0.75\ in$
- Cross-Sectional Root Area = $0.302\ in^2$
- Number of Bolts = 20
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Allowable Stress at Tubeside Design Temperature = $25000\ psi$
- Allowable Stress at Shellside Design Temperature = $25000\ psi$

Gasket Data

- Material = Solid Flat Metal (Iron/Soft Steel)
- Gasket Factor = 5.5
- Gasket Seating Factor = $18000\ psi$
- Inside Diameter = $16.1875\ in$
- Outside Diameter = $17.0625\ in$

Per paragraph 4.7.5.3, as an alternative to the rules in paragraph 4.7.5.1 and 4.7.5.2, the following procedure can be used to determine the required head and flange thickness of a Type D head. This procedure accounts for the continuity between the flange ring and the head, and represents a more accurate method of analysis.

a) STEP 1 – Determine the design pressure and temperature of the flange joint. If the pressure is negative, a negative value must be used for P in all of the equations of this procedure, and

$$P_e = 0.0 \qquad for\ internal\ pressure$$
$$P_e = P \qquad for\ external\ pressure$$

$$Tubeside\ Conditions: P = 213\ psig\ at\ 400°F$$
$$Shellside\ Conditions: P = 305\ psig\ at\ 250°F$$

b) STEP 2 – Determine an initial Type D head configuration geometry (see Figure E4.7.1). The following geometry parameters are required.

1) Flange bore

$$B = \left[16.25 + 2(CAT)\right] = \left[16.25 + 2(0.125)\right] = 16.50\ in$$

2) Bolt circle diameter

$$C = 18.125\ in$$

3) Outside diameter of the flange

$$A = \left[19.625 - 2(CAS)\right] = \left[19.625 - 2(0.125)\right] = 19.375\ in$$

4) Flange thickness, (see Figure E4.7.1)

$$T = T - 2(CAS) = 2.375 - 2(0.125) = 2.125\ in$$

5) Mean head radius, (see Figure 4.7.5)

$$R = \frac{(L + t_{uncorroded} - CAS) + (L + CAT)}{2}$$

$$R = \frac{(16.0 + 0.875 - 0.125) + (16.0 + 0.125)}{2} = 16.4375\ in$$

6) Head thickness

$$t = t - CAT - CAS = 0.875 - 0.125 - 0.125 = 0.625\ in$$

7) Inside depth of flange to the base of the head, (see Figure 4.7.5)

$$q = q - CAS = 1.0762 - 0.125 = 0.9512\ in$$

c) STEP 3 – Select a gasket configuration and determine the location of the gasket reaction, G, and the design bolt loads for the gasket seating, W_g, and operating conditions, W_o, using the rules of paragraph 4.16. Computations for the following parameters are shown in E4.7.1.

$$G = 16.625\ in$$

$$W_g = 141590.0\ lbs$$

$$W_{gs} = 132181.1\ lbs$$

$$W_o = 63442.9\ lbs$$

d) STEP 4 – Determine the geometry parameters

$$h_1 = \frac{(C-G)}{2} = \frac{(18.125-16.625)}{2} = 0.75\ in$$

$$h_2 = \frac{(G-B)}{2} = \frac{(16.625-16.5)}{2} = 0.0625\ in$$

$$d = \frac{(A-B)}{2} = \frac{(19.375-16.5)}{2} = 1.4375\ in$$

$$n = \frac{T}{t} = \frac{2.125}{0.625} = 3.4$$

$$K = \frac{A}{B} = \frac{19.375}{16.5} = 1.1742$$

$$\phi = \arcsin\left[\frac{B}{2R}\right] = \arcsin\left[\frac{16.5}{2(16.4375)}\right] = \begin{Bmatrix} 0.5258\ rad \\ 30.1259\ deg \end{Bmatrix}$$

$$e = q - \frac{1}{2}\left[T - \frac{t}{\cos[\phi]}\right] = 0.9512 - \frac{1}{2}\left[2.125 - \frac{0.625}{\cos[30.1259]}\right] = 0.25\ in$$

$$k_1 = 1 - \left(\frac{1-2v}{2\lambda}\right)\cot[\phi] = 1 - \left[\frac{1-2(0.3)}{2(6.5920)}\right]\cot[30.1259] = 0.9477$$

$$k_2 = 1 - \left(\frac{1+2v}{2\lambda}\right)\cot[\phi] = 1 - \left[\frac{1+2(0.3)}{2(6.5920)}\right]\cot[30.1259] = 0.7907$$

Where,

$$\lambda = \left[3\left(1-v^2\right)\left(\frac{R}{t}\right)^2\right]^{0.25} = \left\{3\left(1-0.3^2\right)\left(\frac{16.4375}{0.625}\right)^2\right\}^{0.25} = 6.5920$$

e) STEP 5 – Determine the shell discontinuity geometry factors

$$C_1 = \frac{0.275n^3t\cdot\ln[K]}{k_1} - e = \left(\frac{0.275(3.4)^3(0.625)\cdot\ln[1.1742]}{0.9477}\right) - (0.25) = 0.8947$$

$$C_2 = \frac{1.1\lambda n^3 t\ln[K]}{Bk_1} + 1 = \left(\frac{1.1(6.5920)(3.4)^3(0.625)\cdot\ln[1.1742]}{(16.5)(0.9477)}\right) + 1 = 2.8293$$

$$C_4 = \frac{\lambda\sin[\phi]}{2}\left(k_2 + \frac{1}{k_1}\right) + \frac{B}{4nd} + \frac{1.65e}{tk_1}$$

$$C_4 = \left[\begin{array}{l}\dfrac{(6.5920)\sin[30.1259]}{2}\left(0.7907 + \dfrac{1}{0.9477}\right) + \\ \dfrac{16.5}{4(3.4)(1.4375)} + \dfrac{1.65(0.25)}{(0.625)(0.9477)}\end{array}\right] = 4.5940$$

$$C_5 = \frac{1.65}{tk_1}\left(1 + \frac{4\lambda e}{B}\right) = \left(\frac{1.65}{(0.625)(0.9477)}\right)\left(1 + \frac{4(6.5920)(0.25)}{(16.5)}\right) = 3.8986$$

f) STEP 6 – Determine the shell discontinuity load factors for the operating and gasket seating conditions.

Operating Condition – Tubeside:

$$C_{3o} = \frac{\pi B^2 P}{4}\left[e\cot[\phi] + \frac{2q(T-q)}{B} - h_2\right] - W_o h_1$$

$$C_{3o} = \left(\frac{\pi(16.5)^2(213)}{4}\right)\left[\begin{array}{l}(0.25)\cot[30.1259] + \\ \dfrac{2(0.9512)(2.125-0.9512)}{(16.5)} - 0.0625\end{array}\right] - 63442.9(0.75)$$

$$C_{3o} = -24643.1908\ in-lbs$$

$$C_{6o} = \frac{\pi B^2 P}{4}\left(\frac{4q - B\cot[\phi]}{4nd} - \frac{0.35}{\sin[\phi]}\right)$$

$$C_{6o} = \frac{\pi(16.5)^2(213)}{4}\left(\frac{4(0.9512) - (16.5)\cot[30.1259]}{4(3.4)(1.4375)} - \frac{0.35}{\sin[30.1259]}\right)$$

$$C_{6o} = -89138.7025\ lbs$$

Operating Condition – Shellside:

$$C_{3o} = \frac{\pi B^2 P}{4}\left[e\cot[\phi] + \frac{2q(T-q)}{B} - h_2\right] - W_o h_1$$

For shellside pressure (external pressure) the operating bolt load, W_o, is set equal to the bolt load required to seat the gasket, defined as W_{gs}. See STEP 3.

$$C_{3o} = \left(\frac{\pi(16.5)^2(-305)}{4}\right)\left[\begin{array}{l}(0.25)\cot[30.1259] + \\ \dfrac{2(0.9512)(2.125-0.9512)}{(16.5)} - 0.0625\end{array}\right] - 132181.1(0.75)$$

$$C_{3o} = -131982.7272\ in-lbs$$

$$C_{6o} = \frac{\pi B^2 P}{4}\left(\frac{4q - B\cot[\phi]}{4nd} - \frac{0.35}{\sin[\phi]}\right)$$

$$C_{6o} = \frac{\pi(16.5)^2(-305)}{4}\left(\frac{4(0.9512)-(16.5)\cot[30.1259]}{4(3.4)(1.4375)} - \frac{0.35}{\sin[30.1259]}\right)$$

$$C_{6o} = 127639.9260\ lbs$$

Gasket Seating Condition:

$$C_{3g} = -W_g h_1 = -(141590.0)(0.75) = -106192.5\ in-lbs$$

$$C_{6g} = 0.0$$

g) STEP 7 – Determine the shell discontinuity force and moment for the operating and gasket condition.

Operating Condition – Tubeside:

$$V_{do} = \frac{C_2 C_{6o} - C_{3o} C_5}{C_2 C_4 - C_1 C_5}$$

$$V_{do} = \frac{(2.8293(-89138.7025)) - (-24643.1908(3.8986))}{(2.8293(4.5940)) - (0.8947(3.8986))} = -16417.5261\ lbs$$

$$M_{do} = \frac{C_1 C_{6o} - C_{3o} C_4}{C_2 C_4 - C_1 C_5}$$

$$M_{do} = \frac{(0.8947(-89138.7025)) - (-24643.1908(4.5940))}{(2.8293(4.5940)) - (0.8947(3.8986))} = 3518.3368\ in-lbs$$

Operating Condition – Shellside:

$$V_{do} = \frac{C_2 C_{6o} - C_{3o} C_5}{C_2 C_4 - C_1 C_5}$$

$$V_{do} = \frac{\left(2.8293\left(127639.9260\right)\right) - \left(-131982.7272\left(3.8986\right)\right)}{\left(2.8293\left(4.5940\right)\right) - \left(0.8947\left(3.8986\right)\right)} = 92082.5091\ lbs$$

$$M_{do} = \frac{C_1 C_{6o} - C_{3o} C_4}{C_2 C_4 - C_1 C_5}$$

$$M_{do} = \frac{\left(0.8947\left(127639.9260\right)\right) - \left(-131982.7272\left(4.5940\right)\right)}{\left(2.8293\left(4.5940\right)\right) - \left(0.8947\left(3.8986\right)\right)} = 75767.4860\ in-lbs$$

Gasket Seating Condition:

$$V_{dg} = \frac{C_2 C_{6g} - C_{3g} C_5}{C_2 C_4 - C_1 C_5} = \frac{\left(2.8293\left(0.0\right)\right) - \left(-106192.5\left(3.8986\right)\right)}{\left(2.8293\left(4.5940\right)\right) - \left(0.8947\left(3.8986\right)\right)} = 43534.5925\ lbs$$

$$M_{dg} = \frac{C_1 C_{6g} - C_{3g} C_4}{C_2 C_4 - C_1 C_5} = \frac{\left(0.8947\left(0.0\right)\right) - \left(-106192.5\left(4.5940\right)\right)}{\left(2.8293\left(4.5940\right)\right) - \left(0.8947\left(3.8986\right)\right)} = 51299.9328\ in-lbs$$

h) STEP 8 – Calculate the stresses in the head and at the head to flange junction using Table 4.7.1 and check the stress criteria for both the operating and gasket conditions.

Calculated Stresses – Operating Conditions – Tubeside:

$$S_{hm} = \frac{PR}{2t} + P_e = \frac{213\left(16.4375\right)}{2\left(0.625\right)} + 0.0 = 2801.0\ psi$$

$$S_{hl} = \frac{PR}{2t} + \frac{V_{do}\cos\left[\phi\right]}{\pi Bt} + P_e$$

$$S_{hl} = \frac{213\left(16.4375\right)}{2\left(0.625\right)} + \frac{\left(-16417.5261\right)\cos\left[30.1259\right]}{\pi\left(16.5\right)\left(0.625\right)} + 0.0 = 2362.6\ psi$$

$$S_{hb} = \frac{6M_{do}}{\pi Bt^2} = \frac{6\left(3518.3368\right)}{\pi\left(16.5\right)\left(0.625\right)^2} = 1042.5\ psi$$

$$S_{hlbi} = S_{hl} - S_{hb} = 2362.6 - 1042.5 = 1320.1\ psi$$

$$S_{hlbo} = S_{hl} + S_{hb} = 2362.6 + 1042.5 = 3405.1\ psi$$

$$S_{fm} = \frac{1}{\pi BT}\left(\frac{\pi B^2 P}{4}\left(\frac{4q}{B} - \cot[\phi]\right) - V_{do}\right)\left(\frac{K^2+1}{K^2-1}\right) + P_e$$

$$S_{fm} = \left(\frac{1}{\pi(16.5)(2.125)}\right)\left(\left(\frac{\pi(16.5)^2(213)}{4}\right)\left(\frac{4(0.9512)}{(16.5)} - \cot[30.1259]\right) - (-16417.5261)\right)\left(\frac{(1.1742)^2+1}{(1.1742)^2-1}\right) + 0.0$$

$$S_{fm} = -2940.2 \ psi$$

$$S_{fb} = \frac{0.525n}{Btk_1}\left(V_{do} - \frac{4M_{do}\lambda}{B}\right)$$

$$S_{fb} = \frac{0.525(3.4)}{(16.5)(0.625)(0.9477)}\left((-16417.5261) - \frac{4(3518.3368)(6.5920)}{(16.5)}\right) = -4025.5 \ psi$$

$$S_{fmbo} = S_{fm} - S_{fb} = -2940.2 - (-4025.5) = 1085.3 \ psi$$

$$S_{fmbi} = S_{fm} + S_{fb} = -2940.2 - (-4025.5) = -6965.7 \ psi$$

Acceptance Criteria – Operating Conditions – Tubeside:

$$\{S_{hm} = 2801.0 \ psi\} \le \{S_{ho} = 18200 \ psi\} \qquad True$$

$$\{S_{hl} = 2362.6 \ psi\} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi\} \qquad True$$

$$\left\{\begin{matrix} S_{hlbi} = 1320.1 \ psi \\ S_{hlbo} = 3405.1 \ psi \end{matrix}\right\} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi\} \qquad True$$

$$\{S_{fm} = -2940.2 \ psi\} \le \{S_{fo} = 20500 \ psi\} \qquad True$$

$$\left\{\begin{matrix} S_{fmbo} = 1085.3 \ psi \\ S_{fmbi} = -6965.7 \ psi \end{matrix}\right\} \le \{1.5S_{fo} = 1.5(20500) = 30750 \ psi\} \qquad True$$

Calculated Stresses – Operating Conditions – Shellside:

$$S_{hm} = \frac{PR}{2t} + P_e = \frac{(-305)(16.4375)}{2(0.625)} + (-305) = -4315.8 \ psi$$

$$S_{hl} = \frac{PR}{2t} + \frac{V_{do}\cos[\phi]}{\pi Bt} + P_e$$

$$S_{hl} = \frac{(-305)(16.4375)}{2(0.625)} + \frac{(92082.5091)\cos[30.1259]}{\pi(16.5)(0.625)} + (-305) = -1857.4 \ psi$$

$$S_{hb} = \frac{6M_{do}}{\pi Bt^2} = \frac{6(75767.4860)}{\pi(16.5)(0.625)^2} = 22451.2 \ psi$$

$$S_{hlbi} = S_{hl} - S_{hb} = -1857.4 - 22451.2 = -24308.6 \ psi$$

$$S_{hlbo} = S_{hl} + S_{hb} = -1857.4 + 22451.2 = 20593.8 \ psi$$

$$S_{fm} = \frac{1}{\pi BT}\left(\frac{\pi B^2 P}{4}\left(\frac{4q}{B} - \cot[\phi]\right) - V_{do}\right)\left(\frac{K^2+1}{K^2-1}\right) + P_e$$

$$S_{fm} = \left(\frac{1}{\pi(16.5)(2.125)}\right)\left(\begin{array}{l}\left(\frac{\pi(16.5)^2(-305)}{4}\right) \\ \left(\frac{4(0.9512)}{(16.5)} - \cot[30.1259]\right) - \\ (92082.5091)\end{array}\right)\left(\frac{(1.1742)^2+1}{(1.1742)^2-1}\right) + (-305)$$

$$S_{fm} = -4.7 \ psi$$

$$S_{fb} = \frac{0.525n}{Btk_1}\left(V_{do} - \frac{4M_{do}\lambda}{B}\right)$$

$$S_{fb} = \frac{0.525(3.4)}{(16.5)(0.625)(0.9477)}\left(92082.5091 - \frac{4(75767.4860)(6.5920)}{(16.5)}\right) = -5296.4 \ psi$$

$$S_{fmbo} = S_{fm} - S_{fb} = (-4.7) - (-5296.4) = 5291.7 \ psi$$

$$S_{fmbi} = S_{fm} + S_{fb} = (-4.7) + (-5296.4) = -5301.1 \ psi$$

Acceptance Criteria – Operating Conditions – Shellside:

$$\{S_{hm} = -4315.8 \ psi\} \le \{S_{ho} = 18200 \ psi\} \qquad True$$

$$\{S_{hl} = -1857.4 \ psi\} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi\} \qquad True$$

$$\left\{\begin{array}{l} S_{hlbi} = -24308.6 \ psi \\ S_{hlbo} = 20593.8 \ psi \end{array}\right\} \le \{1.5S_{ho} = 1.5(18200) = 27300 \ psi\} \qquad True$$

$$\{S_{fm} = -4.7 \ psi\} \le \{S_{fo} = 20500 \ psi\} \qquad True$$

$$\left\{\begin{array}{l} S_{fmbo} = 5291.7\ psi \\ S_{fmbi} = -5301.1\ psi \end{array}\right\} \le \left\{1.5S_{fo} = 1.5(20500) = 30750\ psi\right\} \qquad True$$

Calculated Stresses – Gasket Seating Conditions:

$$S_{hm} = 0.0$$

$$S_{hl} = \frac{V_{dg}\cos[\phi]}{\pi Bt} = \frac{(43534.5925)\cos[30.1259]}{\pi(16.5)(0.625)} = 1162.2\ psi$$

$$S_{hb} = \frac{6M_{dg}}{\pi Bt^2} = \frac{6(51299.9328)}{\pi(16.5)(0.625)^2} = 15201.1\ psi$$

$$S_{hlbi} = S_{hl} - S_{hb} = 1162.2 - 15201.1 = -14038.9\ psi$$

$$S_{hlbo} = S_{hl} + S_{hb} = 1162.2 + 15201.1 = 16363.3\ psi$$

$$S_{fm} = \frac{1}{\pi BT}\left(-V_{dg}\right)\left(\frac{K^2+1}{K^2-1}\right)$$

$$S_{fm} = \left(\frac{1}{\pi(16.5)(2.125)}\right)(-43534.5925)\left(\frac{(1.1742)^2+1}{(1.1742)^2-1}\right) = -2482.2\ psi$$

$$S_{fb} = \frac{0.525n}{Btk_1}\left(V_{dg} - \frac{4M_{dg}\lambda}{B}\right)$$

$$S_{fb} = \frac{0.525(3.4)}{(16.5)(0.625)(0.9477)}\left(43534.5925 - \frac{4(51299.9328)(6.5920)}{(16.5)}\right) = -7021.9\ psi$$

$$S_{fmbo} = S_{fm} - S_{fb} = -2482.2 - (-7021.9) = 4539.7\ psi$$

$$S_{fmbi} = S_{fm} + S_{fb} = -2482.2 + (-7021.9) = -9504.1\ psi$$

Acceptance Criteria – Gasket Seating Conditions:

$$\left\{S_{hm} = 0.0\ psi\right\} \le \left\{S_{hg} = 18200\ psi\right\} \qquad True$$

$$\left\{S_{hl} = 1162.2\ psi\right\} \le \left\{1.5S_{hg} = 1.5(21300\ psi) = 31950\ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_{hlbi} = -14038.9\ psi \\ S_{hlbo} = 16363.3\ psi \end{array}\right\} \le \left\{1.5S_{hg} = 1.5(21300\ psi) = 31950\ psi\right\} \qquad True$$

$$\left\{S_{fm} = -2482.2\ psi\right\} \le \left\{S_{fg} = 24000\ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_{fmbo} = 4539.7\ psi \\ S_{fmbi} = -9504.1\ psi \end{array}\right\} \le \left\{1.5S_{fg} = 1.5(24000) = 36000\ psi\right\} \qquad True$$

Since the calculated stresses in both the head and flange ring are shown to be within the acceptance criteria, for both internal pressure (tubeside conditions) and external pressure (shellside conditions), and gasket seating, the proposed Type D spherically dished bolted cover is adequately designed.

4.8 Quick-Actuating (Quick Opening) Closures

4.8.1 Example E4.8.1 – Review of Requirements for Quick-Actuating Closures

A plant engineer is tasked with developing a design specification for an air filter vessel to be equipped with a quick-actuating closure that is to be constructed in accordance with ASME B&PV Code, Section VIII, Division 2 (VIII-2). As part of developing the design specification, the following items need to be considered.

a) Scope

Specific calculation methods are not given in paragraph 4.8. However, both general and specific design requirements are provided.

b) General Design Requirements

Quick-actuating closures shall be designed such that:

1) The locking elements will be engaged prior to or upon application of the pressure and will not disengage until the pressure is released.
2) The failure of a single locking component while the vessel is pressurized will not:
   i) Cause or allow the closure to be opened or leaked; or
   ii) Result in the failure of any other locking component or holding element; or
   iii) Increase the stress in any other locking or holding element by more than 50% above the allowable stress of the component.
3) All locking components can be verified to be fully engaged by visual observation or other means prior to application if pressure to the vessel.
4) When installed:
   i) It may be determined by visual external observation that the holding elements are in satisfactory condition.
   ii) All vessels shall be provided with a pressure-indicating device visible from the operating area and suitable to detect pressure at the closure.

c) Specific Design Requirements

Quick-actuating closures that are held in position by positive locking devices and that are fully released by partial rotation or limited movement of the closure itself or the locking mechanism and any closure that is other than manually operates shall be so designed that when the vessel is installed the following conditions are met:

1) The closure and its holding elements are fully engaged in their intended operating position before pressure can be applied in the vessel.
2) Pressure tending to force the closure open or discharge the contents clear of the vessel shall be released before the closure can be fully opened for access.

The designer shall consider the effects of cyclic loading, other loadings, and mechanical wear on the holding and locking components.

d) Alternative Designs for Manually Operated Closures

Quick-actuating closures that are held in position by a locking mechanism designed for manual operation shall be designed such that if an attempt is made to open the closure when the vessel is under pressure, the closure will leak prior to full disengagement of the locking components and release of the closure. Any leakage shall be directed away from the normal position of the operator.

e) Supplementary Requirements

Annex 4.B provides additional design information for the Manufacturer and provides installation.

4.9 Braced and Stayed Surfaces

4.9.1 Example E4.9.1 – Braced and Stayed Surfaces

Determine the required thickness for a flat plate with welded staybolts considering the following design condition. Verify that the welded staybolts are adequately designed. See Figure E4.9.1

Vessel Data:

- Plate Material = $SA-516,\ Grade\ 70$
- Design Conditions = $100\ psig@300°F$
- Staybolt Material = $SA-675,\ Grade\ 70$
- Staybolt Diameter = $1.5\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress Plate Material = $22400\ psi@300°F$
- Allowable Stress Staybolt Material = $20600\ psi@300°F$
- Staybolt Pattern = $Equilateral\ Triangle$
- Staybolt Pitch = $p_s = p_{horizontal} = p_{diagonal} = 15.0\ in$

Using the procedure in paragraph 4.9, calculate the required thickness of the flat plate, the load carried by each staybolt, and the required diameter of the staybolt.

Paragraph 4.9.2, the minimum required thickness for braced and stayed flat plates and those parts that, by these rules, require staying as flat plates or staybolts of uniform diameter symmetrically spaced, shall be calculated by the following equation.

Assume, $C = 2.2$ from Table 4.9.1 with the Welded Staybolt Construction per Figure 4.9.1 Detail (c).

$$t = p_s\sqrt{\frac{P}{SC}} = 15.0\sqrt{\frac{100.0}{22400(2.2)}} = 0.6757\ in$$

Paragraph 4.9.3, the required area of a staybolt or stay as its minimum cross section, usually located at the root of the thread, exclusive of any corrosion allowance, shall be obtained by dividing the load on the staybolt computed in accordance with paragraph 4.9.3.2 by the allowable tensile stress value for the staybolt material, multiplying the result by 1.10.

The area supported by a staybolt or stay shall be computed on the basis of the full pitch dimensions, with a deduction for the area occupied by the stay. The load carried by a stay is the product of the area supported by the stay and the maximum allowable working pressure.

a) The area of the flat plate supported by the staybolt, A_p, is calculated as follows.

$$A_p = \left(p_{horizontal}\cdot p_{diagonal}\cdot\cos[\theta]\right) - A_{sb} = \left(15.0(15.0)\cdot\cos[30]\right) - 1.7671 = 193.0886\ in^2$$

Where,

$$\theta = 30\ deg, \qquad See\ Figure\ E4.9.1$$

$$A_{sb} = \frac{\pi}{4}(1.5)^2 = 1.7671\ in^2$$

b) The load carried by the staybolt, L_{sb}, is calculated as follows.

$$L_{sb} = A_p \cdot P = 193.0886(100.0) = 19308.9\ lbs$$

c) The required area of the staybolt, A_{rsb}, is calculated as follows.

$$A_{rsb} = 1.10\left(\frac{L_{sb}}{S_{sb}}\right) = 1.10\left(\frac{19308.9}{20600}\right) = 1.0311\ in^2$$

Since $\left\{A_{sb} = 1.7671\ in^2\right\} > \left\{A_{rsb} = 1.0311\ in^2\right\}$, the staybolt is adequately designed.

Paragraph 4.9.4.1, welded-in staybolts may be use provided the following requirements are satisfied.

d) The configuration is in accordance with the typical arrangements shown in Figure 4.9.1.

$$Construction\ \ per\ \ Figure\ 4.9.1(c) \qquad\qquad satisfied$$

e) The required thickness of the plate shall not exceed 38 mm (1.5 in).

$$t \le 1.5\ in \qquad t = 0.6757\ in \qquad\qquad satisfied$$

f) The maximum pitch shall not exceed 15 times the diameter of the staybolt.

$$p_s \le 15(d_{sb}) \quad 15.0 \le 15(1.5) = 22.5\ in \qquad\qquad satisfied$$

g) The size of the attachment welds is not less than that shown in Figure 4.9.1.

$$Full\ \ Penetration\ Weld\ \ per\ Figure\ 4.9.1(c) \qquad\qquad satisfied$$

h) The allowable load on the welds shall not exceed the product of the weld area (based on the weld dimension parallel to the staybolt), the allowable tensile stress of the material being welded, and a weld joint factor of 60%.

$$\left\{L_{sb} = 19308.9\ lbs\right\} \le \left\{L_a = 39356.2\ lbs\right\} \qquad\qquad satisfied$$

Where,

$$L_a = E\left(t \cdot \pi d_{sb}\right) S_{sb} = 0.6\left(0.6757 \cdot \left(\pi(1.5)\right)\right) 20600 = 39356.2\ lbs$$

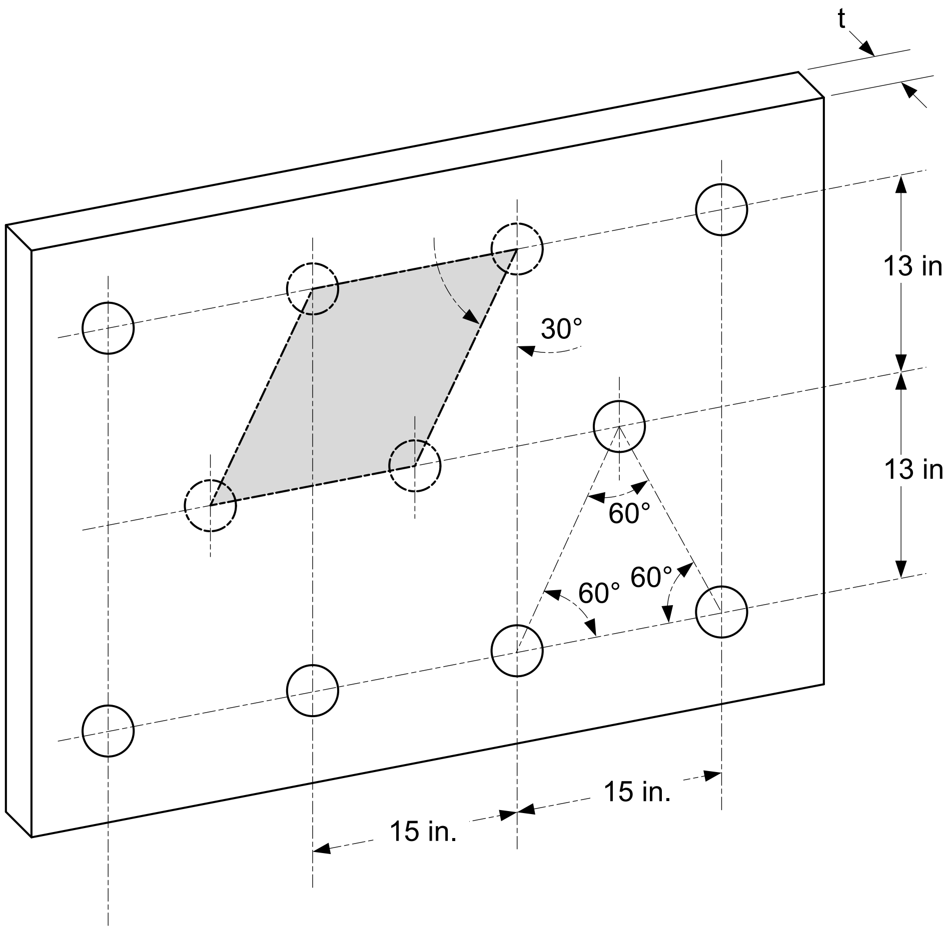


Figure E4.9.1 - Stayed Plate Detail

4.10 Ligaments

4.10.1 Example E4.10.1 – Ligaments

Determine the ligament efficiency and corresponding efficiency to be used in the design equations of paragraph 4.3 for a group of tube holes in a cylindrical shell as shown in Figure E4.10.1.

Using the procedure in paragraph 4.10, calculate the ligament efficiency for the group of tube holes. As shown in Figure E4.10.1, three ligaments are produced; longitudinal, circumferential, and diagonal.

Paragraph 4.10.2.1.c, when the adjacent longitudinal rows are drilled as described in paragraph (b), diagonal and circumferential ligaments shall also be examined. The least equivalent longitudinal ligament efficiency shall be used to determine the minimum required thickness and the maximum allowable working pressure.

> Considering only pressure loading, the circumferential ligament can be half as strong as the longitudinal ligament. This is because the circumferential ligament is subject to longitudinal stress which is essentially half of circumferential stress. By inspection, the circumferential ligament is greater than the longitudinal ligament and thus will the not govern the design. Therefore, the circumferential ligament efficiency is not explicitly calculated.

Paragraph 4.10.2.1.d, when a cylindrical shell is drilled for holes so as to form diagonal ligaments, as shown in Figure E4.10.1, the efficiency of these ligaments shall be determined by paragraph 4.10, Figures 4.10.5 or 4.10.6. Figure 4.10.5 is used when either or both longitudinal and circumferential ligaments exist with diagonal ligaments. The procedure to determine the ligament efficiency is as follows.

a) STEP 1 – Compute the value of p^*/p_1 .

$$Diagonal\ Pitch,\ p^* = 3.75\ in$$

$$Unit\ Length\ of\ Ligament,\ p_1 = 4.5\ in$$

$$\frac{p^*}{p_1} = \frac{3.75}{4.5} = 0.8333$$

b) STEP 2 – Compute the efficiency of the longitudinal ligament in accordance with Figure 4.10.5, Note 4.

$$E_{long} = 100\left(\frac{p_1 - d}{p_1}\right) = 100\left(\frac{4.5 - 2.25}{4.5}\right) = 50\%$$

Where,

$$Diameter\ of\ Tube\ Holes,\ d = 2.25\ in$$

c) STEP 3 – Compute the diagonal efficiency in accordance with Figure 4.10.5, Note 2.

$$E_{diag} = \frac{J + 0.25 - \left(1 - 0.01 \cdot E_{long}\right)\sqrt{0.75 + J}}{0.00375 + 0.005J}$$

$$E_{diag} = \frac{0.6944 + 0.25 - \left(1 - 0.01(50)\right)\sqrt{(0.75 + 0.6944)}}{0.00375 + 0.005(0.6944)} = 47.56\%$$

Where, $J = \left(\dfrac{p^*}{p_1}\right)^2 = \left(\dfrac{3.75}{4.5}\right)^2 = 0.6944$

Alternatively, STEP 3 can be replaced with the following procedure.

STEP 3 (Alternate) – Enter Figure 4.10.5 at the vertical line corresponding to the value of the longitudinal efficiency, E_{long}, and follow this line vertically to the point where it intersects the diagonal line representing the ratio of the value of p^*/p_1. Then project this point horizontally to the left, and read the diagonal efficiency of the ligament on the scale at the edge of the diagram.

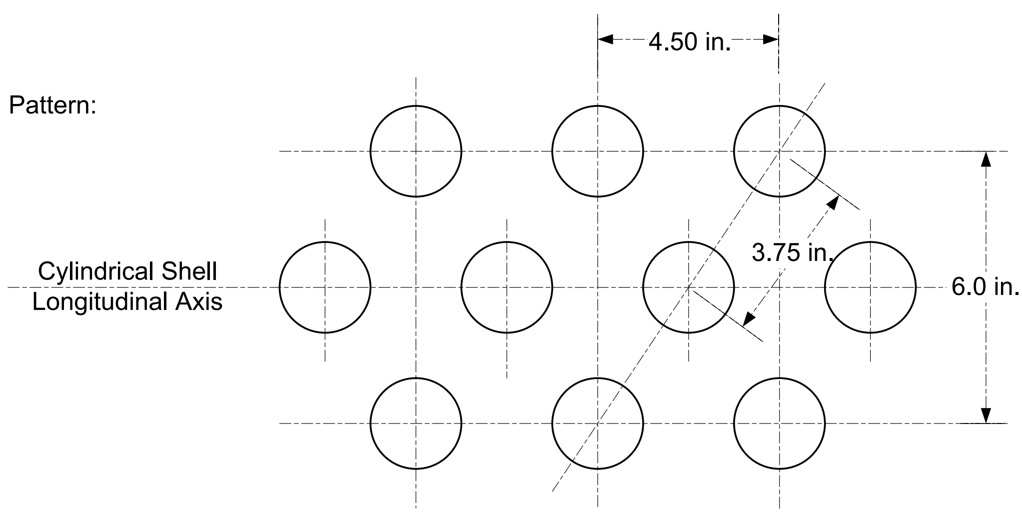
$$E_{diag} \approx 47.5\%$$

d) STEP 4 – The minimum shell thickness and the maximum allowable working pressure shall be based on the ligament that has the lower efficiency.

$$E = \min\left[E_{long}, E_{diag}\right] = \min\left[50\%, 47.5\%\right] = 47.5\%$$

Paragraph 4.10.3, when ligaments occur in cylindrical shells made from welded pipe or tubes and their calculated efficiency is less than 85% (longitudinal) or 50% (circumferential), the efficiency to be used in paragraph 4.3 to determine the minimum required thickness is the calculated ligament efficiency. In this case, the appropriate stress value in tension may be multiplied by the factor 1.18.

- Installation Pattern:



- All Finished Hole Diameters are 2.25 in.

Figure E4.10.1 - Installation Pattern

4.11 Jacketed Vessels

4.11.1 Example E4.11.1 – Jacketed Vessel

Design a jacketed vessel to be installed on the outside diameter of a section of a tower in accordance with Figure 4.11.1, Type 1.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $350\ psig @ 300°F$
- Vessel ID = $90.0\ in$
- Nominal Thickness = $1.125\ in$
- Allowable Stress = $22400\ psi$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0

Jacket Data:

- Jacket Type = $Figure\ 4.11.1,\ Type\ 1$
- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $150\ psig @ 400°F$
- Yield Stress at Design Temperature = $32500\ psi$
- Minimum Ultimate Tensile Strength = $70000\ psi$
- Jacket ID = $96.0\ in$
- Allowable Stress = $22400\ psi$
- Corrosion Allowance = $0.125\ in$
- Weld Joint Efficiency = 1.0

Notes:

1. Jacket closure will be made using closure members in per Table 4.11.2, Detail 6.
2. Full penetration welds will be used in the closure, satisfying paragraph 4.11.3.2.

Establish the corroded dimensions.

$$R_j = 48.0 + Corrosion\ \ Allowance = 48.0 + 0.125 = 48.125\ in$$

$$ID\ of\ Jacket = 2(48.125) = 96.25\ in$$

$$OD\ of\ Inner\ \ Shell = 90 + 2(1.125 - 0.125) = 92.0\ in$$

$$t_s = 1.125 - 2(Corrosion\ \ Allowance) = 1.125 - 2(0.125) = 0.8750\ in$$

$$R_s = \frac{OD\ of\ Inner\ \ Shell}{2} = \frac{92.0}{2} = 46.0\ in$$

a) Paragraph 4.11.2: Design of Jacketed Shells and Jacketed Heads

1) Paragraph 4.11.2.1, determine the required thickness of the jacket using Equation (4.3.1).

$$t_{rj} = R_j\left(\exp\left[\frac{P_j}{S_jE}\right]-1\right) = 48.125\left(\exp\left[\frac{150}{22400(1.0)}\right]-1\right) = 0.3233\ in$$

$$t_{rj} = t_{rj} + Corrosion\ \ Allowance = 0.3233 + 0.125 = 0.4483\ in$$

Therefore, use a jacket plate with a wall thickness of $t_j = 0.5\ in$.

b) Paragraph 4.11.3: Design of Closure Member of Jacket to Vessel

1) Paragraph 4.11.3.1, the design of jacket closure members shall be in accordance with Table 4.11.1 and the additional requirements of paragraph 4.11.3.

2) Paragraph 4.11.3.2, radial welds in closure members shall be butt-welded joints through the full thickness of the member.

3) Paragraph 4.11.3.3, partial penetration and fillet welds are permitted when both of the following requirements are satisfied.

iii) The material of construction satisfies the following equation,

$$\left\{\frac{S_{yT}}{S_u} = \frac{32500}{70000} = 0.464\right\} \le 0.625 \qquad\qquad True$$

iv) The component is not in cyclic service.

4) Determine maximum jacket space, j, to ensure that proposed jacket is acceptable.

$$j_{specified} = \frac{(ID\ of\ Jacket)-(OD\ of\ Inner\ \ Shell)}{2} = \frac{(96.25-92.0)}{2} = 2.125\ in$$

The maximum of j is determined from Table 4.11.1, Detail 6.

$$j = \left(\frac{2S_ct_s^2}{P_jR_j}\right)-\left(\frac{(t_s+t_j)}{2}\right) = \left(\frac{2(22400)(0.875)^2}{150(48.125)}\right)-\left(\frac{0.875+0.5}{2}\right) = 4.0640\ in$$

Since, $\left\{j_{specified} = 2.125\ in\right\} \le \left\{j = 4.0640\ in\right\}$, the design is acceptable.

5) Determine thickness of jacket closures. From Table 4.11.1, Detail 6, Figure (c).

$$t_{rc} = 1.414\sqrt{\frac{P_jR_sj}{S_c}} = 1.414\sqrt{\frac{150(48.125)(2.125)}{22400}} = 1.1701\ in$$

$$t_{rc} = t_{rc} + Corrosion\ \ Allowance = 1.1701 + 0.125 = 1.2951\ in$$

Therefore, use an end closure plate with a wall thickness of $t_c = 1.3125\ in$.

6) Determine minimum required weld sizes, see Table 4.11.1 Detail 6(c).

Jacket to closure weld Table 4.11.1 Detail (c):

- To be full penetration with backing strip.
- Fillet weld to be equal to t_j as a minimum.

Closure to shell weld (a full penetration weld is to be used), see Table 4.11.1 Detail 5(c).

$$t_c = t_{rc} - Corrosion\ \ Allowance = 1.3125 - 0.125 = 1.3\ in$$

$$t_s = 0.875$$

$$\{Y = a + b\} \geq \left\{\min\left[1.5t_c,\ 1.5t_s\right] = \min\left[1.5\left(1.3\right),\ 1.5\left(0.875\right)\right] = 1.3125\right\}$$

$$Z = Y - \frac{t_s}{2} = 1.3125 - \frac{0.875}{2} = 0.8750\ in$$

And,

$$\{a,\ b\} \geq \left\{\min\left[6\ mm(1/4\ in),\ t_c,\ t_s\right] = \min\left[0.25,\ 1.25,\ 0.875\right] = 0.25\ in\right\}$$

4.11.2 Example E4.11.2 – Half-Pipe Jacket

Design a half-pipe jacket for a section of a tower in accordance with paragraph 4.11.6 using the information shown below.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $350\ psig@300°F$
- Vessel ID = $90.0\ in$
- Nominal Thickness = $1.125\ in$
- Allowable Stress = $22400\ psi$
- Corrosion Allowance = $0.125\ in$
- Applied Axial Force = $78104.2\ lbs$
- Applied Net Section Bending Moment = $4.301E+06\ in-lbs$

Half-Pipe Jacket Data:

- Material = $SA-106,\ Grade\ B$
- Design Conditions = $150\ psig@400°F$
- Yield Stress at Design Temperature = $29900\ psi$
- Minimum Ultimate Tensile Strength = $60000\ psi$
- Jacket ID = $NPS\ 4\ (STD\ WT) \rightarrow 0.237\ in$
- Allowable Stress = $20600\ psi$
- Weld Joint Efficiency = 1.0
- Corrosion Allowance = $0.0\ in$
- Fillet Weld Leg = $0.375\ in$

Establish the corroded dimensions.

Vessel:

$$D_0 = 90.0 + 2t_s = 90.0 + 2(0.125) = 90.25\ in$$
$$t_s = 1.125 - Corrosion\ \ Allowance = 1.125 - 0.125 = 1.0\ in$$
$$D = 90.0 + 2(Corrosion\ \ Allowance) = 90.0 + 2(0.125) = 0.25\ in$$

Half-Jacket:

$$D_{pj} = 4.5 - 2t_j = 4.5 - 2(0.237) = 4.026\ in$$
$$r_p = \frac{D_{pj}}{2} = \frac{4.026}{2} = 2.013\ in$$

Evaluate half-pipe jacket per paragraph 4.11.6.

a) Paragraph 4.11.6.1, the rules in this section are applicable for the design of half-pipe jackets constructed of NPS 2, NPS 3, or NPS 4 pipes and subject to internal pressure loading.

$$Half-Pipe\ Jacket \rightarrow NPS\ 4 \qquad\qquad True$$

b) Paragraph 4.11.6.2:

1) The fillet weld attaching the half-pipe jacket to the vessel shall have a throat thickness not less than the smaller of the jacket or shell thickness.

$$throat_f = 0.707 \cdot leg = 0.707(0.375) = 0.265\ in$$

$$\left\{throat_f = 0.265\ in\right\} \geq \left\{\min\left[0.237,\ (1.125-0.125)\right]\ in = 0.237\ in\right\} \qquad True$$

2) The requirements of paragraph 4.11.3.3 shall be satisfied. Paragraph 4.11.3.3, partial penetration and fillet welds are permitted when both of the following requirements are satisfied.

i) The material of construction satisfies the following equation, $SA-106,\ Grade\ B$

$$\left\{\frac{S_{yT}}{S_u} = \frac{32500}{70000} = 0.464\right\} \leq 0.625 \qquad True$$

ii) The component is not in cyclic service. $True$

c) Calculate the minimum required thickness for the $NPS\ 4\ STD\ WT$ half-pipe jacket.

$$t_{rp} = \frac{P_j r_p}{0.85S_j - 0.6P_j} = \frac{150(2.0130)}{0.85(20600) - 0.6(150)} = 0.0173\ in$$

Since $\left\{t_j = 0.237\ in\right\} \geq \left\{t_{rp} = 0.0173\ in\right\}$, the thickness of STD WT pipe is acceptable for the half-pipe jacket.

d) Calculate maximum permissible pressure in the half-pipe, P_{jpm}, to verify that $P_j < P_{jpm}$.

$$P_{jpm} = \frac{F_p}{K_p}$$

$$F_p = \min\left[(1.5S - S^*),\ 1.5S\right]$$

$$K_p = C_1 + C_2D^{0.5} + C_3D + C_4D^{1.5} + C_5D^2 + C_6D^{2.5} + C_7D^3 + C_8D^{3.5} + C_9D^4 + C_{10}D^{4.5}$$

In order to compute P_{jpm}, the parameter S^* defined as the actual longitudinal stress in the shell must be computed. This stress may be computed using the equations in paragraph 4.3.10.2. However, since this is a thin shell, the thin-wall equations for a cylindrical shell will be used.

$$S^* = Pressure\ Stress + Axial\ Stress \pm Bending\ Stress$$

$$S^* = \frac{PD}{4t_s} + \frac{F}{A} \pm \frac{Mc}{I}$$

$$S^* = \begin{cases} \dfrac{350(90.25)}{4(1.0)} + \dfrac{-78104.2}{286.6703} + \dfrac{4.301E+06\left(\dfrac{92.25}{2}\right)}{298408.1359} = 8269.2283\ psi \\ \dfrac{350(90.25)}{4(1.0)} + \dfrac{-78104.2}{286.6703} - \dfrac{4.301E+06\left(\dfrac{92.25}{2}\right)}{298408.1359} = 6959.6156\ psi \end{cases}$$

Where,

$$I = \frac{\pi}{64}\left(D_o^{\,4} - D^4\right) = \frac{\pi}{64}\left((92.25)^4 - (90.25)^4\right) = 298408.1359\ in^4$$

$$A = \frac{\pi}{4}\left(D_o^{\,2} - D^2\right) = \frac{\pi}{4}\left((92.25)^2 - (90.25)^2\right) = 286.6703\ in^2$$

Therefore,

$$F_p = \min\left[\left(1.5S - S^*\right),\ 1.5S\right]$$

$$F_p = \min\left[\left(1.5 \cdot 22400 - 8269.2283\right), 1.5 \cdot 22400\right] = 25330.7717\ psi$$

Coefficients for K_p formula obtained from Table 4.11.3 for NPS 4 and shell thickness, $t_s = 1.0\ in$.

$$C_1 = -2.5016604E+02, \quad C_2 = 1.7178270E+02, \quad C_3 = -4.6844914E+01$$

$$C_4 = 6.6874346E+00, \quad C_5 = -5.2507555E-01, \quad C_6 = 2.1526948E-02$$

$$C_7 = -3.6091550E-04, \quad C_8 = C_9 = C_{10} = 0.0$$

With the a vessel diameter, $D = 90.25\ in$, the value of K_p is calculated as,

$$K_p = C_1 + C_2D^{0.5} + C_3D + C_4D^{1.5} + C_5D^2 + C_6D^{2.5} + C_7D^3 + C_8D^{3.5} + C_9D^4 + C_{10}D^{4.5}$$

$$K_p = 11.2903$$

Therefore, the maximum permissible pressure in the half-pipe is calculated as,

$$P_{jpm} = \frac{F_p}{K_p} = \frac{25330.7717}{11.2903} = 2243.6\ psi$$

Since $\left\{P_{jpm} = 2243.6\ psi\right\} \geq \left\{P_j = 150\ psi\right\}$, the half-pipe design is acceptable.

4.12 NonCircular Vessels

4.12.1 Example E4.12.1 – Type 1

Using the data shown below, design a Type 1 non-circular pressure vessel per paragraph 4.12.7.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $400\ psig @ 500°F$
- Inside Length (Short Side) = $7.125\ in$
- Inside Length (Long Side) = $9.25\ in$
- Overall Vessel Length = $40.0\ in$
- Thickness (Short Side) = $1.0\ in$
- Thickness (Long Side) = $1.0\ in$
- Thickness (End Plate) = $0.75\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $20600\ psi$
- Weld Joint Efficiency (Corner Joint) = 1.0
- Tube Outside Diameter = $1.0000\ in$
- Tube Pitch = $2.3910\ in$

Adjust variable for corrosion.

$$h = 9.25 + 2(Corrosion\ \ Allowance) = 9.25 + 2(0.125) = 9.50\ in$$
$$H = 7.125 + 2(Corrosion\ \ Allowance) = 7.125 + 2(0.125) = 7.375\ in$$
$$t_1 = 1.0 - Corrosion\ \ Allowance = 1.0 - 0.125 = 0.875\ in$$
$$t_2 = 1.0 - Corrosion\ \ Allowance = 1.0 - 0.125 = 0.875\ in$$
$$t_5 = 0.75 - Corrosion\ \ Allowance = 0.75 - 0.125 = 0.625\ in$$

Evaluate per paragraph 4.12.

Paragraph 4.12.2, General Design Requirements

Paragraph 4.12.2.7 – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. Vessels with aspect ratios less than four may be designed in accordance with the provisions of Part 5.

$$Aspect\ \ Ratio = \frac{L_v}{h} = \frac{40.0}{9.5} = 4.21$$

Paragraph 4.12.2.9

The openings in this noncircular vessel meet the requirements of paragraph 4.5.2.

Paragraphs 4.12.3, 4.12.4.4, and 4.12.5.5

These paragraphs are not applicable to this design.

Paragraph 4.12.6, Weld Joint Factors and Ligament Efficiency

Paragraph 4.12.6.1 – The non-circular vessel is constructed with corner joints typical of paragraph 4.2. Therefore, the weld joint efficiencies E_m and E_b are set to 1.0 at stress calculation locations in the corners of the vessel. Since there are no welds or hole pattern in the short side plates of the vessel, the weld joint efficiencies E_m and E_b are set to 1.0 for these stress calculation locations. For the stress calculation locations on the long side plates that do not contain welded joints, but do contain a hole pattern, the weld joint efficiencies E_m and E_b are set equal to the ligament efficiencies e_m and e_b, respectively.

Paragraph 4.12.6.3 – It is assumed that the holes drilled in the long side plates (tube sheet and plug sheet) are of uniform diameter. Therefore, e_m and e_b shall be the same value and calculated in accordance with paragraph 4.10.

$$e_m = e_b = \frac{p-d}{p} = \frac{2.3910-1.0}{2.3910} = 0.5818$$

Paragraph 4.12.7, Design Procedure

a) STEP 1 – The design pressure and temperature are listed in the information given above.
b) STEP 2 – The vessel to be designed is a Type 1 vessel, see Figure 4.12.1.
c) STEP 3 – The vessel configuration and wall thicknesses of the pressure containing plates are listed in the information given above.
d) STEP 4 – Determine the location of the neutral axis from the inside and outside surfaces. Since the section under evaluation does not have stiffeners, but has uniform diameter holes, then $c_i = c_o = t/2$ where t is the thickness of the plate.

$$c_i = c_o = \frac{t_1}{2} = \frac{t_2}{2} = \frac{0.875}{2} = 0.4375\ in$$

e) STEP 5 – Determine the weld joint factor and ligaments efficiencies, as applicable (see paragraph 4.12.6) and determine the factors E_m and E_b.
f) STEP 6 – Complete the stress calculation for the selected noncircular vessel Type (see Table 4.12.1), and check the acceptance criteria.

For non-circular vessel Type 1, the applicable table for stress calculations is Table 4.12.2 and the corresponding details are shown in Figure 4.12.1.

Calculate the equation constants:

$$b = 1.0\ (unit\ width)$$

$$J_{2s} = 1.0$$

$$J_{3s} = 1.0$$

$$J_{2l} = 1.0$$

$$J_{3l} = 1.0$$

$$I_1 = \frac{bt_1^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558\ in^4$$

$$I_2 = \frac{bt_2^3}{12} = \frac{1.0(0.875)^3}{12} = 0.0558\ in^4$$

$$\alpha = \frac{H}{h} = \frac{7.375}{9.5} = 0.7763$$

$$K = \frac{I_2}{I_1}\alpha = \left(\frac{0.0558}{0.0558}\right)0.7763 = 0.7763$$

Calculate the membrane and bending stresses at the Critical Locations of Maximum Stress.

The membrane stress on the short side plate:

$$S_m^s = \frac{Ph}{2(t_1)E_m} = \frac{400(9.5)}{2(0.875)(1.0)} = 2171.4\ psi$$

The bending stress at Location C on the short side plate:

$$S_{bi}^{sC} = -S_{bo}^{sC}\left(\frac{c_i}{c_o}\right) = \frac{PbJ_{2s}c_i}{12I_1E_b}\left[-1.5H^2 + h^2\left(\frac{1+\alpha^2 K}{1+K}\right)\right]$$

$$S_{bi}^{sC} = \frac{400(1.0)(1.0)(0.4375)}{12(0.0558)(1.0)}\left[-1.5(7.375)^2 + (9.5)^2\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$

$$S_{bi}^{sC} = -1831.7\ psi$$

$$S_{bo}^{sC} = -S_{bi}^{sC}\left(\frac{c_o}{c_i}\right)1831.7 = -(-1831.7)\left(\frac{0.4375}{0.4375}\right) = 1831.7\ psi$$

The bending stress at Location B on the short side plate:

$$S_{bi}^{sB} = -S_{bo}^{sB}\left(\frac{c_i}{c_o}\right) = \frac{Pbh^2J_{3s}c_i}{12I_1E_b}\left[\frac{1+\alpha^2 K}{1+K}\right]$$

$$S_{bi}^{sB} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(0.0558)(1.0)}\left[\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$

$$S_{bi}^{sB} = 19490.8\ psi$$

$$S_{bo}^{sB} = -S_{bi}^{sB}\left(\frac{c_o}{c_i}\right) = -19490.8\left(\frac{0.4375}{0.4375}\right) = -19490.8\ psi$$

The membrane stress on the long side plate:

$$S_m^l = \frac{PH}{2t_2E_m} = \frac{400(7.375)}{2(0.875)(0.5818)} = 2897.4\ psi$$

The bending stress at Location A on the long side plate:

$$S_{bi}^{lA} = -S_{bo}^{lA}\left(\frac{c_i}{c_o}\right) = \frac{Pbh^2 J_{2l} c_i}{12 I_2 E_b}\left[-1.5 + \left(\frac{1+\alpha^2 K}{1+K}\right)\right]$$

$$S_{bi}^{lA} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(1.0)(0.0558)(0.5818)}\left[-1.5 + \left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$

$$S_{bi}^{lA} = -27310.9 \ psi$$

$$S_{bo}^{lA} = -S_{bi}^{lA}\left(\frac{c_o}{c_i}\right) = -(-27310.9)\left(\frac{0.4375}{0.4375}\right) = 27310.9 \ psi$$

The bending stress at Location B on the long side plate:

$$S_{bi}^{lB} = -S_{bo}^{lB}\left(\frac{c_i}{c_o}\right) = \frac{Pbh^2 J_{3l} c_i}{12 I^2 E_b}\left[\frac{1+\alpha^2 K}{1+K}\right]$$

$$S_{bi}^{lB} = \frac{400(1.0)(9.5)^2(1.0)(0.4375)}{12(0.0558)(1.0)}\left[\left(\frac{1+(0.7763)^2(0.7763)}{1+0.7763}\right)\right]$$

$$S_{bi}^{lB} = 19490.8 \ psi$$

$$S_{bo}^{lB} = -S_{bi}^{lB}\left(\frac{c_o}{c_i}\right) = -19490.8\left(\frac{0.4375}{0.4375}\right) = -19490.8 \ psi$$

Acceptance Criteria – Critical Locations of Maximum Stress.

$$\left\{S_m^s = 2171.4 \ psi\right\} \le \left\{S = 20600 \ psi\right\} \qquad True$$

$$\begin{Bmatrix} S_m^s + S_{bi}^{sC} = 2171.4 + (-1831.7) = 339.7 \ psi \\ S_m^s + S_{bo}^{sC} = 2171.4 + 1831.7 = 4003.1 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m^s + S_{bi}^{sC} = 339.7 \ psi \\ S_m^s + S_{bo}^{sC} = 4003.1 \ psi \end{Bmatrix} \le \left\{1.5S = 1.5(20600) = 30900 \ psi\right\} \qquad True$$

$$\begin{Bmatrix} S_m^s + S_{bi}^{sB} = 2171.4 + 19490.8 = 21662.2 \ psi \\ S_m^s + S_{bo}^{sB} = 2171.4 + (-19490.8) = -17319.4 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m^s + S_{bi}^{sB} = 21662.2 \ psi \\ S_m^s + S_{bo}^{sB} = -17319.4 \ psi \end{Bmatrix} \le \left\{1.5S = 1.5(20600) = 30900 \ psi\right\} \qquad True$$

$$\left\{S_m^l = 2897.4 \ psi\right\} \le \left\{S = 20600 \ psi\right\} \qquad True$$

$$\begin{Bmatrix} S_m^l + S_{bi}^{lA} = 2897.4 + (-27310.9) = -24413.5 \ psi \\ S_m^l + S_{bo}^{lA} = 2897.4 + 27310.9 = 30208.3 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m^l + S_{bi}^{lA} = -24413.5 \ psi \\ S_m^l + S_{bo}^{lA} = 30208.3 \ psi \end{Bmatrix} \le \{1.5S = 1.5(20600) = 30900 \ psi\} \qquad True$$

$$\begin{Bmatrix} S_m^l + S_{bi}^{lB} = 2897.4 + 19490.8 = 22388.2 \ psi \\ S_m^l + S_{bo}^{lB} = 2897.4 + (-19490.8) = -16593.4 \ psi \end{Bmatrix}$$

$$\begin{Bmatrix} S_m^l + S_{bi}^{lB} = 22388.2 \ psi \\ S_m^l + S_{bo}^{lB} = -16593.4 \ psi \end{Bmatrix} \le \{1.5S = 1.5(20600) = 30900 \ psi\} \qquad True$$

Since the acceptance criteria are satisfied, the design is complete. The vessel meets the requirements as designed and no further iterations are necessary.

4.12.2 Example E4.12.2 – Type 4

Using the data shown below, design a Type 4 non-circular pressure vessel per paragraph 4.12.7. The stiffeners are attached with continuous fillets welds on both sides of the member (Category E, Type 10) and satisfy the requirements of paragraph 4.2, Figure 4.2.2.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $50\,psig\,@\,200°F$
- Inside Length (Short Side) = $30.0\,in$
- Inside Length (Long Side) = $60.0\ in$
- Overall Vessel Length = $240.0\ in$
- Unstiffened Span Length = $12.0\ in$
- Thickness (Short Side) = $0.4375\ in$
- Thickness (Long Side) = $0.4375\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $23200\ psi$
- Weld Joint Efficiency = 1.0
- Yield Stress at Design Temperature = $34800\ psi$
- Modulus of Elasticity at Design Temperature = $28.8E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$

Stiffener Data:

- Material = $SA-36$
- Allowable Stress = $22000\ psi$
- Stiffener Yield Stress at Design Temperature = $33000\,psi$
- Modulus of Elasticity at Design Temperature = $28.8E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$
- Stiffener Cross Sectional Area = $3.83\ in^2$
- Stiffener Moment of Inertia = $11.3\ in^4$
- Stiffener Height = $4.125\ in$
- Stiffener Centerline Distance (Short Side) = $34.125\ in$
- Stiffener Centerline Distance (Long Side) = $64.125\ in$

Required variables.

$$h = 60.0 \ in$$
$$H = 30.0 \ in$$
$$t_1 = 0.4375 \ in$$
$$t_2 = 0.4375 \ in$$

Evaluate per paragraph 4.12.

Paragraph 4.12.2, General Design Requirements

Paragraph 4.12.2.3.c – For a vessel with reinforcement, when the reinforcing member does not have the same allowable stress as the vessel, the total stress shall be determined at the inside and outside surfaces of each component of the composite section. The total stresses at the inside and outside surfaces shall be compared to the allowable stress.

i) For locations of stress below the neutral axis, the bending equation used to compute the stress shall be that considered acting on the inside surface.
ii) For locations of stress above the neutral axis, the bending equation used to compute the stress shall be that considered acting on the outside surface.

Paragraph 4.12.2.7 – The design equations in this paragraph are based on vessels in which the ratio of the length of the vessel to the long side or short side lengths (aspect ratio) is greater than four. Vessels with aspect ratios less than four may be designed in accordance with the provisions of Part 5.

$$Aspect \ Ratio = \frac{L_v}{h} = \frac{240.0}{60.0} = 4.0$$

Paragraph 4.12.2.9

There are no specified openings for this example problem.

Paragraph 4.12.3, Requirements for Vessels with Reinforcement

Paragraph 4.12.3.1 – Design rules are provided for Type 4 configurations where the welded-on reinforcement members are in a plane perpendicular to the long axis of the vessel. All reinforcement members attached to two opposite plates shall have the same moment of inertia.

Paragraph 4.12.3.5 – Reinforcing members shall be placed on the outside of the vessel and shall be attached to the plates of the vessel by welding on each side of the reinforcing member. For continuous reinforcement, the welding may be continuous or intermittent.

Paragraph 4.12.3.6 – The maximum distance between reinforcing members is computed in paragraph 4.12.3 and are covered in STEP 3 of the Design Procedure in paragraph 4.12.7.

Paragraphs 4.12.4 and 4.12.5

These paragraphs are not applicable to this design.

Paragraph 4.12.6, Weld Joint Factors and Ligament Efficiency

Paragraph 4.12..1 – The non-circular vessel is constructed with corner joints typical of paragraph 4.2. Therefore, the weld joint efficiencies E_m and E_b are set to 1.0 at stress calculation locations in the corners of the vessel. Since there are no welds or hole pattern in either the short side plates or long side plates of the vessel, the weld joint efficiencies E_m and E_b are set to 1.0 for these stress calculation locations.

Paragraph 4.12.7, Design Procedure

a) STEP 1 – The design pressure and temperature are listed in the information given above.

b) STEP 2 – The vessel to be designed is a Type 4 vessel, see Figure 4.12.4.

c) STEP 3 – The vessel configuration and wall thicknesses of the pressure containing plates are listed in the information given above. The vessel has stiffeners; therefore, calculate the maximum spacing and size of the stiffeners per paragraph 4.12.3.

Paragraph 4.12.3.6.a – The maximum distance between and reinforcing member centerlines is given by Equation (4.12.1). In the equations for calculating stresses for reinforced noncircular vessels, the value of p shall be the sum of one-half the distances to the next reinforcing member on each side.

For the short side plate, where $\{H = 30.0\ in\} \geq \{p = 12.0\ in\}$,

$$p_1 = t_1\sqrt{\frac{SJ_1}{P}} = 0.4375\sqrt{\frac{(22000)2.2647}{50}} = 13.8105\ in$$

Where,

$$J_1 = -0.26667 + \frac{24.222}{(\beta_{1\max})} - \frac{99.478}{(\beta_{1\max})^2} + \frac{194.59}{(\beta_{1\max})^3} - \frac{169.99}{(\beta_{1\max})^4} + \frac{55.822}{(\beta_{1\max})^5}$$

$$J_1 = -0.26667 + \frac{24.222}{(2.2558)} - \frac{99.478}{(2.2558)^2} + \frac{194.59}{(2.2558)^3} - \frac{169.99}{(2.2558)^4} + \frac{55.822}{(2.2558)^5}$$

$$J_1 = 2.2647$$

$$\beta_{1\max} = \min\left[\max\left[\beta_1, \frac{1}{\beta_1}\right], 4.0\right] = \min\left[\max\left[2.2558, \frac{1}{2.2558}\right], 4.0\right] = 2.2558$$

$$\beta_1 = \frac{H}{p_{b1}} = \frac{30.0000}{13.2988} = 2.2558 \qquad (for\ rectangular\ vessels)$$

$$p_{b1} = t_1\sqrt{\frac{2.1S}{P}} = t_1\sqrt{\frac{2.1S}{P}} = 0.4375\sqrt{\frac{2.1(22000)}{50}} = 13.2988\ in$$

For the long side plate, where $\{h = 60.0\ in\} \geq \{p = 12.0\ in\}$,

$$p_2 = t_2\sqrt{\frac{SJ_2}{P}} = 0.4375\sqrt{\frac{(22000)2.0000}{50}} = 12.9783\ in$$

Where,

$$J_2 = -0.26667 + \frac{24.222}{(\beta_{1\max})} - \frac{99.478}{(\beta_{1\max})^2} + \frac{194.59}{(\beta_{1\max})^3} - \frac{169.99}{(\beta_{1\max})^4} + \frac{55.822}{(\beta_{1\max})^5}$$

$$J_2 = -0.26667 + \frac{24.222}{(4.0)} - \frac{99.478}{(4.0)^2} + \frac{194.59}{(4.0)^3} - \frac{169.99}{(4.0)^4} + \frac{55.822}{(4.0)^5}$$

$$J_2 = 2.0024 \rightarrow However,\ J_2\ is\ limited\ to\ 2.0000$$

$$\beta_{2\max} = \min\left[\max\left[\beta_2, \frac{1}{\beta_2}\right], 4.0\right] = \min\left[\max\left[4.5117, \frac{1}{4.5117}\right], 4.0\right] = 4.0$$

$$\beta_2 = \frac{h}{p_{b2}} = \frac{60.0000}{13.2988} = 4.5117 \qquad (for\ rectangular\ vessels)$$

$$p_{b2} = t_2\sqrt{\frac{2.1S}{P}} = t_2\sqrt{\frac{2.1S}{P}} = 0.4375\sqrt{\frac{2.1(22000)}{50}} = 13.2988\ in$$

Therefore,

$$p = \min[p_1, p_2] = \min[13.8105, 12.9783] = 12.9783\ in$$

Since $\{p_{design} = 12.0\ in\} < \{p_{allow} = 12.9783\ in\}$, the design is acceptable.

Paragraph 4.12.3.6.b – The allowable effective widths of shell plate, w_1 and w_2 shall not be greater than the value given by equation (4.12.16) or Equation (4.12.17), nor greater than the actual value of p if this value is less than that computed in paragraph 4.12.3.6.a. One half of w shall be considered to be effective on each side of the reinforcing member centerline, but the effective widths shall not overlap. The effective width shall not be greater than the actual width available.

$$w_1 = \min[p, \min[w_{\max}, p_1]] = \min[12.0, \min[14.1552, 13.8105]] = 12.0\ in$$

$$w_2 = \min[p, \min[w_{\max}, p_2]] = \min[12.0, \min[14.1552, 12.9783]] = 12.0\ in$$

Where,

$$w_{\max} = \frac{t_1\Delta}{\sqrt{S_y}}\left(\frac{E_y}{E_{ya}}\right) = \frac{t_2\Delta}{\sqrt{S_y}}\left(\frac{E_y}{E_{ya}}\right) = \frac{0.4375(6000)}{\sqrt{33000}}\left(\frac{28.8E+06}{29.4E+06}\right) = 14.1552\ in$$

$$\Delta = 6000\sqrt{psi} \qquad From\ Table\ 4.12.14$$

Paragraph 4.12.3.6.c – At locations, other than in the corner regions where the shell plate is in tension, the effective moments of inertia, I_{11} and I_{21}, of the composite section (reinforcement and shell plate acting together) shall be computed based on the values of w_1 and w_2 computed in paragraph 4.12.3.6.b.

NOTE – A composite structure may include the use of two or more different materials, each carrying a part of the load. Unless all the various materials used have the same Modulus of Elasticity, the evaluation of the composite section will need to consider the ratio of the moduli. Although the material specifications for the shell plate and stiffeners are different, their Moduli of Elasticity are the same; therefore, no adjustment to the procedure to calculate the composite section moment of inertia is required.

Calculate the short side stiffener/plate composite section neutral axis as follows see Figure E4.12.2.

$$\bar{y}=\frac{A_{stif}\left(t_1+\frac{h_s}{2}\right)+A_{plate}\left(\frac{t_1}{2}\right)}{\left(A_{stif}+A_{plate}\right)}$$

$$\bar{y}=\frac{3.83\left(0.4375+\frac{4.125}{2}\right)+0.4375(12.0)\left(\frac{0.4375}{2}\right)}{\left(3.83+0.4375(12.0)\right)}=1.1810\ in$$

Calculate the short side composite section moment of inertia, I_{11}, using parallel axis theorem.

$$I_{11}=I_{stif}+A_{stif}\left(t_1+\frac{h_s}{2}-\bar{y}\right)^2+\frac{w_1(t_1)^3}{12}+w_1(t_1)\left(\bar{y}-\frac{t_1}{2}\right)^2$$

$$I_{11}=\left\{\begin{array}{l}11.3+3.83\left(0.4375+\frac{4.125}{2}-1.1810\right)^2+\\ \frac{12.0(0.4375)^3}{12}+12.0(0.4375)\left(1.1810-\frac{0.4375}{2}\right)^2\end{array}\right\}=22.9081\ in^4$$

Since the stiffener is continuous around the vessel with a consistent net section, the plate thicknesses of the short side and long side are equal, $t_1=t_2$, the pitch of stiffeners are equal, $w_1=w_2$, it follows that $\bar{y}$ for the short side and long side plates are equal and $I_{11}=I_{21}$.

d) STEP 4 – Determine the location of the neutral axis from the inside and outside surfaces. If the section under evaluation has stiffeners, then c_i and c_o are determined from the cross section of the combined plate and stiffener section using strength of materials concepts.

For the short side plate,

$$c_i=\bar{y}=1.1810\ in$$

$$c_o=t_1+h_s-\bar{y}=0.4375+4.125-1.1810=3.3815\ in$$

For the long side plate,

$$c_i=\bar{y}=1.1810\ in$$

$$c_o=t_2+h_s-\bar{y}=0.4375+4.125-1.1810=3.3815\ in$$

The reinforcing member does not have the same allowable stress as the vessel; therefore, the stress at the interface of the components of the composite section shall be determined. Since the interface between components is oriented below the composite section neutral axis, the bending equation used to compute the stress shall be that considered acting on the inside surface. The distance between the composite section neutral axis and the interface of the components is calculated as follows.

For the short side and long side plates, respectively,

$$c_{i(interface)} = \bar{y} - t_1 = 1.1810 - 0.4375 = 0.7435 \ in$$

$$c_{i(interface)} = \bar{y} - t_2 = 1.1810 - 0.4375 = 0.7435 \ in$$

e) STEP 5 – Determine the weld joint factor and ligaments efficiencies, as applicable (see paragraph 4.12.6), and determine the factors E_m and E_b.

$$E_m = E_b = 1.0$$

f) STEP 6 – Complete the stress calculation for the selected noncircular vessel Type (see Table 4.12.1), and check the acceptance criteria.

For non-circular vessel Type 4, the applicable table for stress calculations is Table 4.12.5 and the corresponding details are shown in Figure 4.12.4.

Calculate the equation constants:

$$\alpha_1 = \frac{H_1}{h_1} = \frac{H + 2(t_1) + h_s}{h + 2(t_2) + h_s} = \frac{30 + 2(0.4375) + 4.125}{60 + 2(0.4375) + 4.125} = 0.5385$$

$$k = \frac{I_{21}}{I_{11}}\alpha_1 = \left(\frac{22.9081}{22.9081}\right)0.5385 = 0.5385$$

Calculate the Composite Section membrane and bending stresses at the Critical Locations of Maximum Stress.

<u>The membrane stress on the short side plate:</u>

$$S_m^s = \frac{Php}{2(A_1 + t_1 p)E_m} = \frac{50(60.0)12.0}{2(3.83 + 0.4375(12.0))1.0} = 1982.4 \ psi$$

<u>The bending stress at Location C, on the short side plate:</u>

$$S_{bi}^{sC} = -S_{bo}^{sC}\left(\frac{c_i}{c_o}\right) = \frac{Ppc_i}{24 I_{11} E_b}\left[-3H^2 + 2h^2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{sC} = \frac{50(12.0)1.1810}{24(22.9081)1.0}\left[-3(30.0)^2 + 2(60.0)^2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{sC} = 3493.6 \ psi$$

$$S_{bo}^{sC} = -S_{bi}^{sC}\left(\frac{c_o}{c_i}\right) = -3493.6\left(\frac{3.3815}{1.1810}\right) = -10003.1 \ psi$$

<u>The bending stress at Location B, on the short side plate:</u>

$$S_{bi}^{sB} = -S_{bo}^{sB}\left(\frac{c_i}{c_o}\right) = \frac{Ph^2 pc_i}{12 I_{11} E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{sB} = \frac{50(60.0)^2(12.0)(1.1810)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right]$$

$$S_{bi}^{sB} = 6973.5 \ psi$$

$$S_{bo}^{sB} = -S_{bi}^{sB}\left(\frac{c_o}{c_i}\right) = -6973.5\left(\frac{3.3815}{1.1810}\right) = -19966.9 \ psi$$

The membrane stress on the long side plate:

$$S_m^l = \frac{PHp}{2(A_2 + t_2 p)E_m} = \frac{50(30.0)(12.0)}{2(3.83+0.4375(12.0))1.0} = 991.2 \ psi$$

The bending stress at Location A, on the long side plate:

$$S_{bi}^{lA} = -S_{bo}^{lA}\left(\frac{c_i}{c_o}\right) = \frac{Ph^2 pc_i}{24 I_{21} E_b}\left[-3+2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{lA} = \frac{50(60.0)^2(12.0)(1.1810)}{24(22.9081)1.0}\left[-3+2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{lA} = -6946.0 \ psi$$

$$S_{bo}^{lA} = -S_{bi}^{lA}\left(\frac{c_o}{c_i}\right) = -(-6946.0)\left(\frac{3.3815}{1.1810}\right) = 19888.2 \ psi$$

The bending stress at Location B, on the long side plate:

$$S_{bi}^{lB} = -S_{bo}^{lB}\left(\frac{c_i}{c_o}\right) = \frac{Ph^2 pc_i}{12 I_{21} E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{lB} = \frac{50(60.0)^2(12.0)(1.1810)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right]$$

$$S_{bi}^{lB} = 6973.5 \ psi$$

$$S_{bo}^{lB} = -S_{bi}^{lB}\left(\frac{c_o}{c_i}\right) = -6973.5\left(\frac{3.3815}{1.1810}\right) = -19966.9 \ psi$$

Calculate the bending stresses at the interface of the shell plate and stiffener at the Critical Locations of Maximum Stress.

The bending stress at Location C, on the short side plate:

$$S_{bi}^{sC} = \frac{Ppc_i}{24I_{11}E_b}\left[-3H^2 + 2h^2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{sC} = \frac{50(12.0)(0.7435)}{24(22.9081)1.0}\left[-3(30.0)^2 + 2(60.0)^2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right]$$

$$S_{bi}^{sC} = 2199.4\ psi$$

The bending stress at Location B, on the short side plate:

$$S_{bi}^{sB} = \frac{Ph^2 pc_i}{12I_{11}E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{sB} = \frac{50(60.0)^2(12.0)(0.7435)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] = 4390.2\ psi$$

The bending stress at Location A, on the long side plate:

$$S_{bi}^{lA} = \frac{Ph^2 pc_i}{24I_{21}E_b}\left[-3 + 2\left(\frac{1+\alpha_1^2 k}{1+k}\right)\right]$$

$$S_{bi}^{lA} = \frac{50(60.0)^2(12.0)(0.7435)}{24(22.9081)1.0}\left[-3 + 2\left(\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right)\right] = -4372.9\ psi$$

The bending stress at Location B, on the long side plate:

$$S_{bi}^{lB} = \frac{Ph^2 pc_i}{12I_{21}E_b}\left[\frac{1+\alpha_1^2 k}{1+k}\right]$$

$$S_{bi}^{lB} = \frac{50(60.0)^2(12.0)(0.7435)}{12(22.9081)1.0}\left[\frac{1+(0.5385)^2(0.5385)}{1+0.5385}\right] = 4390.2\ psi$$

Acceptance Criteria – Critical Locations of Maximum Stress: The stiffener allowable stress, S_{stif}, is used for the membrane stress and membrane plus bending stress for the outside fiber stress acceptance criteria, while the plate allowable stress, S, is used for the membrane plus bending stress for inside fiber allowable stress criteria.

$$\left\{S_m^s = 1982.4\ psi\right\} \le \left\{S = 22000\ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_m^s + S_{bi}^{sC} = 1982.4 + 3493.5 = 5476.0\ psi \\ S_m^s + S_{bo}^{sC} = 1982.4 + (-10003.1) = -8020.7\ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(23200) = 34800\ psi \\ 1.5S = 1.5(22000) = 33000\ psi \end{array}\right\} True$$

$$\left\{\begin{array}{l} S_m^s + S_{bi}^{sB} = 1982.4 + 6973.5 = 8955.9\ psi \\ S_m^s + S_{bo}^{sB} = 1982.4 + (-19966.9) = -17984.5\ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(23200) = 34800\ psi \\ 1.5S = 1.5(22000) = 33000\ psi \end{array}\right\} True$$

$$\left\{S_m^l = 991.2\ psi\right\} \le \left\{S = 22000\ psi\right\} \qquad True$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lA} = 991.2 + (-6946.0) = -5954.8 \ psi \\ S_m^l + S_{bo}^{lA} = 991.2 + 19888.2 = 20879.4 \ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(23200) = 34800 \ psi \\ 1.5S = 1.5(22000) = 33000 \ psi \end{array}\right\} True$$

$$\left\{\begin{array}{l} S_m^l + S_{bi}^{lB} = 991.2 + 6973.5 = 7964.7 \ psi \\ S_m^l + S_{bo}^{lB} = 991.2 + (-19966.9) = -18975.7 \ psi \end{array}\right\} \le \left\{\begin{array}{l} 1.5S = 1.5(23200) = 34800 \ psi \\ 1.5S = 1.5(22000) = 33000 \ psi \end{array}\right\} True$$

The allowable stress of the shell plate and stiffener is limited by the stiffener. Therefore, at the interface of the shell plate and stiffener, the allowable stress used in the acceptance criteria is that of the stiffener.

$$\left\{S_m^s + S_{bi}^{sC} = 1982.4 + 2199.4 = 4181.8 \ psi\right\} \le \left\{1.5S = 1.5(22000) = 33000 \ psi\right\} \quad True$$

$$\left\{S_m^s + S_{bi}^{sB} = 1982.4 + 4390.2 = 6372.6 \ psi\right\} \le \left\{1.5S = 1.5(22000) = 33000 \ psi\right\} \quad True$$

$$\left\{S_m^l + S_{bi}^{lA} = 991.2 + (-4372.9) = -3381.7 \ psi\right\} \le \left\{1.5S = 1.5(22000) = 33000 \ psi\right\} \quad True$$

$$\left\{S_m^l + S_{bi}^{lB} = 991.2 + 4390.2 = 5381.4 \ psi\right\} \le \left\{1.5S = 1.5(22000) = 33000 \ psi\right\} \quad True$$

The acceptance criteria for membrane and membrane plus bending stresses are satisfied at all locations; therefore the design is complete.

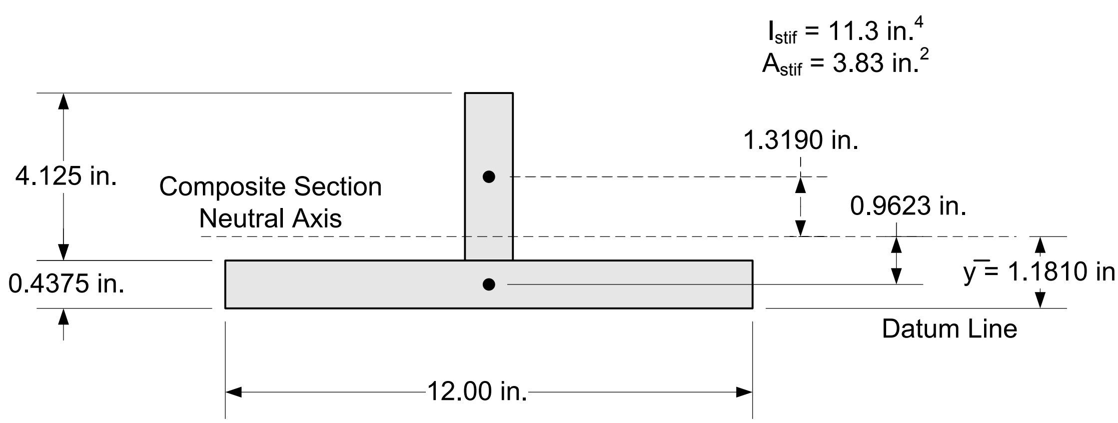


Figure E4.12.2 - Composite Section Details

4.13 Layered Vessels

4.13.1 Example E4.13.1 – Layered Cylindrical Shell

Determine the required total thickness of the layered cylindrical shell for the following design conditions. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with Part 7, paragraph 7.4.11.

Vessel Data:

- Material = $SA-724,\ Grade\ B$
- Design Conditions = $5400\ psig@300°F$
- Inside Diameter = $84.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $39600\ psi$
- Weld Joint Efficiency = 1.0
- Thickness of each layer = $0.3125\ in$

In accordance with paragraph 4.13.4.1, determine the total thickness of the layered cylindrical shell using paragraph 4.3.3.

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{84}{2}\left(\exp\left[\frac{5400}{39600(1.0)}\right]-1\right) = 6.1361\ in$$

The required thickness for all layers is $6.1361\ in$

4.13.2 Example E4.13.2 – Layered Hemispherical Head

Determine the required total thickness of the layered hemispherical head for the following design conditions. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with Part 7, paragraph 7.4.11.

Vessel Data:

- Material = $SA-724,\ Grade\ B$
- Design Conditions = $5400\ psig@300°F$
- Inside Diameter = $84.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $39600\ psi$
- Weld Joint Efficiency = 1.0
- Thickness of each layer = $0.3125\ in$

In accordance with paragraph 4.13.4.1, determine the total thickness of the layered hemispherical head using paragraph 4.3.5.

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right]-1\right) = \frac{84}{2}\left(\exp\left[\frac{0.5(5400)}{39600(1.0)}\right]-1\right) = 2.9635\ in$$

The required thickness for all layers is $2.9635\ in$

4.13.3 Example E4.13.3 – Maximum Permissible Gap in a Layered Cylindrical Shell

Determine the maximum permissible gap between any two layers in accordance with paragraph 4.13.12.3 for the cylindrical shell in Example Problem E4.13.1. The vessel has a corrosion resistant internal liner. Examination requirements are to be in accordance with Part 7, paragraph 7.4.11.

Vessel Data:

- Material = $SA-302,\ Grade\ B$
- Design Conditions = $4800\ psi\ @\ 300^{o}F$
- Inside Diameter = $84.0\ in$
- Corrosion Allowance = $0.0\ in$
- Allowable Stress = $30300\ psi$
- Weld Joint Efficiency = 1.0
- Thickness of each layer = $0.3125\ in$
- Number of layers = 20
- Specified design cycles in the UDS = 1000
- Stress amplitude at 1000 cycles = $80600\ psi$
- Elastic modulus = $28.3E+06\ psi$

In accordance with paragraph 4.13.12.3, determine the maximum permissible gap between any two layers, consider the outermost layer for the value of R_g.

$$h = 0.55\left(N - 0.5 - \frac{P}{S_m}\right)\frac{R_g S_m}{E_y}$$

$$h = 0.55\left(3.3324 - 0.5 - \frac{5400}{39600}\right)\frac{(45.125)(5400)}{28.3E+06} = 0.0128\ in$$

Where,

$$N = \frac{2}{K_c}\left(\frac{S_a}{S_m}\right) = \frac{2}{1.2216}\left(\frac{80600}{39600}\right) = 3.3324$$

With,

$$K_c = \sqrt{\frac{4S_a}{3S_m} + 0.25} - 0.5 = \sqrt{\frac{4(80600)}{3(39600)} + 0.25} - 0.5 = 1.2216$$

$$R_g = \frac{84 + 20(0.3125)}{2} = 45.125\ in$$

The maximum permissible gap is $0.0128\ in$.

4.14 Evaluation of Vessels Outside of Tolerance

4.14.1 Example E4.14.1 – Shell Tolerances

A pressure vessel is constructed from NPS 30 long seam welded pipe. During construction, examination of the vessel shell indicates peaking at the long seam weld. The shell tolerances do not satisfy the fabrication tolerances given in paragraph 4.3.2 and 4.4.4. The Determine if the design may be qualified using paragraph 4.14.1.

<u>Vessel Data</u>

- Material = $SA-106,\ Grade\ B$
- Design Conditions = $325\ psig @ 600°F$
- Pipe Outside Diameter = $30\ in$
- Wall Thickness = $0.5\ in$
- Joint Efficiency = $100\ \%$
- Corrosion Allowance = $0.063\ in$
- Allowable Stress = $17900\ psi @ 600°F$
- Material Yield Strength = $26800\ psi @ 600°F$

<u>Examination Data</u>

- Peaking distortion δ = $0.33\ in$

The vessel is not in cyclic operation based on the screening criteria performed in accordance with paragraph 5.5.2.

The user has agreed to permit the assessment procedures in API 579-1/ASME FFS-1 to be used to qualify the design. When API 579-1/ASME FFS-1 is used for the assessment, a Remaining Strength Factor of 0.95 shall be used in the calculations unless another value is agreed to by the user. However, the Remaining Strength Factor shall not be less than 0.90. A fatigue analysis is not required.

The assessment procedure for evaluation peaking at a weld joint is provided in API 579-1/ASME FFS-1, Part 8, paragraph 8.3.4.2. The step-by-step procedure from this paragraph is shown below.

a) STEP 1 – Identify the component and weld misalignment type (see Part 8 Table 8.10 API 579-1/ASME FFS-1) and determine the following variables as applicable (see Figures 8.2, 8.3, and 8.4 Part 8 API 579-1/ASME FFS-1). The weld misalignment is identified as peaking which occurs on a longitudinal weld seam. The following data is required for the assessment:

$$D_o = 30\ in \qquad LOSS = 0.0\ in$$
$$t_{nom} = 0.5\ in \qquad FCA = 0.063\ in$$
$$P = 325\ psig \qquad \delta = 0.33\ in$$
$$E_y = 26.5E+06\ psi \qquad S_a = 17900\ psi$$
$$\nu = 0.3 \qquad H_f = 3.0$$

b) STEP 2 – Determine the wall thickness to be used in the assessment

$$t_c = t_{nom} - LOSS - FCA = 0.5 - 0.0 - 0.063 = 0.437\ in$$

c) STEP 3 – Determine the membrane stress based on the current design pressure, see API 579-1/ASME FFS-1 Annex A, Equation (A.11). Note that API 579-1/ASME FFS-1 still uses the membrane stress equation from Section VIII, Division 1.

$$R = \frac{D_o}{2} - t_c = \frac{30}{2} - 0.437 = 14.563 \ in$$

$$MA = 0.0 \ in$$

$$Y_{B31} = 0.4$$

$$\sigma_m^c = \frac{P}{E}\left(\frac{R}{t_c} + 0.6\right) = \frac{325}{1.0}\left(\frac{14.563}{0.437} + 0.6\right) = 11025.606 \ psi$$

d) STEP 4 – Calculate the ratio of the induced bending stress to the applied membrane stress using the equations in Part 8 Table 8.10 of API 579-1/ASME FFS-1 based on local peaking.

$$S_p = \sqrt{\frac{12\left(1-\nu^2\right)PR^3}{E_y t_c^3}} = \sqrt{\frac{12\left(1-(0.3)^2\right)(325)(14.563)^3}{(26.5E+06)(0.437)^3}} = 2.2263$$

$$\frac{\delta}{R} = \frac{0.33}{(14.563)} = 0.02266$$

From Figure 8.13, with $\left\{\begin{matrix} S_p = 2.2263 \\ \dfrac{\delta}{R} = 0.02266 \end{matrix}\right\} \Rightarrow C_f \approx 0.84$

$$R_b^{clja} = \frac{(6)(0.33)}{(0.437)}(0.84) = 3.81$$

$$R_b = R_b^{cljc} + R_b^{clja} = 0.0 + 3.81$$

$$R_{bs} = -1.0$$

e) STEP 5 – Determine the remaining strength factors, use $H_f = 3.0$ (the bending stress due to the peaking is evaluated as a secondary stress).

$$RSF = \min\left[\frac{(3.0)(17900)}{(11025.606)(1+3.81)+(0.0)(1+(-1.0))}, \ 1.0\right] = \min[1.01257, \ 1.0] = 1$$

f) STEP 6 – Evaluate the results.

$$\{RSF = 1.0\} \geq \{RSF_a = 0.95\} \qquad True$$

The Level 2 Assessment Criterion is satisfied; therefore the peaking angle is acceptable for the specified design conditions.

4.14.2 Example E4.14.2 – Shell Tolerances and Fatigue Evaluation

Determine if the vessel in the Example Problem E4.14.2 can operate for 1200 cycles at 325 psig.

In accordance with paragraph 4.14.1, a fatigue analysis may be performed using API 579-1/ASME FFS-1. The fatigue analysis procedure is given in Part 8, paragraph 8.4.3.8. A fatigue analysis may be performed since the vessel satisfies the Level 2 criterion for the assessment of the weld misalignment as shown in Example Problem E4.14.2.

The step-by-step procedure from API 579-1/ASME FFS-1, Part 8, paragraph 8.3.4.2 is shown below.

a) STEP 1 – Determine the nature of the loading, the associated membrane stress and the number of operating cycles.
   - The loading consists of pressure loading.
   - From Example E4.14.2, the circumferential membrane stress is $\sigma_m = 11025.606$ psi.
   - The desired number of operating cycles is 1500.

b) STEP 2 – Determine the ratio of the induced bending stress to the membrane stress, R_b resulting from weld misalignment.

$$R_b^{cljc} = 0.0 \qquad \left(\mathit{since\ centerline\ offset\ is\ not\ present}\right)$$

$$R_b^{clja} = 3.81$$

$$R_b^{or} = 0.0 \qquad \left(\mathit{since\ neither\ general\ or\ arbitrary\ out\text{-}of\text{-}roundness\ is\ present}\right)$$

$$R_b = R_b^{cljc} + R_b^{clja} + R_b^{or} = 0.0 + 3.81 + 0.0 = 3.81$$

c) STEP 3 – Using the loading history and membrane stress from STEP 1 and R_b from STEP 2, calculate the stress range for the fatigue analysis using Part 8 Table 8.12 of API 579-1/ASME FFS-1.

   From Table 8.12, for a cylinder with a longitudinal weld joint with weld misalignment:

$$\Delta\sigma_m = \sigma_m = 11498 \ psi$$

$$\Delta\sigma_b = \sigma_m\left(R_b^{cljc} + R_b^{clja} + R_b^{or}\right) = (11025.606)(0.0 + 3.81 + 0.0) = 42007 \ psi$$

   The fatigue strength reduction factor will be applied when computing the alternating stress range; therefore, set $K_f = 1.0$ in the equation for ΔS_P.

$$\Delta S_P = \sigma_m\left(1 + R_b^{cljc} + R_b^{clja} + R_b^{or}\right)\left(K_f\right)$$

$$\Delta S_P = (11025.606)(1 + 0.0 + 3.81 + 0.0)(1.0) = 55033 \ psi$$

d) STEP 4 – Compute the number of allowed cycles using the stress range determined in STEP 3.
   Table 8.12 references Annex B1, paragraph B1.5 of API 579-1/ASME FFS-1. Paragraph B1.5 provides three methods for determining the permissible number of cycles:

   1) Elastic Stress Analysis and Equivalent Strength in accordance with paragraph B1.5.3
   2) Elastic-Plastic Stress Analysis and Equivalent Strain in accordance with paragraph B1.5.4
   3) Elastic Stress Analysis and Structural Stress in accordance with paragraph B1.5.5

Since an elastic-plastic stress analysis has not been conducted, the permitted number of cycles will be determined using Methods 1 and 3. In both cases the stresses considered consist of those due to pressure loading, stresses from supplementary loads and thermal gradients are considered negligible.

Method 1 per API 579-1/ASME FFS-1, Annex B1, paragraph B1.5.3

For a fatigue assessment using an elastic stress analysis and equivalent stresses, STEPS 1 through 3 in paragraph B1.5.3.1 of API 579-1/ASME FFS-1 are similar to STEPS 1 through 3 in paragraph 8.4.3.8 with the exception that the elastic stress range is calculated from the stress tensors and that the stress state from both mechanical and thermal loading are considered. For this example problem the stress range due to thermal loading is considered negligible and the mechanical loading consists of internal pressure. Thus the stress range is given by STEP 3 and is 54372 psi.

Paragraph B1.5.3, STEP 4 – Determine the effective alternating stress from Equation (B1.30), modified to ignore cyclic thermal stress, (i.e., $\Delta S_{LT} = 0.0$):

$$S_{alt} = \frac{K_f \cdot K_e \cdot \Delta S_P}{2}$$

The fatigue strength reduction factor, K_f, is determined from Table B1.10 based on type of weld and the quality level determined from Table B.11. The quality level in Table B1.11 is based on the type of inspection performed on the weld. For the vessel material, the specification called for full volumetric and full visual examination, but neither MT nor PT were performed on the weld. Thus from Table B1.11 the quality level is 4. The weld being assessed is a full penetration weld. For a full penetration weld inspected to quality level 4, Table B1.10 stipulated a weld fatigue reduction factor of $K_f = 2.0$.

The factor K_e is a fatigue penalty factor that may be determined from Equations (B1.31) to (B1.33) depending on the value of the stress range ΔS_P compared to the permitted primary plus secondary stress range, S_{PS}. The value of S_{PS} is the larger of three times the allowable stress at temperature or two times the material yield strength at the average temperature during a stress cycle. The allowable stress at temperature, S_a, equals 17900 psi and the yield strength for the SA-106 Grade B material, S_y, equals 26800 psi at 600° F and 35,000 at ambient temperature. The average yield stress during the cycle is thus 30,900 psi.

$$S_{PS} = \max\left[3.0S_a, 2S_y\right] = \max\left[(3)(17900), (2)(30900)\right] = 61800\ psi$$

Compare the value of ΔS_P to S_{PS}:

$$\left\{\Delta S_P = 55033\right\} \le \left\{S_{PS} = 61800\right\} \qquad (True)$$

Therefore from Equation (B1.31), $K_e = 1$

$$S_{alt} = \frac{K_f \cdot K_e \cdot \Delta S_P}{2} = \frac{(2)(1)(55033)}{2} = 55033\ psi$$

Paragraph B1.5.3, STEP 5 – Determine the permitted number of cycles, N, for the alternating stress computed in STEP 4 and the smooth bar fatigue curves as provided in Annex F, paragraph F.6.2.1 of API 579-1/ASME FFS-1. For temperatures not in the creep range, the permitted number of cycles is given by the following equations.

$$N = (10)^X$$

where

$$X = \frac{C_1 + C_3 Y + C_5 Y^2 + C_7 Y^3 + C_9 Y^4 + C_{11} Y^5}{1 + C_2 Y + C_4 y^2 + C_6 Y^3 + C_8 Y^4 + C_{10} Y^5}$$

$$Y = \left(\frac{S_{alt,k}}{C_{us}}\right)\left(\frac{E_{FC}}{E_T}\right)$$

The values of the coefficients C_i are given in Table F.13 for low allow steels where $\sigma_{UTS} \le 80$ ksi. Examining Table F.13, it is noted that the values of C_6 through C_{11} all equal zero.

Substituting the values for $S_{alt} = 55.305\ ksi$, $C_{us} = 1$, $E_T = 28.3E+03\,ksi\,@\ 335°F$, and $E_{FC} = 28.3E+03\,ksi$:

$$Y = \left(\frac{S_{alt,k}}{C_{us}}\right)\left(\frac{E_{FC}}{E_T}\right) = \left(\frac{55.033}{1.0}\right)\left(\frac{28.3E+03}{28.3E+03}\right) = 55.033\ ksi =$$

Substituting the values for C_1 through C_5, $S_{alt} = 54.372$:

$$X = \frac{7.999502 + (1.50085E-01)(55.033) + (-5.263661E-05)(55.033)^2}{1 + (5.832491E-02)(55.033) + (1.273659E-04)(55.033)^2} = 3.5519$$

And the number of permissible cycles is:

$$N = (10)^{3.5519} = 3564\ cycles$$

Method 3 per API 579-1/ASME FFS-1, Annex B1, paragraph B1.5.5

Paragraph B1.5.3, STEP 1 – Determine the load history for the component, considering all significant operating loads. The load applied to the pipe consists of internal pressure, P, of 325 psig.

Paragraph B1.5.3, STEP 2 – For the weld joint subject to fatigue evaluation determine the individual number of stress-strain cycles. The desired number of cycles, N, is 1500.

Paragraph B1.5.3, STEP 3 – Determine the elastically calculated membrane and bending stress normal to the hypothetical crack plane at the start and end of the cycle. Using this data calculate the membrane and bending stress ranges between the time of maximum and minimum stress for the cycle. From Example Problem 1 the maximum membrane stress for the cycle occurs at a pressure of 325 psig, and the minimum membrane stress for the cycle occurs at zero pressure. Similarly, the maximum bending stress for the cycle occurs at a pressure of 325 psig, and the minimum bending stress for the cycle occurs at zero pressure. The values of the two stress ranges given by Equations (B1.46) through (B1.50) are:

$$\Delta\sigma_m = {}^m\sigma_m^e - {}^n\sigma_m^e = 11.025 - 0 = 11.025\ ksi$$

$$\Delta\sigma_b = {}^m\sigma_b^e - {}^n\sigma_b^e = (R_b)\left({}^m\sigma_m^e - {}^n\sigma_m^e\right) = (3.81)(11.025-0) = 42.0\ ksi$$

$$\sigma_{\max} = \max\left[\left({}^m\sigma_m^e + {}^m\sigma_b^e\right), \left({}^n\sigma_m^e + {}^n\sigma_b^e\right)\right]$$

$$\sigma_{\max} = \max\left[(11.025+42.0), (0+0)\right] = 53.025\ ksi$$

$$\sigma_{\min} = \min\left[\left({}^m\sigma_m^e + {}^m\sigma_b^e\right), \left({}^n\sigma_m^e + {}^n\sigma_b^e\right)\right]$$

$$\sigma_{\min} = \min\left[(11.025+42.0), (0+0)\right] = 0\ ksi$$

$$\sigma_{mean} = \frac{\sigma_{\max}^e + \sigma_{\min}^e}{2} = \frac{53.025+0}{2} = 26.512\ ksi$$

Paragraph B1.5.3, STEP 4 – Determine the elastically calculated structural stress range, $\Delta\sigma^e$, for the cycle using Equation (B1.51)

$$\Delta\sigma^e = \Delta\sigma_m^e + \Delta\sigma_b^e = 11.025 + 42.0 = 53.025\ ksi$$

Paragraph B1.5.3, STEP 5 – Determine the elastically calculated structural strain, $\Delta\varepsilon^e$, from the elastically calculated structural stress range, $\Delta\sigma^e$, using Equation (B1.52) and the elastic modulus for the material at the average temperature of 335° F.

$$\Delta\varepsilon^e = \frac{\Delta\sigma^e}{E_{ya}} = \frac{53.025}{28.0(10)^3} = 1.8940(10)^{-3}$$

Determine the stress range, $\Delta\sigma$, and strain range, $\Delta\varepsilon$, by correcting the elastically computed values of $\Delta\sigma^e$ and $\Delta\varepsilon^e$ by solving Equations (B1.53) and (B1.54), these equations are shown below, simultaneously (this is an application of Neuber's Rule).

$$\Delta\sigma \cdot \Delta\varepsilon = \Delta\sigma^e \cdot \Delta\varepsilon^e$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{E_{ya}} + 2\left(\frac{\Delta\sigma}{K_{css}}\right)^{\frac{1}{n_{css}}}$$

The parameters K_{css} and n_{css} are determined from Table F.8 in Annex F for the average temperature during the cycle. The values for Carbon Steel (0.75 in. – weld metal) are:

$$T = 70^o F \qquad n_{css} = 0.110 \qquad K_{css} = 100.8$$

$$T = 390^o F \qquad n_{css} = 0.118 \qquad K_{css} = 99.6$$

The average temperature of the cycle is 335° F. Therefore, the values of n_{css} and K_{css} can be conservatively set at the value given for 390° F, or:

$$n_{css} = 0.118$$
$$K_{css} = 99.6$$

Substituting the above values into the simultaneous equations previously shown (i.e. Neuber's Rule), results in the following simultaneous equations.

$$\Delta\sigma \cdot \Delta\varepsilon = \Delta\sigma^e \cdot \Delta\varepsilon^e = (55.305)\left(1.97581(10)^{-3}\right) = 1.004(10)^{-1}$$

$$\Delta\varepsilon = \frac{\Delta\sigma}{28.0E+03} + 2\left(\frac{\Delta\sigma}{99.6}\right)^{\frac{1}{0.118}}$$

The solution of these equations is:

$$\Delta\sigma = 40.6896\ ksi$$
$$\Delta\varepsilon = 2.46785(10)^{-3}$$

Modify the value of $\Delta\sigma$ is modified or low-cycle fatigue using Equation (B1.55),

$$\Delta\sigma = \left(\frac{E_{ya}}{1-\nu^2}\right)\Delta\varepsilon = \left(\frac{28.0E+03}{1-(0.3)^2}\right)\left(2.46785(10)^{-3}\right) = 75.9340\ ksi$$

Paragraph B1.5.3, STEP 6 – Compute the equivalent structural stress range ΔS_{ess} using Equation (B1.56) where the input parameters are as follows:

$$\Delta\sigma = 75.9340\ ksi$$

$$t_{ess} = 0.625\ in \qquad (since\ the\ component\ thickness\ is\ \ t_c = 0.437 \le\ 0.625\ in)$$

$$R_b = \frac{|\Delta\sigma_b|}{|\Delta\sigma_m| + |\Delta\sigma_b|} = \frac{42.0}{11.025 + 42.0} = 0.7921$$

$$I^{\frac{1}{m_{ss}}} = \frac{1.23 - 0.364R_b - 0.17R_b^2}{1.007 - 0.306R_b - 0.178R_b^2}$$

$$I^{\frac{1}{m_{ss}}} = \frac{1.23 - 0.364(0.7921) - 0.17(0.7921)^2}{1.007 - 0.306(0.7921) - 0.178(0.7921)^2} = 1.2789$$

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} = \frac{0}{53.025} = 0$$

$$f_M = 1.0 \qquad \left(since\ R = 0 \le 0;\ see\ Equations\ (B1.63)\ and\ (B1.64)\right)$$

$$m_{ss} = 3.6$$

$$\Delta S_{ess} = \frac{\Delta\sigma}{t_{ess}^{\left(\frac{2-m_{ss}}{2 \cdot m_{ss}}\right)} \cdot I^{\frac{1}{m_{ss}}} \cdot f_M} = \frac{75.9340}{(0.625)^{\left(\frac{2-3.6}{2 \cdot 3.6}\right)} \cdot (1.2789) \cdot (1)} = 53.4882\ ksi$$

Paragraph B1.5.3, STEP 7 – Determine the permitted number of cycles, N, using the value of ΔS_{ess} from STEP 6 and the welded component fatigue curves in Annex F. The welded component fatigue curves are represented in Annex F by Equation (F.218):

$$N = \frac{f_1}{f_E}\left(\frac{f_{MT} \cdot C}{\Delta S_{ess}}\right)^{\frac{1}{h}}$$

The adjustment factors set as follows.

$$f_1 = 1.0 \qquad \left(\textit{fatigue improvement techniques have not been used}\right)$$

$$f_E = 4.0 \qquad \left(\textit{the process fluid is considered mildly aggressive}\right)$$

$$f_{MT} = \frac{E_{ACS}}{E_T} = \frac{2.94E+04}{2.80E+04} = 1.0500$$

From Table F.29, for a lower 99% prediction interval (-3σ), the values of C and h for low alloy steel are, $C = 818.3$ and $h = 0.3195$, respectively. The number of cycles is computed as follows.

$$N = \frac{f_1}{f_E}\left(\frac{f_{MT} \cdot C}{\Delta S_{ess}}\right)^{\frac{1}{h}} = \frac{1.0}{4.0}\left(\frac{(1.0500)(818.3)}{53.4882}\right)^{\frac{1}{0.3195}} = 1486\ cycles$$

Paragraph B1.5.3, STEP 8 – Evaluate the component by comparing the number of permitted cycles to the number of desired cycles:

$$\{N = 1486\} \geq \{Specified\ \ Design\ \ Cycles = 1200\} \qquad (True)$$

The component is acceptable for cyclic operation at the specified design conditions.

In summary, the fatigue life is satisfied by both Method 1 and Method 3, or

Method 1:

$$\{N = 3303\} \geq \{Specified\ \ Design\ \ Cycles = 1200\} \qquad (True)$$

Method 3:

$$\{N = 1486\} \geq \{Specified\ \ Design\ \ Cycles = 1200\} \qquad (True)$$

4.14.3 Example E4.14.3 – Local Thin Area

For the vessel in Example Problem 1, an arch strike was removed during fabrication by blend grinding that has resulted in a region of local metal loss. If the region of local metal loss is made to conform to the requirements of Paragraph 4.14.2.2 by blend grinding, determine whether the local thin area is acceptable using Part 5 of API 579-1/ASME FFS-1.

Vessel Data

- Material = SA-106, Grade B
- Pipe Outside Diameter = $30\ in$
- Wall Thickness = $0.5\ in$
- Design Conditions = $325\ psig@600\ °F$
- Joint Efficiency = $100\,\%$
- Future Corrosion Allowance = $0.063\ in$
- Supplemental Loads = Negligible
- The vessel is not in cyclic service (subject to less than 150 cycles)

Examination Data

Based on inspection data, the initial thickness profile in the longitudinal direction has a length s of 8.0 in and a uniform measured thickness of 0.36 in. The critical thickness profile in the circumferential direction has a length c of 10.0 in with the same uniform thickness. The region of local metal loss is located 45 in away from the nearest structural discontinuity and is the only region of local metal loss found in the vessel during inspection.

The user has agreed to permit the assessment procedures in API 579-1/ASME FFS-1 to be used to qualify the design. When API 579-1/ASME FFS-1 is used in the assessment, a Remaining Strength Factor of 0.98 shall be used in the calculations unless another value is agreed to by the user.

The assessment procedure for evaluation of a local thin area is provided in API 579-1/ASME FFS-1, Part 5, paragraph 5.4.2.2. The step-by-step procedure is shown below.

a) STEP 1 – Determine the CTP (Critical Thickness Profiles) (See Inspection Data above).

b) STEP 2 – Determine the wall thickness to be used in the assessment using equation (5.3).

$$t_{nom} = 0.5\ in$$

$$LOSS = 0.0\ in$$

$$FCA = 0.063\ in$$

$$t_c = t_{nom} - LOSS - FCA = 0.5 - 0.0 - 0.063 = 0.437\ in$$

c) STEP 3 – Determine the minimum measured thickness, t_{mm} , and the dimension, s , for the longitudinal CTP. There is only one LTA in the vessel; therefore, the spacing criteria in Part 4, paragraph 4.3.3.3.f.3 of API 579-1/ASME FFS-1 do not apply.

$$t_{mm} = 0.36\ in$$

$$s = 8.0\ in$$

d) STEP 4 – Determine the remaining thickness ratio and the longitudinal flaw length parameter, λ using equations (5.5) and (5.6) of API 579-1/ASME FFS-1.

$$R_t = \frac{t_{mm} - FCA}{t_c} = \frac{0.36 - 0.063}{0.437} = 0.6796$$

$$D = 30 - 2t_{nom} + 2(LOSS + FCA) = 30 - 2(0.5) + 2(0.0 + 0.063) = 29.176 \; in$$

$$\lambda = \frac{1.285s}{\sqrt{Dt_c}} = \frac{1.285(8)}{\sqrt{29.176(0.437)}} = 2.881$$

e) STEP 5 – Check the limiting flaw size criteria for a Level 1 Assessment using equations (5.7), (5.8), and (5.9).

$$\{R_t = 0.6796\} \geq 0.20 \qquad True$$

$$\{t_{mm} - FCA = 0.36 - 0.063 = 0.297 \; in\} \geq 0.10 \; in \qquad True$$

$$\{L_{msd} = 45 \; in\} \geq \left\{1.8\sqrt{Dt_c} = 1.8\sqrt{29.126(0.437)} = 6.422 \; in\right\} \qquad True$$

f) STEP 6 – Check the criteria for a groove-like flaw. This step is not applicable because the region of localized metal loss is categorized as an LTA.

g) STEP 7 – Determine the $MAWP$ for the component (see A.3.4) using equations (A.10), (A.16), and (A.22) of API 579-1/ASME FFS-1.

$$R = \frac{D}{2} = \frac{29.176}{2} = 14.563 \; in$$

$$MAWP^C = \frac{SEt_c}{R + 0.6t_c} = \frac{(17900)(1.0)(0.437)}{(14.563) + 0.6(0.437)} = 527.64 \; psi$$

$$MAWP^L = \frac{2SE(t_c - t_{sl})}{R - 0.4(t_c - t_{sl})} = \frac{(2)(17900)(1.0)(0.437 - 0.0)}{(14.563) - 0.4(0.437 - 0.0)} = 1087.32 \; psi$$

$$MAWP = \min[527.64, 1087.32] = 527.64 \; psi$$

h) STEP 8 – Evaluate the longitudinal extent of the flaw.

From Figure 5.6 with $\left\{\begin{array}{l}\lambda = 2.881 \\ R_t = 0.6796\end{array}\right\}$, the longitudinal extent of the flaw is acceptable. Using Table 5.2 and equation (5.11) of API 579-1/ASME FFS-1:

$$M_t = 1.857 + \frac{(2.881 - 2.5)}{(3.0 - 2.5)}(2.103 - 1.857) = 2.044$$

$$\left(RSF = \frac{R_t}{1 - \frac{1}{M_t}(1 - R_t)} = \frac{0.6796}{1 - \frac{1}{2.044}(1 - 0.6796)} = 0.8059\right) < (RSF_a = 0.9)$$

Since $RSF < RSF_a$, the reduced $MAWP$ can be calculated using equation (2.2)

$$MAWP_r = MAWP\frac{RSF}{RSF_a} = (527.64)\left(\frac{0.8059}{0.98}\right) = 433.9\ psi$$

$$\{MAWP_r = 433.9\ \ psi\} > \{P_{Design} = 325\ \ psi\}$$

The longitudinal extent of the flaw is acceptable.

i) STEP 9 – Evaluate circumferential extent of the flaw.

1) STEP 9.1 – From the circumferential CTP, determine λ_c using equation (5.12).

$$c = 10.0\ in$$

$$\lambda_C = \frac{1.285c}{\sqrt{Dt_c}} = \frac{1.285(10.0)}{\sqrt{(29.126)(0.437)}} = 3.6018$$

2) STEP 9.2 – Check the following conditions (equations (5.13) to (5.17)).

$$\{\lambda_c = 3.6018\} \le 9 \qquad True$$

$$\left\{\frac{D}{t_c} = \frac{29.176}{0.437} = 66.650\right\} \ge 20 \qquad True$$

$$0.7 \le \{RSF = 0.8059\} \le 1.0 \qquad True$$

$$0.7 \le \{E_L = 1\} \le 1.0 \qquad True$$

$$0.7 \le \{E_C = 1\} \le 1.0 \qquad True$$

3) STEP 9.3 – Calculate tensile strength factor using equation (5.18),

$$TSF = \frac{E_C}{2 \cdot RSF}\left(1 + \frac{\sqrt{4 - 3E_L^2}}{E_L}\right) = \frac{1}{2(0.8059)}\left(1 + \frac{\sqrt{4 - 3(1)^2}}{1}\right) = 1.2408$$

From Figure 5.8 with $\left\{\begin{matrix} \lambda_C = 3.6018 \\ R_t = 0.6796 \end{matrix}\right\}$, the circumferential extent of the flaw is acceptable with a $TSF = 1.2408$.

The Level 1 Assessment Criteria are satisfied. Therefore, the vessel is acceptable for operation at the specified design conditions.

4.15 Supports and Attachments

4.15.1 Example E4.15.1 – Horizontal Vessel, Zick's Analysis

Determine if the stresses in the horizontal vessel induced by the proposed saddle supports are with acceptable limits. The vessel is supported by two symmetric equally spaced saddles welded to the vessel, without reinforcing plates or stiffening rings. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined. See Figure E4.15.1

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $2074\ psig @ 175°F$
- Outside Diameter = $66.0\ in$
- Thickness = $3.0\ in$
- Corrosion Allowance = $0.125\ in$
- Formed Head Type = $2:1\ Elliptical$
- Head Height (Based on OD) = $16.5\ in$
- Allowable Stress = $23500\ psi$
- Weld Joint Efficiency = 1.0
- Shell Tangent to Tangent Length = $292.0\ in$

Saddle Data:

- Material = $SA-516,\ Grade\ 70$
- Saddle Center Line to Head Tangent Line = $41.0\ in$
- Saddle Contact Angle = $123.0\ deg$
- Width of Saddles = $8.0\ in$
- Vessel Load per Saddle = $50459.0\ lbs$

Adjust variables for corrosion and calculate the mean shell radius.

$$ID = 60.0 + 2(Corrosion\ \ Allowance) = 60.0 + 2(0.125) = 60.25\ in$$

$$t = 3.0 - Corrosion\ \ Allowance = 3.0 - 0.125 = 2.875\ in$$

$$R_m = \frac{OD + ID}{4} = \frac{66.0 + 60.25}{4} = 31.5625\ in$$

Use the procedure described in paragraph 4.15.3.

Paragraph 4.15.3.1,

a) The stress calculation method is based on linear elastic mechanics and covers modes of failure by excessive deformation and elastic instability.

b) Saddle supports for horizontal vessels shall be configured to provide continuous support for at least one-third of the shell circumference, or $\theta = 120.0\ deg$. Since the contact angle specified is $123.0\ deg$, the geometry is acceptable.

Paragraph 4.15.3.2,

c) The vessel is composed of a cylindrical shell with formed heads at each end that is supported by two equally spaced saddle supports. The moment at the saddle, M_1, the moment at the center of the vessel, M_2, and the shear force at the saddle, T, may be computed if the distance between the saddle centerline and head tangent line satisfies the following limit.

$$\{a = 41.0\} \le \{0.25L = 0.25(292.0) = 73.0\} \qquad \textit{Satisfied}$$

<u>Bending Moment at the Saddle</u>

$$M_1 = -Qa\left(1 - \frac{1 - \left(\frac{a}{L}\right) + \frac{R_m^2 - h_2^2}{2aL}}{1 + \frac{4h_2}{3L}}\right)$$

$$M_1 = -(50459.0)(41.0)\left(1 - \frac{1 - \left(\frac{41.0}{292.0}\right) + \frac{(31.5625)^2 - (16.5)^2}{2(41.0)(292.0)}}{1 + \frac{4(16.5)}{3(292.0)}}\right) = -356913.7\ in-lbs$$

<u>Bending Moment at the Center of the Vessel</u>

$$M_2 = \frac{QL}{4}\left(\frac{1 + \frac{2\left(R_m^2 - h_2^2\right)}{L^2}}{1 + \frac{4h_2}{3L}} - \frac{4a}{L}\right)$$

$$M_2 = \frac{50459.0(292.0)}{4}\left(\frac{1 + \frac{2\left[(31.5625)^2 - (16.5)^2\right]}{(292.0)^2}}{1 + \frac{4(16.5)}{3(292.0)}} - \frac{4(41.0)}{292.0}\right) = 1414775.7\ in-lbs$$

<u>Shear Force at the Saddle</u>

$$T = \frac{Q(L - 2a)}{L + \frac{4h_2}{3}} = \frac{50459.0\left[292.0 - 2(41.0)\right]}{292.0 + \frac{4(16.5)}{3}} = 33746.5\ lbs$$

Paragraph 4.15.3.3,

d) The longitudinal membrane plus bending stresses in the cylindrical shell between the supports are given by the following equations.

At the top of shell:

$$\sigma_1 = \frac{PR_m}{2t} - \frac{M_2}{\pi R_m^2 t} = \frac{2074(31.5625)}{2(2.875)} - \frac{1414775.7}{\pi(31.5625)^2(2.875)} = 11227.2 \ psi$$

Note: A load combination that includes zero internal pressure and the vessel full of contents would provide the largest compressive stress at the top of the shell, and should be checked as part of the design.

At the bottom of the shell:

$$\sigma_2 = \frac{PR_m}{2t} + \frac{M_2}{\pi R_m^2 t} = \frac{2074(31.5625)}{2(2.875)} + \frac{1414775.7}{\pi(31.5625)^2(2.875)} = 11541.7 \ psi$$

e) The longitudinal stresses in the cylindrical shell at the support location are given by the following equations. The values of these stresses depend on the rigidity of the shell at the saddle support. The cylindrical shell may be considered as suitably stiffened if it incorporates stiffening rings at, or on both sides of the saddle support, or if the support is sufficiently close defined as $a \le 0.5R_m$ to the elliptical head.

$$\{a = 41.0\} > \{0.5R_m = 0.5(31.5625) = 15.7813\} \qquad \textit{Not Satisfied}$$

Therefore, for an unstiffened shell, calculate the maximum values of longitudinal membrane plus bending stresses at the saddle support as follows.

At points A and B in Figure 4.15.5:

$$\sigma_3^* = \frac{PR_m}{2t} - \frac{M_1}{K_1 \pi R_m^2 t} = \frac{2074(31.5625)}{2(2.875)} - \frac{-356913.7}{0.0682(\pi)(31.5625)^2(2.875)} = 11740.5 \ psi$$

Where the coefficient K_1 is found in Table 4.15.1,

$$K_1 = \frac{\Delta + \sin\Delta \cdot \cos\Delta - \dfrac{2\sin^2\Delta}{\Delta}}{\pi\left(\dfrac{\sin\Delta}{\Delta} - \cos\Delta\right)} = \frac{1.4181 + \sin[1.4181] \cdot \cos[1.4181] - \dfrac{2\sin^2[1.4181]}{1.4181}}{\pi\left(\dfrac{\sin[1.4181]}{1.4181} - \cos[1.4181]\right)}$$

$$K_1 = 0.1114$$

$$\Delta = \frac{\pi}{6} + \frac{5\theta}{12} = \frac{\pi}{6} + \frac{5\left[(123.0)\left(\dfrac{\pi}{180}\right)\right]}{12} = 1.4181 \ rad$$

At the bottom of the shell:

$$\sigma_4^* = \frac{PR_m}{2t} + \frac{M_1}{K_1^* \pi R_m^2 t} = \frac{2074(31.5625)}{2(2.875)} + \frac{-356913.7}{0.2003(\pi)(31.5625)^2(2.875)}$$

$$\sigma_4^* = 11186.4 \ psi$$

Where the coefficient K_1^* is found in Table 4.15.1,

$$K_1^* = \frac{\Delta + \sin\Delta \cdot \cos\Delta - \frac{2\sin^2\Delta}{\Delta}}{\pi\left(1 - \frac{\sin\Delta}{\Delta}\right)} = \frac{1.4181 + \sin[1.4181]\cdot\cos[1.4181] - \frac{2\sin^2[1.4181]}{1.4181}}{\pi\left(1 - \frac{\sin[1.4181]}{1.4181}\right)}$$

$$K_1^* = 0.2003$$

f) Acceptance Criteria:

$$|\sigma_1| \le SE \rightarrow \{|11227.2| \ psi\} \le \{23500(1.0) = 23500 \ psi\} \qquad True$$

$$|\sigma_2| \le SE \rightarrow \{|11541.7| \ psi\} \le \{23500(1.0) = 23500 \ psi\} \qquad True$$

$$|\sigma_3^*| \le SE \rightarrow \{|11740.5| \ psi\} \le \{23500(1.0) = 23500 \ psi\} \qquad True$$

$$|\sigma_4^*| \le SE \rightarrow \{|11186.4| \ psi\} \le \{23500(1.0) = 23500 \ psi\} \qquad True$$

Since all calculated stresses are positive (tensile), the compressive stress check per paragraph 4.15.3.3.c.2 is not required.

Paragraph 4.15.3.4,

The shear stress in the cylindrical shell without stiffening ring(s) that is not stiffened by a formed head, $\{a = 41.0 \ in\} > \{0.5R_m = 0.5(31.5625) = 15.7813 \ in\}$, is calculated as follows.

$$\tau_2 = \frac{K_2 T}{R_m t} = \frac{1.1229(33746.5)}{31.5625(2.875)} = 417.6 \ psi$$

Where the coefficient K_2 is found in Table 4.15.1,

$$K_2 = \frac{\sin\alpha}{\pi - \alpha + \sin\alpha \cdot \cos\alpha} = \frac{\sin[1.9648]}{\pi - (1.9648) + \sin[1.9648]\cdot\cos[1.9648]} = 1.1229$$

$$\alpha = 0.95\left(\pi - \frac{\theta}{2}\right) = 0.95\left(\pi - \frac{123.0\left(\frac{\pi}{180}\right)}{2}\right) = 1.9648 \ rad$$

Acceptance Criteria, where $C = 0.8$ for ferritic materials:

$$|\tau_2| \le CS \rightarrow \{|417.6| \ psi\} \le \{0.8(23500) = 18800 \ psi\} \qquad True$$

Paragraph 4.15.3.5,

g) Maximum circumferential bending moment - the distribution of the circumferential bending moment at the saddle support is dependent on the use of stiffeners at the saddle location. For a cylindrical shell without a stiffening ring, the maximum circumferential bending moment is shown in Figure 4.15.6 Sketch (a) and is calculated as follows.

$$M_\beta = K_7 Q R_m = (0.0504)(50459.0)(31.5625) = 80267.7\ in-lbs$$

Where the coefficient K_7 is found in Table 4.15.1,

When $\dfrac{a}{R_m} \geq 1.0$, $K_7 = K_6$

$$\left\{\frac{a}{R_m} = \frac{41.0}{31.5625} = 1.2990\right\} \geq 1.0 \rightarrow K_7 = K_6 = 0.0504$$

$$K_6 = \left(\frac{\dfrac{3\cos\beta}{4}\left(\dfrac{\sin\beta}{\beta}\right)^2 - \dfrac{5\sin\beta\cos^2\beta}{4\beta} + \dfrac{\cos^3\beta}{2} - \dfrac{\sin\beta}{4\beta} + \dfrac{\cos\beta}{4} - \beta\sin\beta\left[\left(\dfrac{\sin\beta}{\beta}\right)^2 - \dfrac{1}{2} - \dfrac{\sin 2\beta}{4\beta}\right]}{2\pi\left[\left(\dfrac{\sin\beta}{\beta}\right)^2 - \dfrac{1}{2} - \dfrac{\sin 2\beta}{4\beta}\right]} \right)$$

$$K_6 = \left(\frac{\begin{gathered}\dfrac{3\cos[2.0682]}{4}\left(\dfrac{\sin[2.0682]}{2.0682}\right)^2 - \dfrac{5\sin[2.0682]\cos^2[2.0682]}{4(2.0682)} + \\ \dfrac{\cos^3[2.0682]}{2} - \dfrac{\sin[2.0682]}{4(2.0682)} + \dfrac{\cos[2.0682]}{4} - \\ (2.0682)\sin[2.0682]\left[\left(\dfrac{\sin[2.0682]}{2.0682}\right)^2 - \dfrac{1}{2} - \left(\dfrac{\sin[2(2.0682)]}{4(2.0682)}\right)\right]\end{gathered}}{2\pi\left[\left(\dfrac{\sin[2.0682]}{2.0682}\right)^2 - \dfrac{1}{2} - \left(\dfrac{\sin[2(2.0682)]}{4(2.0682)}\right)\right]} \right) = 0.0504$$

$$\beta = \pi - \frac{\theta}{2} = \pi - \frac{123.0\left(\dfrac{\pi}{180}\right)}{2} = 2.0682\ rad$$

h) Width of cylindrical shell - the width of the cylindrical shell that contributes to the strength of the cylindrical shell at the saddle location shall be determined as follows.

$$x_1, x_2 \le 0.78\sqrt{R_m t} = 0.78\sqrt{31.5625(2.875)} = 7.4302\ in$$

If the width $(0.5b + x_1)$ extends beyond the limit of a, as shown in Figure 4.15.2, then the width x_1 shall be reduced such as not to exceed a.

$$\left\{(0.5b + x_1) = 0.5(8.0) + 7.4302 = 11.4302\ in\right\} \le \left\{a = 41.0\ in\right\} \qquad \textit{Satisfied}$$

i) Circumferential stresses in the cylindrical shell without stiffening ring(s).

1) The maximum compressive circumferential membrane stress in the cylindrical shell at the base of the saddle support shall be calculated as follows.

$$\sigma_6 = \frac{-K_5 Q k}{t(b + x_1 + x_2)} = \frac{-0.7492(50459.0)(0.1)}{2.875(8.0 + 7.4302 + 7.4302)} = -57.5\ psi$$

Where the coefficient K_5 is found in Table 4.15.1,

$$K_5 = \frac{1 + \cos\alpha}{\pi - \alpha + \sin\alpha \cdot \cos\alpha} = \frac{1 + \cos[1.9648]}{\pi - (1.9648) + \sin[1.9648] \cdot \cos[1.9648]} = 0.7492$$

$$k = 0.1 \qquad \textit{when the vessel is welded to the saddle support}$$

2) The circumferential compressive membrane plus bending stress at Points G and H of Figure 4.15.6 Sketch (a) is determined as follows.

iii) If $L \ge 8R_m$, then the circumferential compressive membrane plus bending stress shall be computed using Equation (4.15.24).

Since $\left\{L = 292.0\ in\right\} \ge \left\{8R_m = 8(31.5625) = 252.5\ in\right\}$, the criterion is satisfied.

$$\sigma_7 = \frac{-Q}{4t(b + x_1 + x_2)} - \frac{3K_7 Q}{2t^2}$$

$$\sigma_7 = \frac{-(50459.0)}{4(2.875)(8 + 7.4302 + 7.4302)} - \frac{3(0.0504)(50459.0)}{2(2.875)^2} = -653.4\ psi$$

3) The stresses at σ_6 and σ_7 may be reduced by adding a reinforcement or wear plate at the saddle location that is welded to the cylindrical shell.

A wear plate was not specified in this example.

j) Circumferential stress in the cylindrical shell with a stiffening ring along the plane of the saddle support.

Stiffeners were not specified in the example.

k) Circumferential stress in the cylindrical shell with stiffening rings on both sides of the saddle support.

Stiffeners were not specified in the example.

l) Acceptance Criteria:

$$\{|\sigma_6| = |57.5|\ psi\} \le \{S = 23500\ psi\} \qquad True$$

$$\{|\sigma_7| = |653.4|\ psi\} \le \{1.25S = 1.25(23500) = 29375\ psi\} \qquad True$$

Paragraph 4.15.3.6,

The horizontal force at the minimum section at the low point of the saddle is given by Equation (4.15.42). The saddle shall be designed to resist this force.

$$F_h = Q\left(\frac{1+\cos\beta - 0.5\sin^2\beta}{\pi - \beta + \sin\beta\cdot\cos\beta}\right)$$

$$F_h = (50459.0)\left(\frac{1+\cos[2.0682] - 0.5\sin^2[2.0682]}{\pi - (2.0682) + \sin[2.0682]\cdot\cos[2.0682]}\right) = 10545.1\ lbs$$

Note: The horizontal splitting force is equal to the sum of all of the horizontal reactions at the saddle due to the weight loading of the vessel. The splitting force is used to calculate tension stress and bending stress in the web of the saddle. The following provides one possible method of calculating the tension and bending stress in the web and its acceptance criteria. However, other methods may also be deemed acceptable by the Manufacturer and accepted by the Authorized Inspector.

The membrane stress is given by,

$$\left\{\sigma_t = \frac{F_h}{A_s}\right\} \le \{0.6S_y\}$$

where A_s is the cross-sectional area of the web at the low point of the saddle with units of in^2, and S_y is the yield stress of the saddle material with units of psi.

The bending stress is given by,

$$\left\{\sigma_b = \frac{F_h\cdot d\cdot c}{I}\right\} \le \{0.66S_y\}$$

where d is the moment arm of the horizontal splitting force, measured from the center of gravity of the saddle arc to the bottom of the saddle baseplate with units of in, c is the distance from the centroid of the saddle composite section to the extreme fiber with units of in, I is the moment of inertia of the composite section of the saddle with units of in^4, and S_y is the yield stress of the saddle material with units of psi.

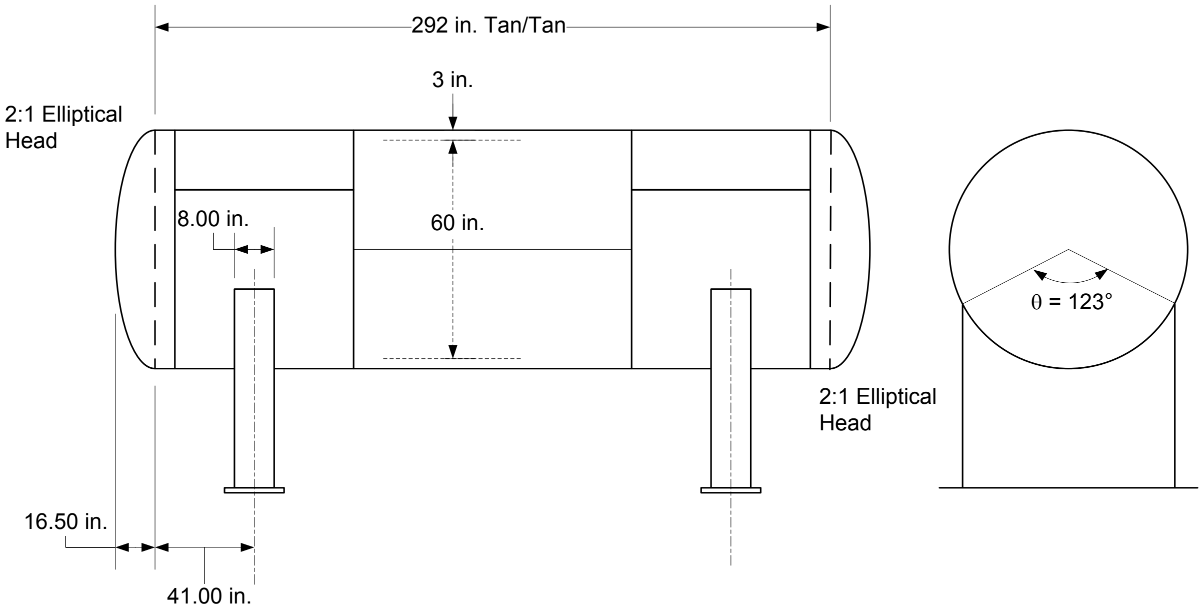


Figure E4.15.1 - Saddle Details

4.15.2 Example E4.15.2 – Vertical Vessel, Skirt Design

Determine if the proposed cylindrical vessel skirt is adequately designed considering the following loading conditions.

Skirt Data:

- Material = $SA-516,\ Grade\ 70$
- Design Temperature = $300°F$
- Skirt Inside Diameter = $150.0\ in$
- Thickness = $0.625\ in$
- Length of Skirt = $147.0\ in$
- Allowable Stress at Design Temperature = $22400\ psi$
- Modulus of Elasticity at Design Temperature = $28.3E+06\ psi$
- Yield Strength at Design Temperature = $33600\ psi$
- Axial Force, Weight = $-363500\ lbs$
- Axial Force, Appurtenance Live Loading = $-85700\ lbs$
- Bending Moment, Appurtenance Loading = $90580\ in-lbs$
- Bending Moment, Earthquake Loading = $18550000\ in-lbs$
- Bending Moment, Wind Loading = $29110000\ in-lbs$

Adjust variable for corrosion and determine outside dimensions.

$$D = 150.0 + 2(Corrosion\ Allowance) = 150.0 + 2(0.0) = 150.0\ in$$

$$R = 0.5D = 0.5(150.0) = 75.0\ in$$

$$t = 0.625 - Corosion\ Allowance = 0.625 - 0.0 = 0.625\ in$$

$$D_o = 150.0 + 2(Uncorroded\ Thickness) = 150.0 + 2(0.625) = 151.25\ in$$

$$R_o = 0.5D_o = 0.5(151.25) = 75.625\ in$$

Evaluate per paragraph 4.15.4 with reference to paragraph 4.3.10.

The loads transmitted to the base of the skirt are given in the Table E4.15.2.2. Note that this table is given in terms of the load parameters and load combinations shown in Table 4.1.1 and Table 4.1.2. (Table E4.15.2.1 of this example). As shown in Table E4.15.2.1, the acceptance criteria is that the general primary membrane stress for each load case must be less than or equal to the allowable stress at the specified design condition.

In accordance with paragraph 4.3.10.2, the following procedure shall be used to design cylindrical, spherical, and conical shells subjected to internal pressure plus supplemental loads of applied net section axial force, bending moment, and torsional moment. By inspection of the results shown in Table E4.15.2.3, Load Case 5 is determined to be a potential governing load case. The pressure, net section axial force, and bending moment at the location of interest for Load Case 5 are:

$$0.9P + P_s = 0.0 \ psi$$
$$F_5 = -363500 \ lbs$$
$$M_5 = 17466000 \ in-lbs$$
$$M_{t5} = 0.0 \ in-lbs$$

Determine applicability of the rules of paragraph 4.3.10 based on satisfaction of the following requirements.

The section of interest is at least $2.5\sqrt{Rt}$ away from any major structural discontinuity.

$$2.5\sqrt{Rt} = 2.5\sqrt{(75.0)(0.625)} = 17.1163 \ in$$

Shear force is not applicable.

The shell R/t ratio is greater than 3.0, or:

$$\left\{R/t = \frac{75.0}{0.625} = 120.0\right\} > 3.0 \qquad\qquad True$$

a) STEP 1 – Calculate the membrane stress. For the skirt, weld joint efficiency is set as $E = 1.0$. Note, that the maximum bending stress occurs at $\theta = 0.0 \ deg$.

$$\sigma_{\theta m} = \frac{P}{E(D_o - D)} = \frac{P}{E(151.25 - 150.0)} = 0.0 \ psi$$

$$\sigma_{sm} = \frac{1}{E}\left(\frac{PD^2}{D_o^2 - D^2} + \frac{4F}{\pi\left(D_o^2 - D^2\right)} \pm \frac{32MD_o\cos[\theta]}{\pi\left(D_o^4 - D^4\right)}\right)$$

$$\sigma_{sm} = \frac{1}{1.0}\left(0.0 + \frac{4(-363500)}{\pi\left((151.25)^2 - (150.0)^2\right)} \pm \frac{32(17466000)(151.25)\cos[0.0]}{\pi\left((151.25)^4 - (150.0)^4\right)}\right)$$

$$\sigma_{sm} = \begin{Bmatrix} -1229.0724 + 1574.7814 = 345.7090 \ psi \\ -1229.0724 - 1574.7814 = -2803.8538 \ psi \end{Bmatrix}$$

$$\tau = \frac{16M_tD_o}{\pi\left(D_o^4 - D^4\right)} = 0.0 \ psi$$

b) STEP 2 – Calculate the principal stresses.

$$\sigma_1 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} + \sqrt{(\sigma_{\theta m} - \sigma_{sm})^2 + 4(\tau)^2}\right)$$

$$\sigma_1 = \begin{Bmatrix} 0.5\left(0.0 + 345.7090 + \sqrt{(0.0 - 345.7090)^2 + 4(0.0)^2}\right) = 345.7090 \ psi \\ 0.5\left(0.0 + (-2803.8538) + \sqrt{(0.0 - (-2803.8538))^2 + 4(0.0)^2}\right) = 0.0 \ psi \end{Bmatrix}$$

$$\sigma_2 = 0.5\left(\sigma_{\theta m} + \sigma_{sm} - \sqrt{\left(\sigma_{\theta m} - \sigma_{sm}\right)^2 + 4\left(\tau\right)^2}\right)$$

$$\sigma_2 = \begin{Bmatrix} 0.5\left(0.0 + 345.7090 - \sqrt{\left(0.0 - 345.7090\right)^2 + 4\left(0.0\right)^2}\right) = 0.0 \; psi \\ 0.5\left(0.0 + \left(-2803.8538\right) - \sqrt{\left(0.0 - \left(-2803.8538\right)\right)^2 + 4\left(0.0\right)^2}\right) = -2803.8538 \; psi \end{Bmatrix}$$

$$\sigma_3 = \sigma_r = -0.5P = 0.0 \; psi$$

c) STEP 3 – Check the allowable stress acceptance criteria.

$$\sigma_e = \frac{1}{\sqrt{2}}\left[\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right]^{0.5} \le SE$$

$$\sigma_e = \begin{Bmatrix} \frac{1}{\sqrt{2}}\begin{bmatrix} \left(345.7090 - 0.0\right)^2 + \\ \left(0.0 - 0.0\right)^2 + \left(0.0 - 345.7090\right)^2 \end{bmatrix}^{0.5} = 345.7090 \; psi \\ \frac{1}{\sqrt{2}}\begin{bmatrix} \left(0.0 - \left(-2803.8538\right)\right)^2 + \\ \left(\left(-2803.8538\right) - 0.0\right)^2 + \left(0.0 - 0.0\right)^2 \end{bmatrix}^{0.5} = 2803.8538 \; psi \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_e = 345.7 \; psi \\ \sigma_e = 2803.9 \; psi \end{Bmatrix} \le \left\{SE = 22400 \; psi\right\}$$

Since the equivalent stress is less than the acceptance criteria, the shell section is adequately designed considering Load Case 5.

d) STEP 4 – For cylindrical and conical shells, if the axial membrane stress, σ_{sm} is compressive, then Equation (4.3.45) shall be satisfied where F_{xa} is evaluated using paragraph 4.4.12.2 with $\lambda = 0.15$.

$$\sigma_{sm} \le F_{xa}$$

Since σ_{sm} is compressive, $\left\{\sigma_{sm} = -2803.8538 \; psi < 0\right\}$, a buckling check is required.

VIII-2, paragraph 4.4.12.2.b – Axial Compressive Stress Acting Alone.

In accordance with paragraph 4.4.12.2.b, the value of F_{xa} is calculated as follows, with $\lambda = 0.15$.

The design factor FS used in paragraph 4.4.12.2.b is dependent on the predicted buckling stress F_{ic} and the material's yield strength, S_y as shown in paragraph 4.4.2. An initial calculation is required to determine the value of F_{xa} by setting $FS = 1.0$, with $F_{ic} = F_{xa}$. The initial value of F_{ic} is then compared to S_y as shown in paragraph 4.4.2 and the value of FS is determined. This computed value of FS is then used in paragraph 4.4.12.2.b.

For $\lambda_c = 0.15$, (Local Buckling)

$$F_{xa} = \min\left[F_{xa1},\, F_{xa2}\right]$$

$$\frac{D_o}{t} = \frac{151.25}{0.625} = 242.0$$

$$M_x = \frac{L}{\sqrt{R_o t}} = \frac{147.0}{\sqrt{75.625(0.625)}} = 21.3818$$

Since $135 < \dfrac{D_o}{t} \le 600$, calculate F_{xa1} as follows with an initial value of $FS = 1.0$.

$$F_{xa1} = \frac{466S_y}{FS\left(331 + \dfrac{D_o}{t}\right)} = \frac{466(33600)}{1.0\left(331 + \dfrac{151.25}{0.625}\right)} = 27325.6545\ psi$$

The value of F_{xa2} is calculated as follows with an initial value of $FS = 1.0$.

$$F_{xa2} = \frac{F_{xe}}{FS}$$

$$F_{xe} = \frac{C_x E_y t}{D_o}$$

Since $\dfrac{D_o}{t} \le 1247$, calculate C_x as follows:

$$C_x = \min\left[\frac{409\bar{c}}{\left(389 + \dfrac{D_o}{t}\right)},\ 0.9\right]$$

Since $M_x \ge 15$, calculate $\bar{c}$ as follows:

$$\bar{c} = 1.0$$

$$C_x = \min\left[\frac{409(1.0)}{389 + \dfrac{151.25}{0.625}},\ 0.9\right] = 0.6482$$

Therefore,

$$F_{xe} = \frac{0.6482(28.3E+06)(0.625)}{151.25} = 75801.9008 \ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{75801.9008}{1.0} = 75801.9008 \ psi$$

$$F_{xa} = \min[27325.6545, \ 75801.9008] = 27325.6545 \ psi$$

With a value of $F_{ic} = F_{xa} = 27325.6545$, in accordance with paragraph 4.4.2, the value of FS is determined as follows.

Since $\{0.55S_y = 0.55(33600) = 18480\} \le \{F_{ic} = 27325.6545\} \le \{S_y = 33600\}$,

$$FS = 2.407 - 0.741\left(\frac{F_{ic}}{S_y}\right) = 2.407 - 0.741\left(\frac{27325.6545}{33600}\right) = 1.8044$$

Using this computed value of $FS = 1.8044$ in paragraph 4.4.12.2.b, F_{xa} is calculated as follows.

$$F_{xa1} = \frac{466S_y}{FS\left(331+\frac{D_o}{t}\right)} = \frac{466(33600)}{1.8044\left(331+\frac{151.25}{0.625}\right)} = 15143.9007 \ psi$$

$$F_{xa2} = \frac{F_{xe}}{FS} = \frac{75801.9008}{1.8044} = 42009.4773 \ psi$$

$$F_{xa} = \min[15143.9007, \ 42009.4773] = 15143.9007 \ psi$$

Compare the calculated axial compressive membrane stress, σ_{sm} to the allowable axial compressive membrane stress, F_{xa} per following criteria

$$\{\sigma_{sm} = 2803.9 \ psi\} \le \{F_{xa} = 15143.9 \ psi\} \qquad True$$

Therefore, local buckling due to axial compressive membrane stress is not a concern.

Table E4.15.2.1 - Design Loads and Load Combinations from VIII-2

Table 4.1.1 – Design Loads

Design Load Parameter	Description
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)
P_s	Static head from liquid or bulk materials (e.g. catalyst)
D	Dead weight of the vessel, contents, and appurtenances at the location of interest, including the following: • Weight of vessel including internals, supports (e.g. skirts, lugs, saddles, and legs), and appurtenances (e.g. platforms, ladders, etc.) • Weight of vessel contents under operating and test conditions • Refractory linings, insulation • Static reactions from the weight of attached equipment, such as motors, machinery, other vessels, and piping • Transportation Loads (The static forces obtained as equivalent to the dynamic loads experienced during normal operation of a transport vessel – see paragraph 1.2.1.2.b)
L	• Appurtenance Live loading • Effects of fluid flow, steady state or transient • Loads resulting from wave action
E	Earthquake loads (see ASCE 7 for the specific definition of the earthquake load, as applicable)
W	Wind Loads (See 4.1.5.3.b)
S	Snow Loads
F	Loads due to Deflagration

Table 4.1.2 – Design Load Combinations

Design Load Combination (1)	General Primary Membrane Allowable Stress (2)
$P+P_s+D$	S
$P+P_s+D+L$	S
$P+P_s+D+S$	S
$0.9P+P_s+D+0.75L+0.75S$	S
$0.9P+P_s+D+(0.6W\ or\ 0.7E)$	S
$0.9P+P_S+D+0.75(0.6W\ or\ 0.7E)+0.75L+0.75S$	S
$0.6D+(0.6W\ or\ 0.7E)$ (3)	S
P_s+D+F	See Annex 4.D

Notes
1) The parameters used in the Design Load Combination column are defined in Table 4.1.1.
2) S is the allowable stress for the load case combination (see paragraph 4.1.5.3.c)
3) This load combination addresses an overturning condition for foundation design. It does not apply to design of anchorage (if any) to the foundation. Refer to ASCE/SEI 7-10, 2.4.1 Exception 2 for an additional reduction to W that may be applicable.

Table E4.15.2.2 - Design Loads (Net-Section Axial Force and Bending Moment) at the Location of Interest

Design Load Parameter	Description	Magnitude of Pressure, Force and Moment
P	Internal or External Specified Design Pressure (see paragraph 4.1.5.2.a)	$P = 0.0$
P_s	Static head from liquid or bulk materials (e.g. catalyst)	$P_s = 0.0$
D	The dead weight of the vessel including skirt, contents, and appurtenances at the location of interest	$D_F = -363500\ lbs$ $D_M = 0.0\ in-lbs$
L	Appurtenance live loading and effects of fluid flow	$L_F = -85700\ lbs$ $L_M = 90580\ in-lbs$
E	Earthquake loads	$E_F = 0.0\ lbs$ $E_M = 18550000\ in-lbs$
W	Wind Loads	$W_F = 0.0\ lbs$ $W_M = 29110000\ in-lbs$
S	Snow Loads	$S_F = 0.0\ lbs$ $S_M = 0.0\ in-lbs$
F	Loads due to Deflagration	$F_F = 0.0\ lbs$ $F_M = 0.0\ in-lbs$

Based on these loads, the shell is required to be designed for the load case combinations shown in Table E4.15.2.3. Note that this table is given in terms of the load combinations shown in Table 4.1.2 (Table E4.15.2.1 of this example).

Table E4.15.2.3 - Load Case Combination at the Location of Interest

Load Case	Design Load Combination	Magnitude of Pressure, Force and Moment	General Primary Membrane Allowable Stress
1	$P+P_s+D$	$P+P_s=0.0\ psi$ $F_1=-363500\ lbs$ $M_1=0.0\ in-lbs$	S
2	$P+P_s+D+L$	$P+P_s=0.0\ psi$ $F_2=-449200\ lbs$ $M_2=90580\ in-lbs$	S
3	$P+P_s+D+S$	$P+P_s=0.0\ psi$ $F_3=-363500\ lbs$ $M_3=0.0\ in-lbs$	S
4	$0.9P+P_s+D+0.75L+0.75S$	$0.9P+P_s=0.0\ psi$ $F_4=-427775\ lbs$ $M_4=67935\ in-lbs$	S
5	$0.9P+P_s+D+(0.6W\ or\ 0.7E)$	$0.9P+P_s=0.0\ psi$ $F_5=-363500\ lbs$ $M_5=17466000\ in-lbs$	S
6	$\left(\begin{array}{l}0.9P+P_S+D+0.75(0.6W\ or\ 0.7E)+\\0.75L+0.75S\end{array}\right)$	$0.9P+P_s=0.0\ psi$ $F_6=-427775\ lbs$ $M_6=13167435\ in-lbs$	S
7	$0.6D+(0.6W\ or\ 0.7E)$ Anchorage is included in the design. Therefore, consideration of this load combination is not required.	$F_7=-218100\ lbs$ $M_7=17466000\ in-lbs$	S
8	P_s+D+F	$P_s=0.0\ psi$ $F_8=-363500\ lbs$ $M_8=0.0\ in-lbs$	See Annex 4.D

4.16 Flanged Joints

4.16.1 Example E4.16.1 – Integral Type

Determine if the stresses in the heat exchanger girth flange are with acceptable limits, considering the following design conditions. The flange is of an integral type and is attached to a cylindrical shell with a Category C, Type 1 butt weld and has been 100% radiographically examined. See Figure E4.16.1.

General Data:

- Cylinder Material = $SA-516,\ Grade\ 70$
- Design Conditions = $135\ psig@650°F$
- Allowable Stress at Design Temperature = $18800\ psi$
- Allowable Stress at Ambient Temperature = $25300\ psi$
- Corrosion Allowance = $0.125\ in$

Flange Data

- Material = $SA-105$
- Allowable Stress at Design Temperature = $17800\ psi$
- Allowable Stress at Ambient Temperature = $24000\ psi$
- Modulus of Elasticity at Design Temperature = $26.0E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$

Bolt Data

- Material = $SA-193,\ Grade\ B7$
- Allowable Stress at Design Temperature = $25000\ psi$
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Diameter = $0.75\ in$
- Number of Bolts = 44
- Root area = $0.302\ in^2$

Gasket Data

- Material = Flat Metal Jacketed (Iron/Soft Steel)
- Gasket Factor = 3.75
- Seating Stress = $7600\ psi$
- Inside Diameter = $29.0\ in$
- Outside Diameter = $30.0\ in$

Evaluate the girth flange in accordance with paragraph 4.16.

Paragraph 4.16.6, Design Bolt Loads. The procedure to determine the bolt loads for the operating and gasket seating conditions is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

$$P = 135\ psig\ at\ 650°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 4.16.1.

$$m = 3.75$$
$$y = 7600\ psi$$

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5\left(GOD - GID\right) = 0.5\left(30.0 - 29.0\right) = 0.500\ in$$

From Table 4.16.3, Facing Sketch Detail 2, Column II,

$$b_o = \frac{w + 3N}{8} = \frac{0.125 + 3\left(0.500\right)}{8} = 0.2031\ in$$

Where,

$$w = raised\ nubbin\ width = 0.125\ in$$

For $\{b_o = 0.2031\ in\} \le \{0.25\ in\}$,

$$b = b_o = 0.2031\ in$$
$$G = mean\ diameter\ of\ the\ gasket\ contact\ face$$
$$G = 0.5\left(30.0 + 29.0\right) = 29.5\ in$$

d) STEP 4 – Determine the design bolt load for the operating condition.

$$W_o = \frac{\pi}{4} G^2 P + 2b\pi GmP \qquad for\ non-self-energized\ gaskets$$

$$W_o = \frac{\pi}{4}\left(29.5\right)^2\left(135\right) + 2\left(0.2031\right)\pi\left(29.5\right)\left(3.75\right)\left(135\right) = 111329.5\ lbs$$

e) STEP 5 – Determine the design bolt load for the gasket seating condition.

$$W_g = \left(\frac{A_m + A_b}{2}\right) S_{bg} = \left(\frac{5.7221 + 13.2880}{2}\right) 25000 = 237626.3\ lbs$$

Where,

$$A_b = \left(Number\ of\ bolts\right)\left(Root\ area\ of\ one\ bolt\right) = 44\left(0.302\right) = 13.2880\ in^2$$

$$A_m = \max\left[\left(\frac{W_o + F_A + \dfrac{4M_E}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right] = \max\left[\left(\frac{111329.5 + 0.0 + 0.0}{25000}\right), \left(\frac{143052.5}{25000}\right)\right]$$

$$A_m = \max\left[4.4532,\ 5.7221\right] = 5.7221\ in^2$$

And,

$$W_{gs} = \pi bG\left(C_{us} y\right) \qquad for\ non-self-energized\ gaskets$$

$$W_{gs} = \pi\left(0.2031\right)\left(29.5\right)\left(1.0\left(7600\right)\right) = 143052.5\ lbs$$

$F_A = 0$ and $M_E = 0$ since there are no externally applied net-section axial forces or bending moments.

Paragraph 4.16.7, Flange Design Procedure. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings. The procedure incorporates both a strength check and a rigidity check for flange rotation.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint and the external net-section axial force, F_A, and bending moment, M_E.

$$P = 135\ psig\ at\ 650°F$$

$$F_A = 0$$

$$M_E = 0$$

b) STEP 2 – Determine the design bolt loads for operating condition W_o, and the gasket seating condition W_g, and the corresponding actual bolt load area A_b, from paragraph 4.16.6.

$$W_o = 111329.5\ lbs$$

$$W_g = 2237626.3\ lbs$$

$$A_b = 13.288\ in^2$$

c) STEP 3 – Determine an initial flange geometry, see Figure E4.16.1, in addition to the information required to determine the bolt load, the following geometric parameters are required.

1) Flange bore

$$B = \left[26.0 + 2\left(Corrosion\ Allowance\right)\right] = \left[26.0 + 2\left(0.125\right)\right] = 26.25\ in$$

2) Bolt circle diameter

$$C = 31.25\ in$$

3) Outside diameter of the flange

$$A = 32.875\ in$$

4) Flange thickness

$$t = 1.625 - 0.1875 = 1.4375\ in$$

5) Thickness of the hub at the large end

$$g_1 = \left(0.5\left(Hub\ OD\ at\ Back\ of\ Flange - Uncorroded\ Bore\right) - Corrosion\ Allowance\right)$$

$$g_1 = \left(0.5\left(27.625 - 26.0\right) - 0.125\right) = 0.6875\ in$$

6) Thickness of the hub at the small end

$$g_0 = \left(Hub\ Thickness\ at\ Cylinder\ Attachment - Corrosion\ Allowance\right)$$

$$g_0 = \left(0.4375 - Corrosion\ Allowance\right) = \left(0.4375 - 0.125\right) = 0.3125\ in$$

7) Hub length

$$h = 2.125\ in$$

d) STEP 4 – Determine the flange stress factors using the equations in Table 4.16.4 and Table 4.16.5.

$$K = \frac{A}{B} = \frac{32.875}{26.25} = 1.2524$$

$$Y = \frac{1}{K-1}\left[0.66845 + 5.71690\left(\frac{K^2 \log_{10} K}{K^2 - 1}\right)\right]$$

$$Y = \frac{1}{1.2524-1}\left[0.66845 + 5.71690\frac{(1.2524)^2 \log_{10}\left[1.2524\right]}{(1.2524)^2 - 1}\right] = 8.7565$$

$$T = \frac{K^2\left(1 + 8.55246\log_{10} K\right) - 1}{\left(1.04720 + 1.9448K^2\right)\left(K-1\right)} = \frac{(1.2524)^2\left(1 + 8.55246\log_{10}\left[1.2524\right]\right) - 1}{\left(1.04720 + 1.9448(1.2524)^2\right)\left(1.2524 - 1\right)} = 1.8175$$

$$U = \frac{K^2\left(1 + 8.55246\log_{10} K\right) - 1}{1.36136\left(K^2 - 1\right)\left(K-1\right)} = \frac{(1.2524)^2\left(1 + 8.55246\log_{10}\left[1.2524\right]\right) - 1}{1.36136\left((1.2524)^2 - 1\right)\left(1.2524 - 1\right)} = 9.6225$$

$$Z = \frac{\left(K^2 + 1\right)}{\left(K^2 - 1\right)} = \frac{\left((1.2524)^2 + 1\right)}{\left((1.2524)^2 - 1\right)} = 4.5180$$

$$h_o = \sqrt{Bg_0} = \sqrt{(26.25)(0.3125)} = 2.8641\ in$$

$$X_g = \frac{g_1}{g_0} = \frac{0.6875}{0.3125} = 2.2000$$

$$X_h = \frac{h}{h_o} = \frac{2.125}{2.8641} = 0.7419$$

$$F = \begin{pmatrix} 0.897697 - 0.297012\ln\left[X_g\right] + 9.5257\left(10^{-3}\right)\ln\left[X_h\right] + \\ 0.123586\left(\ln\left[X_g\right]\right)^2 + 0.0358580\left(\ln\left[X_h\right]\right)^2 - 0.194422\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right) - \\ 0.0181259\left(\ln\left[X_g\right]\right)^3 + 0.0129360\left(\ln\left[X_h\right]\right)^3 - \\ 0.0377693\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right)^2 + 0.0273791\left(\ln\left[X_g\right]\right)^2\left(\ln\left[X_h\right]\right) \end{pmatrix}$$

$$F = \begin{pmatrix} 0.897697 - 0.297012\ln[2.20] + 9.5257\left(10^{-3}\right)\ln[0.7419] + 0.123586\left(\ln[2.20]\right)^2 + \\ 0.0358580\left(\ln[0.7419]\right)^2 - 0.194422\left(\ln[2.20]\right)\left(\ln[0.7419]\right) - \\ 0.0181259\left(\ln[2.20]\right)^3 + 0.0129360\left(\ln[0.7419]\right)^3 - \\ 0.0377693\left(\ln[2.20]\right)\left(\ln[0.7419]\right)^2 + 0.0273791\left(\ln[2.20]\right)^2\left(\ln[0.7419]\right) \end{pmatrix}$$

$$F = 0.7695$$

$$For\ 0.5 \le X_h \le 2.0,$$

$$V = \begin{pmatrix} 0.0144868 - \frac{0.135977}{X_g} - \frac{0.0461919}{X_h} + \frac{0.560718}{X_g^2} + \frac{0.0529829}{X_h^2} + \\ \frac{0.244313}{X_g X_h} + \frac{0.113929}{X_g^3} - \frac{0.00929265}{X_h^3} - \frac{0.0266293}{X_g X_h^2} - \frac{0.217008}{X_g^2 X_h} \end{pmatrix}$$

$$V = \begin{pmatrix} 0.0144868 - \frac{0.135977}{2.20} - \frac{0.0461919}{0.7419} + \frac{0.560718}{(2.20)^2} + \frac{0.0529829}{(0.7419)^2} + \\ \frac{0.244313}{(2.20)(0.7419)} + \frac{0.113929}{(2.20)^3} - \frac{0.00929265}{(0.7419)^3} - \frac{0.0266293}{(2.20)(0.7419)^2} - \frac{0.217008}{(2.20)^2(0.7419)} \end{pmatrix}$$

$$V = 0.1577$$

$$f = \max\left[1.0, \left(\frac{\begin{array}{c} 0.0927779 - 0.0336633X_g + 0.964176X_g^2 + \\ 0.0566286X_h + 0.347074X_h^2 - 4.18699X_h^3 \end{array}}{\begin{array}{c} 1 - 5.96093\left(10^{-3}\right)X_g + 1.62904X_h + \\ 3.49329X_h^2 + 1.39052X_h^3 \end{array}}\right)\right]$$

$$f = \max\left[1.0, \left(\frac{\begin{array}{c} 0.0927779 - 0.0336633(2.20) + 0.964176(2.20)^2 + \\ 0.0566286(0.7419) + 0.347074(0.7419)^2 - 4.18699(0.7419)^3 \end{array}}{\begin{array}{c} 1 - 5.96093\left(10^{-3}\right)(2.20) + 1.62904(0.7419) + \\ 3.49329(2.20)^2 + 1.39052(0.7419)^3 \end{array}}\right)\right]$$

$$f = 1.0$$

$$d = \frac{Ug_0^2 h_o}{V} = \frac{(9.6225)(0.3125)^2(2.8641)}{0.1577} = 17.0665\ in$$

$$e = \frac{F}{h_o} = \frac{0.7695}{2.8641} = 0.2687$$

$$L = \frac{te+1}{T} + \frac{t^3}{d} = \frac{1.4375(0.2687)+1}{1.8175} + \frac{(1.4375)^3}{17.0665} = 0.9368$$

e) STEP 5 – Determine the flange forces.

$$H_D = \frac{\pi}{4}B^2P = \frac{\pi}{4}(26.25)^2(135) = 73060.4\ lbs$$

$$H = \frac{\pi}{4}G^2P = \frac{\pi}{4}(29.5)^2(135) = 92271.5\ lbs$$

$$H_T = H - H_D = 92271.5 - 73060.4 = 19211.1\ lbs$$

$$H_G = W_o - H = 111329.5 - 92227.5 = 19058.0\ lbs$$

f) STEP 6 – Determine the flange moment for the operating condition using Equation (4.16.14) or Equation (4.16.15), as applicable. When specified by the user or his designated agent, the maximum bolt spacing (B_{smax}) and the bolt spacing correction factor (B_{sc}) shall be applied in calculating the flange moment for internal pressure using the equations in Table 4.16.11. The flange moment M_o for the operating condition and the flange moment M_g for the gasket seating condition without correction for bolt spacing $B_{sc} = 1$ is used for the calculation of the rigidity index in Step 10. In these equations, h_D, h_T, and h_G are determined from Table 4.16.6. For integral and loose type flanges, the moment M_{oe} is calculated using Equation (4.16.16) where and I and I_p in this equation are determined from Table 4.16.7.

For internal pressure,

$$M_o = abs\left[\left(\left(H_D h_D + H_T h_T + H_G h_G\right)B_{sc} + M_{oe}\right)F_s\right]$$

$$M_o = abs\left[\left(\left(73060.4(2.1563) + 19211.1(1.6875) + 19058.0(0.875)\right)1.0 + 0.0\right)1.0\right]$$

$$M_o = 206634.6\ in - lbs$$

From Table 4.16.11, the maximum bolt spacing and the bolt spacing correction factor are calculated as follows.

$$B_{smax} = 2a + \frac{6t}{m+0.5} = 2(0.75) + \frac{6(1.4375)}{3.75+0.5} = 3.5294\ in$$

$$B_{sc} = \sqrt{\frac{B_s}{2a+t}} = \sqrt{\frac{0.7102}{2(0.75)+1.4375}} = 0.4917$$

The actual bolt spacing is determined using the following equation.

$$B_s = \frac{C}{No.\ of\ bolts} = \frac{31.25}{44} = 0.7102\ in$$

Since $\{B_s = 0.7102\ in\} \le \{B_{smax} = 3.5294\ in\}$, the value of $B_{sc} = 1.0$.

From Table 4.16.6,

$$h_D = \frac{C - B - g_1}{2} = \frac{31.25 - 26.25 - 0.6875}{2} = 2.1563\ in$$

$$h_G = \frac{C - G}{2} = \frac{31.25 - 29.5}{2} = 0.875\ in$$

$$h_T = \frac{1}{2}\left[\frac{C - B}{2} + h_G\right] = \frac{1}{2}\left[\frac{31.25 - 26.25}{2} + 0.875\right] = 1.6875\ in$$

Since $F_A = 0$ and $M_E = 0$, the flange cross-section bending moment of inertia, I, and polar moment of inertia, I_p, need not be calculated; and the flange design moment calculation for net-section bending moment and axial force supplemental loads, $M_{oe} = 0$. Additionally, $F_s = 1.0$ for non-split rings.

g) STEP 7 – Determine the flange moment for the gasket seating condition using Equation (4.16.17) or Equation (4.16.18), as applicable.

For internal pressure,

$$M_g = \frac{W_g(C - G)F_s}{2} = \frac{(237626.3)(31.25 - 29.5)(1.0)}{2} = 207923.0\ in - lbs$$

h) STEP 8 – Determine the flange stresses for the operating and gasket seating conditions using the equations in Table 4.16.8.

Note: As provided in paragraph 4.16.12 for the definition of B, if $B < 20g_1$, the designer may substitute the value of B_1 for B in the equation for S_H, where,

For integral flanges when $f \ge 1.0$,

$$B_1 = B + g_o$$

Since $\{B = 26.25\ in\} \ge \{20g_1 = 20(0.6875) = 13.75\ in\}$, the value of $B = 26.25\ in$ will be used in the equation for S_H.

Operating Condition

$$S_H = \frac{fM_o}{Lg_1^2 B} = \frac{(1.0)(206634.6)}{(0.9368)(0.6875)^2(26.25)} = 17777.9\ psi$$

$$S_R = \frac{(1.33te+1)M_o}{Lt^2B} = \frac{\left[(1.33)(1.4375)(0.2687)+1\right](206634.6)}{(0.9368)(1.4375)^2(26.25)} = 6155.4\ psi$$

$$S_T = \frac{YM_o}{t^2B} - ZS_R = \frac{(8.7565)(206634.6)}{(1.4375)^2(26.25)} - 4.5180(6155.4) = 5547.0\ psi$$

Gasket Seating Condition

$$S_H = \frac{fM_g}{Lg_1^2B} = \frac{(1.0)(207923.0)}{(0.9368)(0.6875)^2(26.25)} = 17888.8\ psi$$

$$S_R = \frac{(1.33te+1)M_g}{Lt^2B} = \frac{\left[(1.33)(1.4375)(0.2687)+1\right](207923.0)}{(0.9368)(1.4375)^2(26.25)} = 6193.8\ psi$$

$$S_T = \frac{YM_g}{t^2B} - ZS_R = \frac{(8.7565)(207923.0)}{(1.4375)^2(26.25)} - 4.5180(6193.8) = 5581.5\ psi$$

i) STEP 9 – Check the flange stress acceptance criteria. The criteria below shall be evaluated. If the stress criteria are satisfied, go to STEP 10. If the stress criteria are not satisfied, re-proportion the flange dimensions and go to STEP 4.

Allowable normal stress – The criteria to evaluate the normal stresses for the operating and gasket seating conditions are shown in Table 4.16.9, (for integral type flanges).

Operating Condition

$$S_H \le \min\left[1.5S_{fo},\ 2.5S_{no}\right]$$

$$\left\{S_H = 17777.9\ psi\right\} \le \left\{\min\left[1.5(17800),\ 2.5(18800)\right] = 26700\ psi\right\} \qquad True$$

$$\left\{S_R = 6155.4\ psi\right\} \le \left\{S_{fo} = 17800\ psi\right\} \qquad True$$

$$\left\{S_T = 5547.0\ psi\right\} \le \left\{S_{fo} = 17800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_H+S_R)}{2} = \frac{(17777.9+6155.4)}{2} = 11966.7\ psi\right\} \le \left\{S_{fo} = 17800\ psi\right\} \qquad True$$

$$\left\{\frac{(S_H+S_T)}{2} = \frac{(17777.9+5547.0)}{2} = 11662.5\ psi\right\} \le \left\{S_{fo} = 17800\ psi\right\} \qquad True$$

Gasket Seating Condition

$$S_H \le \min\left[1.5S_{fg},\ 2.5S_{ng}\right]$$

$$\left\{S_H = 17888.8\ psi\right\} \le \left\{\min\left[1.5(24000),\ 2.5(25300)\right] = 36000\ psi\right\} \qquad True$$

$$\left\{S_R = 6193.8\ psi\right\} \le \left\{S_{fg} = 24000\ psi\right\} \qquad True$$

$$\{S_T = 5581.5\ psi\} \le \{S_{fg} = 24000\ psi\} \qquad True$$

$$\left\{\frac{(S_H + S_R)}{2} = \frac{(17888.8 + 6193.8)}{2} = 12041.3\ psi\right\} \le \{S_{fg} = 24000\ psi\} \qquad True$$

$$\left\{\frac{(S_H + S_T)}{2} = \frac{(17888.8 + 5581.5)}{2} = 11735.2\ psi\right\} \le \{S_{fg} = 24000\ psi\} \qquad True$$

j) STEP 10 – Check the flange rigidity criterion in Table 4.16.10. If the flange rigidity criterion is satisfied, then the design is complete. If the flange rigidity criterion is not satisfied, then re-proportion the flange dimensions and go to STEP 3.

<u>Operating Condition</u>

$$J = \frac{52.14VM_o}{LE_{yo}g_0^2K_Rh_o} \le 1.0$$

$$\left\{J = \frac{52.14(0.1577)(206634.6)}{(0.9368)(26.0E+06)(0.3125)^2(0.3)(2.8641)} = 0.8313\right\} \le 1.0 \qquad True$$

Where, $K_R = 0.3\ for\ integral\ flanges$

<u>Gasket Seating Condition</u>

$$J = \frac{52.14VM_g}{LE_{yg}g_o^2K_Rh_o} \le 1.0$$

$$\left\{J = \frac{52.14(0.1577)(207923.0)}{(0.9368)(29.4E+06)(0.3125)^2(0.3)(2.8641)} = 0.7398\right\} \le 1.0 \qquad True$$

Where, $K_R = 0.3\ for\ integral\ flanges$

Since the acceptance criteria are satisfied, the design is complete.

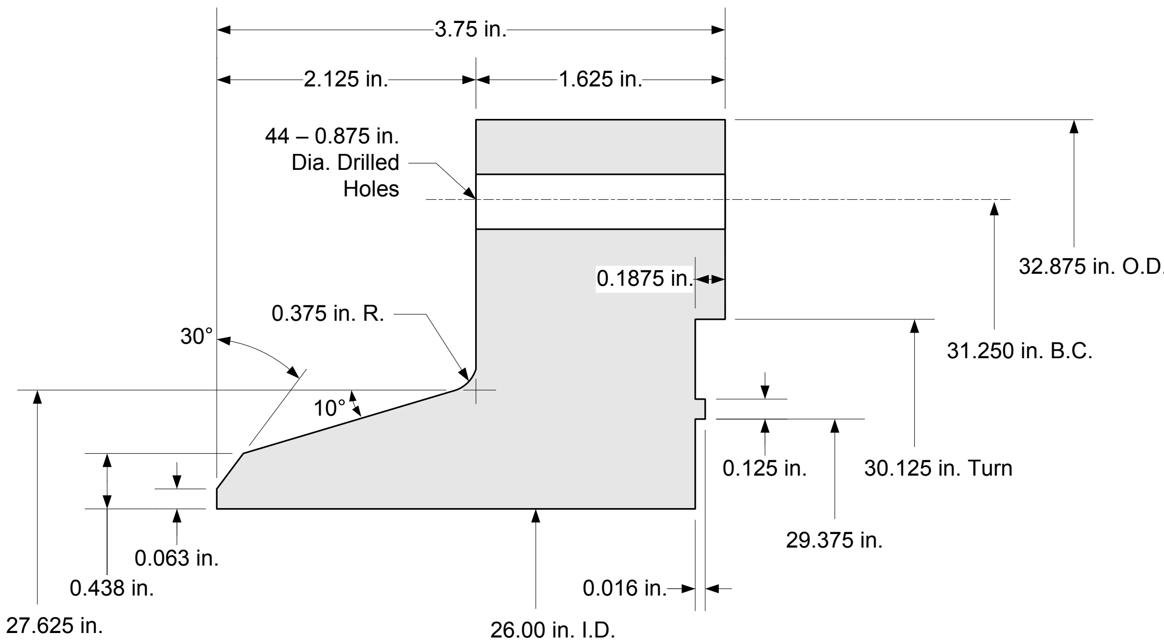


Figure E4.16.1 - Flanged Joints

Note: The blind flange bolted to the heat exchanger girth flange is evaluated in Example Problem E4.6.1.

4.16.2 Example E4.16.2 – Loose Type

Determine if the stresses in the ASME B16.5, Class 300, NPS 20 Slip–on Flange are with acceptable limits, considering the following design conditions. The flange is of a loose type with hub and is attached to a cylindrical shell with Category C, Type 10 fillet welds. See Table 4.2.9, Detail 1.

General Data:

- Cylinder Material = $SA-516,\ Grade\ 70$
- Design Conditions = $450\ psig@650°F$
- Allowable Stress at Design Temperature = $18800\ psi$
- Allowable Stress at Ambient Temperature = $25300\ psi$
- Corrosion Allowance = $0.0\ in$

Flange Data

- Material = $SA-105$
- Allowable Stress at Design Temperature = $17800\ psi$
- Allowable Stress at Ambient Temperature = $24000\ psi$
- Modulus of Elasticity at Design Temperature = $26.0E+06\ psi$
- Modulus of Elasticity at Ambient Temperature = $29.4E+06\ psi$

Bolt Data

- Material = $SA-193,\ Grade\ B7$
- Allowable Stress at Design Temperature = $25000\ psi$
- Allowable Stress at Ambient Temperature = $25000\ psi$
- Diameter = $1.25\ in$
- Number of Bolts = 24
- Root area = $0.929\ in^2$

Gasket Data

- Material = Kammprofile
- Gasket Factor = 2.0
- Seating Stress = $2500\ psi$
- Inside Diameter = $20.875\ in$
- Outside Diameter = $22.875\ in$

Evaluate the flange in accordance with paragraph 4.16.

Paragraph 4.16.6, Design Bolt Loads. The procedure to determine the bolt loads for the operating and gasket seating conditions is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

$$P = 450\ psig\ at\ 650°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 4.16.1.

$$m = 2.0$$

$$y = 2500\ psi$$

Note: Table 4.16.1 provides a list of many commonly used gasket materials and contact facings with suggested design values of m and y that have generally proved satisfactory in actual service when using effective seating width b given in Table 4.16.3. The design values and other details given in this table are suggested only and are not mandatory.

For this example, gasket manufacturer's suggested m and y values were used.

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.5\left(GOD - GID\right) = 0.5\left(22.875 - 20.875\right) = 1.0\ in$$

From Table 4.16.3, Facing Sketch Detail 1a, Column II,

$$b_o = \frac{N}{2} = \frac{1.0}{2} = 0.500\ in$$

For $\{b_o = 0.500\ in\} > \{0.25\ in\}$,

$$b = 0.5C_{ul}\sqrt{\frac{b_o}{C_{ul}}} = 0.5\left(1.0\right)\sqrt{\frac{0.500}{1.0}} = 0.3536\ in$$

$$G = G_C - 2b = 22.875 - 2\left(0.3536\right) = 22.1678\ in$$

Where,

$$C_{ul} = 1.0,\ for\ US\ Custumary\ Units$$

$$G_C = \min\left[Gasket\ OD,\ Flange\ Face\ OD\right] = \min\left[22.875,\ 23.0\right] = 22.875\ in$$

d) STEP 4 – Determine the design bolt load for the operating condition.

$$W_o = \frac{\pi}{4}G^2P + 2b\pi GmP \qquad for\ non-self-energized\ gaskets$$

$$W_o = \frac{\pi}{4}\left(22.1678\right)^2\left(450\right) + 2\left(0.3536\right)\pi\left(22.1678\right)\left(2.0\right)\left(450\right) = 218005.0\ lbs$$

e) STEP 5 – Determine the design bolt load for the gasket seating condition.

$$W_g = \left(\frac{A_m + A_b}{2}\right) S_{bg} = \left(\frac{8.7202 + 22.2960}{2}\right) 25000 = 387702.5\ lbs$$

Where,

$$A_b = (Number\ of\ bolts)(Root\ area\ of\ one\ bolt) = 24(0.929) = 22.2960\ in^2$$

$$A_m = \max\left[\left(\frac{W_o + F_A + \frac{4M_E}{G}}{S_{bo}}\right), \left(\frac{W_{gs}}{S_{bg}}\right)\right] = \max\left[\left(\frac{220219.1 + 0.0 + 0.0}{25000}\right), \left(\frac{61563.7}{25000}\right)\right]$$

$$A_m = \max\left[8.7202,\ 2.4625\right] = 8.7202\ in^2$$

And,

$$W_{gs} = \pi b G (C_{us} y) \qquad for\ non-self-energized\ gaskets$$

$$W_{gs} = \pi (0.3536)(22.1678)(1.0(2500)) = 61563.7\ lbs$$

$F_A = 0$ and $M_E = 0$ since there are no externally applied net-section axial forces or bending moments.

Paragraph 4.16.7, Flange Design Procedure. The procedure in this paragraph can be used to design circular integral, loose or reverse flanges, subject to internal or external pressure, and external loadings. The procedure incorporates both a strength check and a rigidity check for flange rotation.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint and the external net-section axial force, F_A, and bending moment, M_E.

$$P = 450\ psig\ at\ 650°F$$

$$F_A = 0$$

$$M_E = 0$$

b) STEP 2 – Determine the design bolt loads for operating condition W_o, and the gasket seating condition W_g, and the corresponding actual bolt load area A_b, from paragraph 4.16.6.

$$W_o = 218005.0\ lbs$$

$$W_g = 387702.5\ lbs$$

$$A_b = 22.2960\ in^2$$

c) STEP 3 – Determine an initial flange geometry, in addition to the information required to determine the bolt load, the following geometric parameters are required. The flange is an ASME B16.5, Class 300, NPS 20 Slip–on Flange.

1) Flange bore

$$B = 20.20\ in$$

2) Bolt circle diameter

$$C = 27.0\ in$$

3) Outside diameter of the flange

$$A = 30.5\ in$$

4) Flange thickness

$$t = 2.44\ in$$

5) Thickness of the hub at the large end

$$g_1 = 1.460\ in$$

6) Thickness of the hub at the small end

$$g_0 = 1.460\ in$$

7) Hub length

$$h = 1.25\ in$$

d) STEP 4 – Determine the flange stress factors using the equations in Table 4.16.4 and Table 4.16.5.

$$K = \frac{A}{B} = \frac{30.5}{20.20} = 1.5099$$

$$Y = \frac{1}{K-1}\left[0.66845 + 5.71690\left(\frac{K^2 \log_{10} K}{K^2 - 1}\right)\right]$$

$$Y = \frac{1}{1.5099-1}\left[0.66845 + 5.71690\frac{(1.5099)^2 \log_{10}[1.5099]}{(1.5099)^2 - 1}\right] = 4.8850$$

$$T = \frac{K^2\left(1+8.55246\log_{10} K\right)-1}{\left(1.04720+1.9448K^2\right)(K-1)} = \frac{(1.5099)^2\left(1+8.55246\log_{10}[1.5099]\right)-1}{\left(1.04720+1.9448(1.5099)^2\right)(1.5099-1)} = 1.7064$$

$$U = \frac{K^2\left(1+8.55246\log_{10} K\right)-1}{1.36136\left(K^2-1\right)(K-1)} = \frac{(1.5099)^2\left(1+8.55246\log_{10}[1.5099]\right)-1}{1.36136\left((1.5099)^2-1\right)(1.5099-1)} = 5.3681$$

$$Z = \frac{\left(K^2+1\right)}{\left(K^2-1\right)} = \frac{\left((1.5099)^2+1\right)}{\left((1.5099)^2-1\right)} = 2.5627$$

$$h_o = \sqrt{Bg_0} = \sqrt{(20.20)(1.46)} = 5.4307\ in$$

$$X_g = \frac{g_1}{g_0} = \frac{1.460}{1.460} = 1.0$$

$$X_h = \frac{h}{h_o} = \frac{1.25}{5.4307} = 0.2302$$

$$F_L = \left(\frac{\left\{ \begin{array}{l} 0.941074 + 0.176139\left(\ln\left[X_g\right]\right) - 0.188556\left(\ln\left[X_h\right]\right) + \\ 0.0689847\left(\ln\left[X_g\right]\right)^2 + 0.523798\left(\ln\left[X_h\right]\right)^2 - \\ 0.513894\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right) \end{array} \right\}}{\left\{ \begin{array}{l} 1 + 0.379392\left(\ln\left[X_g\right]\right) + 0.184520\left(\ln\left[X_h\right]\right) - \\ 0.00605208\left(\ln\left[X_g\right]\right)^2 - 0.00358934\left(\ln\left[X_h\right]\right)^2 + \\ 0.110179\left(\ln\left[X_g\right]\right)\left(\ln\left[X_h\right]\right) \end{array} \right\}} \right)$$

$$F_L = \left(\frac{\left\{ \begin{array}{l} 0.941074 + 0.176139\left(\ln\left[1.0\right]\right) - 0.188556\left(\ln\left[0.2302\right]\right) + \\ 0.0689847\left(\ln\left[1.0\right]\right)^2 + 0.523798\left(\ln\left[0.2302\right]\right)^2 - \\ 0.513894\left(\ln\left[1.0\right]\right)\left(\ln\left[0.2302\right]\right) \end{array} \right\}}{\left\{ \begin{array}{l} 1 + 0.379392\left(\ln\left[1.0\right]\right) + 0.184520\left(\ln\left[0.2302\right]\right) - \\ 0.00605208\left(\ln\left[1.0\right]\right)^2 - 0.00358934\left(\ln\left[0.2302\right]\right)^2 + \\ 0.110179\left(\ln\left[1.0\right]\right)\left(\ln\left[0.2302\right]\right) \end{array} \right\}} \right)$$

$$F_L = 3.2556$$

$$For\ 0.1 \le X_h \le 0.25,$$

$$\ln\left[V_L\right] = \left(\begin{array}{l} 6.57683 - 0.115516X_g + 1.39499\sqrt{X_g}\left(\ln\left[X_g\right]\right) + \\ 0.307340\left(\ln\left[X_g\right]\right)^2 - 8.30849\sqrt{X_g} + 2.62307\left(\ln\left[X_g\right]\right) + \\ 0.239498X_h\left(\ln\left[X_h\right]\right) - 2.96125\left(\ln\left[X_h\right]\right) + \dfrac{7.035052\left(10^{-4}\right)}{X_h} \end{array} \right)$$

$$\ln\left[V_L\right] = \left(\begin{array}{l} 6.57683 - 0.115516(1.0) + 1.39499\sqrt{1.0}\left(\ln 1.0\right) + \\ 0.307340\left(\ln\left[1.0\right]\right)^2 - 8.30849\sqrt{1.0} + 2.62307\left(\ln\left[1.0\right]\right) + \\ 0.239498(0.2302)\left(\ln\left[0.2302\right]\right) - 2.96125\left(\ln\left[0.2302\right]\right) + \dfrac{7.035052\left(10^{-4}\right)}{0.2302} \end{array} \right)$$

$$\ln\left[V_L\right] = 2.4244$$

$$V_L = \exp\left[2.4244\right] = 11.2955$$

$$f = 1.0$$

$$d = \frac{Ug_0^2 h_o}{V_L} = \frac{(5.3681)(1.460)^2(5.4307)}{11.2955} = 5.5014\ in$$

$$e = \frac{F_L}{h_o} = \frac{3.2556}{5.4307} = 0.5995$$

$$L = \frac{te+1}{T} + \frac{t^3}{d} = \frac{2.44(0.5995)+1}{1.7064} + \frac{(2.44)^3}{5.5014} = 4.0838$$

e) STEP 5 – Determine the flange forces.

$$H_D = \frac{\pi}{4}B^2P = \frac{\pi}{4}(20.20)^2(450) = 144213.2\ lbs$$

$$H = \frac{\pi}{4}G^2P = \frac{\pi}{4}(22.1678)^2(450) = 173679.1\ lbs$$

$$H_T = H - H_D = 173679.1 - 144213.2 = 29465.9\ lbs$$

$$H_G = W_o - H = 218005.0 - 173679.1 = 44325.9\ lbs$$

f) STEP 6 – Determine the flange moment for the operating condition using Equation (4.16.14) or Equation (4.16.15), as applicable. When specified by the user or his designated agent, the maximum bolt spacing (B_{smax}) and the bolt spacing correction factor (B_{sc}) shall be applied in calculating the flange moment for internal pressure using the equations in Table 4.16.11. The flange moment M_o for the operating condition and the flange moment M_g for the gasket seating condition without correction for bolt spacing $B_{sc} = 1$ is used for the calculation of the rigidity index in Step 10. In these equations, h_D, h_T, and h_G are determined from Table 4.16.6. For integral and loose type flanges, the moment M_{oe} is calculated using Equation (4.16.16) where and I and I_p in this equation are determined from Table 4.16.7.

For internal pressure,

$$M_o = abs\left[\left(\left(H_Dh_D + H_Th_T + H_Gh_G\right)B_{sc} + M_{oe}\right)F_s\right]$$

$$M_o = abs\left[\left(\left(144213.2(3.40) + 29465.9(2.9081) + 44325.9(2.4161)\right)1.0 + 0.0\right)1.0\right]$$

$$M_o = 683110.5\ in-lbs$$

From Table 4.16.11, the maximum bolt spacing and the bolt spacing correction factor are calculated as follows.

$$B_{smax} = 2a + \frac{6t}{m+0.5} = 2(1.25) + \frac{6(2.44)}{2.0+0.5} = 8.3560\ in$$

$$B_{sc} = \sqrt{\frac{B_s}{2a+t}} = \sqrt{\frac{1.125}{2(1.25)+2.44}} = 0.4772$$

The actual bolt spacing is determined using the following equation.

$$B_s = \frac{C}{No.\ of\ bolts} = \frac{27.0}{24} = 1.125\ in$$

Since $\{B_s = 1.125\ in\} \leq \{B_{smax} = 8.3560\ in\}$, the value of $B_{sc} = 1.0$.

From Table 4.16.6,

$$h_D = \frac{C-B}{2} = \frac{27.0-20.20}{2} = 3.40\ in$$

$$h_G = \frac{C-G}{2} = \frac{27.0-22.1678}{2} = 2.4161\ in$$

$$h_T = \frac{h_D+h_G}{2} = \frac{3.40+2.4161}{2} = 2.9081\ in$$

Since $F_A = 0$ and $M_E = 0$, the flange cross-section bending moment of inertia, I, and polar moment of inertia, I_p, need not be calculated; and the flange design moment calculation for net-section bending moment and axial force supplemental loads, $M_{oe} = 0$. Additionally, $F_s = 1.0$ for non-split rings.

g) STEP 7 – Determine the flange moment for the gasket seating condition using Equation (4.16.17) or Equation (4.16.18), as applicable.

For internal pressure,

$$M_g = \frac{W_g(C-G)F_s}{2} = \frac{(387702.5)(27.0-22.1678)(1.0)}{2} = 936728.0\ in-lbs$$

h) STEP 8 – Determine the flange stresses for the operating and gasket seating conditions using the equations in Table 4.16.8.

Note: As provided in paragraph 4.16.12 for the definition of B, if $B < 20g_1$, the designer may substitute the value of B_1 for B in the equation for S_H, where,

For integral flanges when $f < 1.0$ and for loose type flanges,

$$B_1 = B + g_1 = 20.20 + 1.460 = 21.66\ in$$

Since $\{B = 20.20\ in\} < \{20g_1 = 20(1.460) = 29.2\ in\}$, the value of $B_1 = 21.66\ in$ will be used in the equation for S_H.

<u>Operating Condition</u>

$$S_H = \frac{fM_o}{Lg_1^2B} = \frac{(1.0)(683110.5)}{(4.0838)(1.460)^2(21.66)} = 3622.9\ psi$$

$$S_R = \frac{(1.33te+1)M_o}{Lt^2B} = \frac{[(1.33)(2.44)(0.5995)+1](683110.5)}{(4.0838)(2.44)^2(20.20)} = 4096.9 \ psi$$

$$S_T = \frac{YM_o}{t^2B} - ZS_R = \frac{(4.8850)(683110.5)}{(2.44)^2(20.20)} - 2.5627(4096.9) = 17248.4 \ psi$$

Gasket Seating Condition

$$S_H = \frac{fM_g}{Lg_1^2B} = \frac{(1.0)(936728.0)}{(4.0838)(1.460)^2(21.66)} = 4968.0 \ psi$$

$$S_R = \frac{(1.33te+1)M_g}{Lt^2B} = \frac{[(1.33)(2.44)(0.5995)+1](936728.0)}{(4.0838)(2.44)^2(20.20)} = 5617.9 \ psi$$

$$S_T = \frac{YM_g}{t^2B} - ZS_R = \frac{(4.8850)(936728.0)}{(2.44)^2(20.20)} - 2.5627(5617.9) = 23652.3 \ psi$$

i) STEP 9 – Check the flange stress acceptance criteria. The criteria below shall be evaluated. If the stress criteria are satisfied, go to STEP 10. If the stress criteria are not satisfied, re-proportion the flange dimensions and go to STEP 4.

Allowable normal stress – The criteria to evaluate the normal stresses for the operating and gasket seating conditions are shown in Table 4.16.9, (for loose type flanges with a hub).

Operating Condition

$$\{S_H = 3622.9 \ psi\} \le \{\min[1.5(17800), 2.5(18800)] = 26700 \ psi\} \qquad True$$

$$\{S_R = 4096.9 \ psi\} \le \{S_{fo} = 17800 \ psi\} \qquad True$$

$$\{S_T = 17248.4 \ psi\} \le \{S_{fo} = 17800 \ psi\} \qquad True$$

$$\left\{\frac{(S_H + S_R)}{2} = \frac{(3622.9 + 4096.9)}{2} = 3859.9 \ psi\right\} \le \{S_{fo} = 17800 \ psi\} \qquad True$$

$$\left\{\frac{(S_H + S_T)}{2} = \frac{(3622.9 + 17248.4)}{2} = 10435.7 \ psi\right\} \le \{S_{fo} = 17800 \ psi\} \qquad True$$

Gasket Seating Condition

$$\{S_H = 4968.0 \ psi\} \le \{\min[1.5(24000), 2.5(25300)] = 36000 \ psi\} \qquad True$$

$$\{S_R = 5617.9 \ psi\} \le \{S_{fg} = 24000 \ psi\} \qquad True$$

$$\{S_T = 23652.3 \ psi\} \le \{S_{fg} = 24000 \ psi\} \qquad True$$

$$\left\{\frac{(S_H+S_R)}{2}=\frac{(4968.0+5617.9)}{2}=5293.0\ psi\right\}\le\left\{S_{fg}=24000\ psi\right\}\qquad True$$

$$\left\{\frac{(S_H+S_T)}{2}=\frac{(4968.0+23652.3)}{2}=14310.2\ psi\right\}\le\left\{S_{fg}=24000\ psi\right\}\qquad True$$

j) STEP 10 – Check the flange rigidity criterion in Table 4.16.10. If the flange rigidity criterion is satisfied, then the design is complete. If the flange rigidity criterion is not satisfied, then re-proportion the flange dimensions and go to STEP 3.

Operating Condition

$$J=\frac{52.14V_LM_o}{LE_{yo}g_0^2K_Rh_o}\le 1.0$$

$$\left\{J=\frac{52.14(11.2955)(683110.5)}{(4.0838)(26.0E+06)(1.460)^2(0.2)(5.4307)}=1.6366\right\}\le 1.0\ \ Not\ Satisfied$$

Where, $K_R=0.2\ for\ loose\ type\ flanges$.

Gasket Seating Condition

$$J=\frac{52.14V_LM_g}{LE_{yg}g_o^2K_Rh_o}\le 1.0$$

$$\left\{J=\frac{52.14(11.2955)(936728.0)}{(4.0838)(29.4E+06)(1.460)^2(0.2)(5.4307)}=1.9847\right\}\le 1.0\ \ Not\ Satisfied$$

Where, $K_R=0.2\ for\ loose\ type\ flanges$.

Since the flange rigidity criterion is not satisfied for either the operating condition or the gasket seating condition, the flange dimensions should be re-proportioned and the design procedure shall be performed beginning with STEP 3.

NOTE: Although the proposed ASME B16.5 slip-on flange is shown not to satisfy the flange rigidity acceptance criteria of VIII-2 paragraph 4.16 Design Rules for Flanged Joints, Table 4.16.10, the ASME B16.5–2009, permits the use of an ASME Class 300 flange to operate at a pressure of 550 psi for a coincident temperature of 650°F.

4.17 Clamped Connections

4.17.1 Example E4.17.1 – Flange and Clamp Design Procedure

Using the data shown below, determine if the clamp design meets the design requirements of Section VIII, Division 2.

Data (Refer to Figure E4.17.1)

- Design Conditions = $3000\ psi@200°F$
- Corrosion Allowance = $0.0\ in$

Clamp

- Material = $SA-216,\ Grade\ WCB$
- Inside Diameter = $43.75\ in$
- Thickness = $7.625\ in$
- Width = $28.0\,in$
- Gap = $14.0\ in$
- Lug height = $15.0\ in$
- Lug Width = $28.0\ in$
- Lip Length = $2.75\ in$
- Radial Distance from Connection Centerline to Bolts = $32.25\ in$
- Distance from W to the point where the clamp lug joins the clamp body = $3.7\ in$
- Allowable Stress @ Design Temperature = $22000\ psi$
- Allowable Stress @ Ambient Temperature = $24000\ psi$

Hub

- Material = $SA-105$
- Inside Diameter = $18.0\ in$
- Pipe End Neck Thickness = $12.75\ in$
- Shoulder End Neck Thickness = $12.75\ in$
- Shoulder Thickness = $7.321\ in$
- Shoulder Height = $2.75\ in$
- Friction Angle = $5\ deg$
- Shoulder Transition Angle = $10\ deg$
- Allowable Stress @ Design Temperature = $22000\ psi$
- Allowable Stress @ ambient Temperature = $24000\ psi$

Bolt Data

- Material = $SA-193,\ Grade\ B7$
- Allowable Stress @ Design Temperature = $23000\ psi$
- Allowable Stress @ Gasket Temperature = $23000\ psi$
- Diameter = $1.75\ in$
- Number of Bolts = 2
- Root area = $1.980\ in^2$

Gasket Data

- Material = Self Energizing O-ring Type
- Gasket Reaction Location = $19.0\ in$
- Gasket Factor = 0
- Seating Stress = $0\ psi$

Determine the Design Bolt Loads

The procedure to determine the bolt loads for the operating and gasket seating conditions is given in Part 4, paragraph 4.17.4.2, and is shown below.

a) STEP 1 – Determine the design pressure and temperature of the flanged joint

$$P = 3000\ psig\ at\ 200°F$$

b) STEP 2 – Select a gasket and determine the gasket factors m and y from Table 4.16.1.

$$m = 0.0 \qquad for\ self-energized\ gaskets$$
$$y = 0.0$$

c) STEP 3 – Determine the width of the gasket, N, basic gasket seating width, b_o, the effective gasket seating width, b, and the location of the gasket reaction, G.

$$N = 0.0 \qquad for\ self-energized\ gaskets\ per\ paragraph\ 4.16.12$$

From Table 4.16.3, Effective Gasket Width (not required because gasket is self-energized)

$$b_o = \frac{N}{2} = \frac{0.0}{2} = 0.0\ in$$

For $b_o \le 0.25\ in$,

$$b = b_o = 0.0\ in$$

Therefore, from paragraph 4.16.6, the location of the gasket reaction is calculated as follows.

$$G = mean\ diameter\ of\ the\ gasket\ contact\ face$$
$$G = 19.0\ in$$

d) STEP 4 - Determine the flange forces for the bolt load calculation.

$$H = 0.785G^2P = 0.785(19.0)^2(3000) = 850155.0\ lbs$$
$$H_p = 0.0 \qquad (for\ self-energized\ gaskets)$$
$$H_m = 0.0 \qquad (for\ self-energized\ gaskets)$$

e) STEP 5 – Determine the design bolt load for the operating condition.

$$W_o = \frac{2}{\pi}(H + H_p)\tan[\phi - \mu] = \frac{2}{\pi}(850155.0 + 0)\cdot\tan[10-5] = 47351.0941\ lbs$$

f) STEP 6 – Determine the minimum required total bolt load for the gasket seating and assembly conditions.

$$W_{g1}=\frac{2}{\pi}H_m\tan[\phi+\mu]=\frac{2}{\pi}(0.0)\tan[10+5]=0.0\ lbs$$

$$W_{g2}=\frac{2}{\pi}\left(H+H_p\right)\tan[\phi+\mu]=\frac{2}{\pi}(850155+0)\tan[10+5]=145020.9308\ lbs$$

g) STEP 7 - Determine the design bolt load for the gasket seating and assembly conditions.

$$W_g=\left(A_m+A_b\right)S_{bg}=(3.1526+3.96)23000=163589.8\ lbs$$

Where,

$$A_m=\max\left[\frac{W_o}{2S_{bo}},\ \frac{W_{g1}}{2S_{bg}},\ \frac{W_{g2}}{2S_{bg}}\right]$$

$$A_m=\max\left[\frac{47531.0941}{2(23000)},\ \frac{0}{2(23000)},\ \frac{145020.9308}{2(23000)}\right]=3.1526\ in^2$$

The actual bolt area is calculated as follows (using two 1.75 in diameter bolts).

$$A_b=(Number\ of\ bolts)(Root\ area\ of\ one\ bolt)=2(1.980)=3.96\ in^2$$

Alternatively, if controlled bolting (e.g., bolt tensioning or torque control) techniques are used to assemble the clamp, assembly design bolt load may be calculated as follows.

$$W=2A_m\cdot S_a=2(3.1526)23000=145019.6\ lbs$$

Note: This calculation is shown for informational purposes only and will not be used in the example problem.

Flange Design Procedure:

Refer to paragraph 4.17.5

a) STEP 1 – Determine the design pressure and temperature of the flange joint.

See above data.

b) STEP 2 – Determine an initial flange and clamp geometry see Figures 4.17.1(c) and 4.17.2(a), and Figure E4.17.1 of this example.

c) STEP 3 – Determine the design bolt loads for operating condition, W_o, and the gasket seating and assembly condition, W_g, from paragraph 4.17.4.2.

$$W_o=47351.0941\ lbs$$

$$W_g=163589.8\ lbs$$

d) STEP 4 – Determine the flange forces, H, H_p, and H_m from paragraph 4.17.4.2, and:

$$H = 850155.0 \; lbs$$

$$H_p = 0.0 \; lbs$$

$$H_m = 0.0 \; lbs$$

$$H_D = 0.785B^2P = 0.785(18.0)^2(3000) = 763020.0 \; lbs$$

$$H_G = \frac{1.571W_o}{\tan[\phi+\mu]} - (H + H_p) = \frac{1.571(47531.0941)}{\tan[10+5]} - (850155.0 + 0.0) = -571477.7323 \; lbs$$

$$H_T = H - H_D = 850155.0 - 763020.0 = 87135.0 \; lbs$$

e) STEP 5 – Determine the flange moment for the operating condition.

$$M_o = M_D + M_G + M_T + M_F + M_P + M_R$$

$$M_o = 5961093.75 + 0 + 1214444.063 + 0 + 25957.3484 - 254998.8042$$

$$M_o = 6946496.357 \; in-lbs$$

Where,

$$M_D = H_D\left[\frac{C-(B+g_1)}{2}\right] = 763020.0\left[\frac{46.375-(18.0+12.75)}{2}\right] = 5961093.75 \; lbs$$

$$M_G = H_G h_G = -571477.7323(0.0) = 0.0 \; lbs$$

$$M_T = H_T\left[\frac{C}{2} - \frac{(B+G)}{4}\right] = 87135.0\left[\frac{46.375}{2} - \frac{(18.0+19.0)}{4}\right] = 1214444.063 \; in-lbs$$

$$M_F = H_D\left(\frac{g_1-g_0}{2}\right) = 763020.0\left(\frac{12.75-12.75}{2}\right) = 0.0 \; in-lbs$$

$$M_P = PBT\pi\left(\frac{T}{2} - \bar{h}\right) = 3000(18)(7.321)\pi\left(\frac{7.321}{2} - 3.6396\right) = 25957.3484 \; lbs$$

$$M_R = 1.571W_o\left(\bar{h} - T + \frac{(C-N)\tan[\phi]}{2}\right)$$

$$M_R = 1.571(47351.0941)\left(3.6396 - 7.321 + \frac{(46.375-43.5)\tan[10]}{2}\right)$$

$$M_R = -254998.8042 \; lbs$$

And,

$$A = B + 2(g_1 + g_2) = 18.0 + 2(12.75 + 2.75) = 49.0 \; in$$

$$N = B + 2g_1 = 18 + 2(12.75) = 43.5\ in$$

$$C = \frac{(A + C_i)}{2} = \frac{(49 + 43.75)}{2} = 46.375\ in$$

$$\bar{h} = \frac{T^2 g_1 + h_2^2 g_2}{2(Tg_1 + h_2 g_2)} = \frac{(7.321)^2 12.75 + (7.0785)^2 2.75}{2(7.321(12.75) + 7.0785(2.75))} = 3.6396\ in$$

$$h_2 = T - \frac{g_2 \tan[\phi]}{2} = 7.321 - \frac{2.75 \tan[10]}{2} = 7.0786\ in$$

f) STEP 6 – Determine the flange moment for the gasket seating condition

$$M_g = \frac{0.785 W_g (C - G)}{\tan[\phi + \mu]} = \frac{0.785(163589.8)(46.375 - 19.0)}{\tan[10 + 5]} = 13119810.24\ \ in - lbs$$

g) STEP 7 – Determine the hub factors

$$F_H = 1 + \frac{1.818}{\sqrt{Bg_1}}\left[T - \bar{h} + \frac{3.305 I_h}{g_1^2 (0.5B + \bar{g})}\right]$$

$$F_H = 1 + \frac{1.818}{\sqrt{18(12.75)}}\left[7.321 - 3.6396 + \frac{3.305(498.4148)}{(12.75)^2 (0.5(18.0) + 7.7123)}\right]$$

$$F_H = 1.5146$$

$$I_h = \frac{g_1 T^3}{3} + \frac{g_2 h_2^3}{3} - (g_2 h_2 + g_1 T)\bar{h}^2$$

$$I_h = \frac{12.75(7.321)^3}{3} + \frac{2.75(7.0785)^3}{3} - (2.75(7.0786) + 12.75(7.321))(3.6396)^2$$

$$I_h = 498.4148\ in^4$$

$$\bar{g} = \frac{Tg_1^2 + h_2 g_2 (2g_1 + g_2)}{2(Tg_1 + h_2 g_2)} = \frac{7.321(12.75)^2 + 7.0786(2.75)(2(12.75) + 2.75)}{2(7.321(12.75) + 7.0786(2.75))}$$

$$\bar{g} = 7.7123\ in$$

h) STEP 8 – Determine the reaction shear force at the hub neck for the operating condition.

$$Q_o = \frac{1.818 M_o}{F_H \sqrt{Bg_1}} = \frac{1.818(6946496.357)}{1.5146\sqrt{(18.0)(12.75)}} = 550389.8215\ lbs$$

i) STEP 9 – Determine the reaction shear force at the hub neck for the gasket seating condition.

$$Q_g = \frac{1.818 M_g}{F_H \sqrt{Bg_1}} = \frac{1.818(13119810.24)}{1.5146\sqrt{(18.0)(12.75)}} = 1039518.29\ lbs$$

j) STEP 10 – Determine the clamp factors.

$$e_b = B_c - \frac{C_i}{2} - l_c - X = 32.25 - \frac{43.75}{2} - 2.75 - 2.7009 = 4.9241\ in$$

$$X = \frac{\left(\frac{C_w}{2} - \frac{C_t}{3}\right)C_t^2 - 0.5\left(C_w - C_g\right)l_c^2}{A_c} = \frac{\left(\frac{28.0}{2} - \frac{7.625}{3}\right)(7.625)^2 - 0.5(28.0 - 14.0)(2.75)^2}{227.0577}$$

$$X = 2.7009\ in$$

$$A_c = A_1 + A_2 + A_3 = 97.2188 + 91.3389 + 38.5 = 227.0577\ in^2$$

$$I_c = \left(\frac{A_1}{3} + \frac{A_2}{4}\right)C_t^2 + \frac{A_3 l_c^2}{3} - A_c X^2$$

$$I_c = \left(\frac{97.2188}{3} + \frac{91.3389}{4}\right)(7.625)^2 + \frac{38.5(2.75)^2}{3} - 227.0577(2.7009)^2$$

$$I_c = 1652.4435\ in^4$$

$$A_1 = \left(C_w - 2C_t\right)C_t = \left(28.0 - 2(7.625)\right)7.625 = 97.2188\ in^2$$

$$A_2 = 1.571 C_t^2 = 1.571(7.625)^2 = 91.3389\ in^2$$

$$A_3 = \left(C_w - C_g\right)l_c = (28.0 - 14.0)2.75 = 38.5\ in^2$$

k) STEP 11 – Determine the hub stress correction factor, f, based on g_1, g_0, h, and B using the equations in Table 4.16.4 and 4.16.5 and l_m using the following equation.

$$X_g = \frac{g_1}{g_0} = \frac{12.75}{12.75} = 1$$

$$X_h = \frac{h}{h_o} = 0$$

$$f = \max\left[1.0, \left(\frac{\begin{array}{c}0.0927779 - 0.0336633X_g + 0.964176X_g^2 + \\ 0.0566286X_h + 0.347074X_h^2 - 4.18699X_h^3\end{array}}{\begin{array}{c}1 - 5.96093(10)^{-3}X_g + 1.62904X_h + \\ 3.49329X_h^2 + 1.39052X_h^3\end{array}}\right)\right]$$

$$f = \max\left[1.0, \left(\frac{\begin{array}{c}0.0927779 - 0.0336633(1) + 0.964176(1)^2 + \\ 0.0566286(1) + 0.347074(0) - 4.18699(0)\end{array}}{\begin{array}{c}1 - 5.96093(10)^{-3}1 + 1.62904(0) + \\ 3.49329(0) + 1.39052(0)\end{array}}\right)\right]$$

$$f = \max[1.0, 1.0294] = 1.0294$$

$$l_m = l_c - 0.5(C - C_i) = 2.75 - 0.5(46.375 - 43.75) = 1.4375\ in$$

l) STEP 12 – Determine the flange and clamp stresses for the operating and gasket seating conditions using the equations in Table 4.17.1.

Operating Condition – Location: Flange

Longitudinal Stress:

$$S_{1o} = f\left[\frac{PB^2}{4g_1(B+g_1)} + \frac{1.91M_o}{g_1^2(B+g_1)F_H}\right]$$

$$S_{1o} = 1.0294\left[\frac{3000(18.0)^2}{4(12.75)(18.0+12.75)} + \frac{1.91(6946496.357)}{(12.75)^2(18.0+12.75)1.5146}\right] = 2442.0\ psi$$

Lame Hoop Stress:

$$S_{2o} = P\left(\frac{N^2+B^2}{N^2-B^2}\right) = 3000\left(\frac{(43.5)^2+(18.0)^2}{(13.5)^2-(18.0)^2}\right) = 4239.6\ psi$$

Axial Shear Stress:

$$S_{3o} = \frac{0.75W_o}{T(B+2g_1)\tan[\phi-\mu]} = \frac{0.75(47351.0941)}{7.321(18.0+2(12.75))\tan[10-5]} = 1274.6\ psi$$

Radial Shear Stress:

$$S_{4o} = \frac{0.477Q_o}{g_1(B+g_1)} = \frac{0.477(550389.8215)}{12.75(18+12.75)} = 669.6\ psi$$

Operating Condition – Location: Clamp

Longitudinal Stress:

$$S_{5o} = \frac{W_o}{2C\tan[\phi-\mu]}\left[\frac{1}{C_t}+\frac{3(C_t+2l_m)}{C_t^2}\right]$$

$$S_{5o} = \frac{47351.0941}{2(46.375)\tan[10-5]}\left[\frac{1}{7.625}+\frac{3(7.625+2(1.4375))}{(7.625)^2}\right] = 3926.8\ psi$$

Tangential Stress:

$$S_{6o} = \frac{W_o}{2}\left[\frac{1}{A_c}+\frac{|e_b|\cdot(C_t-X)}{I_c}\right]$$

$$S_{6o} = \frac{47351.0941}{2}\left[\frac{1}{227.0577}+\frac{|4.9241|\cdot(7.625-2.7009)}{1652.4435}\right] = 451.7\ psi$$

Lip Shear Stress:

$$S_{7o} = \frac{1.5W_o}{(C_w-C_g)C\tan[\phi-\mu]} = \frac{1.5(47351.0941)}{(28.0-14.0)(46.375)\tan[10-5]} = 1250.4\ psi$$

Lug Bending Stress:

$$S_{8o} = \frac{3W_oL_a}{L_wL_h^2} = \frac{3(47351.0941)(3.7)}{28.0(15.0)^2} = 83.4\ psi$$

Bearing Stress:

$$S_{9o} = \frac{W_o}{(A-C_i)C\tan[\phi-\mu]} = \frac{47351.0941}{(49.0-43.75)(46.375)\tan[10-5]} = 2223.0\ psi$$

Gasket Seating/Assembly Condition – Location: Flange

Longitudinal Stress:

$$S_{1g} = f\left[\frac{1.91M_g}{g_1^2(B+g_1)F_H}\right] = 1.0294\left|\frac{1.91(13119810.24)}{(12.75)^2(18.0+12.75)(1.5146)}\right| = 3407.1\ psi$$

Lame Hoop Stress:

$$S_{2g} = 0.0$$

Axial Shear Stress:

$$S_{3g} = \frac{0.75W_g}{T(B+2g_1)\tan[\phi+\mu]} = \frac{0.75(163589.8)}{7.321(18.0+2(12.75))\tan[10+5]} = 1437.8\ psi$$

Radial Shear Stress:

$$S_{4g}=\frac{0.477Q_g}{g_1\left(B+g_1\right)}=\frac{0.477\left(1039518.29\right)}{12.75\left(18.0+12.75\right)}=1264.7\ psi$$

Gasket Seating/Assembly Condition– Location: Clamp

Longitudinal Stress:

$$S_{5g}=\frac{W_g}{2C\tan\left[\phi+\mu\right]}\left[\frac{1}{C_t}+\frac{3\left(C_t+2l_m\right)}{C_t^2}\right]$$

$$S_{5g}=\frac{163589.8}{2\left(46.375\right)\tan\left[10+5\right]}\left[\frac{1}{7.625}+\frac{3\left(7.625+2\left(1.4375\right)\right)}{7.625^2}\right]=4429.6\ psi$$

Tangential Stress:

$$S_{6g}=\frac{W_g}{2}\left[\frac{1}{A_c}+\frac{\left|e_b\right|\cdot\left(C_t-X\right)}{I_c}\right]$$

$$S_{6g}=\frac{163589.8}{2}\left[\frac{1}{227.0577}+\frac{\left|4.9241\right|\cdot\left(7.625-2.7009\right)}{1652.4435}\right]=1560.4\ psi$$

Lip Shear Stress:

$$S_{7g}=\frac{1.5W_g}{\left(C_w-C_g\right)C\tan\left[\phi+\mu\right]}=\frac{1.5\left(163589.8\right)}{\left(28.0-14.0\right)\left(46.375\right)\tan\left[10+5\right]}=1410.5\ psi$$

Lug Bending Stress:

$$S_{8g}=\frac{3W_gL_a}{L_wL_h^2}=\frac{3\left(163589.8\right)\left(3.7\right)}{28.0\left(15.0\right)^2}=288.2\ psi$$

Bearing Stress:

$$S_{9g}=\frac{W_g}{\left(A-C_i\right)C\tan\left[\phi+\mu\right]}=\frac{163589.8}{\left(49.0-43.75\right)\left(46.375\right)\tan\left[10+5\right]}=2507.6\ psi$$

m) STEP 13 – Check the flange stress acceptance criteria for the operating and gasket seating conditions are shown in Table 4.17.2.

Operating Condition – Location: Flange

$$\left\{S_{1o}=2442.0\,psi\right\}\le\left\{1.5S_{ho}=33000\,psi\right\}\qquad True$$

$$\left\{S_{2o}=4239.6\ psi\right\}\le\left\{S_{ho}=22000\ psi\right\}\qquad True$$

$$\left\{S_{3o}=1274.6\ psi\right\}<\left\{0.8S_{ho}=17600\ psi\right\}\qquad True$$

$$\left\{S_{4o} = 669.6\ psi\right\} \le \left\{0.8S_{ho} = 17600\ psi\right\} \qquad True$$

Operating Condition – Location: Clamp

$$\left\{S_{5o} = 3926.8\ psi\right\} \le \left\{1.5S_{co} = 33000\ psi\right\} \qquad True$$

$$\left\{S_{6o} = 451.7\ psi\right\} \le \left\{1.5S_{co} = 33000\ psi\right\} \qquad True$$

$$\left\{S_{7o} = 1250.4\ psi\right\} < \left\{0.8S_{co} = 17600\ psi\right\} \qquad True$$

$$\left\{S_{8o} = 83.4\ psi\right\} \le \left\{S_{co} = 22000\ psi\right\} \qquad True$$

$$\left\{S_{9o} = 2223.0\ psi\right\} \le \left\{1.6\min\left[S_{ho},\ S_{co}\right] = 35200\ psi\right\} \qquad True$$

Gasket Seating/Assembly Condition – Location: Flange

$$\left\{S_{1g} = 3407.1\ psi\right\} \le \left\{1.5S_{hg} = 33000\ psi\right\} \qquad True$$

$$\left\{S_{2g} = 0.0\ psi\right\} \le \left\{S_{hg} = 22000\ psi\right\} \qquad True$$

$$\left\{S_{3g} = 1437.8\ psi\right\} \le \left\{0.8S_{hg} = 17600\ psi\right\} \qquad True$$

$$\left\{S_{4g} = 1264.7\ psi\right\} \le \left\{0.8S_{hg} = 17600\ psi\right\} \qquad True$$

Gasket Seating/Assembly Condition – Location: Clamp

$$\left\{S_{5g} = 4429.6\ psi\right\} \le \left\{1.5S_{cg} = 33000\ psi\right\} \qquad True$$

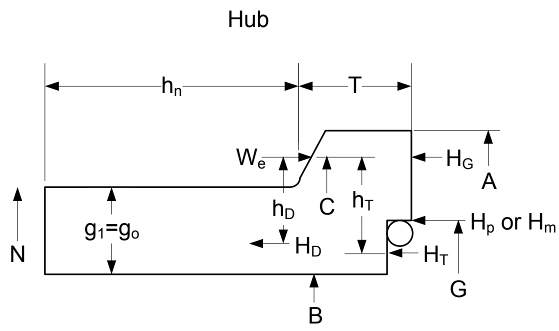
$$\left\{S_{6g} = 1560.4\ psi\right\} \le \left\{1.5S_{cg} = 33000\ psi\right\} \qquad True$$

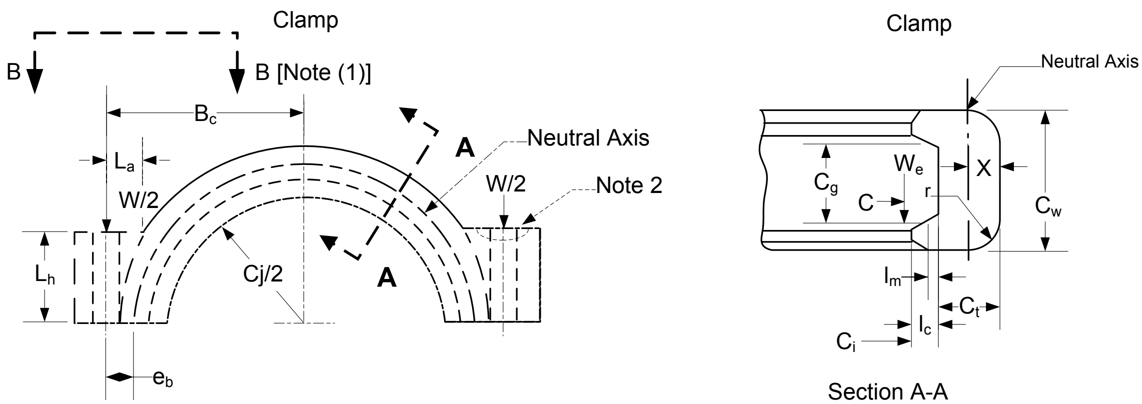
$$\left\{S_{7g} = 1410.5\ psi\right\} \le \left\{0.8S_{cg} = 17600\ psi\right\} \qquad True$$

$$\left\{S_{8g} = 288.2\ psi\right\} < \left\{S_{cg} = 22000\ psi\right\} \qquad True$$

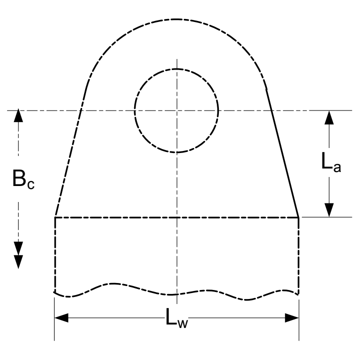
$$\left\{S_{9g} = 2507.6\ psi\right\} < \left\{1.6\min\left[S_{hg},\ S_{cg}\right] = 35200\ psi\right\} \qquad True$$

The proposed hub/clamp assembly is acceptable for the specified design conditions.





Notes:
1) See Figure 4.17.2 for sction B-B
2) Clamp may have spherical depressions at bolt holes to facilitate the use of spherical nuts



(b)
Section B-B

Figure E4.17.1 - Typical Hub and Clamp Configuration

4.18 Tubesheets in Shell and Tube Heat Exchangers

4.18.1 Example E4.18.1 – U-Tube Tubesheet Integral with Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration a as shown in Figure 4.18.4.

- The shell side design conditions are -10 and 60 psig at 500°F.
- The tube side design conditions are -15 and 140 psig at 500°F.
- The tube material is SA-249 S31600 (Stainless Steel 316). The tubes are 0.75 in. outside diameter and 0.065 in. thick and are to be full-strength welded with no credit taken for expansion.
- The tubesheet material is SA-240 S31600 (Stainless Steel 316) with no corrosion allowance on the tube side and no pass partition grooves. The tubesheet outside diameter is 12.939 in. The tubesheet has 76 tube holes on a 1.0 in. square pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 2.25 in., and the radius to the outermost tube hole center is 5.438 in.
- The shell material is SA-312 S31600 (Stainless Steel 316) welded pipe. The shell inside diameter is 12.39 in. and the shell thickness is 0.18 in.
- The channel material is SA-240 S31600 (Stainless Steel 316). The channel inside diameter is 12.313 in. and the channel thickness is 0.313 in.

Data Summary

The data summary consists of those variables from the nomenclature (see paragraph 4.18.15) that are applicable to this configuration.

The data for paragraph 4.18.15 is:

$$c_t = 0\,in.$$

$$d_t = 0.75\,in.$$

$$E = 25.8E10^6\ psi \text{ from Table TM-1 of Section II, Part D at } 500^{\circ}\text{F}$$

$$E_t = 25.8E10^6\ psi \text{ from Table TM-1 of Section II, Part D at } 500^{\circ}\text{F}$$

$$h_g = 0\,in.$$

$$p = 1.0\,in.$$

$$r_o = 5.438\,in.$$

$$S = 18,000\ psi \text{ from Table 5A of Section II, Part D at } 500^{\circ}\text{F}$$

$$S_t = 18,000\ psi \text{ from Table 5A of Section II, Part D at } 500^{\circ}\text{F (for seamless tube, SA-213)}$$

$$t_t = 0.065\,in.$$

$$U_{L1} = 2.25\,in.$$

$$\rho = 0 \text{ for no tube expansion}$$

$$A = 12.939\,in.$$

$$D_c = 12.313\,in.$$

$$D_s = 12.39\,in.$$

$E = 25.8E10^6\ psi$ from Table TM-1 of Section II, Part D at 500° F

$E_c = 25.8E10^6\ psi$ from Table TM-1 of Section II, Part D at 500° F

$E_s = 25.8E10^6\ psi$ from Table TM-1 of Section II, Part D at 500° F

$P_{sd,\max} = 60\ psig$

$P_{sd,\min} = -10\ psig$

$P_{td,\max} = 140\ psig$

$P_{td,\min} = -15\ psig$

$S = 18,000\ psi$ from Table 5A of Section II, Part D at 500° F

$S_c = 18,000\ psi$ from Table 5A of Section II, Part D at 500° F

$S_s = 18,000\ psi$ from Table 5A of Section II, Part D at 500° F (for seamless pipe, SA-312)

$t_c = 0.313\ in.$

$t_s = 0.18\ in.$

$\nu_c = 0.3$

$\nu_s = 0.3$

Calculation Procedure

The calculation procedure for a U-tube heat exchanger tubesheet is given in paragraph 4.18.7. The calculation results are shown for loading case 1 where $P_s = P_{sd,min} = -10\ psig$ and $P_t = P_{td,max} = 140\ psig$, since this case yields the greatest value of σ.

a) STEP 1 – Calculate D_o, μ, $\mu*$, and h'_g from paragraph 4.18.6.4.a.

$D_o = 11.626\ in.$

$L_{L1} = 11.6\ in.$

$A_L = 26.2\ in.^2$

$\mu = 0.25$

$d* = 0.75\ in.$

$p* = 1.15\ in.$

$\mu* = 0.349$

$h'_g = 0\ in.$

b) STEP 2 – Calculate ρ_s, ρ_c, and M_{TS} for configuration a.

$\rho_s = 1.07$

$\rho_c = 1.06$

$M_{TS} = -160\ in.-lb/in.$

c) STEP 3 – Assume a value for the tubesheet thickness, h, and calculate h/p. Determine E^*/E and ν^* from paragraph 4.18.6.4.b and calculate E^*.

$$h = 0.521\,in.$$
$$h/p = 0.521$$
$$E^*/E = 0.445$$
$$\nu^* = 0.254$$
$$E^* = 11.5E10^6\ psi$$

d) STEP 4 – For configuration a, calculate $\beta_s,\ k_s,\ \lambda_s,\ \delta_s,\ \text{and}\ \omega_s$ for the shell and $\beta_c,\ k_c,\ \lambda_c,\ \delta_c,\ \text{and}\ \omega_c$ for the channel.

$$\beta_s = 1.21\,in.^{-1}$$
$$k_s = 33{,}300\,lb.$$
$$\lambda_s = 32.0\times 10^6\ psi$$
$$\delta_s = 7.02\times 10^{-6}\ in.^3/lb$$
$$\omega_s = 0.491\,in.^2$$
$$\beta_c = 0.914\,in.^{-1}$$
$$k_c = 132{,}000\,lb$$
$$\lambda_c = 110\times 10^6\ psi$$
$$\delta_c = 3.99\times 10^{-6}\ in.^3/lb$$
$$\omega_c = 0.756\,in.^2$$

e) STEP 5 – Calculate K and F for configuration a.

$$K = 1.11$$
$$F = 9.41$$

f) STEP 6 – Calculate M^* for configuration a.

$$M^* = -49.4\ \ in.-lb/in.$$

g) STEP 7 – Calculate $M_p,\ M_o,\ \text{and}\ M$.

$$M_p = 568\ in.-lb/in.$$
$$M_o = -463\ in.-lb/in.$$
$$M = 568\ in.-lb/in.$$

h) STEP 8 – Calculate σ and check the acceptance criterion.

$$\sigma = 36{,}000\,psi \le 2S = 36{,}000\,psi$$

i) STEP 9 – Calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If $\left|P_s - P_t\right| \le \dfrac{3.2S\mu h}{D_o}$, then the shear stress is not required to be calculated.

Load Case 1:

$$\left\{\left|P_s - P_t\right| = 150 \ psi\right\} \le \left\{\frac{3.2S\mu h}{D_o} = 645 \ psi\right\}$$

Calculate the average shear stress, τ.

Not required for this example.

j) STEP 10 – For configuration a, calculate $\sigma_{s,m}$, $\sigma_{s,b}$, and σ_s for the shell and $\sigma_{c,m}$, $\sigma_{c,b}$, and σ_c for the channel, and check the acceptance criterion. The shell thickness shall be 0.18 in. for a minimum length of 2.69 in. adjacent to the tubesheet and the channel thickness shall be 0.313 in. for a minimum length of 3.53 in. adjacent to the tubesheet.

$$\sigma_{s,m} = -170 \, psi$$

$$\sigma_{s,b} = -17,600 \, psi$$

$$\sigma_s = 17,700 \, psi \le 1.5S_s = 27,000 \, psi$$

$$\sigma_{c,m} = 1,340 \, psi$$

$$\sigma_{c,b} = 25,300 \, psi$$

$$\sigma_c = 26,600 \, psi \le 1.5S_c = 27,000 \, psi$$

The assumed value for the tubesheet thickness, h, is acceptable and the shell and channel stresses are within the allowable stresses; therefore, the calculation procedure is complete.

4.18.2 Example E4.18.2 – U-Tube Tubesheet Gasketed With Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration d as shown in Figure 4.18.4.

- The shell side design conditions are -15 and 10 psig at 300°F.
- The tube side design condition is 135 psig at 300°F.
- The tube material is SB-111 C44300 (Admiralty). The tubes are 0.625 in. outside diameter and 0.065 in. thick and are to be expanded for the full thickness of the tubesheet.
- The tubesheet material is SA-285, Grade C (K02801) with a 0.125 in. corrosion allowance on the tube side and no pass partition grooves. The tubesheet outside diameter is 20.0 in. The tubesheet has 386 tube holes on a 0.75 in. equilateral triangular pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 1.75 in., and the radius to the outermost tube hole center is 8.094 in.
- The diameter of the shell gasket load reaction is 19.0 in. and the shell flange design bolt load is 147,000 lb.
- The diameter of the channel gasket load reaction is 19.0 in. and the channel flange design bolt load is 162,000 lb.

<u>Data Summary</u>

The data summary consists of those variables from the nomenclature (see paragraph 4.18.15) that are applicable to this configuration.

The data for paragraph 4.18.15 is:

$$c_t = 0.125\,in.$$

$$d_t = 0.625\,in.$$

$$E = 28.3E10^6\ psi \text{ from Table TM-1 of Section II, Part D at } 300^\circ\text{F}$$

$$E_t = 15.4E10^6\ psi \text{ from Table TM-3 of Section II, Part D at } 300^\circ\text{F}$$

$$h_g = 0\,in.$$

$$p = 0.75\,in.$$

$$r_o = 8.094\,in.$$

$$S = 17,700\ psi \text{ from Table 5A of Section II, Part D at } 300^\circ\text{F}$$

$$S_t = 10,000\ psi \text{ from Table 5B of Section II, Part D at } 300^\circ\text{F}$$

$$t_t = 0.065\,in.$$

$$U_{L1} = 1.75\,in.$$

$$\rho = 1.0 \text{ for a full length tube expansion}$$

$A = 20.0\,in.$

$E = 28.3E10^6\ psi$ from Table TM-1 of Section II, Part D at 300˚F

$G_c = 19.0\,in.$

$G_s = 19.0\,in.$

$P_{sd,\max} = 10\ psig$

$P_{sd,\min} = -15\ psig$

$P_{td,\max} = 135\ psig$

$P_{td,\min} = 0\ psig$

$S = 17,700\ psi$ per Table 5A of Section II, Part D at 300˚F

$W^* = 162,000\,lb$ from Table 4.18.6

Calculation Procedure

The calculation procedure for a U-tube heat exchanger tubesheet is given in paragraph 4.18.7. The calculation results are shown for loading case 1 where $P_s = P_{sd,min} = -15\ psig$ and $P_t = P_{td,max} = 135\ psig$ since this case yields the greatest value of σ.

a) STEP 1 – Calculate $D_o, \mu, \mu^*,$ and h'_g from paragraph 4.18.6.4.a.

$$D_o = 16.813\,in.$$
$$L_{L1} = 16.8\,in.$$
$$A_L = 29.4\ in.^2$$
$$\mu = 0.167$$
$$d* = 0.580\,in.$$
$$p* = 0.805\,in.$$
$$\mu* = 0.280$$
$$h'_g = 0\,in.$$

b) STEP 2 – Calculate $\rho_s, \rho_c,$ and M_{TS} for configuration d.

$$\rho_s = 1.13$$
$$\rho_c = 1.13$$
$$M_{TS} = -785\ in.\text{–}lb/in.$$

c) STEP 3 – Assume a value for the tubesheet thickness, h, and calculate h/p. Determine $E*/E$ and $\nu*$ from paragraph 4.18.6.4.b and calculate $E*$.

$$h = 1.28\,in.$$
$$h/p = 1.71$$
$$E*/E = 0.265$$
$$\nu* = 0.358$$
$$E* = 7.50E10^{6}\ psi$$

d) STEP 4 – For configuration d, skip STEP 4 and proceed to STEP 5.

e) STEP 5 – Calculate K and F for configuration d.

$$K = 1.19$$
$$F = 0.420$$

f) STEP 6 – Calculate M^{*} for configuration d.

$$M^{*} = -785\ in.\text{–}lb/in.$$

g) STEP 7 – Calculate M_p, M_o, and M.

$$M_p = -160\ in.\text{–}lb/in.$$
$$M_o = -2,380\ in.\text{–}lb/in.$$
$$M = 2,380\ in.\text{–}lb/in.$$

h) STEP 8 – Calculate σ and check the acceptance criterion.

$$\sigma = 31,200\,psi \le 2S = 35,400\,psi$$

i) STEP 9 – Calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If $\left|P_s - P_t\right| \le \dfrac{3.2S\mu h}{D_o}$, then the shear stress is not required to be calculated.

Load Case 1:

$$\left\{\left|P_s - P_t\right| = 150\ psi\right\} \le \left\{\frac{3.2S\mu h}{D_o} = 639\ psi\right\}$$

Calculate the average shear stress, τ.

Not required for this example.

The assumed value for the tubesheet thickness, h, is acceptable and the calculation procedure is complete.

4.18.3 Example E4.18.3 – U-Tube Tubesheet Gasketed With Shell and Channel

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration d as shown in Figure 4.18.4.

- The shell side design condition is 375 psig at 500°F.
- The tube side design condition is 75 psig at 500°F.
- The tube material is SB-111 C70600 (90/10 copper-nickel). The tubes are 0.75 in. outside diameter and 0.049 in. thick and are to be expanded for one-half of the tubesheet thickness.
- The tubesheet material is SA-516, Grade 70 (K02700) with a 0.125 in. corrosion allowance on the tube side and a 0.1875 in. deep pass partition groove. The tubesheet outside diameter is 48.88 in. The tubesheet has 1,534 tube holes on a 0.9375 in. equilateral triangular pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 2.25 in., and the radius to the outermost tube hole center is 20.5 in.
- The diameter of the shell gasket load reaction is 43.5 in. and the shell flange design bolt load is 675,000 lb.
- The diameter of the channel gasket load reaction is 44.88 in. and the channel flange design bolt load is 584,000 lb.
- The tubesheet shall be designed for the differential design pressure.

Data Summary

The data summary consists of those variables from the nomenclature (see paragraph 4.18.15) that are applicable to this configuration.

The data for paragraph 4.18.15 is:

$$c_t = 0.125\,in.$$

$$d_t = 0.75\,in.$$

$$E = 27.1E10^6\ psi \text{ from Table TM-1 of Section II, Part D at } 500^{\circ}\text{F}$$

$$E_t = 16.6E10^6\ psi \text{ from Table TM-3 of Section II, Part D at } 500^{\circ}\text{F}$$

$$h_g = 0.1875\,in.$$

$$p = 0.9375\,in.$$

$$r_o = 20.5\,in.$$

$$S = 20,600\ psi \text{ from Table 5A of Section II, Part D at } 500^{\circ}\text{F}$$

$$S_t = 8,000\ psi \text{ from Table 5B of Section II, Part D at } 500^{\circ}\text{F}$$

$$t_t = 0.049\,in.$$

$$U_{L1} = 2.25\,in.$$

$$\rho = 0.5 \text{ for tubes expanded for one-half the tubesheet thickness}$$

$A = 48.88\,in.$

$E = 27.1E10^6\ psi$ from Table TM-1 of Section II, Part D at 500°F

$G_c = 44.88\,in.$

$G_s = 43.5\,in.$

$P_{sd,\max} = 375\ psig$

$P_{td,\max} = 75\ psig$

$S = 20,600\ psi$ from Table 5A of Section II, Part D at 500°F

$W^* = 675,000\,lb$ from Table UHX-8.1

Calculation Procedure

The calculation procedure for a U-tube heat exchanger tubesheet is given in paragraph 4.18.7. Since differential pressure design is specified, the calculation results are shown for loading case 3.

a) STEP 1 – Calculate $D_o, \mu, \mu*,$ and h'_g from paragraph 4.18.6.4.a.

$$D_o = 41.75\,in.$$
$$L_{L1} = 41.8\,in.$$
$$A_L = 93.9\,in.^2$$
$$\mu = 0.2$$
$$d* = 0.738\,in.$$
$$p* = 0.971\,in.$$
$$\mu* = 0.240$$
$$h'_g = 0.0625\,in.$$

b) STEP 2 – Calculate $\rho_s, \rho_c,$ and M_{TS} for configuration d.

$$\rho_s = 1.04$$
$$\rho_c = 1.07$$
$$M_{TS} = 2,250\,in.\text{–}lb/in.$$

c) STEP 3 – Assume a value for the tubesheet thickness, h, and calculate h/p. Determine $E*/E$ and $\nu*$ from paragraph 4.18.6.4.b and calculate $E*$.

$$h = 4.15\,in.$$
$$h/p = 4.43$$
$$E*/E = 0.204$$
$$\nu* = 0.407$$
$$E* = 5.54E10^6\ psi$$

d) STEP 4 – For configuration d, skip STEP 4 and proceed to STEP 5.

e) STEP 5 – Calculate K and F for configuration d.

$$K = 1.17$$

$$F = 0.458$$

f) STEP 6 – Calculate $M*$ for configuration d.

$$M^* = 5800 \; in.-lb/in.$$

g) STEP 7 – Calculate $M_p, M_o,$ *and* M.

$$M_p = -1150 \; in.-lb/in.$$

$$M_o = 26,700 \; in.-lb/in.$$

$$M = 26,700 \; in.-lb/in.$$

h) STEP 8 – Calculate σ and check the acceptance criterion.

$$\sigma = 39,900 \; psi \le 2S = 41,200 \; psi$$

i) STEP 9 – Calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If $\left|P_s - P_t\right| \le \dfrac{3.2S\mu h}{D_o}$, then the shear stress is not required to be calculated.

<u>Load Case 3:</u>

$$\left\{\left|P_s - P_t\right| = 300 \;\; psi\right\} \le \left\{\frac{3.2S\mu h}{D_o} = 1311 \;\; psi\right\}$$

Calculate the average shear stress, τ.

Not required for this example.

The assumed value for the tubesheet thickness, h, is acceptable and the calculation procedure is complete.

4.18.4 Example E4.18.4 – U-Tube Tubesheet Gasketed With Shell and Integral with Channel, Extended as a Flange

A U-tube heat exchanger is to be designed with the tubesheet construction in accordance with configuration e as shown in Figure 4.18.4.

- The shell side design condition is 650 psig at 400°F.
- The tube side design condition is 650 psig at 400°F.
- The tube material is SA-179 (K01200). The tubes are 0.75 in. outside diameter and 0.085 in. thick and are to be expanded for the full thickness of the tubesheet.
- The tubesheet material is SA-516, Grade 70 (K02700) with a 0.125 in. corrosion allowance on the tube side and no pass partition grooves. The tubesheet outside diameter is 37.25 in. The tubesheet has 496 tube holes on a 1.0 in. square pattern with one centerline pass lane. The largest center-to-center distance between adjacent tube rows is 1.375 in., and the radius to the outermost tube hole center is 12.75 in.
- The diameter of the shell gasket load reaction is 32.375 in., the shell flange bolt circle is 35 in., and the shell flange design bolt load is 656,000 lb.
- The channel material is SA-516, Grade 70, (K02700). The channel inside diameter is 31 in. and the channel thickness is 0.625 in.

Data Summary

The data summary consists of those variables from the nomenclature (see paragraph 4.18.15) that are applicable to this configuration.

The data for paragraph 4.18.15 is:

$$c_t = 0.125\,in.$$

$$d_t = 0.75\,in.$$

$$E = 27.7E10^6 \; psi \text{ from Table TM-1 of Section II, Part D at } 400^\circ\text{F}$$

$$E_t = 27.7E10^6 \; psi \text{ from Table TM-1 of Section II, Part D at } 400^\circ\text{F}$$

$$h_g = 0\,in.$$

$$p = 1.0\,in.$$

$$r_o = 12.75\,in.$$

$$S = 21,600\; psi \text{ from Table 5A of Section II, Part D at } 400^\circ\text{F}$$

$$S_t = 13,400\; psi \text{ from Table 1A of Section II, Part D at } 400^\circ\text{F}$$

$$t_t = 0.085\,in.$$

$$U_{L1} = 1.375\,in.$$

$$\rho = 1.0 \text{ for full length tube expansion}$$

$A = 37.25\,in.$

$C = 35\,in.$

$D_c = 31\,in.$

$E = 27.7E10^6\ psi$ from Table TM-1 of Section II, Part D at 400˚F

$E_c = 27.7E10^6\ psi$ from Table TM-1 of Section II, Part D at 400˚F

$G_s = 32.375\,in.$

$P_{sd,\max} = 650\ psig$

$P_{sd,\min} = 0\ psig$

$P_{td,\max} = 650\ psig$

$P_{td,\min} = 0\ psig$

$S = 21,600\ psi$ from Table 5A of Section II, Part D at 400˚F

$S_c = 21,600\ psi$ from Table 5A of Section II, Part D at 400˚F

$S_{y,c} = 32,500\ psi$ from Table Y-1 of Section II, Part D at 400˚F

$S_{PS,c} = 65,000\ psi$ (MYS/UTS < 0.7; therefore use $2S_{y,c}$)

$t_c = 0.625\,in.$

$W* = 656,000\,lb$

$\nu_c = 0.3$

Calculation Procedure

The calculation procedure for a U-tube heat exchanger tubesheet is given in paragraph 4.18.7. The calculation results are shown for loading case 2 where $P_s = P_{sd,max} = 650\ psig$ and $P_t = P_{td,min} = 0\ psig$, since this case yields the greatest value of σ.

a) STEP 1 – Calculate $D_o, \mu, \mu*$ and h'_g from paragraph 4.18.6.4.a.

$D_o = 26.25\,in.$

$L_{L1} = 26.3\,in.$

$A_L = 36.1\,in.^2$

$\mu = 0.25$

$d* = 0.636\ in.$

$p* = 1.04\,in.$

$\mu* = 0.385$

$h'_g = 0\,in.$

b) STEP 2 – Calculate ρ_s, ρ_c, and M_{TS} for configuration e.

$$\rho_s = 1.23$$
$$\rho_c = 1.18$$
$$M_{TS} = 16{,}500\ in.\text{–}lb/in.$$

c) STEP 3 – Assume a value for the tubesheet thickness, h, and calculate h/p. Determine E^*/E and ν^* from paragraph 4.18.6.4.b and calculate E^*.

$$h = 3.50\,in.$$
$$h/p = 3.50$$
$$E^*/E = 0.441$$
$$\nu^* = 0.318$$
$$E^* = 12.2E10^6\ psi$$

d) STEP 4 – For configuration e, calculate β_c, k_c, λ_c, δ_c, and ω_c for the channel.

$$\beta_c = 0.409\,in.^{-1}$$
$$k_c = 506{,}000\,lb$$
$$\lambda_c = 7.59\times10^6\ psi$$
$$\delta_c = 1.18\times10^{-5}\ in.^3/lb$$
$$\omega_c = 7.01\,in.^2$$

e) STEP 5 – Calculate K and F for configuration e.

$$K = 1.42$$
$$F = 0.964$$

f) STEP 6 – Calculate M^* for configuration e.

$$M^* = 26{,}900\ in.\text{–}lb/in.$$

g) STEP 7 – Calculate $M_p, M_o,\ and\ M$.

$$M_p = 6830\ in.\text{–}lb/in.$$
$$M_o = 30{,}000\ in.\text{–}lb/in.$$
$$M = 30{,}000\ in.\text{–}lb/in.$$

h) STEP 8 – Calculate σ and check the acceptance criterion.

$$\sigma = 38{,}200\,psi \le 2S = 43{,}200\,psi$$

i) STEP 9 – For each loading case, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If $|P_s - P_t| \le \dfrac{3.2S\mu h}{D_o}$, then the shear stress is not required to be calculated.

Load Case 2:

$$\left\{\left|P_s - P_t\right| = 650 \ psi\right\} \leq \left\{\frac{3.2S\mu h}{D_o} = 2133 \ psi\right\}$$

Calculate the average shear stress, τ .

Not required for this example.

j) STEP 10 – For configuration e, calculate $\sigma_{c,m}, \sigma_{c,b}, \text{ and } \sigma_c$ for the channel, and check the acceptance criterion. The channel thickness shall be 0.625 in. for a minimum length of 7.92 in. adjacent to the tubesheet.

$$\sigma_{c,m} = 0 \, psi$$

$$\sigma_{c,b} = -57,000 \, psi$$

$$\sigma_c = 57,000 \, psi > 1.5S_c = 32,400 \, psi$$

k) STEP 11 – Since the channel stress exceeds the allowable stress, the design must be reconsidered using one of three options.

- Option 1 requires that the tubesheet thickness be increased until the channel stresses calculated in STEP 9 are within the allowable stress for each loading case.
- Option 2 requires that the shell and/or channel thickness be increased until their respective stresses calculated in STEP 9 are within the allowable stress for each loading case.
- Option 3 permits one elastic-plastic calculation for each design. If the tubesheet stress is still within the allowable stress given in STEP 8, the design is acceptable and the calculation procedure is complete. If the tubesheet stress is greater than the allowable stress, the design shall be reconsidered by using Option 1 or 2.

Choose Option 3, configuration e. Since $\sigma_c \leq S_{PS,c} = 65,000 \, psi$ for all loading cases, this option may be used. The calculations for this option are only required for each loading case where $\sigma_c > 1.5S_c = 32,400 \, psi$.

Calculate E_c^* for each loading case where $\sigma_c > 32,400 \, psi$. For this example, E_c^* and the calculations for loading case 2 are shown.

$$E_c^* = 20.1E10^6 \ psi$$

Recalculate k_c and λ_c given in STEP 4 using the applicable reduced effective modulus E_c .

$$k_c = 368,000 \, lb$$

$$\lambda_c = 5.51E10^6 \ psi$$

Recalculate F given in STEP 5.

$$F = 0.848$$

Recalculate M_p, M_o, and M given in STEP 7.

$$M_p = 8{,}130 \; in.\text{–}lb/in.$$
$$M_o = 31{,}400 \; in.\text{–}lb/in.$$
$$M = 31{,}400 \; in.\text{–}lb/in.$$

Recalculate σ given in STEP 8.

$$\sigma = 39{,}800 \, psi \le 2S = 43{,}200 \, psi$$

The assumed value for the tubesheet thickness, h, is acceptable and the calculation procedure is complete.

4.18.5 Example E4.18.5 – Fixed Tubesheet Exchanger, Configuration b, Tubesheet Integral with Shell, Extended as a Flange and Gasketed on the Channel Side

A fixed tubesheet heat exchanger is to be designed with the tubesheet construction in accordance with configuration b as shown in VIII-1, Figure UHX-13.1.

- For the Design Condition, the shell side design pressure is 150 psig at 700°F, and the tube side design pressure is 400 psig at 700°F.
- There is one operating condition. For Operating Condition 1, the shell side design pressure is 150 psig at 700°F, the tube side design pressure is 400 psig at 700°F, the shell mean metal temperature is 550°F, and the tube mean metal temperature is 510°F. For this example, the operating pressures and operating metal temperatures are assumed to be the same as the design values.
- The tube material is SA-214 welded (K01807). The tubes are 1 in. outside diameter, 0.083 in. thick and are to be expanded to 95% of the tubesheet thickness.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet outside diameter is 40.5 in. There are 649 tube holes on a 1.25 in. triangular pattern. There is no pass partition lane, and the outermost tube radius from the tubesheet center is 16.625 in. The distance between the outer tubesheet faces is 168 in. There is no corrosion allowance on the tubesheet.
- The shell material is SA-516, Grade 70 (K02700). The shell inside diameter is 34.75 in. and the thickness is 0.1875 in. There is no corrosion allowance on the shell. The shell contains an expansion joint that has an inside diameter of 38.5 in. and an axial rigidity of 11,388 lb/in. The efficiency of the shell circumferential welded joint (Category B) is 1.0.
- The diameter of the channel flange gasket load reaction is 36.8125 in., the bolt circle diameter is 38.875 in., the design bolt load is 512,937 lb, and the operating condition bolt load is 512,473 lb.

Data Summary - Tubesheet

Tube Layout: Triangular

$h = 3.0625\,in.$

$h_g = 0\,in.$

$c_t = 0\,in.$

$A = 40.5\,in.$

$r_o = 16.625\,in.$

$A_L = 0.0\,in.^2$

$N_t = 649$

$L_t = 168\,in.$

$p = 1.2500\,in.$

$T = 700^\circ \mathrm{F}$

$T_a = 70^\circ \mathrm{F}$

$S = 18,100\,psi$ at T from Table 5A of Section II, Part D

$S_y = 27,200\,psi$ at T

$S_{PS} = 54,400\,psi$ at T

$E = 25.5E6\,psi$ at T from TM-1 of Section II, Part D

$\nu = 0.3$

Data Summary - Tubes

$$P_{td,\max} = 400\ psig$$
$$P_{td,\min} = 0\ psig$$
$$P_{to1} = 400\ psig$$
$$\ell_{tx} = 2.909\ in.$$
$$k = 1$$
$$\ell = 59\ in.$$
$$t_t = 0.083\ in.$$
$$d_t = 1\ in.$$
$$T_t = 700^{\circ}\mathrm{F}$$
$$T_{t,m} = 510^{\circ}\mathrm{F}$$
$$S_t = 10,500\ psi \text{ at } T_t \text{ from Table 1A of Section II, Part D}$$
$$S_{y,t} = 18,600\ psi \text{ at } T_t \text{ from Y-1of Section II, Part D}$$
$$S_{tT} = 10,500\ psi \text{ at } T \text{ from Table 1A of Section II, Part D}$$
$$\alpha_{t,m} = 7.3E-06\ in./in.^{\circ}F \text{ at } T_{t,m}$$
$$E_t = 25,500,000\ psi \text{ at } T_t \text{ from TM-1 of Section II, Part D}$$
$$E_{tT} = 25,500,000\ psi \text{ at } T \text{ from TM-1 of Section II, Part D}$$
$$\upsilon_t = 0.3$$

Note: Since the tubes are welded (SA-214), the tube allowable stresses S_t and S_{tT} can be divided by 0.85 per paragraph 4.18.15.d. This results in adjusted values of $S_t = 12,353\ psi$ and $S_{tT} = 12,353\ psi$.

Data Summary - Shell

$P_{sd,\max} = 150\ psig$

$P_{sd,\min} = 0\ psig$

$P_{so1} = 150\ psig$

$t_s = 0.1875\ in.$

$D_s = 34.75\ in.$

$D_j = 38.5\ in.$

$K_j = 11,388\ lb/in.$

$T_s = 700^{\circ}\mathrm{F}$

$T_{s,m} = 550^{\circ}\mathrm{F}$

$S_s = 18,100\ psi$ at T_s from Table 5A of Section II, Part D

$E_{s,w} = 1.0$

$S_{y,s} = 27,200\ psi$ from Table Y-1 of Section II, Part D

$S_{PS,s} = 54,400\ psi$ at T see $paragraph\ 5.5.6.1.d$

$E_s = 25,500,000\ psi$ from TM-1 of Section II, Part D

$\alpha_{s,m} = 7.3E-06\ in./in./^{\circ}\mathrm{F}$ at $T_{s,m}$

$\nu_s = 0.3$

Data Summary - Channel Flange

$Gasket\ I.D. = 36.3125\ in.$

$Gasket\ O.D. = 37.3125\ in.$

$Mean\ Gasket\ Diameter, G = G_c = 36.8125\ in.$

$Gasket, m, Factor = 3.75$

$Gasket, y, Factor = 7,600\ psi$

$Flange\ Outside\ Diameter = 40.5\ in.$

$Bolt\ Circle, C = 38.875\ in.$

$Bolting\ Data = 68\ bolts,\ 0.75\ in.\ diameter$

$Bolting\ \ Material = \text{SA-193 B7}$

$Bolt\ Load, W_g = 512,937\ lb\ \ per\ VIII-2\ \ paragraph\ 4.16$

$Bolt\ Load, W_o = 512,473\ lb\ \ per\ VIII-2\ \ paragraph\ 4.16$

$W^*\ from\ Table\ 4.18.6$ (see Summary Table for Step 5)

$Gasket\ Monument\ Arm, h_g = \left(C - G_c\right)/2 = 1.03125\ in.$

Calculation Procedure

The tubesheet is extended as a flange. The calculation procedure for a tubesheet flanged extension is given in paragraph 4.18.5.4.

The tubesheet flanged extension required thickness for the operating condition $\left(S = 18{,}100\, psi \text{ at } T\right)$ is:

$$h_r = 1.228\, in.$$

The tubesheet flanged extension required thickness for the gasket seating condition $\left(S = 25{,}300\, psi \text{ at } T_a\right)$ is:

$$h_r = 1.039\ in.$$

The calculation procedure for a Fixed Tubesheet heat exchanger is given in paragraph 4.18.8. The following results are for the design and operating loading cases required to be analyzed (see paragraph 4.18.8.3). This example illustrates the calculation of both the elastic and elastic-plastic solutions.

a) STEP 1 – Calculate $D_o,\ \mu,\ \mu*$ and h'_g from paragraph 4.18.6.4.a.

$$L = 161.875\, in.$$
$$D_o = 34.25\, in.$$
$$a_o = 17.125\, in.$$
$$\rho = 0.95$$
$$d* = 0.8924\, in.$$
$$\mu = 0.2000$$
$$\rho* = 1.2500\, in.$$
$$\mu* = 0.2861$$
$$\rho_s = 1.014598$$
$$\rho_c = 1.074818$$
$$x_s = 0.4467$$
$$x_t = 0.6152$$

b) STEP 2 – Calculate the shell axial stiffness, K_s, tube axial stiffness, K_t, stiffness factors, $K_{s,t}$ and J.

$$K_s = 3{,}241{,}928\, lb/in.$$
$$K_t = 37{,}666\, lb/in.$$
$$K_{s,t} = 0.13262$$
$$J = 0.0035$$

Calculate the shell coefficients β_s, k_s, λ_s, and δ_s.

$$\beta_s = 0.7102\, in.^{-1}$$
$$k_s = 21{,}866\, lb$$
$$\lambda_s = 879{,}437\; psi$$
$$\delta_s = 0.0000536694\, in.^3 / lb$$

Calculate the channel coefficients β_c, k_c, λ_c, and δ_c.

$$\beta_c = 0\, in.^{-1}$$
$$k_c = 0\, lb.$$
$$\lambda_c = 0\; psi$$
$$\delta_c = 0\, in.^3 / lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from paragraph 4.18.6.4.b and calculate E^*.

$$h/p = 2.45$$
$$E^*/E = 0.262993$$
$$\nu^* = 0.363967$$
$$E^* = 6{,}706{,}322\; psi$$

Calculate, X_a, and the parameters from Table 4.18.3.

$$X_a = 3.9630$$
$$Z_d = 0.024609$$
$$Z_v = 0.064259$$
$$Z_m = 0.371462$$
$$Z_a = 6.54740$$
$$Z_w = 0.064259$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.1825$$
$$F = 0.4888$$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} *and* U.

$$\Phi = 0.6667$$
$$Q_1 = -0.022635$$
$$Q_{Z1} = 2.8556$$
$$Q_{Z2} = 6.888$$
$$U = 13.776$$

e) STEP 5 – Calculate γ, ω_s, ω_s^*, ω_c, ω_c^*, and γ_b

Summary Table for Step 5 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	γ	W*
1	0	400	0	512473
2	150	0	0	0
3	150	400	0	512473

Summary Table for Step 5 – Operating Condition 1				
Loading Case	P_s (psi)	P_t (psi)	γ	*W**
1	0	400	-0.047	512937
2	150	0	-0.047	512937
3	150	400	-0.047	512937
4	0	0	-0.047	512937

$$\omega_s = 2.685\, in.^2$$

$$\omega^*_s = -2.6536\, in.^2$$

$$\omega_c = 0\, in.^2$$

$$\omega^*_c = 9.6816\, in.^2$$

$$\gamma_b = -0.06022$$

f) STEP 6 – For each loading case, calculate P_s', P_t', P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

Summary Table for STEP 6 – Design Condition							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	862,002	0	0	230.7	181.9	-399.4
2	-46,387	0	0	0	0	18.7	-21.5
3	-46,387	862,002	0	0	230.7	200.6	-420.9

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	862,002	-1,254	0	230.9	181.9	-400
2	-46,387	0	-1,254	0	230.9	18.7	-22
3	-46,387	862,002	-1,254	0	230.9	200.6	-421.5
4	0	0	-1,254	0	230.9	0	-0.5

g) STEP 7 – Elastic Iteration, calculate Q_2 and Q_3, the tubesheet bending stress, and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition								
Loading Case	Q_2 (lbs)	Q_3	F_m	h'_g (in)	$h-h'_g$ (in)	$\lvert\sigma_{elastic}\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	-7,040.7	0.0976	0.0975	0	3.0625	25,540	27,150	---
2	-319	0.0786	0.0901	0	3.0625	-1,269	27,150	---
3	-7,359.7	0.0966	0.0971	0	3.0625	26,809	27,150	---

Summary Table for STEP 7 – Operating Condition 1								
Loading Case	Q_2 (lbs)	Q_3	F_m	h'_g (in)	$h-h'_g$ (in)	$\lvert\sigma_{elastic}\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	-7,044.2	0.09746	0.09749	0	3.0625	25,569	---	54,400
2	-4,259.3	1.299	0.67047	0	3.0625	9,658	---	54,400
3	-7,363.3	0.09650	0.09711	0	3.0625	26,839	---	54,400
4	-3,940.3	56.627	28.409	0	3.0625	8,838	---	54,400

For Design Loading Cases 1-3 $\left|\sigma_{elastic}\right| < 1.5S$, and for Operating Cases 1-4 $\left|\sigma_{elastic}\right| < S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – For each loading case, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If $\left|P_e\right| \le \dfrac{1.6S\mu h}{a_o}$, then the shear stress is not required to be calculated.

<u>Design Condition 1, Load Case 1:</u>

$$\left\{\left|P_e\right| = 399.4 \ psi\right\} \le \left\{\frac{1.6S\mu h}{a_o} = 1036 \ psi\right\}$$

Therefore, shear stress is not required to be calculated for design condition, case 1. Similarly, the shear stress is not required to be calculated for design conditions, case 2 and case 3, and operating condition cases 1-4.

Summary Table for STEP 8 – Design Condition		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	Not required	14,480
2	Not required	14,480
3	Not required	14,480

Summary Table for STEP 8 – Operating Condition 1		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	Not required	14,480
2	Not required	14,480
3	Not required	14,480
4	Not required	14,480

For all Loading Cases, the shear stress criterion is not required.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.3255\ in.$$

$$F_t = 181.24\,in.$$

$$C_t = 164.5$$

Summary Table for STEP 9 – Design Condition				
Loading Case	F_t, min	$\sigma_{t,1}$ (psi)	F_t, max	$\sigma_{t,2}$ (psi)
1	-1.081	-4024	3.809	7,570
2	-1.011	269	3.658	865
3	-1.077	-3,755	3.801	8,435

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	F_t, min	$\sigma_{t,1}$ (psi)	F_t, max	$\sigma_{t,2}$ (psi)
1	-1.081	-4,028.8	3.807	7,580.9
2	-5.520	-322.2	13.334	2,137
3	-1.078	-3,760	3.8	8,445.5
4	-213.188	-600.4	451.8	1,272.4

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\lvert\sigma_{t,\min}\rvert$ (psi)	F_s	S_{tb} (psi)
1	7,570	12,353	4,024	1.346	5,693.9
2	865	12,353	0	0	0
3	8,435	12,353	3,755	1.349	5,677

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\lvert\sigma_{t,\min}\rvert$ (psi)	F_s	S_{tb} (psi)
1	7,580.9	24,706	4,028.8	1.346	5,690.9
2	2,137.0	24,706	322.2	1.250	6,129.4
3	8,445.5	24,706	3,760	1.350	5,674.9
4	1,272.4	24,706	600.4	1.250	6,129.4

For all Loading Cases $\left|\sigma_{t,\max}\right| < S_t$. The axial tension stress criterion for the tube is satisfied.

For all Loading Cases $\left|\sigma_{t,\min}\right| < S_{tb}$. The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Design Condition				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	26.1	18,100	---	---
2	-760	18,100	---	8,505
3	-738.7	18,100	---	8,508

Summary Table for STEP 10 – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	0.0579	---	36,200	…
2	-786.1	---	36,200	8,505
3	-764.8	---	36,200	8,505
4	-21.2	---	36,200	8,505

k) STEP 11 – For each loading case, calculate the stresses in the shell and/or channel when integral with the tubesheet.

Summary Table for STEP 11 – Design Condition					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	26.1	-42,440	42,466	27,150	---
2	-760	19,214	19,978	27,150	---
3	-738.7	-23,227	23,966	27,150	---

Summary Table for STEP 11 – Operating Condition 1					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	0.0579	-42,484	42,484	---	54,400
2	-786.1	8,633	9,419	---	54,400
3	-764.8	-23,271	24,035	---	54,400
4	-21.2	-10,581	10,602	---	54,400

For Design Loading Cases 1 and 3 $|\sigma_s| > 1.5S_s$, and for Operating Loading Cases 1-4 $|\sigma_s| < S_{PS,s}$. The stress criterion for the shell is not satisfied.

l) STEP 12 – The design shall be reconsidered by using one or a combination of the following options.

- Option 1 – Increase the tubesheet thickness and return to STEP 1.
- Option 2 – Increase the integral shell and/or channel thickness and return to STEP 1
- Option 3 – Perform the elastic-plastic calculation procedures as defined in paragraph 4.18.8.6.

Choose Option 3. Since the total axial stress in the shell σ_s is between $1.5S_s$ and $S_{PS,s}$ for Design Condition Loading Case 1 and 3, the procedure of paragraph 4.18.8.6 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

Summary Results for STEP 12, Elastic Plastic Iteration Results per paragraph 4.18.8.6	
Design Condition Loading Case	1
S^*_s, psi	27,200
$fact_s$	0.776
E^*_s, psi	19,785,000
k_s, lb	16,965
λ_s	0.682E+06
F	0.470
ϕ	0.641
Q_1	-0.0215
Q_{Z1}	2.865
Q_{Z2}	6.941
U	13.882
P_W, psi	232.5
P_{rim}, psi	183.309
P_e, psi	-399.4
Q_2, lb	-7,095
Q_3	0.100
F_m	0.098
$\lvert\sigma\rvert, psi$	25,752

The final calculated tubesheet bending stress of 25,752 psi (Design Loading Case 1) is less than the allowable tubesheet bending stress of 27,150 psi. As such, this geometry meets the requirements of paragraph 4.18.8.6. The intermediate results for the elastic-plastic calculation are shown above.

4.18.6 Example E4.18.6 – Fixed Tubesheet Exchanger, Configuration b, Tubesheet Integral with Shell, Extended as a Flange and Gasketed on the Channel Side

A fixed tubesheet heat exchanger is to be designed with the tubesheet construction in accordance with configuration b as shown in VIII-1, Figure UHX-13.1.

- For the Design Condition, the shell side design pressure is 335 psig at 675°F, and the tube side design pressure is 1040 psig at 650°F.
- There is one operating condition. For Operating Condition 1, the shell side design pressure is 335 psig at 675°F, the tube side design pressure is 1040 psig at 650°F, the shell mean metal temperature is 550°F, and the tube mean metal temperature is 490°F. For this example, the operating pressures and operating metal temperatures are assumed to be the same as the design values.
- The tube material is welded SA-214 (K01807). The tubes are 0.75 in. outside diameter, are 0.083 in. thick, and are to be expanded for a length of 4.374 in.
- The tubesheet material is SA-516, Grade 70 (K02700). The tubesheet outside diameter is 32.875 in. There are 434 tube holes on a 0.9375 in. triangular pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 10.406 in. The distance between the outer tubesheet faces is 144.375 in. There is a 0.125 in. corrosion allowance on both sides of the tubesheet.
- The shell material is SA-516, Grade 70 (K02700). The shell outside diameter is 24 in. and the thickness is 0.5 in. There is a 0.125 in. corrosion allowance on the shell. There is also a shell band 1.25 in. thick, 9.75 in. long with a 0.125 in. corrosion allowance. The shell and shell band materials are the same. The shell contains an expansion joint that has an inside diameter of 29.46 in. and an axial rigidity of 14,759 lb/in. The efficiency of shell circumferential welded joint (Category B) is 0.85.
- The diameter of the channel flange gasket load reaction is 25.625 in., the bolt circle diameter is 30.125 in., the design bolt load is 804,478 lb, and the operating condition bolt load is 804,456 lb.

Data Summary - Tubesheet

Tube Layout: Triangular

$h = 4.75\,in. - 0.125\,in. - 0.125\,in. = 4.5\,in.$

$h_g = 0\,in.$

$c_t = 0.125\,in.$

$A = 32.875\,in.$

$r_o = 10.406\,in.$

$A_L = 0.0\,in.^2$

$N_t = 434$

$L_t = 144.375\,in.$

$p = 0.9375\,in.$

$T = 675^{\circ}F$

$T_a = 70^{\circ}F$

$S = 18,450\,psi$ at T from Table 5A of Section II, Part D

$S_y = 27,700\,psi$ at T from Table Y-1 of Section II, Part D

$S_{PS} = 55,400\,psi$ at T

$E = 25,575,000\,psi$ at T from TM-1 of Section II, Part D

$\nu = 0.3$

Data Summary – Tubes

$P_{td,\max} = 1040\ psig$

$P_{td,\min} = 0\ psig$

$P_{to1} = 1040\ psig$

$\ell_{tx} = 4.374\ in.$

$k = 1$

$\ell = 34\ in.$

$t_t = 0.083\ in.$

$d_t = 0.75\ in.$

$T_t = 675^\circ F$

$T_{t,m} = 490^\circ F$

$S_t = 10,700\ psi$ from Table 1A of Section II, Part D

$S_{y,t} = 18,950\ psi$ from Y-1 of Section II, Part D

$S_{tT} = 10,700\ psi$ at T from Table 1A of Section II, Part D

$\alpha_{t,m} = 7.28E-06\ in./in./^\circ F$ at $T_{t,m}$

$E_t = 25,750,000\ psi$ at T_t from TM-1 of Section II, Part D

$E_{tT} = 25,750,000\ psi$ at T from TM-1 of Section II, Part D

$\nu_t = 0.3$

Since the tubes are welded (SA-214), the tube allowable stresses S_t and S_{tT} can be delivered by 0.85 per paragraph 4.18.15.d. This results in adjusted values of $S_t = 12,588\ psi$ and $S_{tT} = 12,588\ psi$.

Data Summary - Shell

$P_{sd,\max} = 335\ psig$

$P_{sd,\min} = 0\ psig$

$P_{so1} = 335\ psig$

$t_s = 0.5\,in. - 0.125\,in. = 0.375\,in.$

$D_s = 23\,in. + 2(0.125)\,in. = 23.25\,in.$

$D_j = 29.46\,in.$

$K_j = 14,759\,lb/in.$

$T_s = 675^{\circ} F$

$T_{s,m} = 550^{\circ} F$

$E_s = 25,750,000\ psi$ from TM-1 of Section II, Part D

$\alpha_{s,m} = 7.3E-06\ in./in./^{\circ}F$ at $T_{s,m}$

$\nu_s = 0.3$

$t_{s,1} = 1.25\ in. - 0.125\,in. = 1.125\,in.$

$\ell_1 = 9.75\,in. + 0.125\,in. = 9.875\,in.$

$\ell_1' = 9.75\,in. + 0.125\,in. = 9.875\,in.$

$S_{s,1} = 18,450\ psi$ at T_s from Table 5A of Section II, Part D

$S_{y,s,1} = 27,700\ psi$ at T_s from Table Y-1 of Section II, Part D

$S_{PS,s,1} = 55,400\ psi$; see *paragraph* 5.5.6.1.*d*

$E_{s,1} = 25,750,000\ psi$ from TM-1 of Section II, Part D

$E_{s,w} = 0.85$

$\alpha_{s,m,1} = 7.3E-06\,in./in./^{\circ}F$ at $T_{s,m}$

Data Summary - Channel Flange

$$Gasket\ I.D. = 25.125\,in.$$
$$Gasket\ O.D. = 26.125\,in.$$
$$Mean\ Gasket\ Diameter, G = G_c = 25.625\,in.$$
$$Gasket, m, Factor = 6.5$$
$$Gasket, y, Factor = 26,000\ psi$$
$$Flange\ Outside\ Diameter = 32.875\,in.$$
$$Bolt\ Circle, C = 30.125\,in.$$
$$Bolting\ Data = 28\,bolts, 1.375\,in.diameter, SA-193\,B7$$
$$Bolt\ Load, W_g = 808,478\ lb\ \ per\ VIII-2\ \ paragraph\ 4.16$$
$$Bolt\ Load, W_o = 808,456\ lb\ \ per\ VIII-2\ \ paragraph\ 4.16$$
$$W^*\ from\ Table\ 4.18.6\ \text{(see Summary Table for Step 5)}$$
$$Gasket\ Moment\ Arm, h_g = (C-G)/2 = 2.25\,in.$$

Calculation Procedure

The tubesheet is extended as a flange. The calculation procedure for a tubesheet flanged extension is given in paragraph 4.18.5.4.

The tubesheet flanged extension required thickness for the operating condition $(S = 18,450\,psi \text{ at } T)$ is:

$$h_r = 2.704\,in.$$

The tubesheet flanged extension required thickness for the gasket seating condition $(S = 25,300\,psi \text{ at } T_a)$ is:

$$h_r = 2.309\ in.$$

The calculation procedure for a Fixed Tubesheet heat exchanger is given in paragraph 4.18.8. The following results are for the design and operating loading cases required to be analyzed (see paragraph 4.18.8.3). This example illustrates the calculation of both the elastic and elastic-plastic solutions.

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$L = 134.875\,in. + 0.125\,in. + 0.125\,in. = 135.125\,in.$$
$$D_o = 21.562\,in.$$
$$a_o = 10.781\,in.$$
$$\rho = 0.972$$
$$d^* = 0.6392\,in.$$
$$\mu = 0.2$$
$$p^* = 0.9375\,in.$$
$$\mu^* = 0.3182$$
$$\rho_s = 1.078286$$
$$\rho_c = 1.188433$$
$$x_s = 0.4749$$
$$x_t = 0.6816$$

b) STEP 2 – Calculate the shell axial stiffness, K_s, tube axial stiffness, K_t, stiffness factors, $K_{s,t}$ and J.

$$K^*_s = 5,876,500\,lb/in.$$
$$K_t = 33,143\,lb/in.$$
$$K_{st} = 0.40854$$
$$J = 0.0025063$$

Calculate the shell coefficients β_s, k_s, λ_s, and δ_s.

$$\beta_s = 0.3471\,in.^{-1}$$
$$k_s = 2,331,037\,lb$$
$$\lambda_s = 13,497,065\,psi$$
$$\delta_s = 0.0000039653\,in.^3/lb$$

Calculate the channel coefficients β_c, k_c, λ_c, and δ_c.

$$\beta_c = 0\,in.^{-1}$$
$$k_c = 0\,lb$$
$$\lambda_c = 0\,psi$$
$$\delta_c = 0\,in.^3/lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from paragraph 4.18.6.4.b and calculate E^*.

$$h/p = 4.80$$
$$E^*/E = 0.305132$$
$$\nu^* = 0.342304$$
$$E^* = 7{,}803{,}761\ psi$$

Calculate, X_a, and the parameters from Table 4.18.3.

$$X_a = 1.9955$$
$$Z_d = 0.174495$$
$$Z_m = 0.667867$$
$$Z_v = 0.160532$$
$$Z_a = 0.809161$$
$$Z_w = 0.160532$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.5247$$
$$F = 2.0466$$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} *and* U.

$$\Phi = 2.747$$
$$Q_1 = -0.128$$
$$Q_{Z1} = 1.2206$$
$$Q_{Z2} = 0.5952$$
$$U = 1.1904$$

e) STEP 5 – Calculate γ, ω_s, ω_s^*, ω_c, ω_c^*, and γ_b. The following results are those for the corroded condition, elastic solution.

Summary Table for Step 5 – Design Condition				
Loading Case	P_s (psi)	P_t (psi)	γ	W*
1	0	1,040	0	808456
2	335	0	0	0
3	335	1040	0	808456

Summary Table for Step 5 – Operating Condition 1				
Loading Case	P_s (psi)	P_t (psi)	γ	W*
1	0	1040	-0.060	808478
2	335	0	-0.060	808478
3	335	1040	-0.060	808478
4	0	0	-0.060	808478

$$\omega_s = 8.8648\, in.^2 \qquad \omega_c = 0\, in.^2$$

$$\omega^*_s = -8.4947\, in.^2 \qquad \omega^*_c = 8.6591\, in.^2$$

$$\gamma_b = -0.2087$$

f) STEP 6 – For each loading case, calculate P'_s, P'_t, P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

Summary Table for STEP 6 – Design Condition							
Loading Case	P'_s (psi)	P'_t (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	1,017,041	0	0	275	92.2	-1,039.2
2	-167,351	0	0	0	0	29.1	-170.7
3	-167,351	1,017,041	0	0	275	121.4	-1,210.2

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P'_s (psi)	P'_t (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	1,017,041	-2,376	0	275	92.2	-1,041.6
2	-167,351	0	-2,376	0	275	29.1	-173.2
3	-167,351	1,017,041	-2,376	0	275	121.4	-1,212.7
4	0	0	-2,376	0	275	0	-2.1

g) STEP 7 – Elastic Iteration, calculate Q_2 and Q_3, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition								
Loading Case	Q_2 (lbs)	Q_3	F_m	h'_g (in)	$h-h'_g$ (in)	$\lvert\sigma_{elastic}\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	-12,650	0.0815	0.19861	0	4.5	22,335.8	27,675	---
2	-1003.9	-0.027	0.1574	0	4.5	2913	27,675	---
3	-13,654	0.06617	0.1927	0	4.5	25,249.4	27,675	---

Summary Table for STEP 7 – Operating Condition 1								
Loading Case	Q_2 (lbs)	Q_3	F_m	h'_g (in)	$h-h'_g$ (in)	$\lvert\sigma_{elastic}\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	-12,650	0.08101	0.1984	0	4.5	22,367.1	---	55,400
2	-10,477	0.91305	0.5333	0	4.5	9,994.5	---	55,400
3	-13,654	0.06578	0.19264	0	4.5	25,280.7	---	55,400
4	-9,473	75.77	37.935	0	4.5	8,817.0	---	55,400

For Design Loading Cases 1-3 $\left|\sigma_{elastic}\right| < 1.5S$, and for Operating Cases 1-4 $\left|\sigma_{elastic}\right| < S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – For each loading case, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If $\left|P_e\right| \le \dfrac{1.6S\mu h}{a_o}$, then the shear stress is not required to be calculated.

Design Condition 1, Load Case 1:

$$\left\{\left|P_e\right| = 1039.2 \ psi\right\} \le \left\{\frac{1.6S\mu h}{a_o} = 2464 \ psi\right\}$$

Therefore, shear stress is not required to be calculated for design condition, case 1. Similarly, the shear stress is not required to be calculated for design conditions, case 2 and case 3, and operating condition cases 1-4.

Summary Table for STEP 8 – Design Condition		
Loading Case	$\|\tau\|$ (psi)	$0.8S$ (psi)
1	Not required	14,760
2	Not required	14,760
3	Not required	14,760

Summary Table for STEP 8 – Operating Condition 1		
Loading Case	$\|\tau\|$ (psi)	$0.8S$ (psi)
1	Not required	14,760
2	Not required	14,760
3	Not required	14,760
4	Not required	14,760

For all Loading Cases, the shear stress criterion is not required.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.2376\,in.$$
$$F_t = 143.07$$
$$C_t = 163.7755$$

Summary Table for STEP 9 – Design Condition				
Loading Case	F_t, min	$\sigma_{t,1}$ (psi)	F_t, max	$\sigma_{t,2}$ (psi)
1	0.459	-1,120.1	1.487	4,046.9
2	0.59	1258	1.349	1886
3	0.478	137.7	1.468	5,932.7

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	F_t, min	$\sigma_{t,1}$ (psi)	F_t, max	$\sigma_{t,2}$ (psi)
1	0.460	-1,111.8	1.487	4,061.2
2	-0.543	-314.9	2.545	2,902.1
3	0.478	146	1.467	5,947
4	-90.755	-942.9	97.817	1,016.3

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\left\|\sigma_{t,\min}\right\|$ (psi)	F_s	S_{tb} (psi)
1	4,046	12,588.2	1,120.1	2	5,336.3
2	1886	12,588.2	---	---	---
3	5,932	12,588.2	---	---	---

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\left\|\sigma_{t,\min}\right\|$ (psi)	F_s	S_{tb} (psi)
1	4,061	25,176.5	1,111.8	2	5,336.3
2	2,902	25,176.5	---	---	---
3	5,947	25,176.5	---	---	---
4	1,016	25,176.5	942.9	1.25	8,538.1

For all Loading Cases $\left|\sigma_{t,\max}\right| < S_t$. The axial tension stress criterion for the tube is satisfied.

For all Loading Cases $\left|\sigma_{t,\min}\right| < S_{tb}$. The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Main Shell – Design Condition				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	10.4	15,682.5	---	---
2	-1,525.1	15,682.5	----	10,802
3	-1,518.3	15,682.5	---	10,802

Summary Table for STEP 10 – Main Shell – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	-21.4	---	36,900	10,802
2	-1,556.9	---	36,900	10,802
3	-1,550.2	---	36,900	10,802
4	-28.2	---	36,900	10,802

Summary Table for STEP 10 – Shell Band – Design Condition				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	3.4	15,682.5	---	-41040
2	-492.7	15,682.5	---	618
3	-490.5	15,682.5	---	-40422

Summary Table for STEP 10 – Shell Band – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	-6.9	---	36,900	-41074
2	-503	---	36,900	-19412
3	-500.8	---	36,900	-40456
4	-9.1	---	36,900	-20030

k) STEP 11 – For each loading case, calculate the stresses in the shell and/or channel when integral with the tubesheet.

Summary Table for STEP 11 – Design Condition					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	S_s (psi)	$S_{PS,s}$ (psi)
1	3.4	-41,040	41,043	27,675	---
2	-492.7	618	1112	27,675	---
3	-490.5	-40,422	40,912	27,675	---
Summary Table for STEP 11 – Operating Condition 1					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	S_s (psi)	$S_{PS,s}$ (psi)
1	-6.9	-41,074	41,081	---	55,400
2	-503	-19,412	19,915	---	55,400
3	-500.8	-40,456	40,957	---	55,400
4	-9.1	-20,030	20,039	---	55,400

For Design Loading Cases 1 and 3 $|\sigma_s| > 1.5S_s$, and for Operating Loading Cases 1-4 $|\sigma_s| < S_{PS,s}$. The stress criterion for the shell is not satisfied.

l) STEP 12 – The design shall be reconsidered by using one or a combination of the following options.
- Option 1 – Increase the tubesheet thickness and return to STEP 1.
- Option 2 – Increase the integral shell and/or channel thickness and return to STEP 1
- Option 3 – Perform the elastic-plastic calculation procedures as defined in paragraph 4.18.8.6.

Since the total axial stress in the shell σ_s is between $1.5S_{s,1}$ and $S_{PS,s,1}$ for Design Condition Loading Cases 1 and 3, the procedure of paragraph 4.18.8.6 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

Summary Results for STEP 12, Elastic Plastic Iteration Results per paragraph 4.18.8.6		
Design Condition Loading Case	1	3
S^*_s, psi	27,700	27,700
$fact_s$	0.807	0.820
E^*_s, psi	20.789E6	21.1E6
k_s, lb	1.88E6	1.91E6
λ_s	0.109E8	0.111E8
F	1.827	1.842
ϕ	2.453	2.472
Q_1	-0.1196	-0.1202
Q_{Z1}	1.231	1.230
Q_{Z2}	0.640	0.636
U	1.279	1.273
P_W, psi	295.5	294.1
P_{rim}, psi	99.099	129.78
P_e, psi	-1,039.2	-1,210.2
Q_2, lb	-13,592	-14,599
Q_3	0.105	0.087
F_m	0.208	0.201
$\lvert\sigma\rvert, psi$	23,358	26,304

The final calculated tubesheet bending stresses of 23,358 psi (Loading Case 1) and 26,304 psi (Loading Case 3) are less than the allowable tubesheet bending stress of 27,675 psi. As such, this geometry meets the requirements of paragraph 4.18.8.6. The intermediate results for the elastic-plastic calculation are shown above.

4.18.7 Example E4.18.7 – Fixed Tubesheet Exchanger, Configuration a

A fixed tubesheet heat exchanger with the tubesheet construction in accordance with configuration a as shown in VIII-1, Figure UHX-13.1.

- For the Design Condition, the shell side design pressure is 325 psig at 400°F, and the tube side design pressure is 200 psig at 300°F.
- There is one operating condition. For Operating Condition 1, the shell side design pressure is 325 psig at 400°F, the tube side design pressure is 200 psig at 300°F, the shell mean metal temperature is 151°F, and the tube mean metal temperature is 113°F. For this example, the operating pressures and operating metal temperatures are assumed to be the same as the design values.
- The tube material is SA-249, Type 304L (S30403). The tubes are 1 in. outside diameter and are 0.049 in. thick.
- The tubesheet material is SA-240, Type 304L (S30403). The tubesheet outside diameter is 43.125 in. There are 955 tube holes on a 1.25 in. triangular pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 20.125 in. The distance between the outer tubesheet faces is 240 in. The option for the effect of differential radial expansion is not required. There is no corrosion allowance on the tubesheet.
- The shell material is SA-240, Type 304L (S30403). The shell inside diameter is 42 in. and the thickness is 0.5625 in. There is no corrosion allowance on the shell and no expansion joint in the shell. The efficiency of shell circumferential welded joint (Category B) is 0.85.
- The channel material is SA-516, Grade 70 (K02700). The inside diameter of the channel is 42.125 in. and the channel is 0.375 in. thick. There is no corrosion allowance on the channel.

Data Summary - Tubesheet

$$\text{Tube Layout: Triangular}$$
$$h = 1.375\,in.$$
$$h_g = 0\,in.$$
$$c_t = 0\,in.$$
$$A = 43.125\,in.$$
$$r_o = 20.125\,in.$$
$$A_L = 0.0\,in.^2$$
$$N_t = 955$$
$$L_t = 240\,in.$$
$$p = 1.25\,in.$$
$$T = 400^{\circ}F$$
$$T_a = 70^{\circ}F$$
$$S = 15{,}800\,psi \text{ at } T \text{ from Table 5A of Section II, Part D}$$
$$S_y = 17{,}500\,psi \text{ at } T$$
$$S_{PS} = 47{,}400\,psi \text{ at } T$$
$$E = 26{,}400{,}000\,psi \text{ at } T$$
$$\nu = 0.3$$

Data Summary – Tubes

$$P_{td,\max} = 200\ psig$$
$$P_{td,\min} = 0\ psig$$
$$P_{to1} = 200\ psig$$
$$\ell_{tx} = 1.25$$
$$k = 1$$
$$\ell = 48\ in.$$
$$t_t = 0.049\ in.$$
$$d_t = 1\ in.$$
$$T_t = 300^\circ F$$
$$T_{t,m} = 113^\circ F$$
$S_t = 16,700\ psi$ at T_t from Table 5A of Section II, Part D

$S_{y,t} = 19,200\ psi$ at T_t from Y-1 of Section II, Part D

$S_{tT} = 15,800\ psi$ at T from Table 5A of Section II, Part D

$\alpha_{t,m} = 8.65E-06\ in./in./^\circ F$ at $T_{t,m}$

$E_t = 27,000,000\ psi$ at T_t from TM-1 of Section II, Part D

$E_{tT} = 26,400,000\ psi$ at T from TM-1 of Section II, Part D

$$\nu_t = 0.3$$

Data Summary - Shell

Since there is no expansion joint in the shell, $J = 1$ and D_j and K_j need not be defined.

$P_{sd,\max} = 325\ psig$

$P_{sd,\min} = 0\ psig$

$P_{so1} = 325\ psig$

$t_s = 0.5625\ in.$

$D_s = 42\ in.$

$T_s = 400^\circ F$

$T_{s,m} = 151^\circ F$

$S_s = 15,800\ psi$ at T_s from Table 5A of Section II, Part D

$E_{s,w} = 0.85$

$S_{PS,s} = 47,400\ psi$ at T_s

$S_{y,s} = 17,500\ psi$ at T_s

$E_s = 26,400,000\ psi$ from TM-1 of Section II, Part D

$\alpha_{s,m} = 8.802E-06\ in./in./^\circ F$ at $T_{s,m}$

$\nu = 0.3$

Data Summary – Channel

$t_c = 0.375\ in.$

$D_c = 42.125\ in.$

$T_c = 300^\circ F$

$S_c = 22,400\ psi$ at T_c from Table 5A of Section II, Part D

$S_{y,c} = 33,600\ psi$ at T_c

$S_{PS,c} = 67,200\ psi$ at T_c

$E_c = 28,300,000\ psi$ at T_c from TM-1 of Section II, Part D

$\nu_c = 0.3$

Calculation Procedure

The calculation procedure for a Fixed Tubesheet heat exchanger is given in paragraph 4.18.8. The following results are for the design and operating loading cases required to be analyzed (see paragraph 4.18.8.3). This example illustrates the calculation of both the elastic and elastic-plastic solutions.

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$L = 237.25\,in.$$
$$D_o = 41.25\,in.$$
$$a_o = 20.625\,in.$$
$$\rho = 0.9091$$
$$d^* = 0.9111\,in.$$
$$\mu = 0.2$$
$$p^* = 1.25\,in.$$
$$\mu^* = 0.2711$$
$$\rho_s = 1.0182$$
$$\rho_c = 1.0212$$
$$x_s = 0.4388$$
$$x_t = 0.5434$$

b) STEP 2 – Calculate the shell axial stiffness, K_s, tube axial stiffness, K_t, stiffness factors, $K_{s,t}$ and J.

$$K_s = 8,369,456\ lb/in.$$
$$K_t = 16,660\ lb/in.$$
$$K_{st} = 0.526$$
$$J = 1$$

Calculate the shell coefficients $\beta_s,\ k_s,\ \lambda_s,\ \text{and}\ \delta_s$

$$\beta_s = 0.3715\,in.^{-1}$$
$$k_s = 319,712\ lb$$
$$\lambda_s = 50,867,972\ psi$$
$$\delta_s = 25.24E-6\ in.^3/lb$$

Calculate the channel coefficients $\beta_c,\ k_c,\ \lambda_c,\ \text{and}\ \delta_c$.

$$\beta_c = 0.4554\ in.^{-1}$$
$$k_c = 124,461\ lb$$
$$\lambda_c = 22,049,112\ psi$$
$$\delta_c = 35.532E-6\ in.^3/lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from paragraph 4.18.6.4.b and calculate E^*.

$$h/p = 1.10000$$
$$E^*/E = 0.274948$$
$$\nu^* = 0.340361$$
$$E^* = 7.26E6\ psi$$

Calculate, X_a, and the parameters from Table 4.18.3.

$$X_a = 7.0155$$
$$Z_d = 0.00433$$
$$Z_v = 0.02064$$
$$Z_m = 0.2067$$
$$Z_a = 295.63$$
$$Z_w = 0.02064$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.0455$$
$$F = 6.7322$$

Calculate Φ, Q_1, Q_{z1}, Q_{z2} *and* U.

$$\Phi = 9.0236$$
$$Q_1 = -0.058647$$
$$Q_{Z1} = 3.7782$$
$$Q_{Z2} = 10.3124$$
$$U = 20.6248$$

e) STEP 5 – Calculate γ, ω_s, ω_s^*, ω_c, ω_c^*, and γ_b. The following results are those for the corroded condition, elastic solution

Summary Table for Step 5 – Design Condition			
Loading Case	P_s (psi)	P_t (psi)	γ
1	0	200	0
2	325	0	0
3	325	200	0

Summary Table for Step 5 – Operating Condition 1			
Loading Case	P_s (psi)	P_t (psi)	γ
1	0	200	-0.0809
2	325	0	-0.0809
3	325	200	-0.0809
4	0	0	-0.0809

$$\omega_s = 4.6123\ in.^2 \qquad \omega_c = 3.344\ in.^2$$
$$\omega^*_s = -4.5413\ in.^2 \qquad \omega^*_c = -2.6027\ in.^2$$
$$\gamma_b = 0$$

f) STEP 6 – For each loading case, calculate P_s', P_t', P_γ, P_ω, P_W, P_{rim} and effective pressure P_e.

Summary Table for STEP 6 – Design Condition							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	543.7	0	0	0	-25.2	-97
2	613.7	0	0	0	0	71.6	116.8
3	613.7	543.7	0	0	0	46.3	19.8

Summary Table for STEP 6 – Operating Condition 1							
Loading Case	P_s' (psi)	P_t' (psi)	P_γ (psi)	P_ω (psi)	P_W (psi)	P_{rim} (psi)	P_e (psi)
1	0	543.7	-963	0	0	-25.2	-261.1
2	613.7	0	-963	0	0	71.6	-47.3
3	613.7	543.7	-963	0	0	46.3	-144.3
4	0	0	-963	0	0	0	-164.1

g) STEP 7 – Elastic Iteration, calculate Q_2 and Q_3, the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition								
Loading Case	Q_2 (lbs)	Q_3	F_m	h'_g (in)	$h-h'_g$ (in)	$\lvert\sigma_{elastic}\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	181.7	-0.0675	0.03373	0	1.375	16,286	23,700	---
2	-515.1	-0.0794	0.03969	0	1.375	23,084	23,700	---
3	-333.4	-0.138	0.06886	0	1.375	6,798	23,700	---

Summary Table for STEP 7 – Operating Condition 1								
Loading Case	Q_2 (lbs)	Q_3	F_m	h'_g (in)	$h-h'_g$ (in)	$\lvert\sigma_{elastic}\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	181.7	-0.0619	0.03096	0	1.375	40,253	---	47,400
2	-515.1	-0.00749	0.03210	0	1.375	7,566	---	47,400
3	-333.4	-0.0478	0.02389	0	1.375	17,169	---	47,400
4	0	-0.0587	0.02932	0	1.375	23,967	---	47,400

For Design Loading Cases 1-3 $\lvert\sigma_{elastic}\rvert < 1.5S$, and for Operating Cases 1-4 $\lvert\sigma_{elastic}\rvert < S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – For each loading case, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If $\lvert P_e\rvert \le \dfrac{1.6S\mu h}{a_o}$, then the shear stress is not required to be calculated.

<u>Design Condition 1, Load Case 1:</u>

$$\left\{\lvert P_e\rvert = 97 \ psi\right\} \le \left\{\frac{1.6S\mu h}{a_o} = 337 \ psi\right\}$$

Therefore, shear stress is not required to be calculated for design condition, case 1. Similarly, the shear stress is not required to be calculated for design conditions, case 2 and case 3, and operating condition cases 1-4.

Summary Table for STEP 8 – Design Condition		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	Not required	12,640
2	Not required	12,640
3	Not required	12,640

Summary Table for STEP 8 – Operating Condition 1		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	Not required	12,640
2	Not required	12,640
3	Not required	12,640
4	Not required	12,640

For all Loading Cases the shear stress criterion is not required.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.3367\, in.$$

$$F_t = 142.57$$

$$C_t = 166.6$$

Summary Table for STEP 9 – Design Condition				
Loading Case	F_t, min	$\sigma_{t,1}$ (psi)	F_t, max	$\sigma_{t,2}$ (psi)
1	-0.270	-1,289.5	3.558	2,259.3
2	-0.243	1,634.3	3.260	-2,276.6
3	-0.191	360.4	2.123	-78.2

Summary Table for STEP 9 – Operating Condition 1				
Loading Case	F_t, min	$\sigma_{t,1}$ (psi)	F_t, max	$\sigma_{t,2}$ (psi)
1	-0.285	-1,751.2	3.696	8,187.4
2	-0.490	1,141.5	5.057	3,651.5
3	-0.329	-129.1	4.050	5,910.8
4	-0.295	-462.4	3.778	5,928.1

Summary Table for STEP 9 – Design Condition (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\left\|\sigma_{t,\min}\right\|$ (psi)	F_s	S_{tb} (psi)
1	2,259	16,700	1,289.5	1.471	7,468
2	-2,277	16,700	2,276.6	1.620	6,781
3	360.4	16,700	78.2	2	5,493

Summary Table for STEP 9 – Operating Condition 1 (continued)					
Loading Case	$\sigma_{t,\max}$ (psi)	S_t (psi)	$\left\|\sigma_{t,\min}\right\|$ (psi)	F_s	S_{tb} (psi)
1	8,187	33,400	1,751.2	1.402	7,836.5
2	3,652	33,400	---	---	---
3	5,911	33,400	129	1.25	8788
4	5,928	33,400	462.4	1.361	8,072

For all Loading Cases $\left|\sigma_{t,\max}\right| < S_t$. The axial tension stress criterion for the tube is satisfied.

For all Loading Cases $\left|\sigma_{t,\min}\right| < S_{tb}$. The buckling criterion for the tube is satisfied.

j) STEP 10 – For each loading case, calculate the axial membrane stress in each shell section and determine the maximum allowable longitudinal compressive stress.

Summary Table for STEP 10 – Main Shell – Design Condition				
Loading Case	$\sigma_{s,m}$ (psi)	$S_s E_{s,w}$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	1,830.6	13,430	---	---
2	2,287.2	13,430	---	---
3	4,117.8	13,430	---	---

Summary Table for STEP 10 – Main Shell – Operating Condition 1				
Loading Case	$\sigma_{s,m}$ (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)	$S_{s,b}$ (psi)
1	-1,085.9	---	47,400	6,730
2	-629.3	---	47,400	6,730
3	1,201.3	---	47,400	---
4	-2,916.5	---	47,400	6,730

k) STEP 11 – For each loading case, calculate the stresses in the shell and/or channel when integral with the tubesheet.

Summary Table for STEP 11, Shell Results – Design Condition					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	1,830.6	-12,184	14,015	23,700	---
2	2,287.2	27,748	30,036	23,700	---
3	4,117.8	15,564	19,682	23,700	---

Summary Table for STEP 11, Shell Results – Operating Condition 1					
Loading Case	$\sigma_{s,m}$ (psi)	$\sigma_{s,b}$ (psi)	σ_s (psi)	$1.5S_s$ (psi)	$S_{PS,s}$ (psi)
1	-1,085.9	-38,418	39,504	---	47,400
2	-629.3	1,514	2,144	---	47,400
3	1,201.3	-10,670	11,871	---	47,400
4	-2,916.5	-26,234	29,150	---	47,400

For Design Loading Case 2 $|\sigma_s| > 1.5S_s$, and for Operating Cases 1-4 $|\sigma_s| < S_{PS,s}$. The stress criterion for the shell is not satisfied.

Summary Table for STEP 11, Channel Results – Design Condition					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	$S_{PS,c}$ (psi)
1	5,567	28,346	33,913	33,600	---
2	0	-8,492	8,492	33,600	---
3	5,567	19,854	25,420	33,600	---

Summary Table for STEP 11, Channel Results – Operating Condition 1					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	$S_{PS,c}$ (psi)
1	5,567	52,380	57,946	---	67,200
2	0	15,541	15,541	---	67,200
3	5,567	43,887	49,453	---	67,200
4	0	24,033	24,033	---	67,200

For Design Loading Case 1 $|\sigma_c| > 1.5S_c$, and for Operating Cases 1-4 $|\sigma_c| < S_{PS,c}$. The stress criterion for the channel is not satisfied.

l) STEP 12 – The design shall be reconsidered by using one or a combination of the following options.
   - Option 1 – Increase the tubesheet thickness and return to STEP 1.
   - Option 2 – Increase the integral shell and/or channel thickness and return to STEP 1
   - Option 3 – Perform the elastic-plastic calculation procedures as defined in paragraph 4.18.8.6.

Since the total axial stress in the shell σ_s is between $1.5S_s$ and $S_{PS,s}$ for Design Condition Loading Case 2, the procedure of paragraph 4.18.8.6 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the shell occurs.

Since the total axial stress in the channel σ_c is between $1.5S_c$ and $S_{PS,c}$ for Design Condition Loading Case 1, the procedure of paragraph 4.18.8.6 may be performed to determine if the tubesheet stresses are acceptable when the plasticity of the channel occurs. The results are not presented for Design Condition Loading Case 1, because the calculated values of $fact_s$ and $fact_c$ do not exceed 1.0 for this case and further plasticity calculations are not required.

Summary Results for STEP 12, Elastic Plastic Iteration Results per paragraph 4.18.8.6	
Design Condition Loading Case	1
$S*_s, psi$	17,500
$S*_c, psi$	33,600
$fact_s$	0.766
$fact_c$	1.000
$E*_s, psi$	20.2E6
$E*_c, psi$	28.3E6
k_s, lb	24.48E4
λ_s	0.390E+08
F	5.65
ϕ	7.572
Q_1	-0.0538
Q_{Z1}	3.898
Q_{Z2}	11.518
U	23.037
P_W, psi	0
P_{rim}, psi	79.9
P_e, psi	115.4
Q_2, lb	-575
Q_3	-0.0773
F_m	0.0386
$\lvert\sigma\rvert, psi$	22,204

The final calculated tubesheet bending stress is 22,204 psi, which is less than the Code allowable of 23,700 psi. As such, this geometry meets the requirement of paragraph 4.18.8.6. The intermediate results for the elastic-plastic iteration are shown above.

4.18.8 Example E4.18.8 – Stationary Tubesheet Gasketed With Shell and Channel; Floating Tubesheet Gasketed, Not Extended as a Flange

A floating tubesheet exchanger with an immersed floating head is to be designed with configuration a as shown in Figure 4.18.10. The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in Figure 4.18.11. The floating tubesheet is not extended as a flange in accordance with configuration C as shown in Figure 4.18.12. There is no allowance for corrosion.

Data Summary - Data Common to Both Tubesheets

$$P_{sd,\max} = 250\ psig$$
$$P_{sd,\min} = 0\ psig$$
$$P_{td,\max} = 150\ psig$$
$$P_{td,\min} = 0\ psig$$

Data Summary – Tubesheet

The tube layout pattern is triangular with one centerline pass lane

$$N_t = 466$$
$$p = 1\,in.$$
$$r_o = 12.5\,in.$$
$$\rho = 0.8$$
$$U_{L1} = 2.5\,in.$$
$$c_t = 0\,in.$$
$$\nu = 0.31$$
$$E = 27.0E10^6\ psi$$
$$S = 19,000\ psi$$

Data Summary – Tubes

$$d_t = 0.75\,in.$$
$$t_t = 0.083\,in.$$
$$L_t = 256\,in.$$
$$\ell_t = 15.375\,in.$$
$$\nu_t = 0.31$$
$$E_t = 27.0\times 10_6\ psi$$
$$S_t = 13,350\ psi$$
$$S_{y,t} = 20,550\ psi$$

Data Summary – Stationary Tubesheet Data Summary

$$W^* = 211{,}426\,lb \text{ from Table 4.18.6}$$
$$A = 33.071\,in.$$
$$h = 1.75\,in.$$
$$G_s = 29.375\,in.$$
$$a_s = 14.6875\,in.$$
$$G_c = 29.375\,in.$$
$$a_c = 14.6875\,in.$$
$$C = 31.417\,in.$$
$$h_g = 0.197\,in.$$

Data Summary – Floating Tubesheet Data Summary

$$W^* = 26{,}225\,lb \text{ from Table 4.18.6}$$
$$A = 26.89\,in.$$
$$h = 1.75\,in.$$
$$G_1 = 26.496\,in.$$
$$G_c = 26.496\,in.$$
$$a_c = 13.248\,in.$$
$$a_s = 13.248\,in.$$
$$C = 27.992\,in.$$
$$h_g = 0\,in.$$

Calculation Procedure – Stationary Tubesheet

The following results are for the 3 load cases required to be analyzed (see paragraph 4.18.9.3).

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$D_o = 25.75\,in.$$
$$L_{L1} = 25.8\,in.$$
$$A_L = 64.4\,in.^2$$
$$d^* = 0.6567\,in.$$
$$\mu = 0.250$$
$$\mu^* = 0.385$$
$$h_g' = 0.197\,in.$$
$$a_o = 12.875\,in.$$
$$\rho_s = 1.14$$
$$\rho_c = 1.14$$
$$x_s = 0.605$$
$$x_t = 0.760$$

b) STEP 2 – Calculate the shell coefficients $\beta_s,\, k_s,\, \lambda_s,$ and δ_s and the channel coefficients $\beta_c,\, k_c,\, \lambda_c,$ and δ_c.

$$\beta_s = 0\,in.^{-1}$$
$$k_s = 0\,lb$$
$$\lambda_s = 0\ psi$$
$$\delta_s = 0\,in.^3/lb$$
$$\beta_c = 0\,in.^{-1}$$
$$k_c = 0\,lb$$
$$\lambda_c = 0\ psi$$
$$\delta_c = 0\,in.^3/lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from paragraph 4.18.6.4.b and calculate E^*.

$$h/p = 1.75$$
$$E^*/E = 0.404$$
$$\nu^* = 0.308$$
$$E^* = 10.91E6\ psi$$

Calculate, X_a, and the parameters from Table 4.18.3.

$$X_a = 3.61$$
$$Z_d = 0.0328$$
$$Z_v = 0.0787$$
$$Z_m = 0.421$$
$$Z_w = 0.0787$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.28$$
$$F = 0.429$$

Calculate Φ and Q_1.

$$\Phi = 0.561$$
$$Q_1 = 0.0782$$

e) STEP 5 – Calculate ω_s, ω^*_s, ω_c, ω^*_c, γ_b, P^*_s and P^*_c.

$$\omega_s = 0\,in.^2$$
$$\omega^*_s = 1.758\,in.^2$$
$$\omega_c = 0\ in.^2$$
$$\omega^*_c = 1.758\ in.^2$$
$$\gamma_b = 0$$
$$P^*_s = 0\,psi$$
$$P^*_c = 0\,psi$$

f) STEP 6 – Calculate

Summary Table for STEP 6	
Loading Case	P_e (psi)
1	-150
2	250
3	100

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-213	0.0953	0.102	16,400	28,500
2	356	0.0953	0.102	27,400	28,500
3	142	0.0953	0.102	10,900	28,500

For all loading cases $\lvert\sigma\rvert \le 1.5S$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – For each loading case, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If $\lvert P_e\rvert \le \dfrac{1.6S\mu h}{a_o}$, then the shear stress is not required to be calculated.

<u>Load Case 1:</u>

$$\left\{\lvert P_e\rvert = 150 \ psi\right\} \le \left\{\frac{1.6S\mu h}{a_o} = 1033 \ psi\right\}$$

Therefore, shear stress is not required to be calculated for design load case 1. Similarly, the shear stress is not required to be calculated for load case 2 and case 3.

Summary Table for STEP 8		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	Not required	15200
2	Not required	15200
3	Not required	15200

For all loading cases, the shear stress criterion is not required.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.238\, in.$$
$$F_t = 64.7$$
$$C_t = 161$$

Summary Table for STEP 9					
Loading Case	F_s	$\sigma_{t,1}$ (psi)	$\sigma_{t,2}$ (psi)	S_{tb} (psi)	S_t (psi)
1	1.54	-1,716	2,564	10,700	13350
2	1.54	2,609	-4,524	10,700	13350
3	1.54	894	-1,959	10,700	13350

Determine $\sigma_{t,\max} = \max\left[|\sigma_{t,1}|,|\sigma_{t,2}|\right]$ and $\sigma_{t,\min} = \min\left[\sigma_{t,1},\sigma_{t,2}\right]$

For all loading cases $\sigma_{t,\max} < S_t$. The axial tension stress criterion for the tube is satisfied.

For all loading cases $|\sigma_{t,\min}| < S_{tb}$. The buckling criterion for the tube is satisfied.

Calculation Procedure – Floating Tubesheet

The following results are for the 3 load cases required to be analyzed (see paragraph 4.18.9.3).

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$D_o = 25.75\,in.$$
$$L_{L1} = 25.8\,in.$$
$$A_L = 64.4\,in.^2$$
$$d^* = 0.6567\,in.$$
$$\mu = 0.250$$
$$\mu* = 0.385$$
$$h'_g = 0\,in.$$
$$a_o = 12.875\,in.$$
$$\rho_s = 1.03$$
$$\rho_c = 1.03$$
$$x_s = 0.605$$
$$x_t = 0.760$$

b) STEP 2 – Calculate the shell coefficients $\beta_s, k_s, \lambda_s,$ and δ_s and the channel coefficients $\beta_c, k_c, \lambda_c,$ and δ_c.

$$\beta_s = 0\,in.^{-1}$$
$$k_s = 0\,lb$$
$$\lambda_s = 0\,psi$$
$$\delta_s = 0\,in.^3/lb$$
$$\beta_c = 0\,in.^{-1}$$
$$k_c = 0\,lb$$
$$\lambda_c = 0\,psi$$
$$\delta_c = 0\,in.^3/lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν from paragraph 4.18.6.4.b and calculate E^*.

$$h/p = 1.75$$
$$E^*/E = 0.404$$
$$\nu^* = 0.308$$
$$E^* = 10.91E6\ psi$$

Calculate, X_a, and the parameters from Table 4.18.3.

$$X_a = 3.61$$
$$Z_d = 0.0328$$
$$Z_v = 0.0787$$
$$Z_m = 0.421$$
$$Z_w = 0.0787$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.04$$
$$F = 0.0742$$

Calculate Φ and Q_1.

$$\Phi = 0.0971$$
$$Q_1 = 0.0205$$

e) STEP 5 – Calculate ω_s, ω^*_s, ω_c, ω^*_c, γ_b, P^*_s and P^*_c.

$$\omega_s = 0\, in.^2 \qquad \omega_c = 0\, in.^2$$
$$\omega^*_s = 7.06\times10^{-2}\, in.^2 \qquad \omega^*_c = 7.06\times10^{-2}\, in.^2$$
$$\gamma_b = 0$$
$$P^*_s = 0\, psi$$
$$P^*_c = 0\, psi$$

f) STEP 6 – Calculate P_e.

Summary Table for STEP 6	
Loading Case	P_e (psi)
1	-150
2	250
3	100

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-10.2	0.0213	0.0751	9,500	28,500
2	16.9	0.0213	0.0751	15,800	28,500
3	6.78	0.0213	0.0751	6,330	28,500

For all loading cases $\lvert\sigma\rvert < 1.5S$. The bending stress criterion for the tubesheet is satisfied.

The calculation procedure is complete and the unit geometry is acceptable for the given design conditions.

4.18.9 Example E4.18.9 – Stationary Tubesheet Gasketed With Shell and Channel; Floating Tubesheet Integral

A floating tubesheet exchanger with an externally sealed (packed) floating head is to be designed in accordance with configuration b as shown in Figure 4.18.10. The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in Figure 4.18.11. The floating tubesheet is integral with the head in accordance with configuration A as shown in Figure 4.18.12. There is no allowance for corrosion.

Data Summary - Data Common to Both Tubesheets

$$P_{sd,\max} = 150\ psig$$
$$P_{sd,\min} = 0\ psig$$
$$P_{td,\max} = 30\ psig$$
$$P_{td,\min} = 0\ psig$$

Data Summary – Tubesheet

The tube layout pattern is triangular with no pass lanes

$$N_t = 1189$$
$$p = 1.25\ in.$$
$$r_o = 22.605\ in.$$
$$\rho = 0.958$$
$$h_g = 0\ in.$$
$$c_t = 0\ in.$$
$$\nu = 0.32$$
$$E = 14.8E10^6\ psi$$
$$S = 11,300\ psi$$
$$S_y = 31,600\ psi$$
$$S_{PS} = 33,900\ psi \text{ (MYS/UTS } > 0.7\text{; therefore use } 3S\text{)}$$

Data Summary – Tubes

$$d_t = 1.0\ in.$$
$$t_t = 0.049\ in.$$
$$L_t = 144\ in.$$
$$\ell_t = 16\ in.$$
$$\nu_t = 0.32$$
$$E_t = 14.8E10^6\ psi$$
$$S_t = 11,300\ psi$$
$$S_{y,t} = 31,600\ psi$$

Data Summary – Stationary Tubesheet

$$W^* = 288,910\,lb \quad \text{from Table 4.18.6}$$

$$A = 51\,in.$$

$$h = 1.375\,in.$$

$$G_s = 49.71\,in.$$

$$a_s = 24.9\,in.$$

$$G_c = 49.616\,in.$$

$$a_c = 24.8\,in.$$

$$C = 49.5\,in.$$

Data Summary – Floating Tubesheet

$$P_{so1} = 150\,psig$$

$$P_{to1} = 30\,psig$$

$$W^* = 0\,lb \text{ from Table 4.18.6}$$

$$T' = 200^{\circ}F$$

$$T'_c = 235^{\circ}F$$

$$A = 47.625\,in.$$

$$h = 1.375\,in.$$

$$\alpha' = 4.8E-6\ \ in./in./^{\circ}F$$

$$D_c = 47\,in.$$

$$a_c = 23.5\,in.$$

$$a_s = 23.5\,in.$$

$$t_c = 0.3125\,in.$$

$$\nu_c = 0.32$$

$$E_c = 14.8E6\,psi$$

$$S_c = 11,300\,psi$$

$$S_{y,c} = 31,600\,psi$$

$$S_{PS,c} = 33,900\,psi \text{ (MYS/UTS } > \text{ 0.7; therefore use } 3S\text{)}$$

$$\alpha'_c = 4.8E-6\ \ in./in./^{\circ}F$$

Calculation Procedure – Stationary Tubesheet

The following results are for the 3 load cases required to be analyzed (see paragraph 4.18.9.3).

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$D_o = 46.21\, in.$$
$$A_L = 0\, in.^2$$
$$\mu = 0.200$$
$$\mu^* = 0.275$$
$$h'_g = 0\, in.$$
$$a_o = 23.105\, in.$$
$$\rho_s = 1.08$$
$$\rho_c = 1.07$$
$$x_s = 0.443$$
$$x_t = 0.547$$

b) STEP 2 – Calculate the shell coefficients β_s, k_s, λ_s, and δ_s and the channel coefficients β_c, k_c, λ_c, and δ_c.

$$\beta_s = 0\, in.^{-1}$$
$$k_s = 0\, lb$$
$$\lambda_s = 0\, psi$$
$$\delta_s = 0\, in.^3 / lb$$
$$\beta_c = 0\, in.^{-1}$$
$$k_c = 0\, lb$$
$$\lambda_c = 0\, psi$$
$$\delta_c = 0\, in.^3 / lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from paragraph 4.18.6.4.b and calculate E^*.

$$h/p = 1.10$$
$$E^*/E = 0.280$$
$$\nu^* = 0.337$$
$$E^* = 4.149E6\ psi$$

Calculate, X_a, and the parameters from Table 4.18.3.

$$X_a = 8.84$$
$$Z_a = 3161.6$$
$$Z_d = 0.00214$$
$$Z_v = 0.0130$$
$$Z_m = 0.163$$
$$Z_w = 0.013$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.10$$
$$F = 0.233$$

Calculate Φ and Q_1.

$$\Phi = 0.312$$
$$Q_1 = 0.0682$$

e) STEP 5 – Calculate ω_s, ω^*_s, ω_c, ω^*_c, γ_b, P^*_s and P^*_c.

$$\omega_s = 0\,in.^2 \qquad \omega_c = 0\,in.^2$$
$$\omega^*_s = 1.59\,in.^2 \qquad \omega^*_c = 0.961\,in.^2$$
$$\gamma_b = -2.03\times10^{-3}$$
$$P^*_s = 0\ psi$$
$$P^*_c = 0\ psi$$

f) STEP 6 – Calculate P_e.

Summary Table for STEP 6	
Loading Case	P_e (psi)
1	-30
2	-23.6
3	-53.6

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-116	0.0828	0.0594	11,000	16950
2	138	0.0463	0.0442	6,420	16950
3	110	0.0605	0.0499	16,500	16950

For all loading cases $|\sigma| \le 1.5S$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – For each loading case, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If $|P_e| \le \dfrac{1.6S\mu h}{a_o}$, then the shear stress is not required to be calculated.

Load Case 1:

$$\{|P_e| = 30\ psi\} \le \left\{\frac{1.6S\mu h}{a_o} = 215\ psi\right\}$$

Therefore, shear stress is not required to be calculated for design load case 1. Similarly, the shear stress is not required to be calculated for load case 2 and case 3.

Summary Table for STEP 8		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	Not required	9040
2	Not required	9040
3	Not required	9040

For all loading cases the shear stress criterion is not required.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.3367\,in.$$

$$F_t = 47.5235$$

$$C_t = 96.1507$$

Summary Table for STEP 9

Loading Case	F_s	$\sigma_{t,1}$ (psi)	$\sigma_{t,2}$ (psi)	S_{tb} (psi)	S_t (psi)
1	1.25	-525	2,685	11300	11300
2	---	424	2,546	11300	11300
3	1.25	-72	5,104	11300	11300

Determine $\sigma_{t,\max} = \max\left[|\sigma_{t,1}|, |\sigma_{t,2}|\right]$ and $\sigma_{t,\min} = \min\left[\sigma_{t,1}, \sigma_{t,2}\right]$

For all loading cases $\sigma_{t,\max} < S_t$. The axial tension stress criterion for the tube is satisfied.

For all loading cases $|\sigma_{t,\min}| < S_{tb}$. The buckling criterion for the tube is satisfied.

Calculation Procedure – Floating Tubesheet

The following results are for the design and operating loading cases required to be analyzed (see paragraph 4.18.9.3).

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$D_o = 46.2\,in.$$

$$A_L = 0\,in.^2$$

$$\mu = 0.200$$

$$\mu^* = 0.275$$

$$h'_g = 0\,in.$$

$$a_o = 23.1\,in.$$

$$\rho_s = 1.02$$

$$\rho_c = 1.02$$

$$x_s = 0.443$$

$$x_t = 0.547$$

b) STEP 2 – Calculate the shell coefficients β_s, k_s, λ_s, and δ_s and the channel coefficients β_c, k_c, λ_c, and δ_c.

$\beta_s = 0\, in.^{-1}$
$k_s = 0\, lb$
$\lambda_s = 0\, psi$
$\delta_s = 0\, in.^3 / lb$

$\beta_c = 0.471\, in.^{-1}$
$k_c = 39{,}500\, lb$
$\lambda_c = 7.96 \times 10^6\ psi$
$\delta_c = 1.00 \times 10^{-4}\, in.^3 / lb$

c) STEP 3 – Calculate h/p. Determine $E*/E$ and ν from paragraph 4.18.6.4.b and calculate $E*$.

$h/p = 1.10$
$E*/E = 0.280$
$\nu* = 0.337$
$E* = 4.149E6\ psi$

Calculate, X_a, and the parameters from Table 4.18.3.

$X_a = 8.84$ $\qquad Z_v = 0.0130$
$Z_a = 3161.6$ $\qquad Z_m = 0.163$
$Z_d = 0.00214$ $\qquad Z_w = 0.013$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$K = 1.03$
$F = 1.34$

Calculate Φ and Q_1.

$\Phi = 1.80$
$Q_1 = -4.83 \times 10^{-3}$

e) STEP 5 – Calculate $\omega_s,\ \omega*_s,\ \omega_c,\ \omega*_c,\ \gamma_b,\ T_r,\ T*_c,\ P*_s$ and $P*_c$.

$\omega_s = 0\, in.^2$ $\qquad \omega_c = 3.13\, in.^2$
$\omega*_s = 7.87 \times 10^{-2}\, in.^2$ $\qquad \omega*_c = -3.05\, in.^2$
$\gamma_b = 0$
$T_r = 217.5^\circ F$
$T*_c = 226.25^\circ F$
$P*_s = 0\ psi$
$P*_c = 8.27\ psi$

f) STEP 6 – Calculate P_e.

Summary Table for STEP 6 – Design Condition	
Loading Case	P_e (psi)
1	-30
2	-5.17
3	-35.2

Summary Table for STEP 6 – Operating Condition	
Loading Case	P_e (psi)
1	-30
2	-5.17
3	-35.2
4	0

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7 – Design Condition						
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	70.8	-0.0137	0.0228	4,210	16950	---
2	9.12	-0.0114	0.0235	748	16950	---
3	79.9	-0.0133	0.0229	4,950	16950	---

Summary Table for STEP 7 – Operating Condition						
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)	S_{PS} (psi)
1	90.8	-0.0162	0.0220	4,070	---	33900
2	29.1	-0.0259	0.0193	615	---	33900
3	99.9	-0.0155	0.0222	4,810	---	33900
4	20.0	---	---	231	---	33900

For Design Loading Cases 1-3 $\lvert\sigma\rvert \le 1.5S$ and for Operating Cases 1-4 $\lvert\sigma\rvert \le S_{PS}$. The bending stress criterion for the tubesheet is satisfied.

h) STEPS 8 and 9 – For configuration A, skip STEPS 8 and 9 and proceed to STEP 10.

i) STEP 10 – Calculate the stresses in the shell and/or integral channel with the tubesheet.

Summary Table for STEP 10 – Design Condition					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	S_{PS_c} (psi)
1	1,110	9,750	10,900	16950	---
2	0	1,120	1,120	16950	---
3	1,110	10,900	12,000	16950	---

Summary Table for STEP 10 – Operating Condition					
Loading Case	$\sigma_{c,m}$ (psi)	$\sigma_{c,b}$ (psi)	σ_c (psi)	$1.5S_c$ (psi)	S_{PS_c} (psi)
1	1,110	10,600	11,800	---	33900
2	0	2,010	2,010	---	33900
3	1,110	11,800	12,900	---	33900
4	0	890	890	---	33900

For Design Loading Cases 1-3 $\sigma_c \le 1.5S_c$ and for Operating Cases 1-4 $\sigma_c \le S_{PS,c}$. The stress criterion for the shell and/or integral channel with tubesheet is satisfied.

The calculation procedure is complete and the unit geometry is acceptable for the given design conditions.

4.18.10 Example E4.18.10 – Stationary Tubesheet Gasketed With Shell and Channel; Floating Tubesheet Internally Sealed

A floating tubesheet exchanger with an internally sealed floating head is to be designed in accordance with sketch (c) as shown in Figure 4.18.10. The stationary tubesheet is gasketed with the shell and channel in accordance with configuration d as shown in Figure 4.18.11. The floating tubesheet is packed and sealed on its edge in accordance with configuration D as shown in Figure 4.18.12. There is no allowance for corrosion.

Data Summary - Data Common to Both Tubesheets

$$P_{sd,\max} = 150\ psig$$
$$P_{sd,\min} = 0\ psig$$
$$P_{td,\max} = 175\ psig$$
$$P_{td,\min} = 0\ psig$$

Data Summary – Tubesheets

The tube layout pattern is triangular with no pass lanes

$$N_t = 1066$$
$$p = 0.9375\,in.$$
$$r_o = 15.563\,in.$$
$$\rho = 0.88$$
$$h_g = 0\,in.$$
$$c_t = 0\,in.$$
$$\nu = 0.31$$
$$E = 26.5E6\ psi$$
$$S = 15,800\ psi$$

Data Summary - Tubes

$$d_t = 0.75\,in.$$
$$t_t = 0.065\,in.$$
$$L_t = 155.875\,in.$$
$$\ell_t = 20.75\,in.$$
$$\nu_t = 0.31$$
$$E_t = 26.5E6\ psi$$
$$S_t = 15,800\ psi$$
$$S_{y,t} = 17,500\ psi$$

Data Summary - Stationary Tubesheet

$W^{*} = 290,720\,lb$ from Table 4.18.6
$A = 39.875\,in.$
$h = 1.188\,in.$
$G_s = 39.441\,in.$
$a_s = 19.7\,in.$
$G_c = 39.441\,in.$
$a_c = 19.7\,in.$
$C = 41.625\,in.$

Data Summary - Floating Tubesheet

$W^{*} = 0\,lb$ from Table 4.18.6
$A = 36.875\,in.$
$a_c = 18.4375\,in.$
$a_s = 18.4375\,in.$
$h = 1.188\,in.$

Calculation Procedure – Stationary Tubesheet

The following results are for the 3 load cases required to be analyzed (see paragraph 4.18.9.3).

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$D_o = 31.876\,in.$
$A_L = 0\,in.^2$
$\mu = 0.200$
$\mu^* = 0.322$
$h'_g = 0\,in.$
$a_o = 15.938\,in.$
$\rho_s = 1.24$
$\rho_c = 1.24$
$x_s = 0.410$
$x_t = 0.597$

b) STEP 2 – Calculate the shell coefficients $\beta_s, k_s, \lambda_s,$ and δ_s and the channel coefficients $\beta_c, k_c, \lambda_c,$ and δ_c.

$$\beta_s = 0\,in.^{-1}$$
$$k_s = 0\,lb$$
$$\lambda_s = 0\,psi$$
$$\delta_s = 0\,in.^3 / lb$$
$$\beta_c = 0\,in.^{-1}$$
$$k_c = 0\,lb$$
$$\lambda_c = 0\,psi$$
$$\delta_c = 0\,in.^3 / lb$$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from paragraph 4.18.6.4.b and calculate E^*.

$$h/p = 1.27$$
$$E^*/E = 0.338$$
$$\nu^* = 0.316$$
$$E^* = 8.947E6\ psi$$

Calculate, X_a, and the parameters from Table 4.18.3.

$$X_a = 7.40$$
$$Z_a = 482.2$$
$$Z_d = 0.00369$$
$$Z_v = 0.0186$$
$$Z_m = 0.197$$
$$Z_w = 0.0186$$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$$K = 1.25$$
$$F = 0.454$$

Calculate Φ and Q_1.

$$\Phi = 0.597$$
$$Q_1 = 0.202$$

e) STEP 5 – Calculate ω_s, ω^*_s, ω_c, ω^*_c, γ_b, P^*_s and P^*_c.

$$\omega_s = 0\,in.^2 \qquad \omega_c = 0\,in.^2$$

$$\omega^*_s = 8.00\,in.^2 \qquad \omega^*_c = 8.00\,in.^2$$

$$\gamma_b = 0$$

$$P^*_s = 0\ psi$$

$$P^*_c = 0\ psi$$

f) STEP 6 – Calculate P_e.

Summary Table for STEP 6	
Loading Case	P_e (psi)
1	92.9
2	-79.6
3	13.3

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-1,250	0.0962	0.0702	21,900	23700
2	1,070	0.0962	0.0702	18,800	23700
3	-179	0.0962	0.0702	3,130	23700

For all loading cases $\lvert\sigma\rvert \le 1.5S$. The bending stress criterion for the tubesheet is satisfied.

h) STEP 8 – For all loading cases, calculate the average shear stress in the tubesheet at the outer edge of the perforated region, if required. Compare to the allowable.

If $\lvert P_e\rvert \le \dfrac{1.6S\mu h}{a_o}$, then the shear stress is not required to be calculated.

Load Case 1:

$$\left\{\lvert P_e\rvert = 92.9\ \ psi\right\} \le \left\{\frac{1.6S\mu h}{a_o} = 377\ \ psi\right\}$$

Therefore, shear stress is not required to be calculated for design load case 1. Similarly, the shear stress is not required to be calculated for load case 2 and case 3.

Summary Table for STEP 8		
Loading Case	$\lvert\tau\rvert$ (psi)	$0.8S$ (psi)
1	Not required	12640
2	Not required	12640
3	Not required	12640

For all loading cases the shear stress criterion is not required.

i) STEP 9 – For each load case, calculate the axial tube stress and the allowable axial tube stress based on tube buckling.

$$r_t = 0.243\,in.$$
$$F_t = 85.3$$
$$C_t = 173$$

Summary Table for STEP 9					
Loading Case	F_s	$\sigma_{t,1}$ (psi)	$\sigma_{t,2}$ (psi)	S_{tb} (psi)	S_t (psi)
1	1.25	2	-4,647	10,550	15800
2	1.25	-152	3,833	10,550	15800
3	1.25	-150	-814	10,550	15800

Determine $\sigma_{t,\max} = \max\left[\lvert\sigma_{t,1}\rvert, \lvert\sigma_{t,2}\rvert\right]$ and $\sigma_{t,\min} = \min\left[\sigma_{t,1}, \sigma_{t,2}\right]$.

For all loading cases $\sigma_{t,\max} < S_t$. The axial tension stress criterion for the tube is satisfied.

For all loading cases $\lvert\sigma_{t,\min}\rvert < S_{tb}$. The buckling criterion for the tube is satisfied.

Calculation Procedure – Floating Tubesheet

The following results are for the 3 load cases required to be analyzed (see paragraphs 4.18.9.3).

a) STEP 1 – Calculate the parameters from paragraph 4.18.6.4.a.

$$D_o = 31.876\,in.$$
$$A_L = 0\,in.^2$$
$$\mu = 0.200$$
$$\mu^* = 0.322$$
$$h'_g = 0\,in.$$
$$a_o = 15.938\,in.$$

$\rho_s = 1.16$

$\rho_c = 1.16$

$x_s = 0.410$

$x_t = 0.597$

b) STEP 2 – Calculate the shell coefficients β_s, k_s, λ_s, and δ_s and the channel coefficients β_c, k_c, λ_c, and δ_c.

$\beta_s = 0\, in.^{-1}$

$k_s = 0\, lb$

$\lambda_s = 0\, psi$

$\delta_s = 0\, in.^3 / lb$

$\beta_c = 0\, in.^{-1}$

$k_c = 0\, lb$

$\lambda_c = 0\, psi$

$\delta_c = 0\, in.^3 / lb$

c) STEP 3 – Calculate h/p. Determine E^*/E and ν^* from paragraph 4.18.6.4.b and calculate E^*.

$h/p = 1.27$

$E^*/E = 0.338$

$\nu^* = 0.316$

$E^* = 8.947E6\ psi$

Calculate, X_a, and the parameters from Table 4.18.3.

$X_a = 7.40$

$Z_a = 482.2$

$Z_d = 0.00369$

$Z_v = 0.0186$

$Z_m = 0.197$

$Z_w = 0.0186$

d) STEP 4 – Calculate the diameter ratio, K, the coefficient F, and the associated parameters.

$K = 1.16$

$F = 0.295$

Calculate Φ and Q_1.

$$\Phi = 0.388$$
$$Q_1 = 0.139$$

e) STEP 5 – Calculate ω_s, ω^*_s, ω_c, ω^*_c, γ_b, T_r, T^*_c, P^*_s and P^*_c.

$$\omega_s = 0\, in.^2 \qquad \omega_c = 0\, in.^2$$
$$\omega^*_s = 3.37\, in.^2 \qquad \omega^*_c = 3.37\, in.^2$$
$$\gamma_b = 0$$
$$P^*_s = 0\; psi$$
$$P^*_c = 0\; psi$$

f) STEP 6 – Calculate P_e.

Summary Table for STEP 6	
Loading Case	P_e (psi)
1	59.2
2	-50.7
3	8.46

g) STEP 7 – Calculate the tubesheet bending stress and the allowable tubesheet bending stress.

Summary Table for STEP 7					
Loading Case	Q_2 (in-lb/in)	Q_3	F_m	$\lvert\sigma\rvert$ (psi)	$1.5S$ (psi)
1	-548	0.0661	0.0575	11,400	23700
2	469	0.0661	0.0575	9,780	23700
3	-78.2	0.0661	0.0575	1,630	23700

For loading cases $\lvert\sigma\rvert \le 1.5S$. The bending stress criterion for the tubesheet is satisfied.

The calculation procedure is complete and the unit geometry is acceptable for the given design conditions.

4.19 Bellows Expansion Joints

4.19.1 Example E4.19.1 – U-Shaped Un-reinforced Bellows Expansion Joint and Fatigue Evaluation

Check the acceptability of a U-shaped unreinforced bellows expansion joint for the given design conditions.

Design Conditions:

- Pressure (Internal) = $50\ psig\ @650°F$
- Axial Movements in Compression and Extension = $Independent$
- Axial Movement (Compression) = $4.5\ in$
- Axial Movement (Extension) = $0.375\ in$
- Angular Deflection = $None$
- Lateral Deflection = $None$
- Number of Cycles Required = 1000

Bellows:

- Material = $SA-240,\ Type\ 321$
- Allowable Stress = $17900\ psi$
- Yield Strength = $19800\ psi$
- Modulus of Elasticity at Design Temperature = $25.04E+06\ psi$
- Modulus of Elasticity at Room Temperature = $28.26E+06\ psi$
- Inside Diameter of Convolution = $48.0\ in$
- Outside Diameter of Convolution = $52.0\ in$
- Number of Convolutions = 12
- Number of Plies = 1
- Nominal Ply Thickness = $0.048\ in$
- Convolution Pitch = $1.0\ in$
- Mean Radius of Convolution = $0.25\ in$
- Crest Convolution Inside Radius = $0.226\ in$
- Root Convolution Inside Radius = $0.226\ in$
- End Tangent Length = $1.25\ in$
- Installed without Cold Spring = Yes
- Circumferential welds = No

The bellows was formed with a mandrel from a cylinder with an inside diameter of $48.0\ in$ and preformed 100% to the outside of the cylinder. The bellows is in as-formed condition.

Collar:

- Collar = $None$

Cylindrical shell on which the bellows is attached:

- Inside Diameter of Shell = $47.25\ in$
- Thickness of Shell = $0.375\ in$
- Minimum Length of Shell on each Side of the Bellows = $10.5\ in$

Evaluate per paragraph 4.19.

a) STEP 1 – Check applicability of design rules per paragraph 4.19.2.

1) Bellows length must satisfy $Nq \le 3D_b$.

$$\{Nq = 12(1.0) = 12\} \le \{3D_b = 3(48.0) = 144\} \qquad (True)$$

2) Bellows thickness must satisfy $nt \le 0.2\ in$.

$$\{nt = 1(0.048) = 0.048\} \le \{0.2\} \qquad (True)$$

3) Number of plies must satisfy $n \le 5$.

$$\{n = 1\} \le \{5\} \qquad (True)$$

4) Displacement shall be essentially axial.

No angular or lateral deflection is specified, so the condition is satisfied.

5) Material allowable stress must not be governed by time-dependent properties.

The material is $SA-240, Type\ 321$ austenitic stainless, the design temperature is $650°F$ which is less than the time-dependent value of $800°F$; therefore, the condition is satisfied.

6) The length of the cylindrical shell on each side of the bellows shall not be less than $1.8\sqrt{D_s t_s}$.

$$\{10.5\ in\} \ge \left\{1.8\sqrt{D_s t_s} = 1.8\sqrt{(47.25)(0.375)} = 7.577\ in\right\} \qquad (True)$$

b) STEP 2 – Check the applicability of paragraph 4.19.5.2.

1) Check that the following condition is satisfied.

$$\{0.9r_{ir} = 0.9(0.226) = 0.203\} \le \{r_{ic} = 0.226\} \le \{1.1r_{ir} = 1.1(0.226) = 0.249\} \quad (True)$$

2) Torus radius shall satisfy $r_i \ge 3t$.

$$\left\{r_i = \frac{r_{ic} + r_{ir}}{2} = \frac{0.226 + 0.226}{2} = 0.226\ in\right\} \le \{3t = 3(0.048) = 0.144\ in\} \qquad (True)$$

3) Sidewall offset angle shall meet $-15° \le \alpha \le 15°$.

$$\alpha = \text{atan}\left[\left(\frac{q}{2} - 2r_m\right) / (w - 2r_m)\right] = \text{atan}\left[\left(\frac{(1.0)}{2} - 2(0.25)\right) / (2.0 - 2(0.25))\right] = 0\ rad$$

$$-15° \le \{\alpha = 0\} \le 15° \qquad (True)$$

4) Convolution height shall meet $w \le D_b/3$.

$$\left\{ w = \left(\frac{D_o}{2} - \frac{D_b}{2} \right) = \left(\frac{52.0}{2} - \frac{48.0}{2} \right) = 2.0\ in \right\} \le \left\{ \frac{D_b}{3} = \frac{48.0}{3} = 16.0\,in \right\} \qquad (True)$$

c) STEP 3 – Check stresses in bellows at Design Conditions per paragraph 4.19.5.3. Since the bellows are subject to internal pressure, calculations and acceptability criteria are per Table 4.19.1. The following values are calculated.

$$D_m = D_b + w + rt = 48.0 + 2.0 + 1(0.048) = 50.048\ in$$

$$t_p = t\sqrt{\frac{D_b}{D_m}} = 0.048\sqrt{\frac{48.0}{50.048}} = 0.047\ in$$

$$K = \min\left[\left(\frac{L_t}{1.5\sqrt{D_b t}} \right), 10 \right] = \min\left[\left(\frac{1.25}{1.5\sqrt{48.0(0.048)}} \right), 1.0 \right] = 0.549$$

$$A = \left[2\pi r_m + 2\sqrt{\left\{ \frac{q}{2} - 2r_m \right\}^2 + \left\{ w - 2r_m \right\}^2} \right] n t_p$$

$$A = \left[2\pi(0.25) + 2\sqrt{\left\{ \frac{(1.0)}{2} - 2(0.25) \right\}^2 + \left\{ (2.0) - 2(0.25) \right\}^2} \right] (1)(0.047) = 0.215\ in^2$$

Table 4.19.2 is used to determine C_p. The following values are calculated.

$$C_1 = \frac{2r_m}{w} = \frac{2(0.25)}{2.0} = 0.250 \quad \text{with } 0.0 \le C_1 \le 1.0$$

$$C_2 = \frac{1.82 r_m}{\sqrt{D_m t_p}} = \frac{1.82(0.25)}{\sqrt{(50.048)(0.047)}} = 0.297 \quad \text{with } 0.2 \le C_2 \le 4.0$$

The coefficients, $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$ are interpolated.

$$\alpha_0 = 1.000 \qquad \alpha_3 = 0.711$$
$$\alpha_1 = -0.587 \qquad \alpha_4 = 0.662$$
$$\alpha_2 = -0.589 \qquad \alpha_5 = -0.646$$

$$C_p = \alpha_0 + \alpha_1 C_1 + \alpha_2 C_1^2 + \alpha_3 C_1^3 + \alpha_4 C_1^4 + \alpha_5 C_1^5$$

$$C_p = \left\{ \begin{array}{l} 1.000 + (-0.587)(0.25) + (-0.589)(0.25)^2 + \\ (0.711)(0.25)^3 + (0.662)(0.25)^4 + (-0.646)(0.25)^5 \end{array} \right\} = 0.830$$

Calculate Stresses

Circumferential Membrane stress in bellows tangent, due to pressure (S_1).

$$S_1 = \frac{P(D_b + nt)^2 L_t E_b k}{2\left[nt(D_b + nt)L_t E_b + t_c D_c L_c E_c k\right]}$$

$$S_1 = \frac{50(48.0 + 1(0.048))^2 (1.25)(25.04E+06)(0.549)}{2\left[1(0.048)(48.0 + 1(0.048))(1.25)(25.04E+06) + 0\right]} = 13738.7 \ psi$$

Circumferential Membrane stress in bellows end convolutions, due to pressure $(S_{2,E})$.

$$S_{2,E} = \frac{P\left[qD_m + L_t(D_b + nt)\right]}{2(A + nt_p L_t + t_c L_c)} = \frac{50\left[1.0(50.048) + 1.25(48.0 + 1(0.048))\right]}{2(0.215 + 1(0.047)(1.25) + 0)} = 10055.5 \ psi$$

Circumferential Membrane stress in bellows intermediate convolutions, due to pressure $(S_{2,I})$.

$$S_{2,I} = \frac{P_q D_m}{2A} = \frac{50(1.0)(50.048)}{2(0.215)} = 5819.5 \ psi$$

Meridional Membrane stress in bellows due to pressure (S_3).

$$S_3 = \frac{Pw}{2nt_p} = \frac{50(2.0)}{2(1(0.047))} = 1063.8 \ psi$$

Meridional Bending stress in bellows due to pressure (S_4).

$$S_4 = \frac{PC_p}{2n}\left(\frac{w}{t_p}\right)^2 = \frac{50(0.830)}{2(1)}\left(\frac{2.0}{0.047}\right)^2 = 37573.6 \ psi$$

Acceptance Checks

$$\{S_1 = 13738.7 \ psi\} \le \{S = 17900 \ psi\} \qquad (True)$$

$$\{S_{2,E} = 10055.5 \ psi\} \le \{S = 17900 \ psi\} \qquad (True)$$

$$\{S_{2,I} = 5819.5 \ psi\} \le \{S = 17900 \ psi\} \qquad (True)$$

$$\{(S_3 + S_4) = (1063.8 + 37573.6) = 38637.4 \ psi\} \le \{K_m S = 3(17960) = 53700 \ psi\} \qquad (True)$$

Where,

Factor K_m is calculated by:

$$K_m = 1.5Y_{sm} = 1.5(2.0) = 3.0 \qquad (for\ As-Formed\ Bellows)$$

The forming stain ε_f for bellows formed 100% to the outside of the initial cylinder is:

$$\varepsilon_f = \sqrt{\left[\ln\left(1+\frac{2w}{D_b}\right)\right]^2 + \left[\ln\left(1+\frac{nt_p}{2r_m}\right)\right]^2}$$

$$\varepsilon_f = \sqrt{\left[\ln\left(1+\frac{2(2.0)}{(48.0)}\right)\right]^2 + \left[\ln\left(1+\frac{1(0.047)}{2(0.25)}\right)\right]^2} = 0.120$$

The forming method factor K_f for forming with expanding mandrel is:

$$K_f = 1.0$$

Since material $SA-240, Type\ 321$ is an austenitic stainless steel, the yield strength multiplier Y_{sm} is:

$$Y_{sm} = 1 + 9.94(K_f\varepsilon_f) - 7.59(K_f\varepsilon_f)^2 - 2.4(K_f\varepsilon_f)^3 + 2.21(K_f\varepsilon_f)^4$$

$$Y_{sm} = \begin{Bmatrix} 1 + 9.94((1.0)(0.120)) - 7.59((1.0)(0.120))^2 - \\ 2.4((1.0)(0.120))^3 + 2.21((1.0)(0.120))^4 \end{Bmatrix} = 2.083$$

If Y_{sm} is greater than 2.0, then $Y_{sm} = 2.0$.

The bellows meet internal pressure acceptance criteria at design conditions.

d) STEP 4 – Check column instability due to internal pressure per paragraph 4.19.5.4.

$$P_{sc} = \frac{0.34\pi K_b}{N_q} = \frac{0.34\pi(1647.5)}{12(1)} = 146.7\ psi$$

The axial stiffness, K_b, is calculated using Equation 4.19.17.

$$K_b = \frac{\pi E_b D_m}{2(1-\nu_b^2)C_f}\left(\frac{n}{N}\right)\left(\frac{t_p}{w}\right)^3 = \frac{\pi(25.04E+06)(50.048)}{2(1-0.3^2)(1.419)}\left(\frac{1}{12}\right)\left(\frac{0.047}{2.0}\right)^3$$

$$K_b = 1648.7\ \frac{lbs}{in}$$

C_f is calculated using the method described in Table 4.19.3. With $C_2 = 0.297$, interpolate for the coefficients $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$.

$$\alpha_0 = 1.006 \qquad \alpha_3 = 5.719$$
$$\alpha_1 = 2.106 \qquad \alpha_4 = -5.501$$
$$\alpha_2 = -2.930 \qquad \alpha_5 = 2.067$$

$$C_f = \alpha_0 + \alpha_1 C_1 + \alpha_2 C_1^2 + \alpha_3 C_1^3 + \alpha_4 C_1^4 + \alpha_5 C_1^5$$

$$C_f = \begin{Bmatrix} 1.006 + (2.106)(0.25) + (-2.930)(0.25)^2 + \\ (5.719)(0.25)^3 + (-5.501)(0.25)^4 + (2.067)(0.25)^5 \end{Bmatrix} = 1.419$$

$$\{P = 50\ psi\} \le \{P_{sc} = 146.7\ psi\} \qquad (True)$$

The bellows meet columns instability criteria at design conditions.

e) STEP 5 – Check in-plane instability due to internal pressure per paragraph4.19.5.5.

$$P_{si} = \frac{AS_y^*(\pi - 2)}{D_m q\left[1 + 2\delta^2 + \left(1 - 2\delta^2 + 4\delta^4\right)^{0.5}\right]^{0.5}}$$

$$P_{si} = \left(\frac{0.215(45540)(\pi - 2)}{50.048(1.0)\left[1 - 2(2.15)^2 + \left(1 + 2(2.15)^2 + 4(2.15)^4\right)^{0.5}\right]^{0.5}}\right) = 51.2\ psi$$

$$S_y^* = 2.3S_y = 2.3(19800) = 45540\ psi \qquad (for\ As-Formed\ \ Bellows)$$

$$\delta = \frac{S_4}{3S_{2,I}} = \frac{37573.6}{3(5819.5)} = 2.15$$

$$\{P = 50\ psi\} \le \{P_{si} = 51.2\ psi\} \qquad (True)$$

The bellows meet in-plane instability criteria at design conditions.

f) STEP 6 – Perform a fatigue evaluation per paragraph 4.19.5.7

Calculate the equivalent axial displacement range

The following values are calculated. Δq is calculated using the procedure shown in paragraph 4.19.8.

$$\begin{Bmatrix} x_e = 0.375\ in \\ x_c = -4.5\ in \end{Bmatrix} \rightarrow \qquad See\ design\ data$$

$$\Delta q_{x,e} = \frac{x_e}{N} = \frac{0.375}{12} = 0.031\ in$$

$$\Delta q_{x,c} = \frac{x_c}{N} = \frac{4.5}{12} = -0.375\ in$$

Since no lateral or angular movement:

$$\Delta q_{e,1} = \Delta q_{x,e} = 0.031\ in$$

$$\Delta q_{c,1} = \Delta q_{x,c} = -0.375\ in$$

Since the bellows was installed without cold spring and the axial movements in extension and compression are independent (as opposed to concurrent):

Initial Position	*Final Position*
$\Delta q_{e,0} = 0.0\ in$	$\Delta q_{e,1} = +0.031\ in$
$\Delta q_{c,0} = 0.0\ in$	$\Delta q_{c,1} = -0.375\ in$

$$\Delta q = \max\left[\left|\Delta q_{e,1}\right|, \left|\Delta q_{c,1}\right|\right] = \max\left[\left|0.031\right|, \left|-0.375\right|\right] = 0.375\ in$$

C_d is calculated using the method described in Table 4.19.4. With $C_2 = 0.297$, interpolate for the coefficients $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5$.

$\alpha_0 = 1.000$	$\alpha_3 = -3.441$
$\alpha_1 = 1.228$	$\alpha_4 = 3.453$
$\alpha_2 = 1.309$	$\alpha_5 = -1.190$

$$C_d = \alpha_0 + \alpha_1 C_1 + \alpha_2 C_1^2 + \alpha_3 C_1^3 + \alpha_4 C_1^4 + \alpha_5 C_1^5$$

$$C_d = \left\{ \begin{array}{l} 1.000 + (1.228)(0.25) + (1.309)(0.25)^2 + \\ (-3.441)(0.25)^3 + (3.453)(0.25)^4 + (-1.190)(0.25)^5 \end{array} \right\} = 1.347$$

<u>Calculate stresses due to equivalent axial displacement range:</u>

Meridional membrane (S_5).

$$S_5 = \frac{E_b t_p^2 \Delta q}{2w^3 C_f} = \frac{(25.04E+06)(0.047)^2(0.375)}{2(2.0)^3(1.419)} = 913.6\ psi$$

Meridional bending (S_6).

$$S_6 = \frac{5E_b t_p \Delta q}{3w^2 C_d} = \frac{5(25.04E+06)(0.047)(0.375)}{3(2.0)^2(1.347)} = 136516.3\ psi$$

Total stress range due to cyclic displacement (S_t)

$$S_t = 0.7\left[S_3 + S_4\right] + \left[S_5 + S_6\right]$$

$$S_t = 0.7(1063.8 + 37573.6) + (913.6 + 136516.3) = 164476.1\ psi$$

Calculate allowable number of cycles, N_{alw}, using the equations from Table 4.19.5.

$$K_g\left(\frac{E_o}{E_b}\right)S_t = \left\{1.0\left(\frac{28.26E+06}{25.04E+06}\right)(164476.1) = 185626.78\right\} \geq 65000$$

$$N_{alw} = \left(\frac{5.2+06}{K_g\left(\frac{E_0}{E_b}\right)S_t - 38300}\right)^2 = \left(\frac{5.2E+06}{185626.78-38300}\right)^2 = 1246\ cycles$$

$$\left\{N_{alw} = 1246\ cycles\right\} \geq \left\{N_{spe} = 1000\ cycles\right\}$$

The bellows meets fatigue design criteria at design conditions.

The bellows meets all of the design requirements of paragraph 4.19 at design conditions.

4.19.2 Example E4.19.2 – Toroidal Bellows Expansion Joint and Fatigue Evaluation

Check the acceptability of a toroidal bellows for the given design conditions.

Design Conditions:

- Pressure (Internal) = $400\ psig\ @\ 650°F$
- Axial Movements in Compression and Extension = $Independent$
- Axial Displacement (Compression) = $0.25\ in$
- Axial Displacement (Extension) = $0.745\ in$
- Angular Deflection = $None$
- Lateral Deflection = $None$
- Number of Cycles Required = 1000

Bellows:

- Material = $SA-240,\ Type\ 321$
- Allowable Stress = $17900\ psi$
- Modulus of Elasticity at Design Temperature = $25.04E+06\ psi$
- Modulus of Elasticity at Room Temperature = $28.26E+06\ psi$
- Inside Diameter of Bellows = $36.0\ in$
- Mean Diameter of Bellows = $40.0\ in$
- Number of Convolutions = 2
- Convolution Pitch = $4.000\ in$
- Mean Radius of Convolutions = $1.5\ in$
- Number of Plies = 1
- Ply Thickness = $0.078\ in$
- Installed without Cold Spring = Yes
- Circumferential welds = No

The bellows is attached to the shell externally on both sides.

Reinforcing and Tangent Collars:

- Material = $SA-240,\ Type\ 304$
- Allowable Stress = $16200\ psi$
- Modulus of Elasticity at Design Temperature = $25.04E+06\ psi$

Tangent Collars:

- Tangent Collar Joint Efficiency = 1.0
- Tangent Collar Thickness = $0.75\ in$
- Cross Sectional Metal Area of one Tangent Collar = $1.034\ in^2$
- Length from Attachment Weld to the Center of the First Convolution = $2.000\ in$

Reinforcing Collars:

- Reinforcing Collar Joint Efficiency = 1.0
- Reinforcing Collar Thickness = $0.75\ in$
- Overall Length of one Reinforcing Collar = $3.094\ in$
- Cross Sectional Metal Area of one Reinforcing Collar based on Overall Length = $2.068\ in^2$

Cylindrical shell on which the bellows is attached:

- Inside Diameter of Shell = $35.0\ in$
- Thickness of Shell = $0.50\ in$
- Minimum Length of Shell on each Side of the Bellows = $10.5\ in$

Evaluate per paragraph 4.19.

a) STEP 1 – Check applicability of design rules per paragraph 4.19.2.

1) Bellows length must satisfy $Nq \le 3D_b$.

$$\{(N-1)q+2L_d=(2-1)(4.0)+2(2.0)=8\ in\}\le\{3D_b=3(36)=108\ in\} \qquad (True)$$

2) Bellows thickness must satisfy $nt \le 0.2\ in$.

$$\{nt=1(0.078)=0.078\}\le\{0.2\} \qquad (True)$$

3) Number of plies must satisfy $n \le 5$.

$$\{n=1\}\le\{5\} \qquad (True)$$

4) Displacement shall be essentially axial.

No angular or lateral deflection is specified, so the condition is satisfied.

5) Material allowable stress must not be governed by time dependent properties.

The material is $SA-240,\ Type\ 321$ austenitic stainless, the design temperature is $650°F$ which is less than the time-dependent value of $800°F$; therefore, the condition is satisfied.

6) The length of the cylindrical shell on each side of the bellows shall not be less than $1.8\sqrt{D_s t_s}$.

$$\{10.5\ in\}\ge\left\{1.8\sqrt{D_s t_s}=1.8\sqrt{(35.0)(0.50)}=7.530\ in\right\} \qquad (True)$$

b) STEP 2 – Check the applicability of paragraph 4.19.7.2.

1) The type of attachment to the shell shall be the same on both sides.
The bellows is attached to the shell externally on both sides, so the condition is satisfied.

2) Distance L_g shall be less than $0.75r$ in the maximum extended position.

The distance across the inside opening in the neutral position is calculated as follows,

$$L_{g0}=q-(L_{rt}+2nt)=(4.0)-((3.094)+2(1)(0.078))=0.750\ in$$

The only movement is in the axial direction. The maximum opening corresponds to the maximum extension.

$$L_g = L_{g0} + \Delta q_{x,e} = L_{g0} + \frac{x_e}{N} = (0.75) + \left(\frac{0.745}{2}\right) = 1.1225\ in$$

$$\{L_g = 1.1225\ in\} < \{0.75r = 0.75(1.5) = 1.1250\ in\} \qquad (True)$$

4) For internally attached bellows, the length of the shell on each side of the bellows having thickness t_s shall be at least equal to $L_{sm} = \sqrt{D_s t_s}$.

Not applicable

c) STEP 3 – Check stresses in bellows at design conditions per paragraph 4.19.7.3. Since the bellows are subject to internal pressure, calculations and acceptability criteria are per Table 4.19.8.

$$D_c = D_b + 2nt + t_c = 36.0 + 2(1)(0.078) + 0.75 = 36.906\ in$$

$$D_r = D_b + 2nt + t_r = 36.0 + 2(1)(0.078) + 0.75 = 36.906\ in$$

$$t_p = t\sqrt{\frac{D_b}{D_m}} = 0.078\sqrt{\frac{36.0}{40.0}} = 0.074\ in$$

Calculate stresses

Circumferential Membrane stress in end tangent due to internal pressure (S_1).

$$S_1 = \frac{P(D_b + nt)^2 L_d E_b}{2D_c E_c A_{tc}}$$

$$S_1 = \frac{(400)(36.0 + (1)(0.078))^2 (2.0)(25.04E+06)}{2(36.906)(25.04E+06)(1.034)} = 13643.6\ psi$$

Circumferential Membrane stress in collar due to internal pressure (S_1').

$$S_1' = \frac{PD_c L_d}{2A_{tc}} = \frac{(400)(36.906)(2.0)}{2(1.034)} = 14277.0\ psi$$

Circumferential Membrane stress in bellows due to internal pressure (S_2).

$$S_2 = \frac{Pr}{2nt_p} = \frac{400(1.5)}{2(1)(0.074)} = 4054.1\ psi$$

Circumferential Membrane stress in reinforcing collar due to internal pressure (S_2').

Since $\{L_{rt} = 3.094\ in\} \le \left\{\frac{2}{3}\sqrt{D_r t_r} = \frac{2}{3}\sqrt{(36.906)(0.75)} = 3.507\ in\right\}$

$$S_2' = \frac{D_r\,(L_{rt} + L_g + 2nt)}{2\,A_{rt}}\,P$$

$$S_2' = \frac{(36.906)(3.094 + 1.1225 + 2(1)(0.078))}{2(2.068)}\cdot(400) = 15606.6\ psi$$

Meridional Membrane stress in bellows due to internal pressure (S_3).

$$S_3 = \frac{Pr}{nt_p}\left(\frac{D_m - r}{D_m - 2r}\right) = \frac{400(1.5)}{(1)(0.074)}\left(\frac{40.0 - 1.5}{40.0 - 2(1.5)}\right) = 8436.8\ psi$$

Acceptance Checks

$$\{S_1 = 13643.6\ psi\} \le \{S = 17900\ psi\} \qquad (True)$$

$$\{S_1' = 14277.0\ psi\} \le \{C_{wc}S_c = (1)(16200) = 16200\ psi\} \qquad (True)$$

$$\{S_2 = 4054.1\ psi\} \le \{S = 17900\ psi\} \qquad (True)$$

$$\{S_2' = 15606.6\ psi\} \le \{C_{wr}S_r = (1)(16200) = 16200\ psi\} \qquad (True)$$

$$\{S_3 = 8436.8\ psi\} \le \{S = 17900\ psi\} \qquad (True)$$

Therefore, bellows meets internal pressure stress acceptance criteria at design conditions.

d) STEP 4 – Check column instability due to internal pressure per paragraph 4.19.7.4.

The following values are calculated using the procedure shown in Table 4.19.9.

$$C_3 = \frac{6.61r^2}{D_m t_p} = \frac{6.61(1.5)^2}{40.0(0.074)} = 5.024$$

$$B_3 = \frac{0.99916 - 0.091665C_3 + 0.040635C_3^2 - 0.0038483C_3^3 + 0.00013392C_3^4}{1 - 0.1527C_3 + 0.013446C_3^2 - 0.00062724C_3^3 + 1.4374(10)^{-5}C_3^4}$$

$$B_3 = \frac{\begin{bmatrix} 0.99916 - 0.091665(5.024) + 0.040635(5.024)^2 - \\ 0.0038483(5.024)^3 + 0.00013392(5.024)^4 \end{bmatrix}}{\begin{bmatrix} 1 - 0.1527(5.024) + 0.013446\left((5.024)^2\right) - \\ 0.00062724(5.024)^3 + 1.4374(10)^{-5}(5.024)^4 \end{bmatrix}} = 2.315$$

The axial stiffness, K_b, is calculated using Equation 4.19.28.

$$K_b = \frac{E_b D_m B_3}{12\ (1-\nu_b{}^2)}\left(\frac{n}{N}\right)\left(\frac{t_p}{r}\right)^3 = \frac{(25.04E+06)(40.0)(2.315)}{12\left(1-(0.3)^2\right)}\left(\frac{1}{2}\right)\left(\frac{0.074}{1.5}\right)^3$$

$$K_b = 12747.2\ \frac{lb}{in}$$

Calculate allowable internal pressure:

$$P_{sc} = \frac{0.15\pi K_b}{Nr} = \frac{0.15\pi(12747.2)}{2(1.5)} = 2002\ psi$$

Acceptance criteria

$$\{P = 400\ psi\} \leq \{P_{sc} = 2002\ psi\} \qquad (True)$$

The bellows meets column instability criteria at design conditions.

e) STEP 5 – Perform a fatigue evaluation per paragraph 4.19.7.7.

The axial displacement range, Δq is calculated using the procedure shown in paragraph 4.19.8.

$$\left\{\begin{matrix} x_e = 0.745\ in \\ x_c = -0.25\ in \end{matrix}\right\} \rightarrow \quad See\ design\ data$$

$$\Delta q_{x,e} = \frac{x_e}{N} = \frac{0.745}{2} = 0.3725\ in$$

$$\Delta q_{x,c} = \frac{x_c}{N} = \frac{-0.25}{2} = -0.125\ in$$

Since no angular or lateral displacement:

$$\Delta q_{e,1} = \Delta q_{x,e} = +0.3725\ in$$

$$\Delta q_{c,1} = \Delta q_{x,c} = -0.125\ in$$

Since the bellow was installed without cold spring and the axial movements in extension and compression are independent (as opposed to concurrent):

$$\Delta q = \max\left[\left|\Delta q_{e,1}\right|,\ \left|\Delta q_{c,1}\right|\right] = \max\left[|0.3725|,\ |-0.125|\right] = 0.3725\ in$$

Calculate coefficient B_1, B_2 from Table 4.19.9:

$$B_1 = \frac{\left(1.00404 + 0.028725C_3 + 0.18961C_3^2 - 0.00058626C_3^3\right)}{\left(1 + 0.14069C_3 - 0.0052319C_3^2 + 0.00029867C_3^3 - 6.2088(10)^{-6}C_3^4\right)}$$

$$B_1 = \frac{\begin{pmatrix} 1.00404 + 0.028725(5.024) + \\ 0.18961(5.024)^2 - 0.00058626(5.024)^3 \end{pmatrix}}{\begin{pmatrix} 1 + 0.14069(5.024) - 0.0052319(5.024)^2 + \\ 0.00029867(5.024)^3 - 6.2088(10)^{-6}(5.024)^4 \end{pmatrix}} = 3.643$$

$$B_2 = \frac{\left(0.049198 - 0.77774C_3 - 0.13013C_3^2 + 0.080371C_3^3\right)}{\left(1 - 2.81257C_3 + 0.63815C_3^2 + 0.0006405C_3^3\right)}$$

$$B_2 = \frac{\left(0.049198 - 0.77774(5.024) - 0.13013(5.024)^2 + 0.080371(5.024)^3\right)}{\left(1 - 2.81257(5.024) + 0.63815(5.024)^2 + 0.0006405(5.024)^3\right)} = 0.997$$

Calculate meridional stresses due to axial displacement range:

$$S_5(membrane) = \frac{E_b t_p^2 B_1 \Delta q}{34.3r^3} = \frac{(25.04E+06)(0.074)^2(3.643)(0.3725)}{34.3(1.5)^3} = 1607.4\ psi$$

$$S_6(bending) = \frac{E_b t_p B_2 \Delta q}{5.72r^2} = \frac{(25.04E+06)(0.074)^2(0.997)(0.3725)}{5.72(1.5)^2} = 53469.9\ psi$$

Calculate total cycle stress range due to displacement:

$$S_t = 3S_3 + S_5 + S_6 = 3(8436.8) + 1607.4 + 53469.9 = 80387.7\ psi$$

Calculate the allowable number of cycles, N_{alw}, per Table 4.19.10.

$$\left\{K_g\left(\frac{E_o}{E_b}\right)S_t = 1.0\left(\frac{28.26E+06}{25.04E+06}\right)(80387.7) = 90725.1\ psi\right\} \geq \{65000\ psi\}$$

$$N_{alw} = \left(\frac{5.2E+06}{\left(K_g\left(\frac{E_o}{E_b}\right)S_t - 38300\right)}\right)^2 = \left(\frac{5.2E+06}{(90725.1 - 38300)}\right)^2 = 9838\ cycles$$

$$\{N_{alw} = 9838\ cycles\} \geq \{N_{spe} = 1000\ cycles\}$$

The bellows meets fatigue design criteria at design conditions.

The bellows meets all of the design requirements of paragraph 4.19 at design conditions.

PART 5

DESIGN BY ANALYSIS REQUIREMENTS

PART CONTENTS

5.1 General Requirements

5.2 Protection Against Plastic Collapse

5.2.1 Example E5.2.1 – Elastic Stress Analysis

Evaluate the vessel top head and shell region for compliance with respect to the elastic stress analysis criteria for plastic collapse provided in paragraph 5.2.2. Do not include the standard flanges or NPS 6 piping in the assessment for compliance to allowable stresses. Internal pressure is the only load that is to be considered. Relevant design data and geometry are provided below and in Figures E5.2.1-1 and E5.2.1-2.

<u>Vessel Data</u>

- Material – Shell and Heads = SA-516, Grade 70, Normalized
- Material – Forgings = SA-105
- Design Conditions = 420 psig at 125^{o}F
- Corrosion Allowance = 0.125 inches
- PWHT = Yes

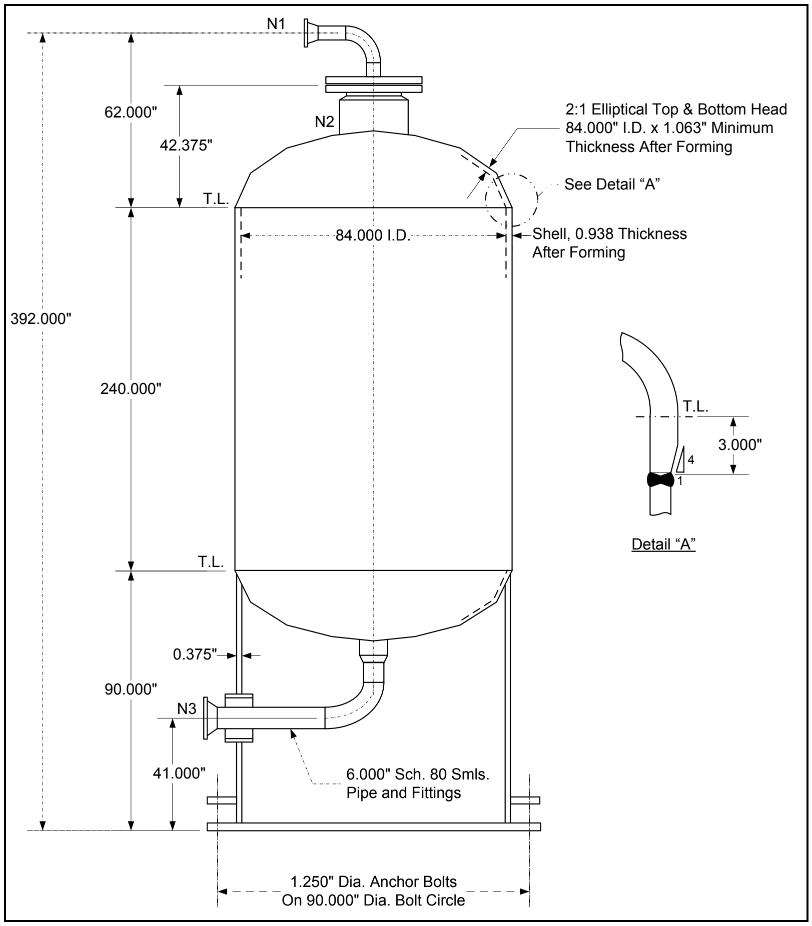


Figure E5.2.1-1 - Vessel Configuration

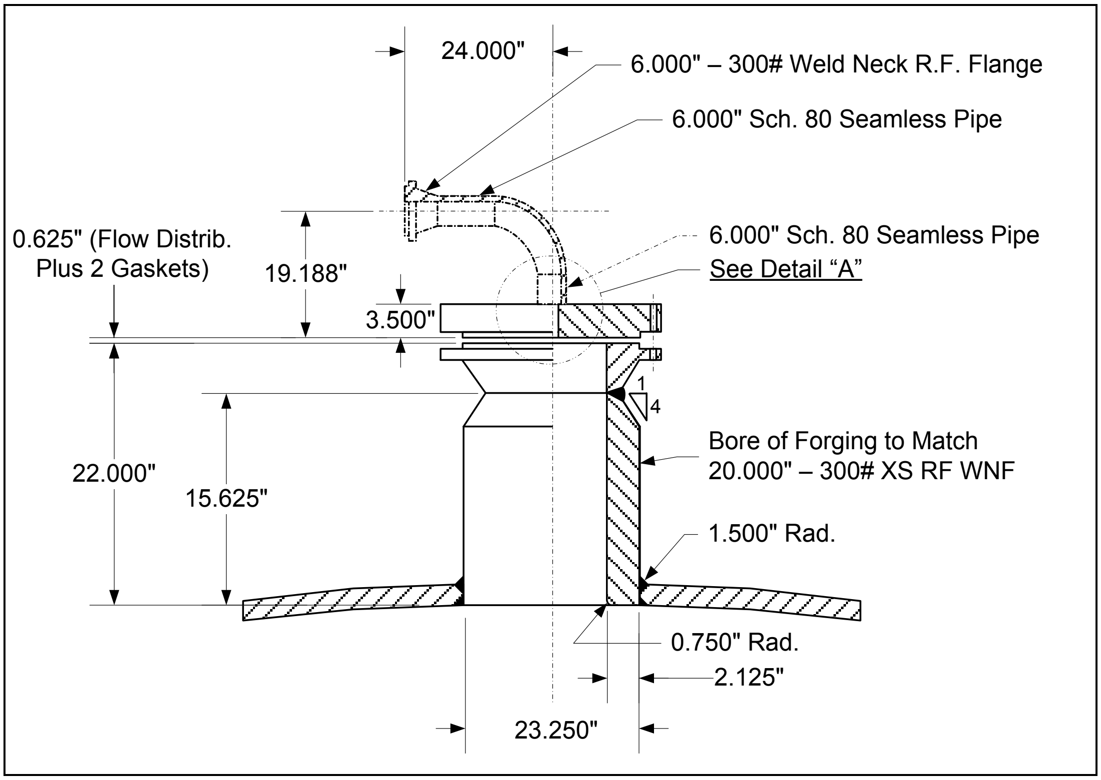


Figure E5.2.1-2 - Vessel Top Inlet Nozzle Geometry

a) STEP 1 – Determine the types of loads acting on the component. In general, separate load cases are analyzed to evaluate "load-controlled" loads such as pressure and externally applied reactions due to weight effects and "strain-controlled" loads resulting from thermal gradients and imposed displacements. The loads to be considered in the design shall include, but not be limited to, those given in Table 5.1. The load combinations that shall be considered for each loading condition shall include, but not be limited to those given in Table 5.3.

The only load to be considered is internal pressure of 420 psig

1) Develop the finite element model.

2) Due to symmetry in geometry and loading, an axisymmetric solid model is generated. To capture proper membrane behavior in the model the shell was extended a distance of $5\sqrt{Rt}$ below the head to shell transition. The FE model is illustrated in Figure E5.2.1-3. The model was generated with the ABAQUS commercial FEA program, version 6.9-1. The geometry was constructed based on the corroded dimensions.

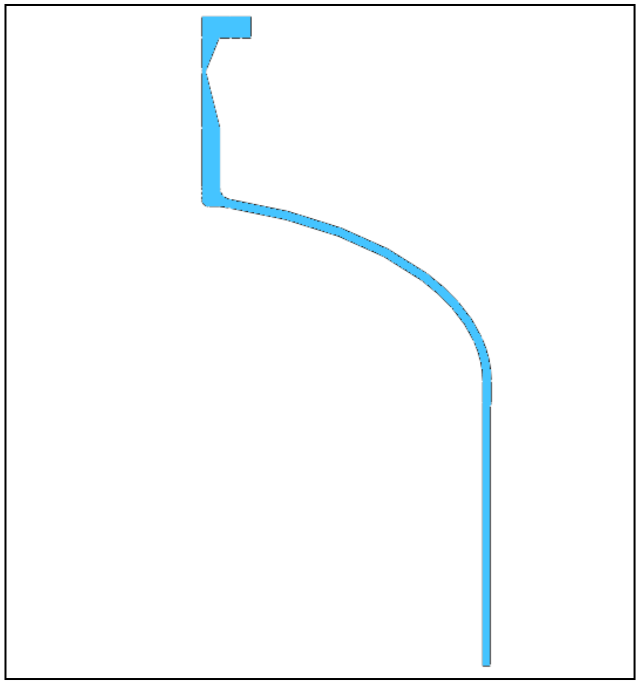


Figure E5.2.1-3 - Axisymmetric FE Model

3) Generate mesh. ABAQUS 8-noded reduced integration elements (CAX8R) are specified for the analysis.

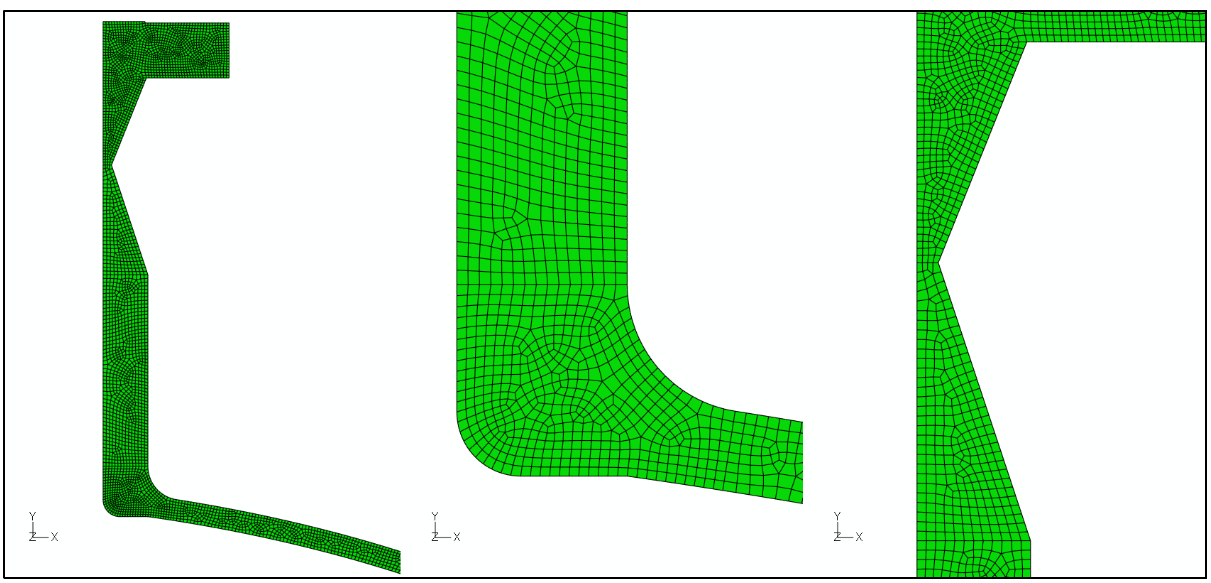


Figure E5.2.1-4 - Mesh of Flange and Nozzle to Head Junction

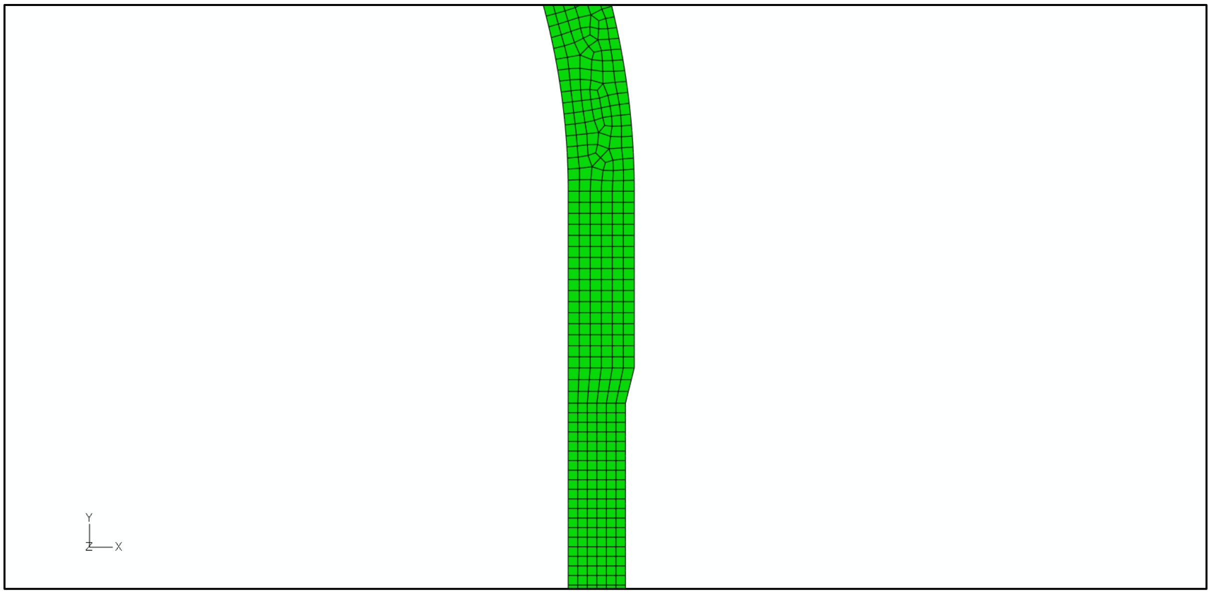


Figure E5.2.1-5 - Mesh of Head to Shell Transition

4) Apply the material properties given below to the appropriate components of the model. Material assignments are depicted in Figure E5.2.1-6. Note that the weld region was assigned the properties of SA-105 material. The difference in modulus between the two materials is negligible, so a single modulus value for the entire model could have been chosen as well.

Component	Material	Modulus of Elasticity (psi)	Poisson Ratio
Weld Neck Flange & Nozzle	SA-105	2.91E+07	0.3
Head and Shell	SA-516-70N	2.88E+07	0.3

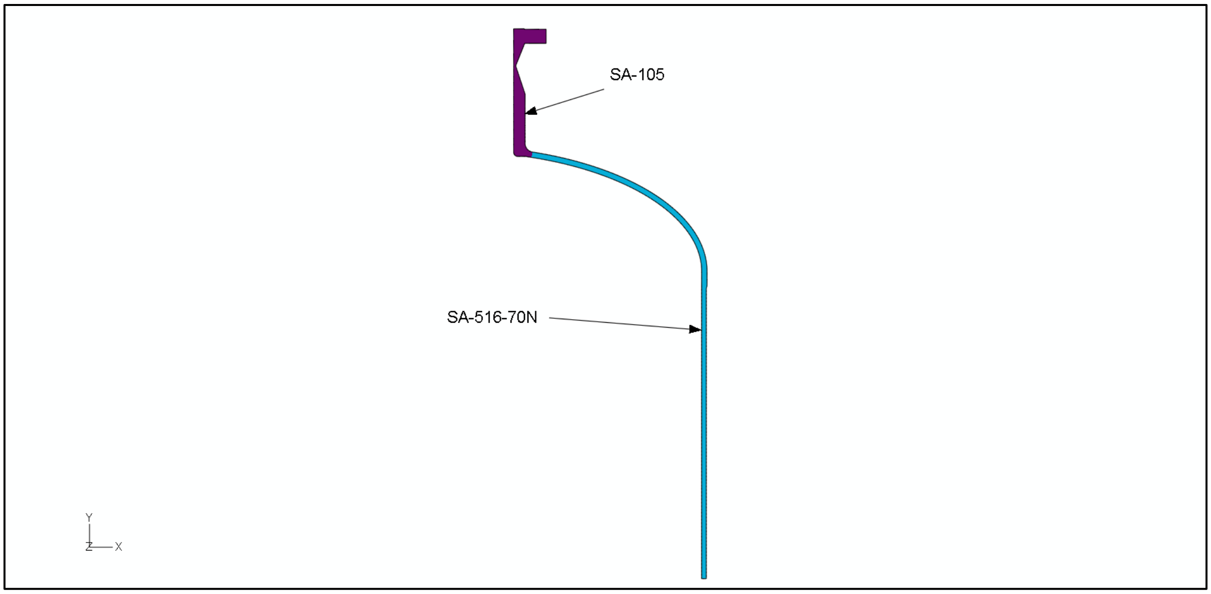


Figure E5.2.1-6 - Material Assignments

5) Apply the internal pressure load to the pressure boundaries of the vessel and an appropriate pressure thrust load to the flange face. Apply the appropriate boundary conditions to the shell edge as per the figure below.

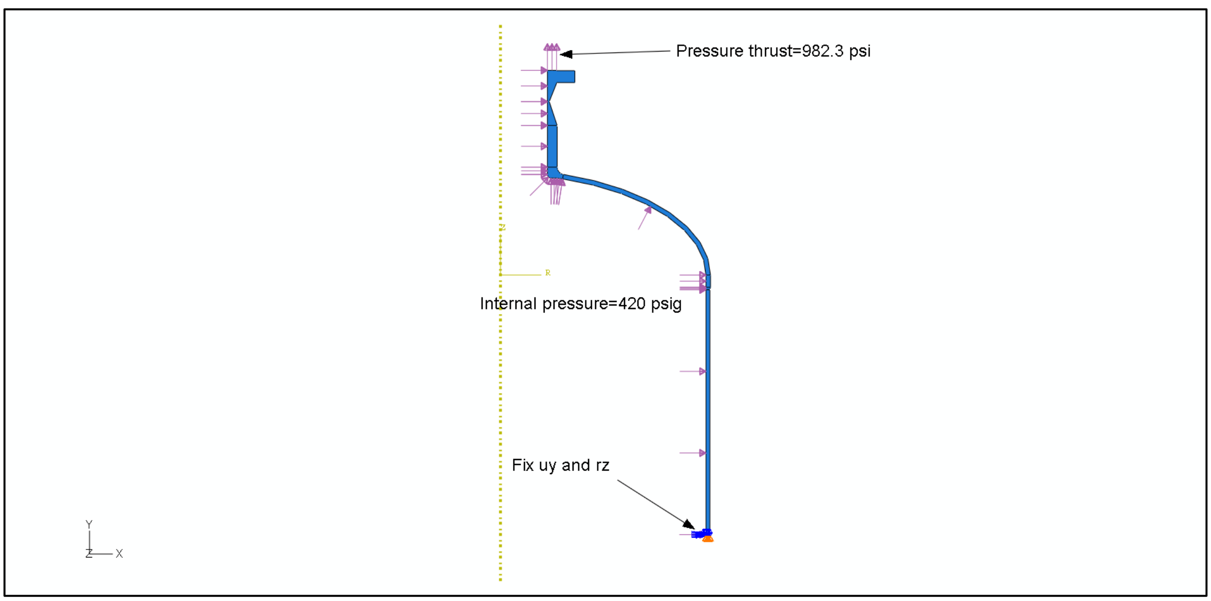


Figure E5.2.1-7 - Load and Boundary Conditions for the FE Model

6) Run analysis and review results. Evaluate displacements and compare calculated reaction force values to hand calculated values.

Figure E5.2.1-8 - Results of Elastic Analysis, von Mises Stress in both Deformed and Non-deformed States

Reaction Force (y-direction):

Calculated (ABAQUS) 2.341E+06 lbf

Hand Calculations 2.341E+06 lbf

Note: Results for Steps 4, 5, and 6 were calculated automatically by analysis routines contained in the ABAQUS commercial FEA software, version 6.9-1. Through wall stress linearization was conducted at critical areas around the pressure boundary to provide data for the routines. The resultant von Mises stresses for P_m, P_L, and P_b stress categories are summarized in Table E5.2.1-1.

Note that per Annex 5.A, bending stresses are calculated only for the local hoop and meridional (normal) component stresses, and not for the local component stress parallel to the SCL or in-plane shear stress.

b) STEP 2 - At the point on the vessel that is being investigated, calculate the stress tensor (six unique components of stress) for each type of load. Assign each of the computed stress tensors to one or to a group of the categories defined below. Assistance in assigning each stress tensor to an appropriate category for a component can be obtained by using Figure 5.1 and Table 5.6. Note that the equivalent stresses Q and F do not need to be determined to evaluate protection against plastic collapse. However, these components are needed for fatigue and ratcheting evaluations that are based on elastic stress analysis (see paragraphs 5.5.3 and 5.5.6, respectively).

1) General primary membrane equivalent stress – P_m

2) Local primary membrane equivalent stress – P_L

3) Primary bending equivalent stress – P_b

4) Secondary equivalent stress – Q

5) Additional equivalent stress produced by a stress concentration or a thermal stress over and above the nominal $(P+Q)$ stress level – F

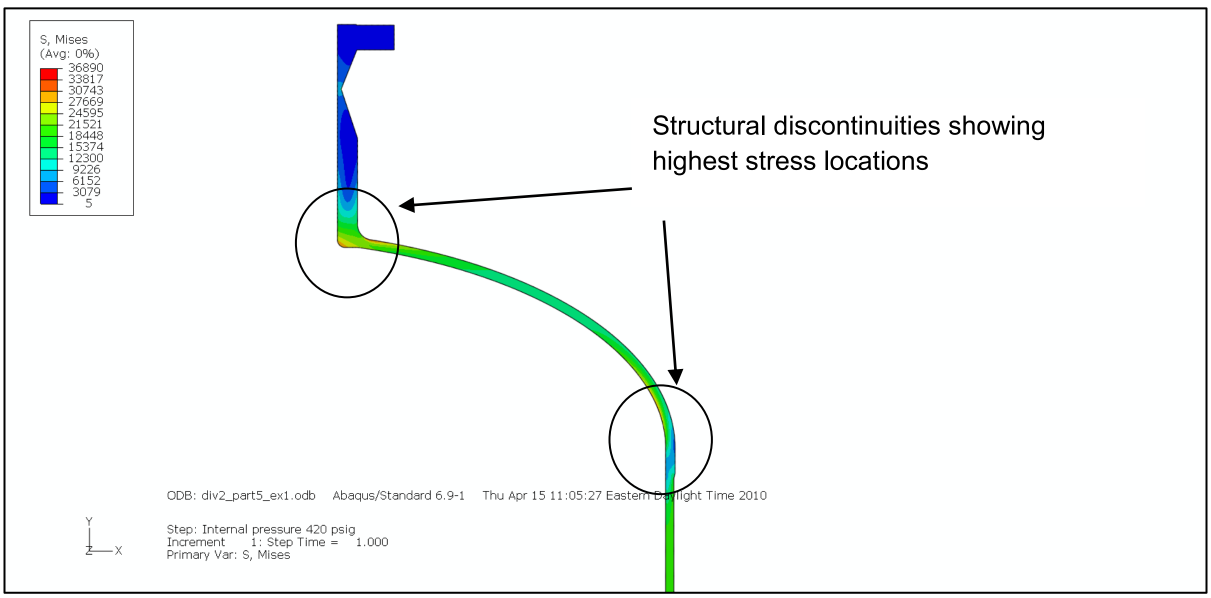


Figure E5.2.1-9 - Critical Locations Through the Vessel Requiring Stress Evaluation

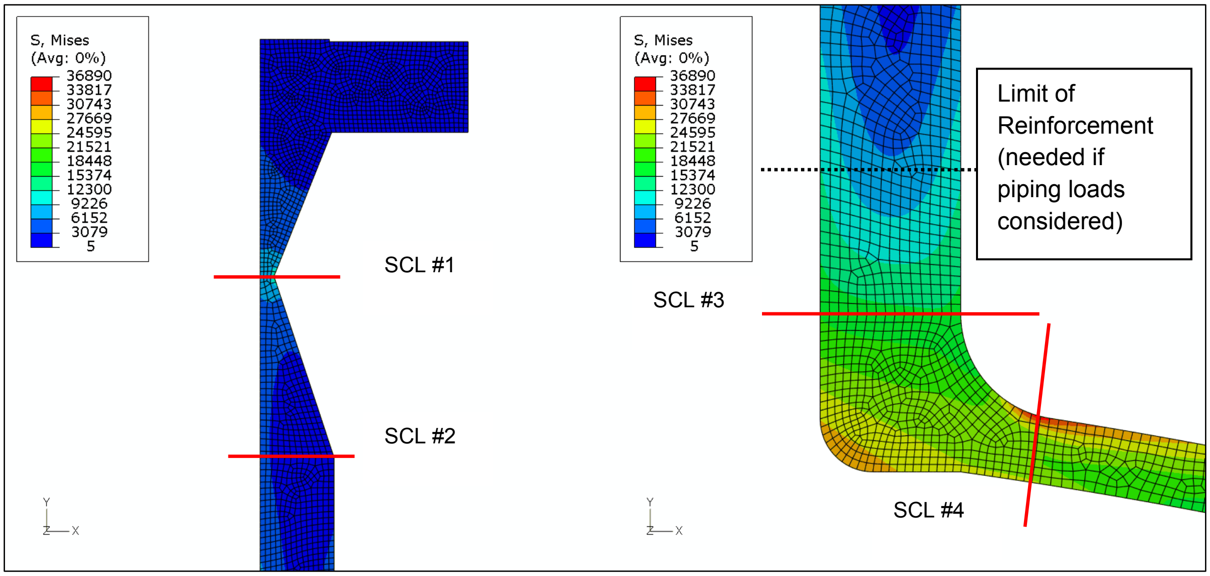


Figure E5.2.1-10 - Stress Classification Lines (SCLs) in the Nozzle to Shell Region

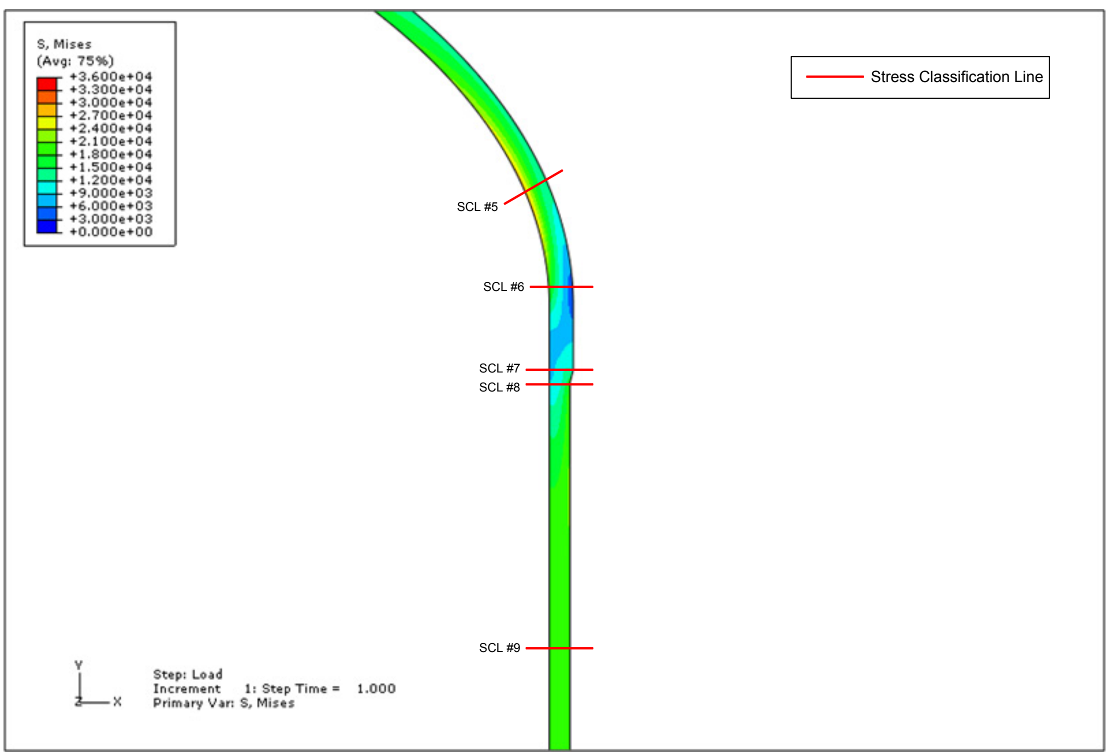


Figure E5.2.1-11 - Stress Classification Lines (SCLs) in the Head to Shell Region and Main Body region

c) STEP 3 – Sum the stress tensors (stresses are added on a component basis) assigned to each equivalent stress category. The final result is a stress tensor representing the effects of all the loads assigned to each equivalent stress category. A detailed stress analysis performed using a numerical method such as finite element analysis typically provides a combination of $P_L + P_b$ and $P_L + P_b + Q + F$ directly.

1) If a load case is analyzed that includes only "load-controlled" loads (e.g. pressure and weight effects), the computed equivalent stresses shall be used to directly represent the P_m, $P_L + P_b$, or $P_L + P_b + Q$. For example, for a vessel subject to internal pressure with an elliptical head; P_m equivalent stresses occur away from the head to shell junction, and P_L and $P_L + P_b + Q$ equivalent stresses occur at the junction.

2) If a load case is analyzed that includes only "strain-controlled" loads (e.g. thermal gradients), the computed equivalent stresses represent Q alone; the combination $P_L + P_b + Q$ shall be derived from load cases developed from both "load-controlled" and "strain-controlled" loads.

3) If the stress in category F is produced by a stress concentration or thermal stress, the quantity F is the additional stress produced by the stress concentration in excess of the nominal membrane plus bending stress. For example, if a plate has a nominal primary membrane equivalent stress of S_e, and has a fatigue strength reduction characterized by a factor K_f, then: $P_m = S_e, P_b = 0$, $Q = 0$, and $F = P_m\left(K_f - 1\right)$. The total equivalent stress is $P_m + F$.

d) STEP 4 – Determine the principal stresses of the sum of the stress tensors assigned to the equivalent stress categories, and compute the equivalent stress using Equation (5.1).

e) STEP 5 – To evaluate protection against plastic collapse, compare the computed equivalent stress to their corresponding allowable values (see paragraph 5.2.2.4). See Table E5.2.1-1 for evaluation results.

Note: All bending stresses exhibited in this example fall under the category of secondary stress, Q; therefore P_b is zero for all cases in this example.

$$P_m \le S$$

$$P_L \le 1.5S$$

$$\left(P_L + P_b\right) \le 1.5S$$

Table E5.2.1-1 - Results of the Elastic Analysis Using Criterion from Step 5 of Paragraph 5.2.2.4 of the 2013 Section VIII, Div 2, Part 5 ASME Code

SCL No.	Location	2013 Div 2		Equivalent Linearized Stresses			Stress Evaluation		
		Material	S_m	P_m	P_L	P_b	$P_m \le S_m$	$P_L \le 1.5S_m$	$P_L + P_b \le 1.5S_m$
1	Flange/nozzle connection	A 105	23300	N/A	6476	N/A	N/A	Pass	Pass
2	Upper nozzle transition	A 105	23300	N/A	1118	N/A	N/A	Pass	Pass
3	Nozzle to shell junction	A 105	23300	N/A	16077	N/A	N/A	Pass	Pass
4	Shell to nozzle junction	A 516-70N	24550	N/A	22232	N/A	N/A	Pass	Pass
5	Elliptical head knuckle	A 516-70N	24550	N/A	18025	N/A	N/A	Pass	Pass
6	Head tangent line	A 516-70N	24550	N/A	9613	N/A	N/A	Pass	Pass
7	Head to shell transition	A 516-70N	24550	N/A	10048	N/A	N/A	Pass	Pass
8	Shell to head transition	A 516-70N	24550	N/A	10985	N/A	N/A	Pass	Pass
9	Shell (away from discontinuities)	A 516-70N	24550	19028	N/A	N/A	Pass	N/A	N/A

5.2.2 Example E5.2.2 – Limit Load Analysis

Evaluate the vessel top head and shell region given in Example Problem E5.2.1 for compliance with respect to the limit load analysis criteria for plastic collapse provided in paragraph 5.2.3.

a) STEP 1 – Develop a numerical model of the component including all relevant geometry characteristics. The model used for the analysis shall be selected to accurately represent the component geometry, boundary conditions, and applied loads.

   The same model was used as in Example Problem E5.2.1, see Figure E5.2.1-3.

b) STEP 2 – Define all relevant loads and applicable load cases. The loads to be considered in the analysis shall include, but not be limited to, those given in Table 5.1.

   The only load to be considered is internal pressure factored according to Table 5.4, $1.5(P) = 630\ psig$. Associated thrust load was factored accordingly.

c) STEP 3 – An elastic-perfectly plastic material model with small displacement theory was used in the analysis. The von Mises yield function and associated flow rule was utilized with a yield strength defining the plastic limit shall equal $1.5S$.

   Material and loading keywords used in the ABAQUS input file are shown in Figure E5.2.2-1.

```
**
** MATERIALS
**
*Material, name=SA105
*Elastic
 2.91e+07, 0.3
*plastic
34950., 0.
*Material, name=SA51670N
*Elastic
 2.878e+07, 0.3
*plastic
36825., 0.
** ----------------------------------------------------------------
**
** STEP: Internal pressure 420 psig
**
*Step, name="Internal pressure 420 psig", nlgeom=no
*Static
0.1, 1., 1e-05, 1.
**
** BOUNDARY CONDITIONS
**
** Name: BC-1 Type: Symmetry/Antisymmetry/Encastre
*Boundary
_PickedSet4, YSYMM
**
** LOADS
**
** Name: Load-1   Type: Pressure
*Dsload
_PickedSurf5, P, 630.
** Name: end thrust   Type: Pressure
*Dsload
_PickedSurf6, P, -1473.45
**
```

Figure E5.2.2-1 - Material Properties and Loading Conditions from ABAQUS Input File

d) STEP 4 – Determine the load case combinations to be used in the analysis using the information from STEP 2 in conjunction with Table 5.4. Each of the indicated load cases shall be evaluated. The effects of one or more loads not acting shall be investigated. Additional load cases for special conditions not included in Table 5.4 shall be considered, as applicable.

No additional load cases are applicable in this example.

e) STEP 5 – Perform a limit-load analysis for each of the load case combinations defined in STEP 4. If convergence is achieved, the component is stable under the applied loads for this load case. Otherwise, the component configuration (i.e. thickness) shall be modified or applied loads reduced and the analysis repeated. Note that if the applied loading results in a compressive stress field within the component, buckling failure should be addressed (see paragraph 5.4).

The results of the analysis are shown in Figure E5.2.2-2, convergence was achieved at the factored load condition; therefore the vessel passes limit load analysis for this load case. Pressure versus displacement for this case is shown in Figure E5.2.2-3.

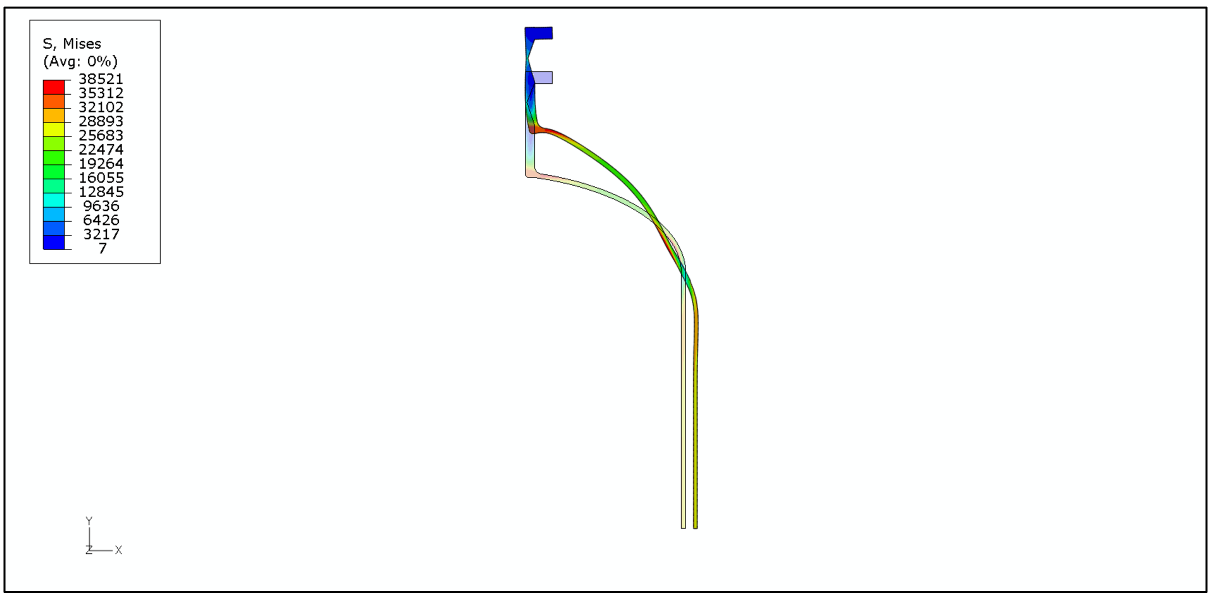


Figure E5.2.2-2 - Results of the Limit load Analysis at Factored Load of 630 psi

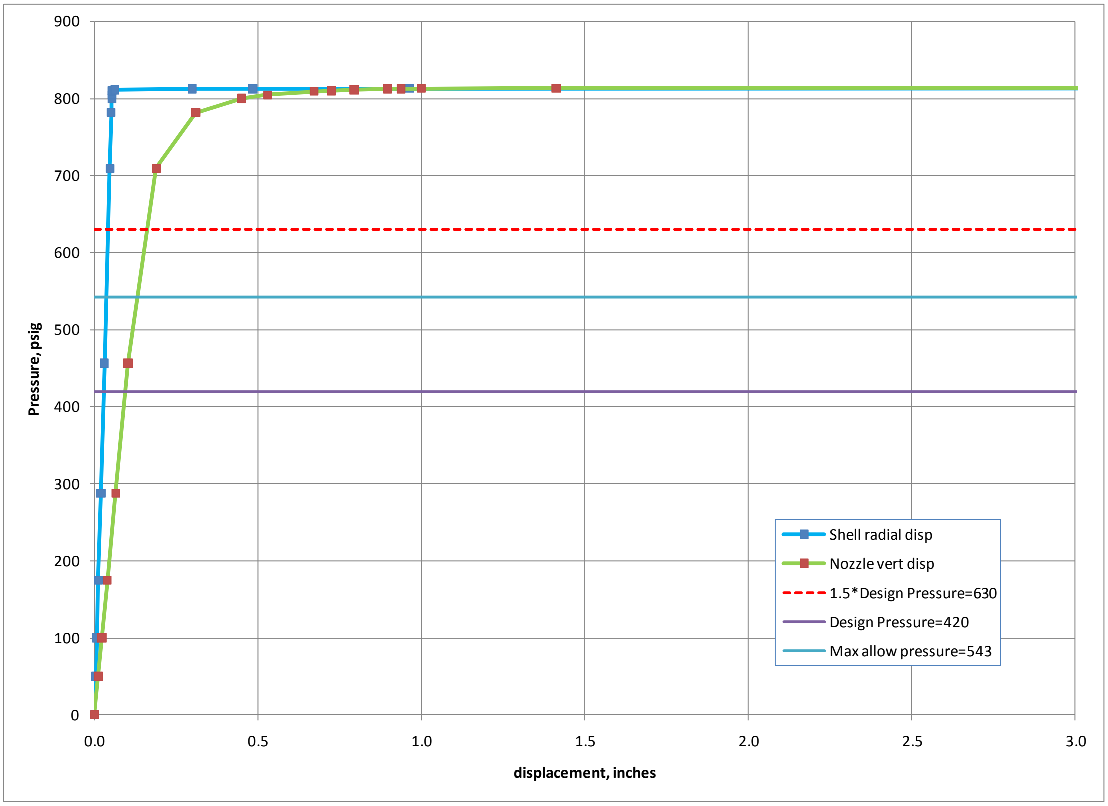


Figure E5.2.2-3 - Pressure vs. Displacement for Limit Load Case

5.2.3 Example E5.2.3 – Elastic-Plastic Analysis

Evaluate the vessel top head and shell region given in Example Problem E5.2.1 for compliance with respect to the elastic-plastic analysis criteria for plastic collapse provided in paragraph 5.2.4.

a) STEP 1 – Develop a numerical model of the component including all relevant geometry characteristics. The model used for the analysis shall be selected to accurately represent the component geometry, boundary conditions, and applied loads. In addition, refinement of the model around areas of stress and strain concentrations shall be provided. The analysis of one or more numerical models may be required to ensure that an accurate description of the stress and strains in the component is achieved.

   The same model was used as in Example Problem E5.2.1, see Figure E5.2.1-3.

b) STEP 2 – Define all relevant loads and applicable load cases. The loads to be considered in the design shall include, but not be limited to, those given in Table 5.1.

   The only load to be considered is internal pressure factored according to Table 5.5 for the global criterion, 2.4(P) = 1008 psig. Associated thrust load was factored accordingly.

c) STEP 3 – An elastic-plastic material model shall be used in the analysis. The von Mises yield function and associated flow rule was utilized. A material model that includes hardening or softening, or an elastic-perfectly plastic model may be utilized. A true stress-strain curve model that includes temperature dependent hardening behavior is provided in Annex 3.D. When using this material model, the hardening behavior shall be included up to the true ultimate stress and perfect plasticity behavior (i.e. the slope of the stress-strain curves is zero) beyond this limit. The effects of non-linear geometry shall be considered in the analysis.

   The true stress-strain curve from Annex 3.D was used for the analysis. The material and loading keywords used in the ABAQUS input file are shown below.

```
** MATERIALS
**
*Material, name=SA105
*Elastic
 2.91e+07, 0.3
*PLASTIC
**
**  Prager Stress-Strain Curve - Units: (psi), (in/in), (F)
**                             Temperature = 125 (F)
**                            Yield stress = 34404.214 (psi)
**               Ultimate Tensile Strength = 69836.572 (psi)
**                     YS Reduction Factor = 1.0000
**                    UTS Reduction Factor = 1.0000
**                         Young's Modulus = 2.910E+07 (psi)
**
  34404.214,      0.0000,    125.0000
  37576.981,   2.035E-03,    125.0000
  40749.748,   5.551E-03,    125.0000
  43922.515,      0.0118,    125.0000
  47095.282,      0.0213,    125.0000
  50268.049,      0.0324,    125.0000
  53440.816,      0.0432,    125.0000
  56613.583,      0.0539,    125.0000
  59786.350,      0.0653,    125.0000
  62959.117,      0.0778,    125.0000
  66131.884,      0.0917,    125.0000
```

```
  69304.651,      0.1073,    125.0000
  72477.418,      0.1245,    125.0000
  75650.185,      0.1436,    125.0000
  78822.952,      0.1647,    125.0000
  81995.719,      0.1877,    125.0000
  85168.486,      0.2129,    125.0000
  88341.253,      0.2404,    125.0000
  91514.020,      0.2702,    125.0000
  94686.787,      0.3024,    125.0000
**
*Material, name=SA51670N
*Elastic
 2.878e+07, 0.3
*PLASTIC
**
**  Prager Stress-Strain Curve - Units: (psi), (in/in), (F)
**                           Temperature = 125 (F)
**                          Yield stress = 36315.560 (psi)
**             Ultimate Tensile Strength = 69836.572 (psi)
**                   YS Reduction Factor = 1.0000
**                  UTS Reduction Factor = 1.0000
**                       Young's Modulus = 2.878E+07 (psi)
**
  36315.560,      0.0000,    125.0000
  39306.562,   1.945E-03,    125.0000
  42297.564,   5.227E-03,    125.0000
  45288.567,      0.0110,    125.0000
  48279.569,      0.0197,    125.0000
  51270.571,      0.0302,    125.0000
  54261.574,      0.0407,    125.0000
  57252.576,      0.0509,    125.0000
  60243.578,      0.0615,    125.0000
  63234.581,      0.0732,    125.0000
  66225.583,      0.0862,    125.0000
  69216.586,      0.1008,    125.0000
  72207.588,      0.1170,    125.0000
  75198.590,      0.1350,    125.0000
  78189.593,      0.1549,    125.0000
  81180.595,      0.1767,    125.0000
  84171.597,      0.2006,    125.0000
  87162.600,      0.2267,    125.0000
  90153.602,      0.2551,    125.0000
  93144.604,      0.2860,    125.0000
** ----------------------------------------------------------------
**
** STEP: Internal pressure factored load of 1008 psig
**
*Step, name="Internal pressure 1008 psig", nlgeom
*Static
0.05, 1., 1e-05, 1.
**
** BOUNDARY CONDITIONS
**
** Name: BC-1 Type: Symmetry/Antisymmetry/Encastre
*Boundary
```

```
_PickedSet4, YSYMM
**
** LOADS
**
** Name: Load-1   Type: Pressure
*Dsload
_PickedSurf5, P, 1008.
** Name: end thrust   Type: Pressure
*Dsload
_PickedSurf6, P, -2357.52
**
```

d) STEP 4 – Determine the load case combinations to be used in the analysis using the information from STEP 2 in conjunction with Table 5.5.

No additional load cases are applicable in this example.

e) STEP 5 – Perform an elastic-plastic analysis for each of the load cases defined in STEP 4. If convergence is achieved, the component is stable under the applied loads for this load case. Otherwise, the component configuration (i.e. thickness) shall be modified or applied loads reduced and the analysis repeated. Note that if the applied loading results in a compressive stress field within the component, buckling may occur, and an evaluation in accordance with paragraph 5.4 may be required.

The results of the analysis are shown in Figure E5.2.3-2, convergence was achieved therefore vessel passes elastic-plastic analysis for this load case. Pressure vs. radial displacement for this case is shown in Figure E5.2.3-3.

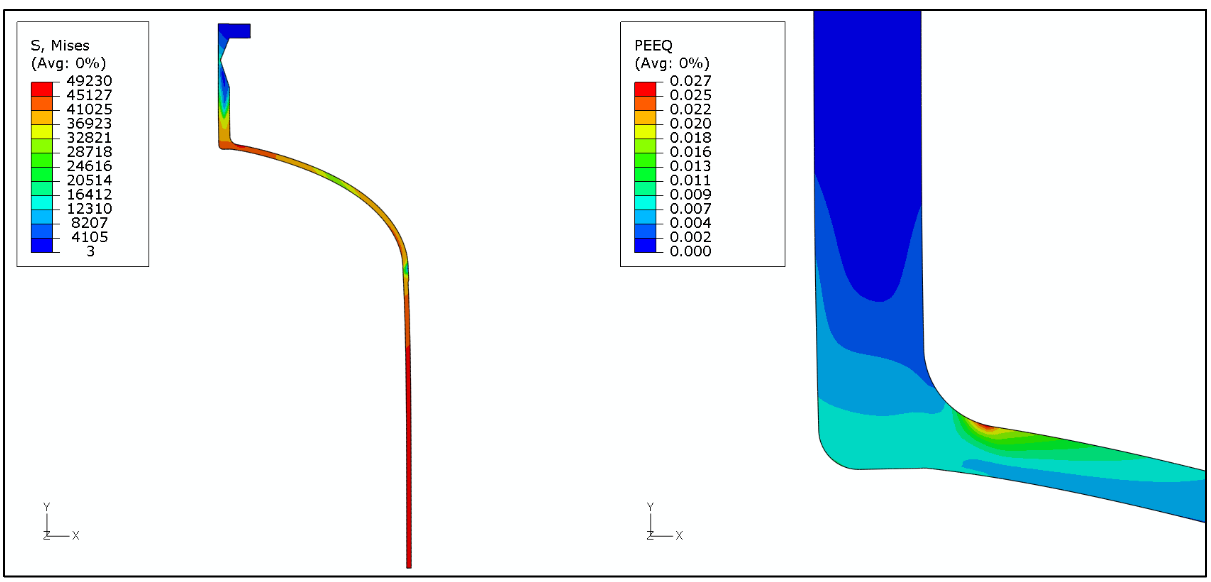


Figure E5.2.3-2 - Results of the Elastic-Plastic Analysis at Factored Load of 1008 psi; vonMises Stress and Equivalent Plastic Strain

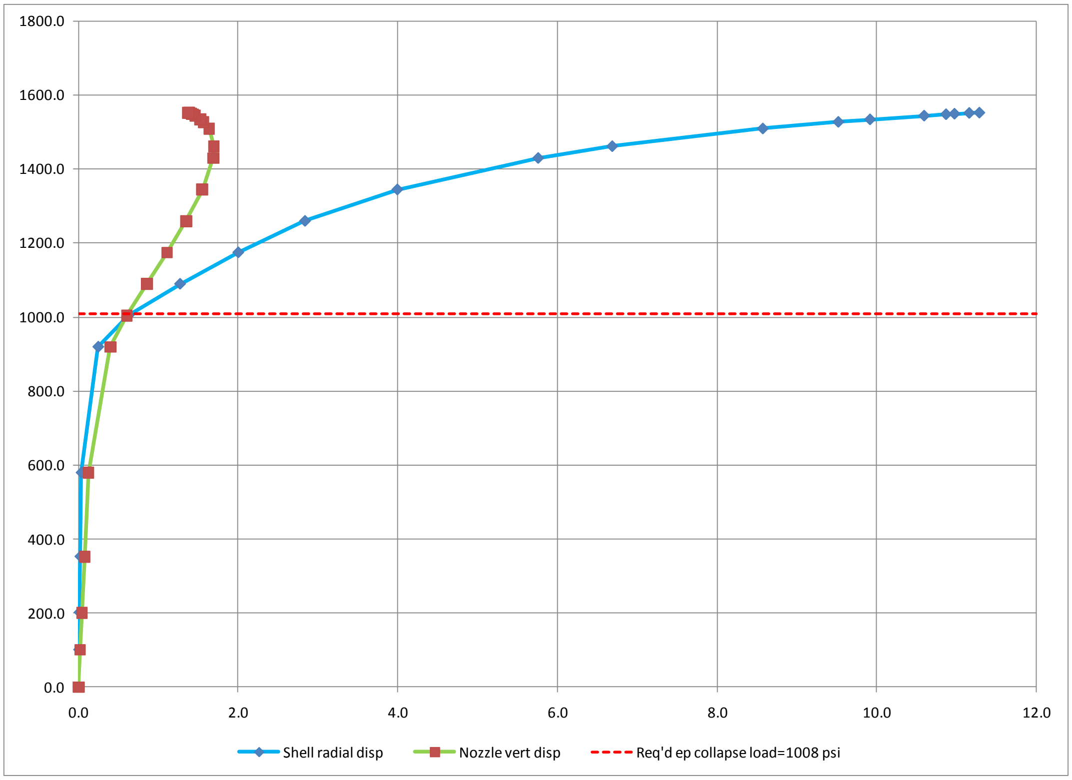


Figure E5.2.3-3 - Pressure vs. Displacement for Elastic-Plastic Case

5.3 Protection Against Local Failure

5.3.1 Overview

Evaluate the vessel top head and shell region given in Example Problem E5.2.1 for compliance with respect to the elastic and elastic-plastic local failure criteria provided in paragraphs 5.3.2 and 5.3.3.

The same model and material conditions were used as in Example Problem E5.2.1.

5.3.2 Example E5.3.2 – Elastic Analysis

In addition to demonstrating protection against plastic collapse, the following elastic analysis criterion shall be satisfied for each point in the component. The sum of the local primary membrane plus bending principal stresses shall be used for checking this criterion.

$$(\sigma_1 + \sigma_2 + \sigma_3) \leq 4S$$

$$4S = 93.2\ ksi$$

The results for Example Problem E5.2.1 (elastic analysis) were evaluated using the above criterion. This analysis revealed that the model satisfies the elastic local failure criterion, see Figure E5.3.2-1. Note that the total (memb+bend+peak) principal stress summation as shown on the contour plot satisfied the allowable limit. Therefore, linearization of the elastic stresses was not necessary to satisfy the elastic local failure check. The elastic-plastic local failure criterion is evaluated in 5.3.3 for illustration since the component was shown to be acceptable based upon the elastic local failure results.

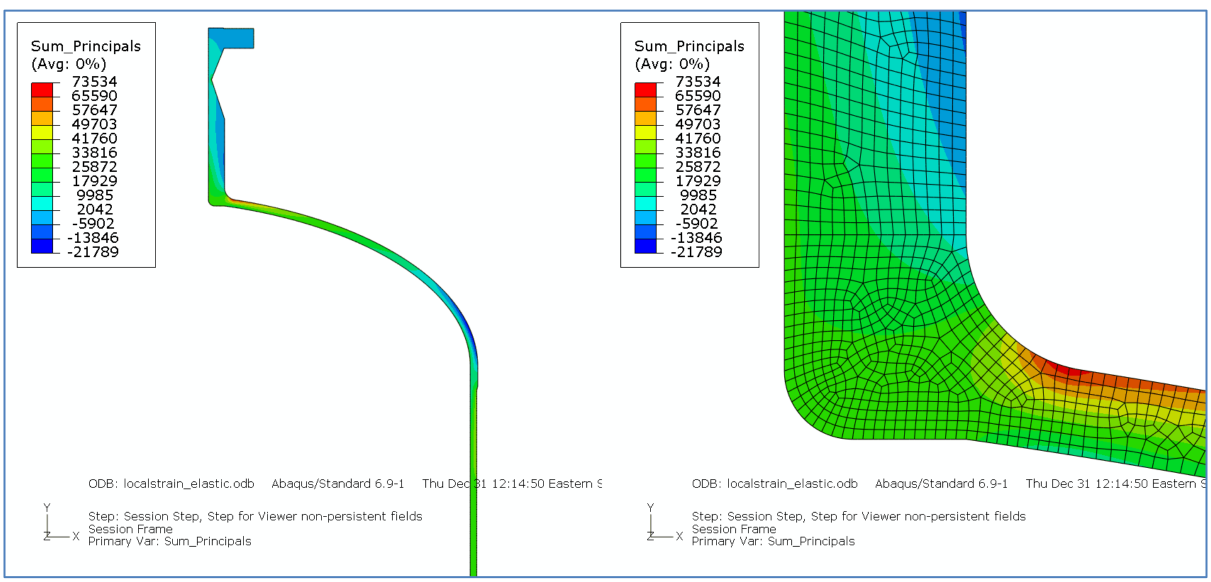


Figure E5.3.2-1 - Local Failure Elastic Analysis Results @ 420 psig

5.3.3 Example E5.3.3 – Elastic-Plastic Analysis

The following procedure shall be used to evaluate protection against local failure for a sequence of applied loads.

a) STEP 1 – Perform an elastic-plastic stress analysis based on the load case combinations for the local criterion given in Table 5.5. The effects of non-linear geometry shall be considered in the analysis.

The same model and material conditions were used as Example Problem E5.2.3. The only load to be considered is internal pressure factored according to Table 5.5 for the local criterion, $1.7(P) = 714\ psig$. Associated thrust load was factored accordingly.

b) STEP 2 – For a location in the component subject to evaluation, determine the principal stresses, σ_1, σ_2, σ_3, the equivalent stress, σ_e, using Equation (5.1) and the total equivalent plastic strain, ε_{peq}.

Values for the principle stresses, equivalent stress and total equivalent plastic strain for each point in the model were extracted from the ABAQUS .odb file. The following example calculation is for one integration point in the model. The full model (all integration points) will be evaluated using the "user defined variable" option available in ABAQUS to create contour plot variables of the strain limit and strain limit ratio.

The principal stresses to be evaluated are shown below.

$$\sigma_1 = 45095\ psi$$
$$\sigma_2 = 34603\ psi$$
$$\sigma_3 = 1118\ psi$$
$$\sigma_e = 39783\ psi$$

c) STEP 3 – Determine the limiting triaxial strain, ε_L, using Equation (5.6) where ε_{Lu}, m_2, and α_{sl} are determined from Table 5.7.

$$\varepsilon_L = \varepsilon_{Lu} \cdot \exp\left[-\left(\frac{\alpha_{sl}}{1+m_2}\right)\left(\left\{\frac{(\sigma_1+\sigma_2+\sigma_3)}{3\sigma_e}\right\}-\frac{1}{3}\right)\right]$$

The strain limit parameters are shown below.

$$R = 0.5429$$
$$\alpha_{sl} = 2.2$$
$$m_2 = 0.2743$$
$$\varepsilon_{Lu} = m_2$$

The computed limit strain is:

$$\varepsilon_L = 0.1515$$

d) STEP 4 – Determine the forming strain ε_{cf} based on the material and fabrication method in accordance with Part 6. If heat treatment is performed in accordance with Part 6, the forming strain may be assumed to be zero.

The forming strain is:

$$\varepsilon_{cf} = 0$$

e) STEP 5– Determine if the strain limit is satisfied. The location in the component is acceptable for the specified load case if Equation (5.7) is satisfied.

$$\varepsilon_{peq} + \varepsilon_{cf} \le \varepsilon_L$$

The total equivalent plastic strain is:

$$\varepsilon_{peq} = 0.002468$$

Since $\varepsilon_{peq} + \varepsilon_{cf}$ is less than ε_L, the strain at this integration point passes the Elastic-Plastic criterion.

A full model contour plot of the strain limit is shown in Figure E5.3.3-1.

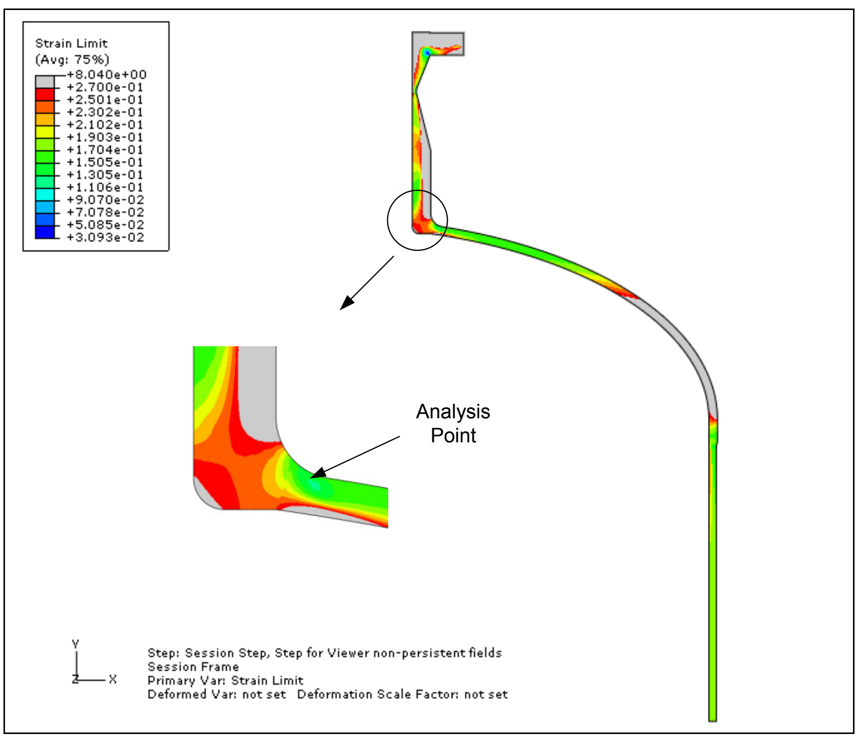


Figure E5.3.3-1 - Contour plot of the Strain Limit, ε_L

A full model contour plot of the equivalent plastic strain is shown in Figure E5.3.3-2

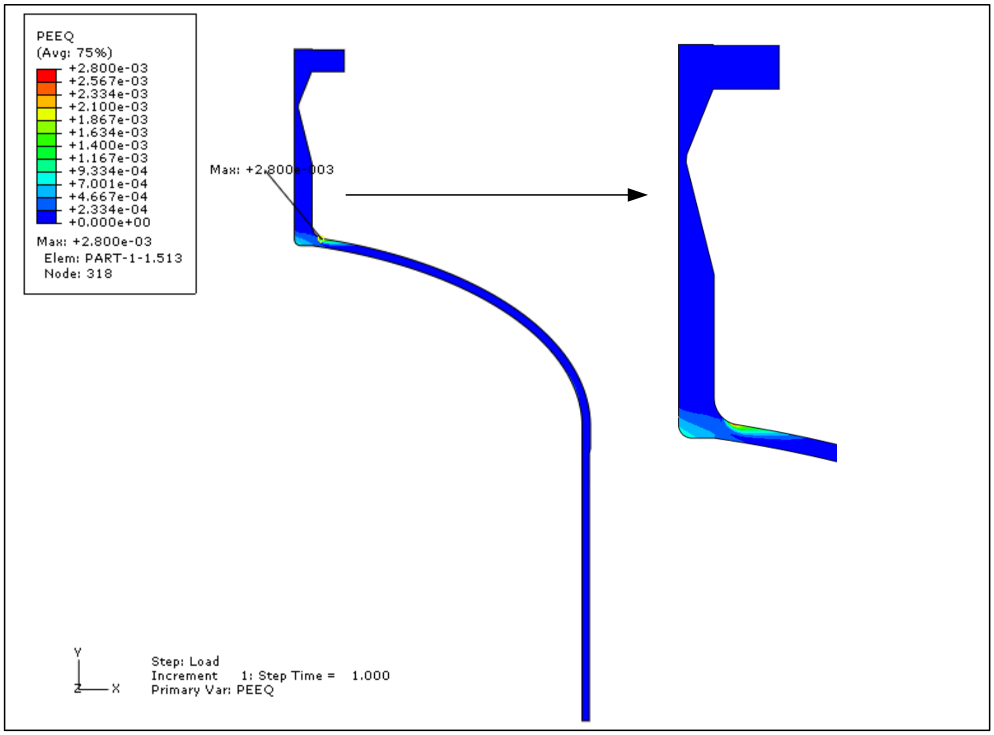


Figure E5.3.3-2 - Contour plot of Equivalent Plastic Strain, ε_{peq}

Full model evaluation of the Elastic-Plastic criterion is shown in Figure 5.3.3-3.

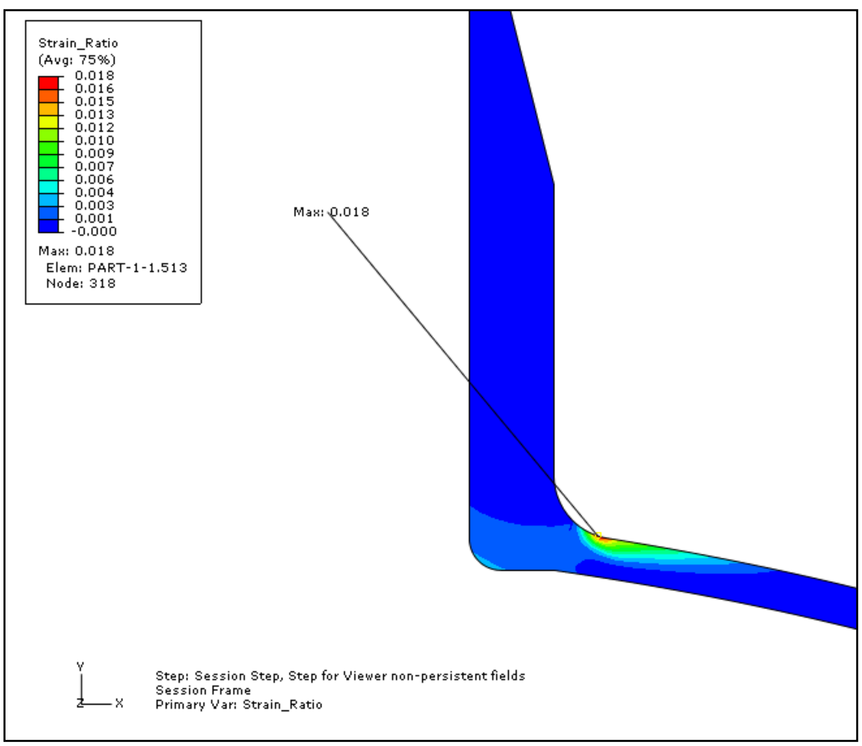


Figure E5.3.3-3 - Elastic-Plastic Local Failure Analysis Results @ 714 psi

Full model evaluation indicates that all integration points meet the criterion of $\varepsilon_{peq} + \varepsilon_{cf} \le \varepsilon_L$. The maximum strain limit ratio $\left[\left(\varepsilon_{peq} + \varepsilon_{cf}\right) / \varepsilon_L\right]$ for this model is 0.018, as indicated in Figure E5.3.3-3. Since this value is less than 1.0 the model passes the elastic-plastic local strain analysis.

5.4 Example E5.4 – Protection Against Collapse from Buckling

Evaluate the following tower, Figure E5.4-1, for compliance with respect to the Type-1 buckling criteria provided in paragraph 5.4.1.2.

- Material – Shell and Heads = SA-516, Grade 70, Normalized
- Design Conditions = -14.7 psig at 300°F
- Corrosion Allowance = 0.125 inches

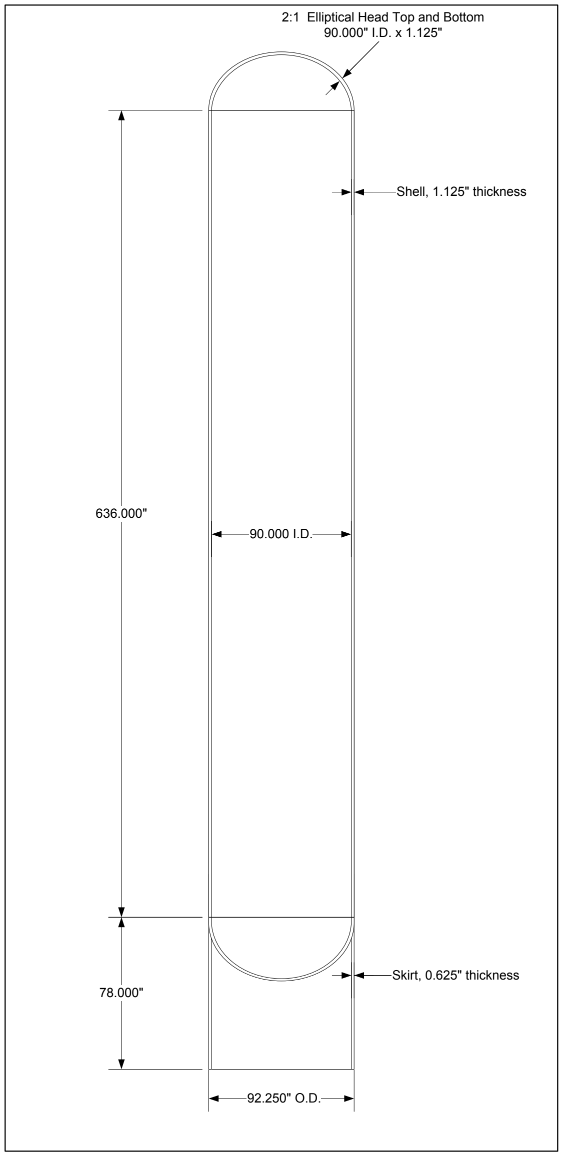


Figure E5.4-1 - Tower Configuration

In addition to evaluating protection against plastic collapse as defined in paragraph 5.2, a design factor for protection against collapse from buckling shall be satisfied to avoid buckling of components with a compressive stress field under applied design loads.

The design factor to be used in a structural stability assessment is based on the type of buckling analysis performed. The following design factor shall be the minimum value for use with shell components when the buckling loads are determined using a numerical solution (i.e. bifurcation buckling analysis or elastic-plastic collapse analysis).

Type 1 – If a bifurcation buckling analysis is performed using an elastic stress analysis without geometric nonlinearities in the solution to determine the pre-stress in the component, a minimum design factor of $\Phi_B = 2/\beta_{cr}$ shall be used (see paragraph 5.4.1.3). In this analysis, the pre-stress in the component is established based on the loading combinations in Table 5.3.

For unstiffened and ring stiffened cylinders and cones under external pressure

$$\beta_{cr} = 0.80$$

$$\Phi_B = \frac{2}{\beta_{cr}} = \frac{2}{0.8} = 2.5$$

a) STEP 1 – Define all relevant loads and applicable load cases. The loads to be considered in the design shall include, but not be limited to, those given in Table 5.3.

   The load case considered in this example includes external pressure of 14.7 psig

b) STEP 2 – Create a finite element model of the tower and apply all relevant loads and boundary conditions.

   1) Create a finite element model of the tower using shell elements, see Figure E5.4-2. Apply appropriate material properties and thicknesses to the components of the tower.

   Note: All possible buckling mode shapes shall be considered in determining the minimum buckling load for the component. Care should be taken to ensure that simplification of the model does not result in exclusion of a critical buckling mode shape.

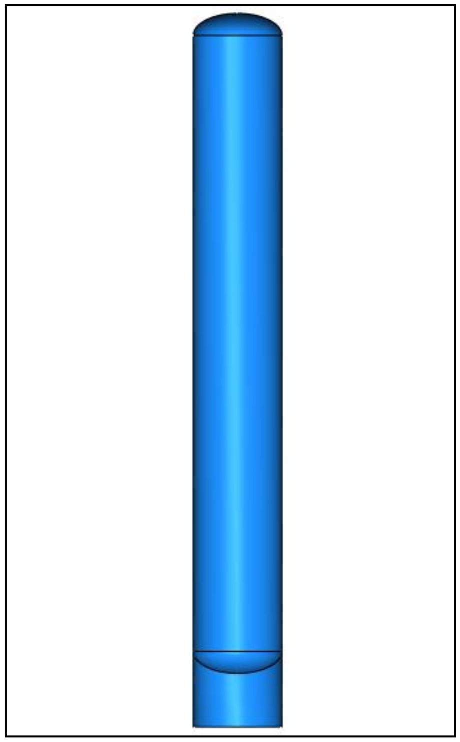


Figure E5.4-2 - Cross-Sectional View of Finite Shell Element Model

2) Generate finite element mesh. For this example, 4-node reduced integration shell elements (ABAQUS - S4R elements) were used, see Figure E5.4-3.

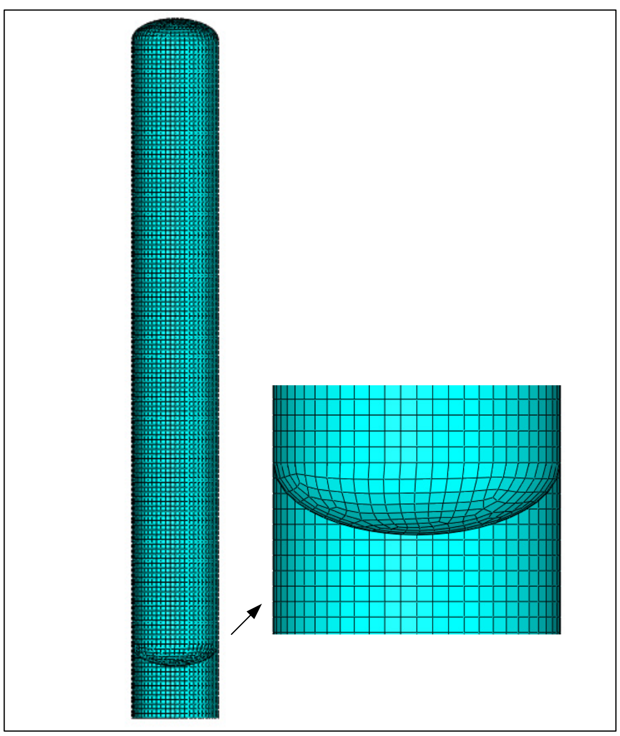


Figure E5.4-3 - Mesh of Tower Components

3) Create a static preload step and apply an external pressure load to the pressure boundary of the tower to be active during this step. Apply a displacement boundary condition to the bottom of the skirt $\left(U_x = U_y = U_z = 0\right)$.

The preload pressure applied=-14.7 psig

4) Create a buckling step and request a minimum of 3 eigenvalues to be solved. Apply a perturbation load to the pressure boundary of the tower to be active in this step, see Figure E5.4-4.

The perturbation pressure applied=-14.7 psig

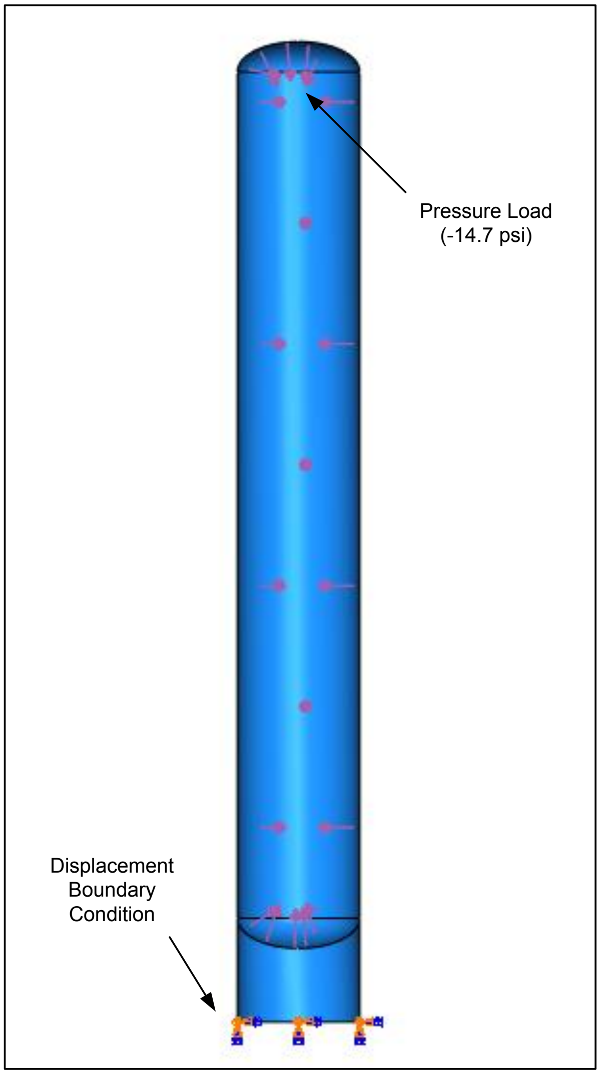


Figure E5.4-4 - Load and Boundary Condition Placement on the Tower

5) Run analysis and review results.

Evaluate mode shapes for proper displacement direction and record calculated eigenvalues, see Figure E5.4-5 for first mode shape and Table E5.4-1 for calculated eigenvalues.

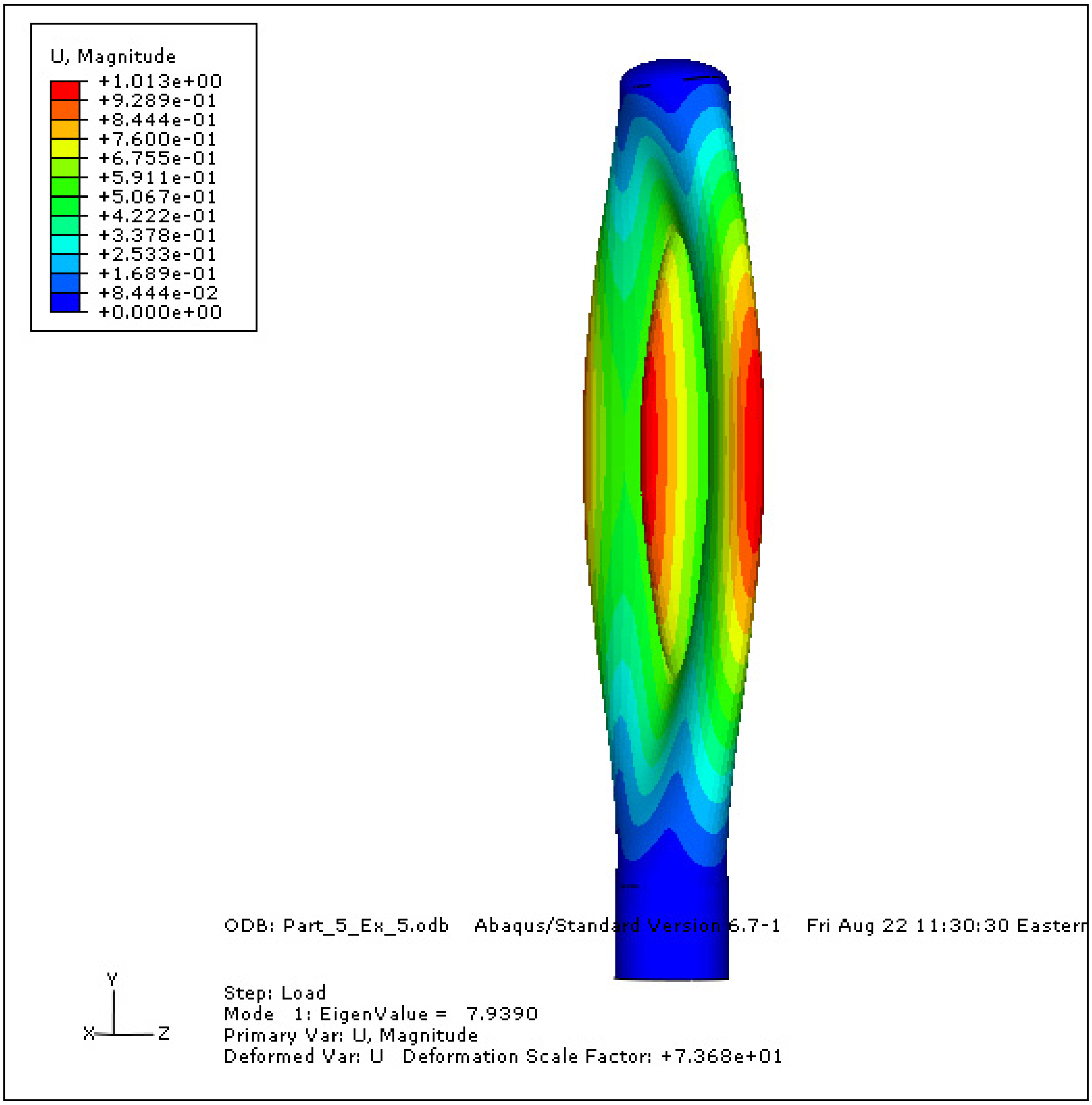


Figure E5.4-5 - First Mode Shape

Table E5.4-1 - Eigenvalue Results

Mode	Eigenvalues
1	7.939
2	7.940
3	14.351

6) Calculate buckling load or maximum allowable vacuum pressure (MAVP) using design factors and the following equation:

$$Buckling\ Load = \frac{("Dead"Loads) + Eigenvalue*("Live"Loads)}{Design\ Factor}$$

"Dead" Loads = Total Load (Preload) before *BUCKLE STEP

"Live" Loads = Incremental (Perturbation) Load in *BUCKLE STEP

$$Buckling\ Load = \frac{-14.7 + 7.939(-14.7)}{2.5} = -52.6\ psig$$

7) Since the maximum allowable external pressure of -52.6 psig is less than the external design pressure of -14.7 psig, the structure is acceptable with respect to buckling under the design conditions.

5.5 Protection Against Failure from Cyclic Loading

5.5.1 Overview

A Fatigue evaluation shall be performed if a component is subject to cyclic operation.

5.5.2 Example E5.5.2 – Fatigue Screening

Evaluate the Vessel top head and shell region given in Example E5.2.1 in accordance with the fatigue screening methodology provided in paragraph 5.5.2.4. The cyclic loading design requirements given in the Users' Design Specification are provided below:

- Operating pressure = 380 psig at 125^{o}F
- Corrosion Allowance = 0.125 inches
- Cyclic Life Requirement = 20000 full pressure cycles
- Number of Shutdowns/Startups = 20

a) STEP 1 - Determine a load history for vessel.

Per the User's Design Specification, a full internal pressure cycle is the only applicable event to be considered. The vessel internal pressure is expected to cycle 20000 times between 0 psig and the operating pressure, 380 psig. This cycling occurs at a constant temperature of 125^{o}F.

b) STEP 2 - Determine the fatigue screening factors, C_1 and C_2.

Per Table 5.10 of the Code, the applicable factors for this assessment are determined for a vessel of Integral Construction. As there are no nozzles or attachments in the knuckle region of a formed head, the factors are:

$$C_1 = 3$$
$$C_2 = 2$$

c) STEP 3 - Determine the number of full range pressure cycles, $N_{\Delta FP}$, and check initial screening criteria.

$$N_{\Delta FP} = \text{Number of Full Range Pressure Cycles + Number of Shutdowns}$$

$$N_{\Delta FP} = 20000 + 20 = 20020\ Cycles$$

To check the initial screening criteria, $N_{\Delta FP} \le N\left(C_1 S\right)$, the number of allowable cycles for a pressure range of $C_1 S$ must be obtained from Annex 3.F, where S is the allowable stress at the design temperature and C_1 is from Step 2. Based on this equation, the screening criteria will be more restrictive for the shell/head material as the higher allowable stress will yield a lower number of cycles. For information, the calculations will be shown for both materials.

The allowable number of cycles for each component is calculated using Equation (3.F.1):

$$N = \left(10\right)^X$$

Where,

$$X = \frac{C_1 + C_3 Y + C_5 Y^2 + C_7 Y^3 + C_9 Y^4 + C_{11} Y^5}{1 + C_2 Y + C_4 y^2 + C_6 Y^3 + C_8 Y^4 + C_{10} Y^5}$$

$$Y = \left(\frac{S_{alt,k}}{C_{us}}\right)\left(\frac{E_{FC}}{E_T}\right)$$

E_T = the material modulus of elasticity at the cycle temperature

E_{FC} = the modulus of elasticity used to establish the design fatigue curve

S_a = the stress amplitude, which is equal to $C_1 S$ for the screening criteria as determined above.

$C_{us} = 1$ (units of stress are in ksi).

For the vessel materials of construction, the coefficients C_i and the modulus E_{FC} are for the smooth bar fatigue curve for carbon steel with cycle temperature below $700°F$ and $\sigma_{uts} \le 80\ ksi$ and are listed in Table 3.F.1. The calculated allowable number of cycles is shown in Table E5.5.2-1 for each vessel region.

Table E5.5.2-1 - Allowable Number of Cycles, $N(C_1 S)$

Component	**Material**	$E_T\ (ksi)$	$E_{FC}\ (ksi)$	$S\ (ksi)$	$S_a\ (ksi)\ C_1 S$	X	$N(C_1 S)$
Nozzle	SA-105	2.910E+04	2.83E+04	23.30	69.90	3.234	1713
Head/Shell	SA-516-70N	2.878E+04	2.83E+04	24.55	73.65	3.156	1431

Check to see if the criteria of $N_{\Delta FP} \le N(C_1 S)$ is satisfied:

$$20020\ cycles \le 1431 cycles \qquad False$$

The criterion is not satisfied, and therefore, a detailed fatigue analysis of the vessel is required. The subsequent screening steps for paragraph 5.5.2.4 of the code need not be evaluated.

5.5.3 Example E5.5.3 – Elastic Stress Analysis, and Equivalent Stresses

Evaluate the vessel top head and shell base metal regions given in Example E5.5.2 in accordance with the fatigue methodology provided in paragraph 5.5.3. Note that the nozzle to head weld is machined and subjected to full volumetric examination and both ID and OD surfaces receive MT/PT and VT. The shell to head weld is in the as-welded condition and the OD surface receive the same inspection as above. The ID surface receives only full volumetric examination.

a) STEP 1 - Determine a load history for vessel.

Per the User's Design Specification (see Example E5.5.2), a full internal pressure cycle is the only applicable event to be considered. The vessel internal pressure will cycle between 0 psig and the operating pressure, 380 psig.

b) STEP 2 - Determine the individual stress-strain cycles and cyclic stress ranges.

Since the only event under consideration is a full internal pressure cycle, the applicable cyclic stress range is between the stress in the vessel at $0\ psig$ internal pressure and the stress in the vessel at $380\ psig$.

c) STEP 3 - Determine the equivalent stress range for the cycle determined in STEP 2.

As thermal loads are not applicable, $\Delta\sigma_{ij,k}^{LT}=0$ and $\Delta S_{LT,k}=0$. Thus, the equations for the component stress range and equivalent stress range reduce to the form of the equations below from paragraph 5.5.3:

$$\Delta\sigma_{ij,k} = {}^{m}\sigma_{ij,k} - {}^{n}\sigma_{ij,k}$$

$$\Delta S_{P,k} = \frac{1}{\sqrt{2}}\left[\begin{array}{l}\left(\Delta\sigma_{11,k}-\Delta\sigma_{22,k}\right)^2+\left(\Delta\sigma_{11,k}-\Delta\sigma_{33,k}\right)^2+\\ \left(\Delta\sigma_{22,k}-\Delta\sigma_{33,k}\right)^2+6\left(\Delta\sigma_{12,k}^2+\Delta\sigma_{13,k}^2+\Delta\sigma_{23,k}^2\right)\end{array}\right]^{0.5}$$

Further, since the start point in the loading cycle is $0\ psig$ internal pressure, the initial stress, ${}^{m}\sigma_{ij,k}$, is 0, and the component stress range equation reduces to $\Delta\sigma_{ij,k}=-{}^{n}\sigma_{ij,k}$, and the equivalent stress range equation becomes:

$$\Delta S_{P,k} = \frac{1}{\sqrt{2}}\left[\begin{array}{l}\left({}^{n}\sigma_{22,k}-{}^{n}\sigma_{11,k}\right)^2+\left({}^{n}\sigma_{33,k}-{}^{n}\sigma_{11,k}\right)^2+\\ \left({}^{n}\sigma_{33,k}-{}^{n}\sigma_{22,k}\right)^2+6\left({}^{n}\sigma_{12,k}^2+{}^{n}\sigma_{13,k}^2+{}^{n}\sigma_{23,k}^2\right)\end{array}\right]^{0.5}$$

The above equation is the von Mises equivalent stress at the end point of the cycle. This stress can be calculated using an elastic finite element analysis of the vessel, applying the loads at the end point of the cycle ($380\ psig$ internal pressure).

If the effect of a weld is not accounted for in the numerical model, a fatigue strength reduction factor, K_f shall be included as per the equation below, and applied to all of the linearized membrane plus bending stress components, unless more detailed test data is available. Recommended values for K_f are provided in Table 5.11.

For the machined nozzle to head weld a K_f of 1.0 is applicable. For the as-welded head to shell weld, K_f values of 3.0 and 1.2 are applicable for the ID and OD surfaces, respectively.

$$\Delta\sigma_{ij,k} = K_f \left({}^{m}\sigma_{ij,k}^{MB} - {}^{n}\sigma_{ij,k}^{MB} \right)$$

Finite Element Model:

The axisymmetric model was taken from Example E5.2.1 (see Figures 5.2.1-3 – 5.2.1-6). The pressure load was modified to the $380\ psig$ operating pressure and the nozzle thrust load was adjusted accordingly. The component geometry was modeled in the corroded condition.

Elastic Analysis Results:

A plot of the equivalent stress range is shown in Figure E5.5.3-1. The maximum stress locations are shown in the figure.

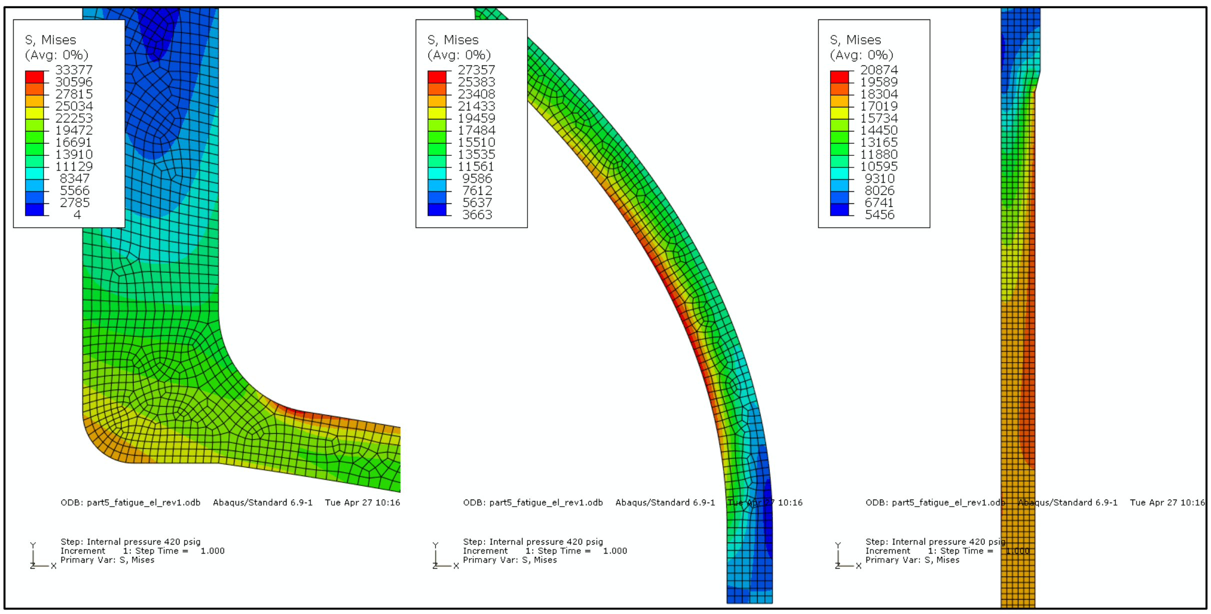


Figure E5.5.3-1 - Equivalent Stress Range (380 psig internal pressure)

The total stress at each of the base metal locations was read directly from the finite element output and is taken as the equivalent stress range $\Delta S_{P,k}$ for the component. Linearized membrane plus bending equivalent stresses were determined to evaluate the need for a fatigue penalty factor and for application of an appropriate fatigue strength reduction factor (FSRF) for weld locations.

In order to determine $K_{e,k}$, the fatigue penalty factor, S_{PS} and $\Delta S_{n,k}$ must first be calculated using the method of Paragraph 5.5.6.1. $\Delta S_{n,k}$ is also used for the application of the FSRF, where $\Delta S_{n,k}$ is the equivalent stress range of the linearized primary membrane, primary bending, and secondary stresses $(P_L + P_B + Q)$ for each location under evaluation. The linearization of stresses for calculation of these stresses was performed using the guidance of Annex 5.A. The Stress Classification Lines used are shown in Figure E5.5.3-2. Results are shown in Table E5.5.3-1.

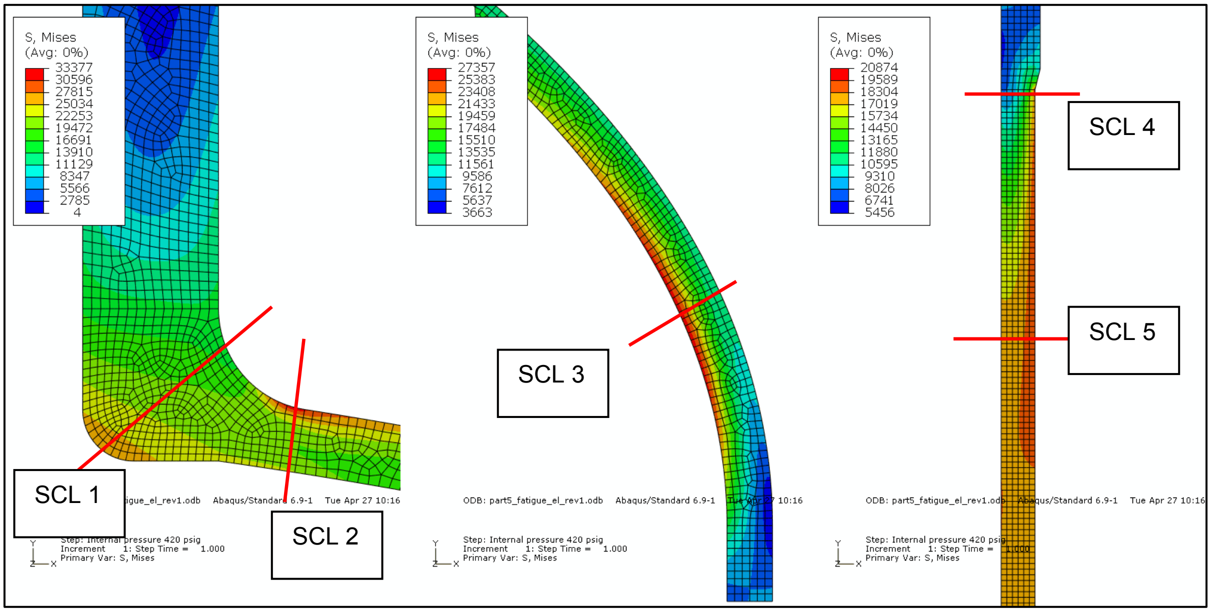


Figure E5.5.3-2 - Stress Classification Lines for Calculation of $\Delta S_{n,k}$

Table E5.5.3-1 - Calculated Values for Primary plus Secondary Equivalent Stress Range

Component	SCL	Location	$\Delta S_{n,k}\ (ksi)$
Nozzle	1	Nozzle ring at inside corner radius	27.060
Nozzle	2	Nozzle transition at OD blend radius	29.877
Head	3	Head knuckle ID	26.821
Shell	4	Head to shell weld ID	7.637
Shell	4	Head to shell weld OD	17.193
Shell	5	Base metal	19.938

d) STEP 4 - Determine the effective alternating equivalent stress amplitude for the cycle, using the stresses calculated in STEP 3.

As noted in STEP 3, there are no thermal effects ($\Delta S_{LT,k} = 0$). Therefore, the alternating stress is calculated as follows:

$$S_{alt,k} = \frac{K_{e,k} \cdot \Delta S_{P,k}}{2}$$

S_{PS} is defined as $\max\left[3S,\ 2S_y\right]$, where S is the material allowable stress at the cycle temperature and S_y is the material yield strength at the cycle temperature $\left(125°F\right)$. Table E5.5.3-2 lists the values for S_{PS} for each component.

Table E5.5.3-2 - Calculated Values for S_{PS}

Component	$S\ (ksi)$	$S_y\ (ksi)$	$S_{PS}\ (ksi)$
Nozzle	23.30	34.90	69.90
Head	24.55	36.85	73.70
Shell	24.55	36.85	73.70

Comparing $\Delta S_{n,k}$ to S_{PS} shows that $\Delta S_{n,k} \le S_{PS}$ for all components, and therefore $K_{e,k} = 1.0$ in each case. The effective alternating equivalent stress amplitudes can then be calculated and are listed in Table E5.5.3-3. Note that the total stress is reported in Table E5.5.3-3 for locations that do not have a fatigue penalty factor applied.

Table E5.5.3-3 - Component Effective Alternating Equivalent Stress Amplitudes $S_{alt,k}$

Component	Location	K_f	$K_{e,k}$	$\Delta S_{P,k}$ (ksi)	$S_{alt,k}$ (ksi)
Nozzle	Nozzle ring at inside corner radius base metal	1.0	1.0	27.263	13.632
Nozzle	Nozzle transition at OD blend radius machined weld	1.0	1.0	31.799	15.900
Head	Head knuckle ID base metal	1.0	1.0	27.356	13.678
Shell	Head to shell weld ID as-welded	3.0	1.0	7.637	11.456
Shell	Head to shell weld OD as-welded	1.2	1.0	17.193	10.316
Shell	base metal	1.0	1.0	19.727	9.863

e) STEP 5 - Determine the permissible number of cycles, N_k, for the alternating equivalent stress computed in STEP 4, using the fatigue curves provided in Annex 3.F.

The allowable number of cycles for each component is calculated using Equation (3.F.1):

$$N = (10)^X$$

Where,

$$X = \frac{C_1 + C_3 Y + C_5 Y^2 + C_7 Y^3 + C_9 Y^4 + C_{11} Y^5}{1 + C_2 Y + C_4 y^2 + C_6 Y^3 + C_8 Y^4 + C_{10} Y^5}$$

$$Y = \left(\frac{S_{alt,k}}{C_{us}}\right)\left(\frac{E_{FC}}{E_T}\right)$$

E_T = the material modulus of elasticity at the cycle temperature

E_{FC} = the modulus of elasticity used to establish the design fatigue curve

$C_{us} = 1$ (units of stress are in ksi).

For the vessel materials of construction, the coefficients C_i and the modulus E_{FC} are for the smooth bar fatigue curve for carbon steel with cycle temperature below $700°F$ and $\sigma_{uts} \le 80\ ksi$ and are listed in Table 3.F.1. The calculated allowable number of cycles is shown in Table E5.5.3-4 for each component.

Table E5.5.3-4 - Allowable Number of Cycles, N_k

Component	Location	$E_T\ (ksi)$	$E_{FC}\ (ksi)$	$S_{alt,k}\ (ksi)$	X	$N_k\ (cycles)$
Nozzle	Nozzle ring at inside corner radius base metal	2.91E+04	2.83E+04	13.632	5.742	5.522E+05
Nozzle	Nozzle transition at OD blend radius machined weld	2.88E+04	2.83E+04	15.900	5.437	2.734E+05
Head	Head knuckle ID base metal	2.88E+04	2.83E+04	13.678	5.708	5.101E+05
Shell	Head to shell weld ID as-welded	2.88E+04	2.83E+04	11.456	6.877	7.527E+06
Shell	Head to shell weld OD as-welded	2.88E+04	2.83E+04	10.316	7.776	5.972E+07
Shell	base metal	2.88E+04	2.83E+04	9.863	8.156	1.433E+08

f) STEP 6 – Determine the fatigue damage for the k^{th} cycle.

The actual number of repetitions of the $k^{th}\ cycle\ (n_k)$ is set to the cyclic life requirement provided in the User's Design Specification, 20020 cycles. The fatigue damage for the k^{th} cycle is then calculated by:

$$D_{f,k} = \frac{n_k}{N_k}$$

This results in calculated fatigue damage for the limiting region (nozzle outside radius) of 0.073. Similarly, a fatigue damage of 0.039 is calculated for the head knuckle.

g) STEP 7 – Repeat STEPs 3 through 6 for each different stress range identified.

The only stress range identified was the full pressure cycle, so no additional ranges need be evaluated.

h) STEP 8 – Compute the accumulated damage using the following equation:

$$D_f = \sum_{k=1}^{M} D_{f,k} \le 1.0$$

There is only one stress range, $D_f = D_{f,k}$ for each region evaluated and $D_f \le 1.0$ for each region. Therefore, all locations meet the cyclic life requirement of the User's Design Specification for the vessel operating conditions.

5.5.4 Example E5.5.4 – Elastic-Plastic Stress Analysis, and Equivalent Strains

Evaluate the vessel top head and shell base metal regions given in Example E5.5.2 in accordance with the fatigue methodology provided in paragraph 5.5.4, using an internal pressure of 1000 psig.

a) STEP 1 - Determine a load history for vessel.

Per the User's Design Specification (see Example E5.5.2), a full internal pressure cycle is the only applicable event to be considered. To better illustrate the method and generate plastic strain in the model, the operating pressure will be taken as1000 psig for this example.

b) STEP 2 - Determine the individual stress-strain cycles.

The only event under consideration is a full internal pressure cycle, from 0 psig to the operating pressure of 1000 psig.

c) STEP 3 – Determine the loadings at the start and end point of the cycle determined in STEP 2.

At the start point of the cycle, no loads are applied. At the end point of the cycle, a load of $1000\ psig$ internal pressure is applied. The loading range for internal pressure is therefore $1000\ psi$.

d) STEP 4 – Perform elastic-plastic stress analysis for the cycle determined in STEP 2.

The Twice-Yield Method (Paragraph 5.5.4.1(b)) will be used for this example. The loading at the start point of the cycle is taken as zero pressure, and the loading at the end point is the loading range determined in STEP 3; in this case 1000 psig internal pressure.

<u>Finite Element Model:</u>

The axisymmetric model was taken from Example E5.2.1 (see Figures E5.2.1-3 – E5.2.1-6). The pressure load was modified to 1000 psig (load at the cycle end point) and the nozzle thrust load was adjusted accordingly.

<u>Material Properties:</u>

The material properties in the model were also modified to include plastic behavior. Per paragraph 5.5.4.1(b), a stabilized cyclic stress-strain range curve was used to model the plasticity.

This form of the curve in Annex 3.D, however, does not match up to the typical input form for a finite-element program, which requires a specific yield stress to separate the elastic and elastic-plastic regions of material behavior. To approximate this yield stress and modify the form of the curve, an offset of plastic strain, ε_{offset}, is assumed and a line is drawn along the elastic slope. The intersection of this line and the cyclic stress-strain curve is taken as the yield stress. This method is described by A. Kalnins in ASME PVP2008-61397, 2008. This point can also be calculated using the equation:

$$\sigma_{yield} = K_{css}(\varepsilon_{offset})^{n_{css}}$$

The cyclic stress-strain curve (strain amplitude versus stress amplitude) then takes the form:

$$\varepsilon_{ta} = \frac{\sigma_a}{E_{ya}} \text{ for } \sigma_a \le \sigma_{yield}$$

$$\varepsilon_{ta} = \frac{\sigma_a}{E_{ya}} + \left[\frac{\sigma_a}{K_{css}}\right]^{\frac{1}{n_{css}}} - \varepsilon_{offset} \text{ for } \sigma_a > \sigma_{yield}$$

For the $SA-516,\ Grade\ 70,\ Normalized$ head and shell, the coefficients were obtained from Table 3.D.2 for Carbon Steel $(0.75\ in - base\ metal)$ since this matches the thickness most closely. Linear interpolation was performed between the values for $70°F$ and $390°F$ to approximate the values for $125°F$. The values used were:

$$n_{CSS} = 0.1290$$
$$K_{CSS} = 109.0781$$

Using an ε_{offset} of 2E-5 (fitting parameter per Annex 3.D), the yield stress for the SA-5-6-70N head and shell is calculated as 27.004 ksi.

The coefficients for the SA-105 nozzle were obtained in a similar fashion, but using the values for Carbon Steel (2.0 inch base metal) since it better matches the nozzle thickness. The values used were:

$$n_{CSS} = 0.1238$$
$$K_{CSS} = 99.0734$$

Using an ε_{offset} of 2E-5, the yield stress for the nozzle material is calculated as 25.965 ksi.

For the Twice-Yield Method, the curves are then converted to the hysteresis loop stress-strain curve form (strain range versus stress range). The plastic strain range is related to the stress range by the following equation:

$$\varepsilon_{pr} = 2\left[\frac{\sigma_r}{2K_{css}}\right]^{\frac{1}{n_{css}}} - 2\varepsilon_{offset}$$

The stress range and plastic strain range used in the analysis are shown in Table E5.5.4-1 for the head and shell, and Table E5.5.4-2 for the nozzle.

Table E5.5.4-1 - Stress range versus plastic strain range SA-516-70N Head and Shell

Stress Range (psi)	Plastic Strain Range
54007.5	0
60000.0	0.0000504
65000.0	0.0001281
70000.0	0.0002586
75000.0	0.0004697
80000.0	0.0008004
85000.0	0.0013045
90000.0	0.0020538
95000.0	0.0031436
100000.0	0.0046976
105000.0	0.0068748
110000.0	0.0098764

Table E5.5.4-2 - Stress range versus plastic strain range SA-105 Nozzle

Stress Range (psi)	Plastic Strain Range
51929.8	0
60000.0	0.0000885
65000.0	0.0002054
70000.0	0.0004065
75000.0	0.0007398
80000.0	0.0012735
85000.0	0.0021037
90000.0	0.0033619
95000.0	0.0052256
100000.0	0.0079296
105000.0	0.0117806
110000.0	0.0171739

Plots of the Mises stress and equivalent plastic strain in the overall model are shown in Figure E5.5.4-1.

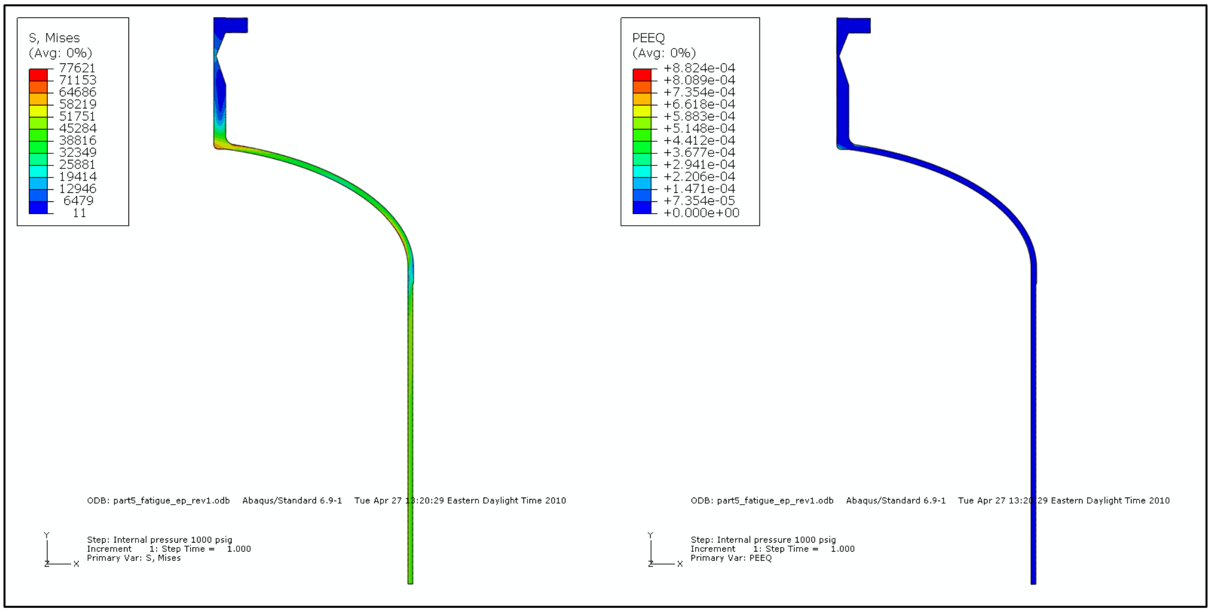


Figure E5.5.4-1 - Mises Stress (psi) and Equivalent Plastic Strain

e) STEP 5 – Calculate the Effective Strain Range for the cycle.

The effective strain range is calculated using Equation (5.41):

$$\Delta\varepsilon_{eff,k} = \frac{\Delta S_{P,k}}{E_{ya,k}} + \Delta\varepsilon_{peq,k}$$

As noted in Paragraph 5.5.4.2(e), because the range of loading is applied in a single step, the equivalent plastic strain range $\Delta\varepsilon_{peq,k}$ and equivalent stress $\Delta S_{P,k}$ can both be obtained directly from the analysis results ("PEEQ" and "S, Mises", variables, respectively, in ABAQUS). Using the elastic-plastic analysis results, the highest stress and strain regions were identified for each component being evaluated. Plots of the equivalent stress and equivalent plastic strain for these three areas (nozzle inside radius, head knuckle, and shell) are shown in Figure E5.5.4-2, Figure E5.5.4-3, and Figure E5.5.4-4. The effective strain range was calculated for multiple points in each area to ensure that the maximum (limiting) strain range was analyzed. Table E5.5.4-3 shows the limiting effective strain range for each component.

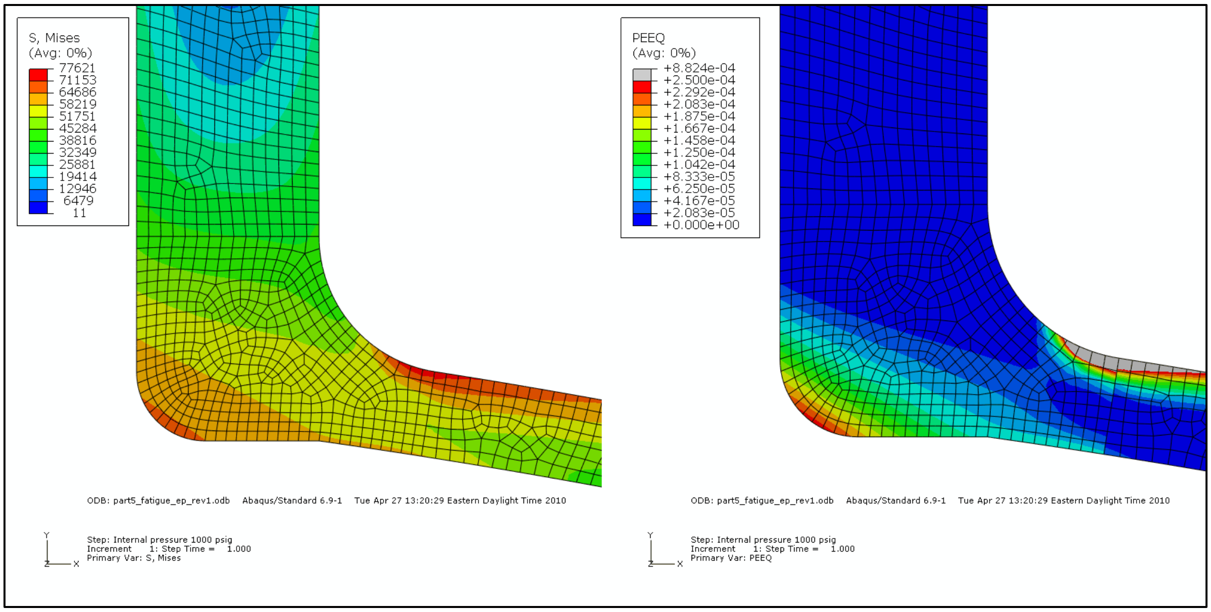


Figure E5.5.4-2 - Mises Stress (psi) and Equivalent Plastic Strain at Nozzle Inner Radius

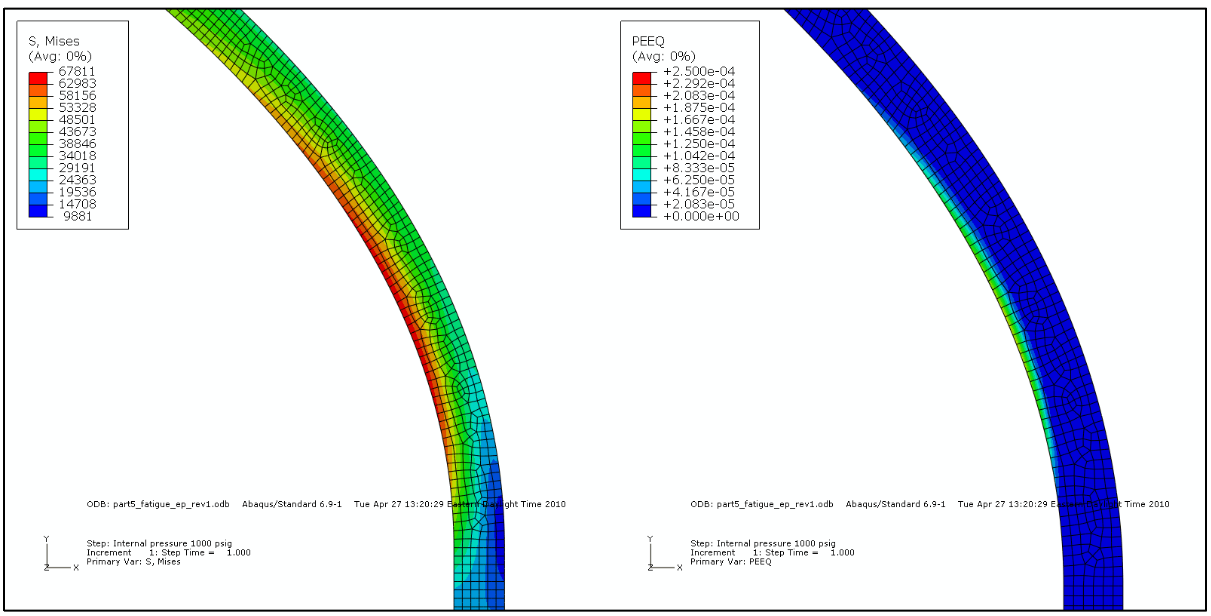


Figure E5.5.4-3 - Mises Stress (psi) and Equivalent Plastic Strain at Head Knuckle

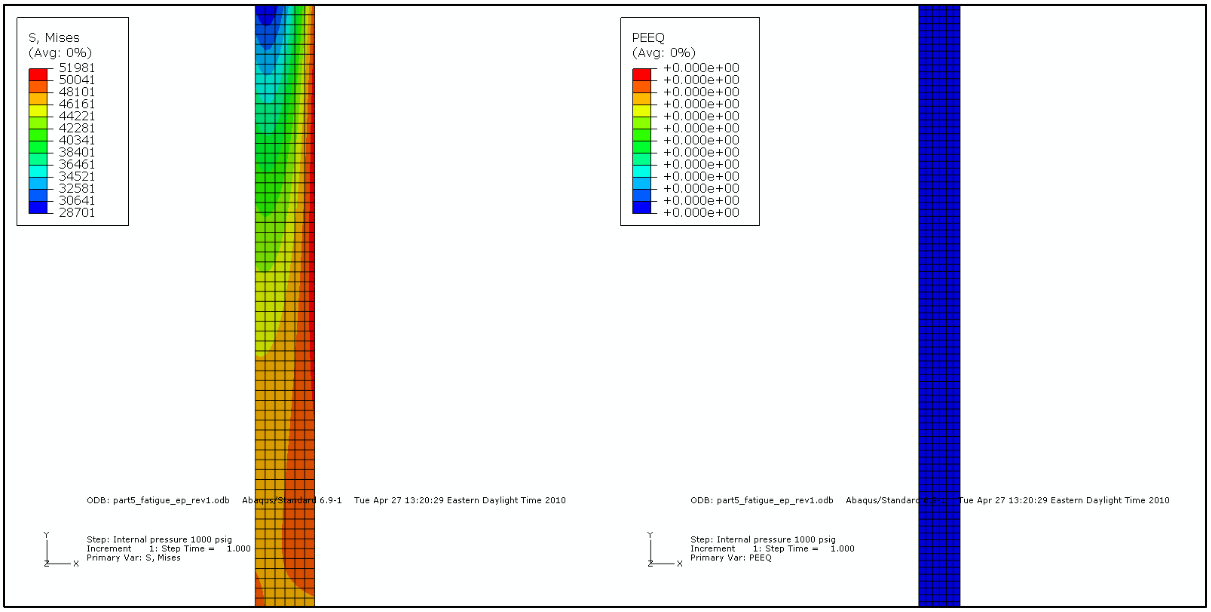


Figure E5.5.4-4 - Mises Stress (psi) and Equivalent Plastic Strain in Shell

Table E5.5.4-3 - Component Limiting Effective Strain Ranges

Component	**Location**	$E_{ya,k}\ (ksi)$	$\Delta S_{P,k}\ (ksi)$	$\Delta\varepsilon_{peq,k}$	$\Delta\varepsilon_{eff,k}$
Nozzle	Inside Radius	$2.91(10)^4$	65.938	$2.431(10)^{-4}$	$2.509(10)^{-3}$
Head	Head knuckle (at ID)	$2.88(10)^4$	67.811	$1.857(10)^{-4}$	$2.540(10)^{-3}$
Shell	OD below head transition	$2.88(10)^4$	51.982	0.0	$1.805(10)^{-3}$

f) STEP 6 – Determine the effective alternating equivalent stress for the cycle.

The effective alternating equivalent stress is calculated using Equation (5.43):

$$S_{alt,k} = \frac{E_{yf} \cdot \Delta\varepsilon_{eff,k}}{2}$$

For the vessel materials of construction, the modulus E_{yf} is for the smooth bar fatigue curve for carbon steel with cycle temperature below $700°F$ and $\sigma_{uts} \le 80\ ksi$ and is given in Annex 3.F, Table 3.F.1. Table E5.5.4-4 shows the limiting effective alternating equivalent stress for each component.

Table E5.5.4-4 - Component Limiting Effective Alternating Equivalent Stress

Component	Location	$E_{yf}\ (ksi)$	$\Delta\varepsilon_{eff,k}$	$S_{alt,k}\ (ksi)$
Nozzle	Inside Radius	$2.91(10)^4$	$2.509(10)^{-3}$	36.506
Head	Head knuckle (at ID)	$2.88(10)^4$	$2.540(10)^{-3}$	36.576
Shell	OD below head transition	$2.88(10)^4$	$1.805(10)^{-3}$	25.991

g) STEP 7 - Determine the permissible number of cycles, N_k, for the alternating equivalent stress computed in STEP 6, using the fatigue curves provided in Annex 3.F.

The allowable number of cycles for each component is calculated using Equation (3.F.1):

The allowable number of cycles for each component is calculated using Equation (3.F.1):

$$N = (10)^X$$

Where,

$$X = \frac{C_1 + C_3 Y + C_5 Y^2 + C_7 Y^3 + C_9 Y^4 + C_{11} Y^5}{1 + C_2 Y + C_4 y^2 + C_6 Y^3 + C_8 Y^4 + C_{10} Y^5}$$

$$Y = \left(\frac{S_{alt,k}}{C_{us}}\right)\left(\frac{E_{FC}}{E_T}\right)$$

E_T = the material modulus of elasticity at the cycle temperature

E_{FC} = the modulus of elasticity used to establish the design fatigue curve

$C_{us} = 1$ (units of stress are in ksi).

For the vessel materials of construction, the coefficients C_i and the modulus E_{FC} are for the smooth bar fatigue curve for carbon steel with cycle temperature below $700°F$ and $\sigma_{uts} \le 80\ ksi$ and are listed in Table 3.F.1. The calculated allowable number of cycles is shown in Table E5.5.4-**5** for each component.

Table E5.5.4-5 - Allowable Number of Cycles, N_k

Component	Location	$E_T\ (ksi)$	$E_{FC}\ (ksi)$	$S_a\ (ksi)$	X	$N_k\ (cycles)$
Nozzle	Inside Radius	$2.91(10)^4$	$2.83(10)^4$	36.506	4.104	12712
Head	Head knuckle (at ID)	$2.88(10)^4$	$2.83(10)^4$	36.576	4.086	12181
Shell	OD below head transition	$2.88(10)^4$	$2.83(10)^4$	25.991	4.557	34658

h) STEP 8 – Determine the fatigue damage for the k^{th} cycle.

The actual number of repetitions of the $k^{th}\ cycle\ (n_k)$ is set to the cyclic life requirement provided in the User's Design Specification, $20020\ cycles$. The fatigue damage for the k^{th} cycle is then calculated by:

$$D_{f,k} = \frac{n_k}{N_k}$$

This results in the calculated fatigue damage for the nozzle inside radius of 1.575. Similarly, a fatigue damage of 1.644 is calculated for the head knuckle and a fatigue damage of 0.577 for the shell.

i) STEP 9 – Repeat STEPs 3 through 8 for each different stress range identified.

The only stress range identified was the full pressure cycle, so no additional ranges need be evaluated.

j) STEP 10 – Compute the accumulated damage using the following equation:

$$D_f = \sum_{k=1}^{M} D_{f,k} \le 1.0$$

Since there is only one stress range, $D_f = D_{f,k}$ for each region evaluated. As $D_f > 1.0$ in the nozzle and head, these components do not meet the specified design cycle life for a $1000\ psig$ operating pressure assumed for this example. For a $1000\ psig$ operating pressure, the design cycle life for the base metal would be limited by the knuckle at 12181 cycles.

5.5.5 Example E5.5.5 – Elastic Stress Analysis, and Structural Stress

Evaluate the vessel top head and shell weld regions gien in Example E5.5.2 in accordance with the fatigue methodology provided in paragraph 5.5.5. Note that the vessel is in non-corrosive service with respect to environmental effects upon the fatigue behavior. However, the analysis was conservatively based on the corroded dimensions as specified in the design requirements. Additionally, the operating pressure is conservatively assumed equal to the design pressure in this example.

a) STEP 1 - Determine a load history for vessel.

Per the User's Design Specification (see Example E5.5.2), a full internal pressure cycle is the only applicable event to be considered. The vessel internal pressure is expected to cycle 20020 times between $0\ psig$ and the operating pressure, $420\ psig$. This includes shutdown and startup conditions.

b) STEP 2 - Determine the individual stress-strain cycles.

Since the only event under consideration is a full internal pressure cycle, the applicable cyclic stress range is between the stress in the vessel at $0\ psig$ internal pressure and the stress in the vessel at $420\ psig$.

c) STEP 3 - Determine the elastically calculated membrane and bending stress normal to the assumed hypothetical crack plane at the start and end points for the cycle determined in Step 2. Using this data, calculate the membrane and bending stress ranges, and the maximum, minimum and mean stress.

Assume the ending time point, assume ${}^{n}t$, for the cycle under consideration is at $0\ psig$ internal pressure; the equations 5.46 through 5.49 reduce as follows:

$$\Delta\sigma^{e}_{m,k} = {}^{m}\sigma^{e}_{m,k} - 0$$

$$\Delta\sigma^{e}_{b,k} = {}^{m}\sigma^{e}_{b,k} - 0$$

$$\sigma_{\max,k} = \left({}^{m}\sigma^{e}_{m,k} + {}^{m}\sigma^{e}_{b,k}\right)$$

$$\sigma_{\min,k} = 0$$

$$\sigma_{mean,k} = \frac{\sigma_{\max,k} + \sigma_{\min,k}}{2}$$

Finite Element Model:

The axisymmetric model was taken from Example E5.5.1 (see Figures 5. E5.2.1-3 – E5.2.1-6). The pressure load for this assessment was $420\ psig$. The model also included nozzle thrust load. This is also the same model used for the Example E5.5.4. It may be useful to compare the fatigue results for each example problem.

Elastic Analysis Results:

A plot of the stress (von Mises) for the model is shown in Figure E5.5.5-1. The three weld locations to be analyzed are indicated on the figure (shell to head, head to top nozzle, nozzle to flange weld). It is assumed, per this analysis method that the hypothetical crack planes initiate at the toe of the welds.

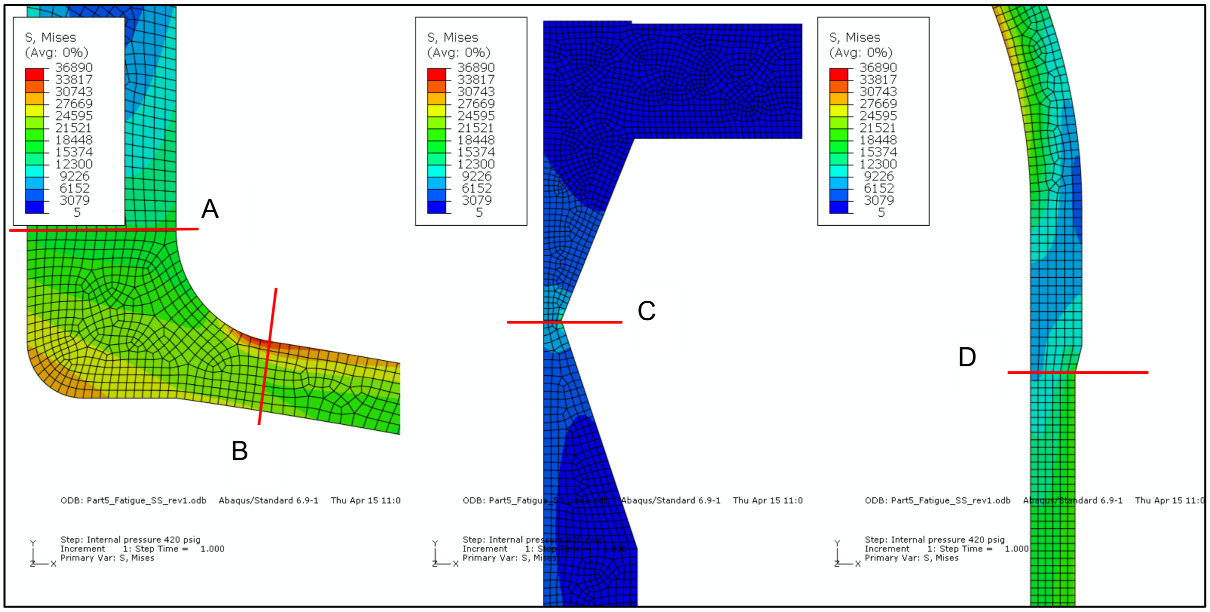


Figure E5.5.5-1 - Stress in Model (420 psig internal pressure)

For each of the theoretical crack locations, the through-wall linearized stress is obtained. From the linearized stress results the membrane and bending stress normal to the crack plane is read directly from the output. For each of the 4 locations, the membrane and bending stress normal to the crack plane are summarized in Table E5.5.5-1 below. Note that the minimum state of stress comprising the stress range is for a zero loading case since the component cycles pressure only.

Table E5.5.5-1 - Linearized Stress Results

Location	Membrane Stress (ksi)	Bending Stress (ksi)
A	0.913	6.694
B	13.925	20.973
C	5.199	13.002
D	10.777	11.019

For each of these locations, calculate the maximum, minimum, and mean stress. The results are summarized in Table E5.5.5-2.

Table E5.5.5-2 - Maximum, Minimum, and Mean Stress Results

Location	Maximum (ksi)	Mean (ksi)	Minimum (ksi)
A	7.607	3.804	0
B	34.898	17.449	0
C	18.201	9.101	0
D	21.796	10.898	0

d) STEP 4 – Determine the elastically calculated structural stress range using equation 5.51.

Because the stress level at time ${}^{n}t$ is zero, the stress range for each location is simply the sum of the membrane and bending stress as the factors ${}^{n}\sigma^{e}_{m,k}$ and ${}^{n}\sigma^{e}_{b,k}$ are equal to zero in equations 5.46 and 5.47. The stress range for each location is summarized in Table E5.5.5-3 using the following:

$$\Delta\sigma^{e}_{k} = \Delta\sigma^{e}_{m,k} + \Delta\sigma^{e}_{b,k}$$

e) STEP 5 – Determine the elastically calculated structural strain, $\Delta\varepsilon^{e}_{k}$, from the elastically calculated structural stress obtained in Step 4.

The elastic structural strain is calculated using equation 5.52:

$$\Delta\varepsilon^{e}_{k} = \frac{\Delta\sigma^{e}_{k}}{E_{ya,k}}$$

Where $E_{ya,k} = 28.8E6\,psi$ (modulus of elasticity for the material at $125°F$) is obtained from ASME Sec. II, Part D.

The corresponding local nonlinear structural stress and strain ranges, $\Delta\sigma_{k}$ and $\Delta\varepsilon_{k}$, respectively, are determined by simultaneously solving Neuber's Rule, Equation (5.53), and a model for the material hysteresis loop stress-strain curve given by Equation (5.54).

$$\Delta\varepsilon_{k} \cdot \Delta\sigma_{k} = \Delta\varepsilon^{e}_{k} \cdot \Delta\sigma^{e}_{k}$$

$$\Delta\varepsilon_{k} = \frac{\Delta\sigma_{k}}{E_{ya,k}} + 2\left(\frac{\Delta\sigma_{k}}{2K_{css}}\right)^{\frac{1}{n_{css}}}$$

The values for K_{css} and n_{css} are obtained from Table 3.D.2 in Annex 3.D. These values used were interpolated from the table data based on temperature. The values used are listed in Table E5.5.5-3 below.

Table E5.5.5-3 - Stress Range Summary

Location	Elastic Ranges		Hysteresis Loop Stress-Strain Factors		Neuber Corrected Stress Range
	$\Delta\sigma_k^e$ (ksi)	$\Delta\varepsilon_k^e$	K_{css} (ksi)	n_{css}	$\Delta\sigma_k$ (ksi)
A	7.6070	2.641E-04	109.078	0.1290	7.6070
B	34.8980	1.212E-03	109.078	0.1290	34.878
C	18.2010	6.320E-04	109.078	0.1290	18.2010
D	21.7960	7.568E-04	109.078	0.1290	21.7950

f) STEP 6 – Compute the equivalent structural stress range parameter for each location A through F.

The out-of-plane shear stresses for these cases are zero; therefore the modified procedure for this step, outlined in 5.5.5.3, need not be employed.

$$\Delta S_{ess,k} = \frac{\Delta\sigma_k}{t_{ess}^{\left(\frac{2-m_{ss}}{2m_{ss}}\right)} \cdot I^{\frac{1}{m_{ss}}} \cdot f_{M,k}}$$

Where,

$$m_{ss} = 3.6$$
$$t_{ess} = 16\ mm\ (0.625\ in.) \qquad for \qquad t \le 16\ mm\ (0.625\ in.)$$
$$t_{ess} = t \qquad for \qquad 16\ mm\ (0.625\ in.) < t < 150\ mm\ (6\ in.)$$
$$t_{ess} = 150\ mm\ (6\ in.) \qquad for \qquad t \ge 150\ mm\ (6\ in.)$$
$$I^{\frac{1}{m_{ss}}} = \frac{1.23 - 0.364R_{b,k} - 0.17R_{b,k}^2}{1.007 - 0.306R_{b,k} - 0.178R_{b,k}^2}$$

$$R_{b,k} = \frac{\left|\Delta\sigma_{b,k}\right|}{\left|\Delta\sigma_{m,k}\right| + \left|\Delta\sigma_{b,k}\right|}$$

$$f_{M,k} = \left(1 - R_k\right)^{\frac{1}{m_{ss}}} \qquad for \qquad \left| \begin{array}{l} \sigma_{mean,k} \geq 0.5 S_{y,k},\ and \\ R_k > 0,\ and \\ \left|\Delta\sigma_{m,k} + \Delta\sigma_{b,k}\right| \leq 2S_{y,k} \end{array} \right.$$

$$f_{M,k} = 1.0 \qquad for \qquad \left| \begin{array}{l} \sigma_{mean,k} < 0.5 S_{y,k},\ or \\ R_k \leq 0,\ or \\ \left|\Delta\sigma_{m,k} + \Delta\sigma_{b,k}\right| > 2S_{y,k} \end{array} \right.$$

$$R_k = \frac{\sigma_{min,k}}{\sigma_{max,k}}$$

The results for this step are summarized in Table E5.5.5-4 for each location. As the minimum stress for the cycle is base on zero $psig$ applied pressure; $\sigma_{\min,k}$ is equal to zero, and therefore R_k is also equal to zero for all locations. Similarly, $f_{M,k}$ is equal to 1.0 based on the R_k value. These values will not be shown in the results table.

Table E5.5.5-4 - Results for Step 6

Location	t_{ess} (in)	R_{bk}	$I^{\frac{1}{m_{ss}}}$	$\Delta S_{ess,k}$ (ksi)
A	2.000	0.880	1.297	6.842
B	0.938	0.601	1.252	27.470
C	0.625	0.714	1.266	12.950
D	0.813	0.506	1.243	16.751

g) STEP 7 – Determine the permissible number of cycles, N_k, based on the equivalent structural stress range parameter computed in Step 6.

The welded joint fatigue curves can be developed using Annex 3.F, paragraph 3.F.2. In this example analysis, the carbon steel material of construction has curves limited to temperatures below $700°F$. As our design temperature is $125°F$, this criteria is met and the curves developed by this Annex may be used.

The design number of allowable design cycles, N, can be computed from the following equations. For this example calculation, it is assume that burr grinding in accordance with Part 6, Figure 6.2 is employed. For these equations $\Delta S_{range} = \Delta S_{ess,k}$ from above.

$$N = \frac{f_I}{f_E}\left(\frac{f_{MT} \cdot C}{\Delta S_{ess,k}}\right)^{\frac{1}{h}}$$

Where,

- The improvement factor is:

$$f_I = 1.0 + 2.5(10)^q \qquad (\mathit{for\ burr\ grinding})$$

$$q = -0.0016\left(\frac{\Delta S_{ess,k}}{C_{us}}\right)^{1.6}$$

- The environmental factor is define as per 3.F.2.2 c); Since this is non-corrosive service, the default value of 4.0 is not applicable.

$$f_E = 1.0$$

- The material correction factor is calculated; In this case of this analysis, the average cycle temperature is very close to $70°F$, therefore E_T will be equal to E_{ACS}, therefore:

$$f_{MT} = \frac{E_T}{E_{ACS}}$$

$$f_{MT} = 1$$

- The factors, C and h, are obtained from Table 3.F.11 for the -3σ curve.

$$C = 818.3$$

$$h = 0.31950$$

For each of the locations, the allowable design cycles are calculated and summarized in Table E5.5.5-5.

Table E5.5.5-5 - Results for Step 7

Location	$\Delta S_{ess,k}$ (ksi)	q	f_I	N $Cycles$	$D_{f,k}$
A	6.842	-0.0347	3.3080	10535490	0.0019
B	27.470	-0.3208	2.1943	90141	0.2221
C	12.950	N/A	N/A	432377	0.0463
D	16.751	N/A	N/A	193177	0.1036

h) STEP 8 – Determine the fatigue damage for the cycle history.

The cycle history specified by the user design specification from Step 1 above, had a requirement for 20020 full pressure cycles (therefore $n_k = 20020$). The fatigue damage factor can be calculated using the following equation.

$$D_{f,k} = \frac{n_k}{N_k}$$

By inspection, it is evident that location B is the controlling welded location, for those analyzed, as the allowable cycles for the stress range calculated is $90141\ cycles$. The damage factor is summarized for each location in Table E5.5.5-5 above.

i) STEPS 9-11 - Assessment of steps 9-11 are not required as there are no other stress ranges other than the 0 to $420\ psig$ operational cycle.

5.5.6 Example E5.5.6 – Protection Against Ratcheting Using Elastic Stress Analysis

Evaluate the vessel top head and shell region given in Example Problem E5.5.2 for compliance with respect to the elastic ratcheting criteria provided in paragraph 5.5.6.

a) STEP 1 – Evaluate the primary plus secondary equivalent stress range, $\Delta S_{n,k}$, for each component.

Note: The finite element model and elastic analysis for the non-weld locations are taken from Example 5.5.3 (See Figure 5.5.3-1 for analysis stress results). For the welded locations, the stress results are taken from Example 5.5.5.

$\Delta S_{n,k}$ is the equivalent stress range of the linearized combination of primary membrane, primary bending, and secondary stresses ($P_L + P_B + Q$) for each location under evaluation. The stress linearization is done as per Annex 5.A of the Code. The Stress Classification Lines used to calculate these values for the base metal locations are shown in Figure E5.5.6-1, and for the weld locations in Figure E5.5.6-2. Results are shown in Table E5.5.6-1.

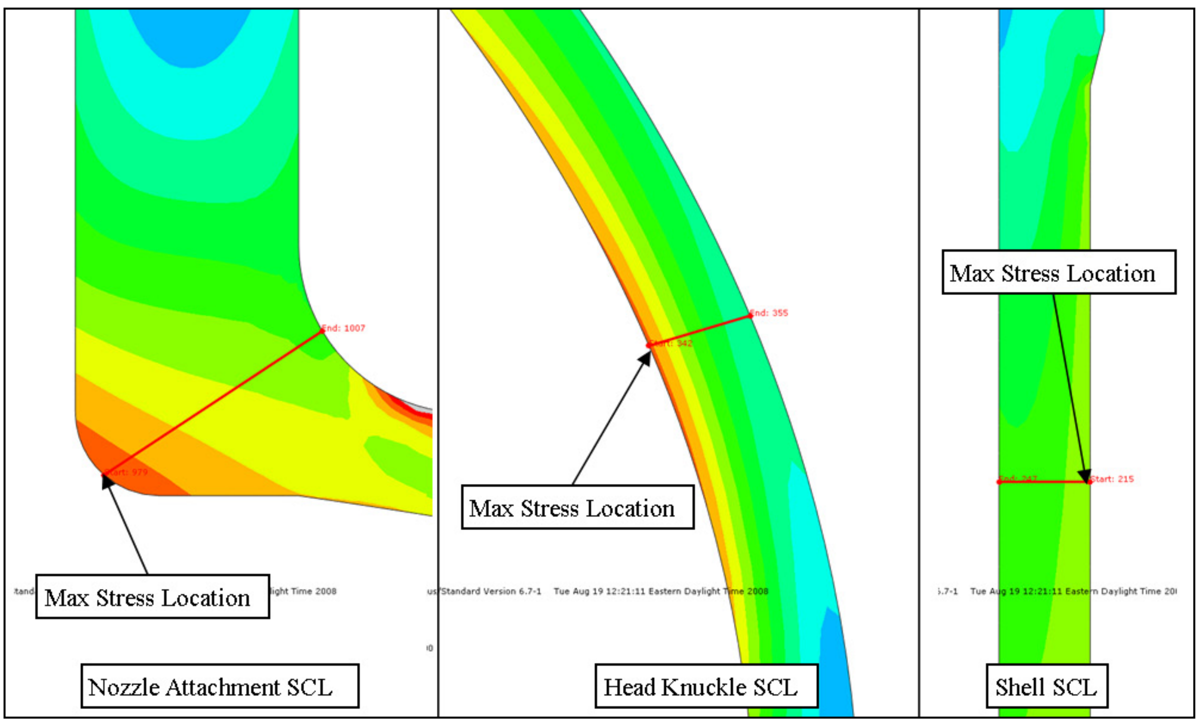


Figure E5.5.6-1 - Stress Classification Lines for Calculation of $\Delta S_{n,k}$ (Base Metal Locations)

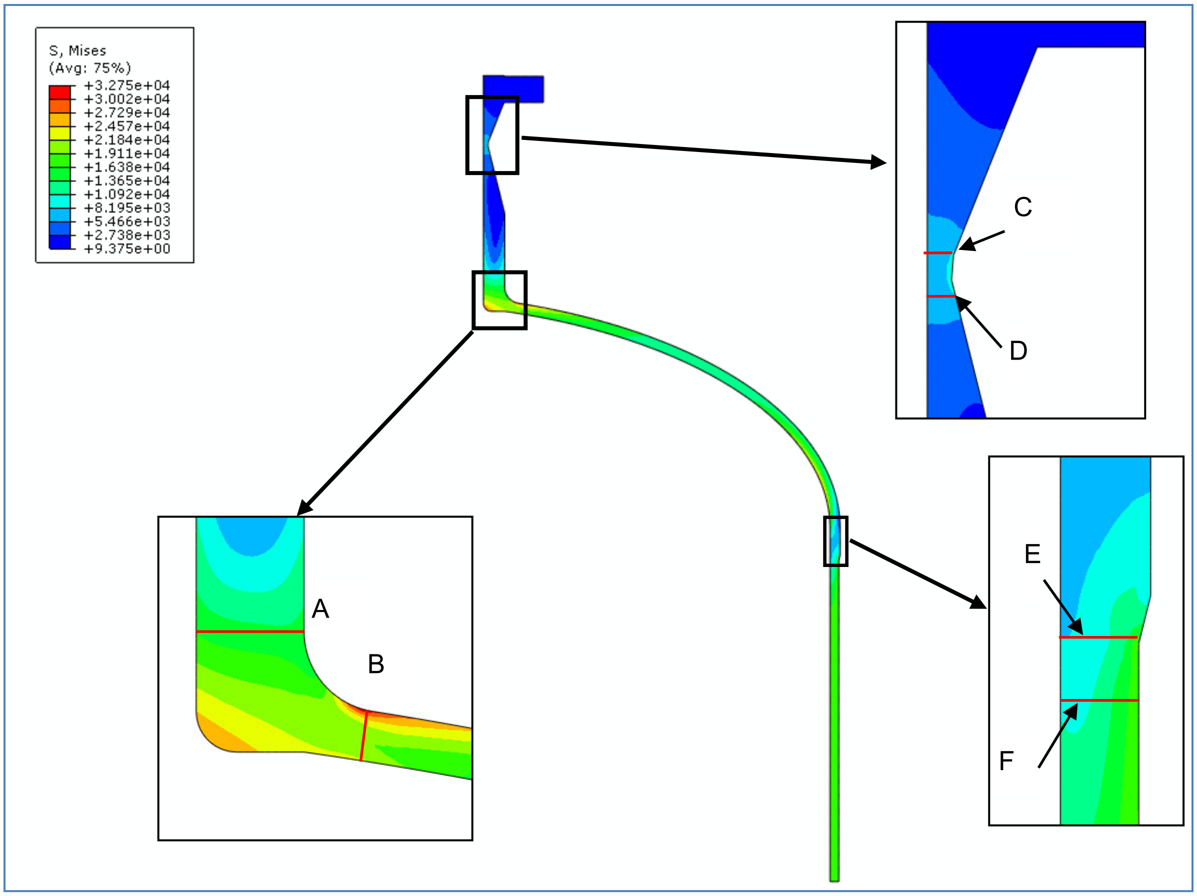


Figure E5.5.6-2 - Stress Classification Lines for Calculation of $\Delta S_{n,k}$ (Weld Locations)

Table E5.5.6-1 - Stress Range Comparison

Component	Location	Type	$\Delta S_{n,k}$ (ksi)
Nozzle	Inside Radius	Base Metal	27.060
Head	Head knuckle (at ID)	Base Metal	26.821
Shell	OD below head transition	Base Metal	19.938
Nozzle	A	Weld	16.41
Head	B	Weld	29.80
Nozzle	C	Weld	8.40
Nozzle	D	Weld	10.47
Shell	E	Weld	16.94
Shell	F	Weld	18.38

b) STEP 2 – Evaluate the limit for the equivalent stress range, S_{PS}.

S_{PS} is defined as $\max\left[3S,\, 2S_y\right]$, where S is the material allowable stress at the cycle temperature and S_y is the material yield strength at the cycle temperature $(125°F)$. Table E5.5.6-2 lists the values for S_{PS} for each component.

Table E5.5.6-2 - Calculated Values for S_{PS}

Component	Location	Type	$S\ (ksi)$	$S_y\ (ksi)$	$S_{PS}\ (ksi)$
Nozzle	Inside Radius	Base Metal	23.3000	34.9000	69.9000
Head	Head knuckle (at ID)	Base Metal	24.5500	36.8500	73.7000
Shell	OD below head transition	Base Metal	24.5500	36.8500	73.7000
Nozzle	A	Weld	23.3000	34.9000	69.9000
Head	B	Weld	24.5500	36.8500	73.7000
Nozzle	C	Weld	23.3000	34.9000	69.9000
Nozzle	D	Weld	23.3000	34.9000	69.9000
Shell	E	Weld	24.5500	36.8500	73.7000
Shell	F	Weld	24.5500	36.8500	73.7000

c) STEP 3 – Compare equivalent stress ranges calculated in STEP 1 to the limits calculated in STEP 2.

Results of the comparison are shown in E5.5.6-3. Since $\Delta S_{n,k} \le S_{PS}$ for all locations, the elastic criteria for protection against racheting is satisfied.

Table E5.5.6-3 - Stress Range Comparison

Component	Location	Type	$\Delta S_{n,k}\ (ksi)$	$S_{PS}\ (ksi)$	$\Delta S_{n,k} \le S_{PS}$
Nozzle	Inside Radius	Base Metal	27.060	69.9000	Yes
Head	Head knuckle (at ID)	Base Metal	26.821	73.7000	Yes
Shell	OD below head transition	Base Metal	19.938	73.7000	Yes
Nozzle	A	Weld	16.41	69.9000	Yes
Head	B	Weld	29.80	73.7000	Yes
Nozzle	C	Weld	8.40	69.9000	Yes
Nozzle	D	Weld	10.47	69.9000	Yes
Shell	E	Weld	16.94	73.7000	Yes
Shell	F	Weld	18.38	73.7000	Yes

5.5.7 Example E5.5.7 – Protection Against Ratcheting Using Elastic-Plastic Stress Analysis

Evaluate the vessel top head and shell region given in Example Problem E5.5.2 for compliance with respect to the elastic-plastic ratcheting criteria provided in paragraph 5.5.7.

a) STEP 1 – Develop a numerical model of the vessel components.

The axisymmetric finite element model geometry was taken from Example E5.2.1 (see Figures E5.2.1-3 – 5.2.1-6).

b) STEP 2 – Define all relevant loads and applicable load cases.

Per the User's Design Specification (see Example E5.5.2), a full internal pressure cycle is the only applicable event to be considered. The vessel internal pressure will cycle between $0\ psig$ and the operating pressure, $380\ psig$.

The pressure load in the finite element model was modified to the $380\ psig$ operating pressure and the nozzle thrust load was adjusted accordingly.

c) STEP 3 – Modify the material model to elastic-perfectly plastic behavior. The effects of non-linear geometry shall be considered.

For each material, the yield strength defining the plastic limit was taken as the yield strength at temperature per Annex 3.D. For $SA-105$ (nozzle) and $SA-516-70N$ (head, shell) the yield strengths were taken as $34.90\ ksi$ and $36.85\ ksi$, respectively. The material section of the ABAQUS input deck is shown below.

```
** MATERIALS
**
*Material, name=SA105
*Elastic
 2.91e+07, 0.3
*Plastic
34900.0,0.
*Material, name=SA516-70N
*Elastic
 2.878e+07, 0.3
*Plastic
36850.0,0.0
**
```

The effects of nonlinear geometry were included in the analysis by setting ABAQUS keyword nlgeom=yes.

d) STEP 4 – Perform an elastic-plastic analysis using the applicable loading from STEP 2.

The elastic-plastic analysis was performed using the $380\ psig$ internal pressure load from STEP 2 and the elastic-perfectly plastic material model from STEP 3. A plot of the Mises stress and equivalent plastic strain under load are shown in Figure E5.5.7-1 below.

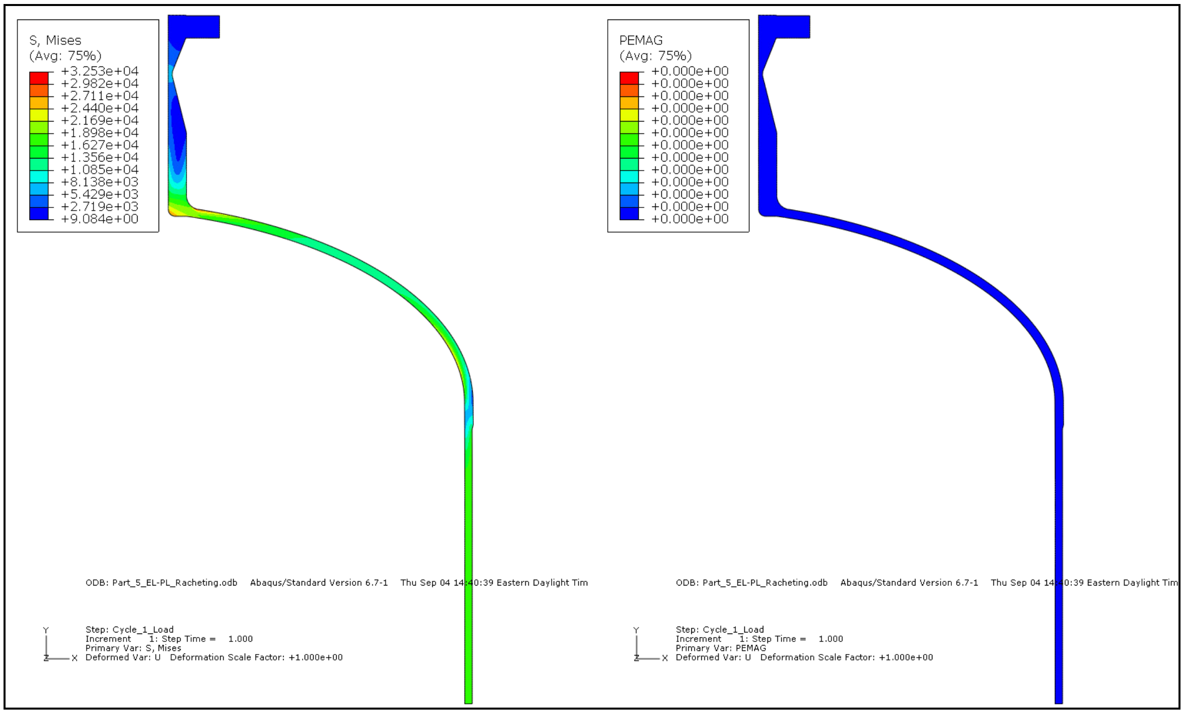


Figure E5.5.7-1 - Stress and Plastic Strain in Model (380 $psig$ internal pressure, 1st cycle)

e) STEP 5 – Continue the elastic-plastic analysis for a minimum of three complete cycles and evaluate using the ratcheting criteria in paragraph 5.5.7.2(e).

Additional loading and unloading steps were added to the finite element analysis until three complete cycles were modeled. Figure E5.5.7-2 shows the Mises stress and plastic strain in the model following the completion of the third cycle.

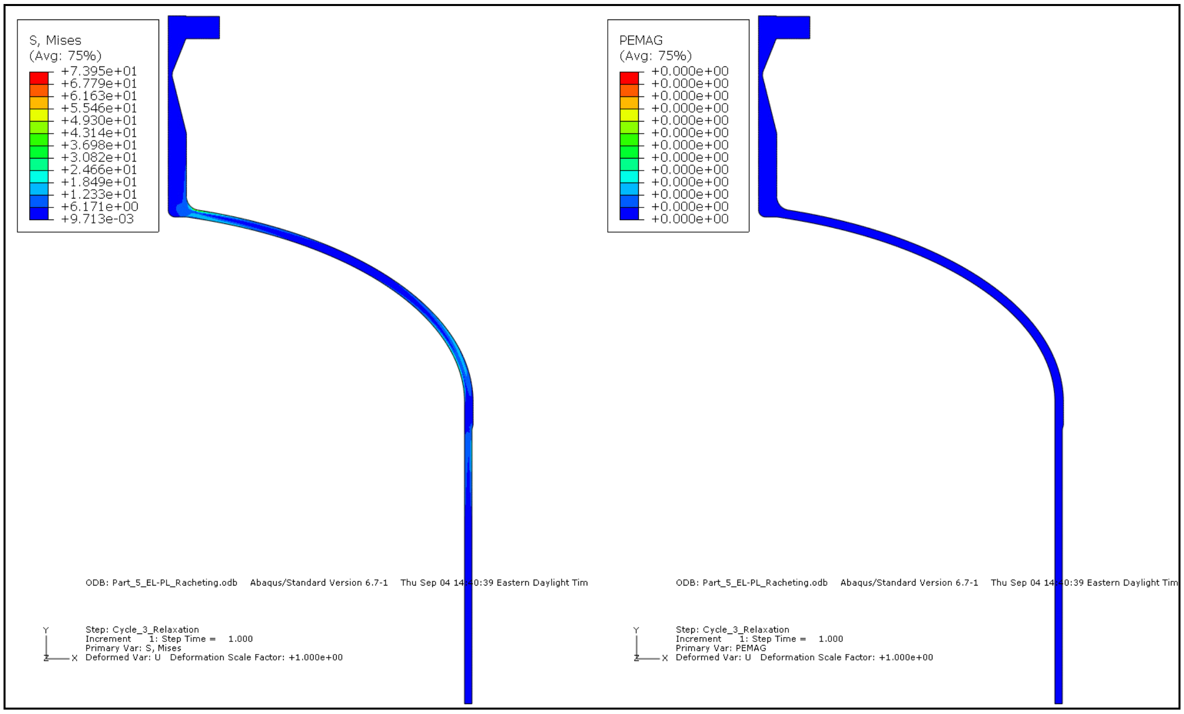


Figure E5.5.7-2 - Stress and Plastic Strain in Model (End of 3rd Cycle)

It can be seen from Figure E5.5.7-2 that zero plastic strains have been incurred in the nozzle, head, or shell. Thus, these components meet the condition detailed in 5.5.7.2(e)(1) and the ratcheting criteria are satisfied.

The vessel components, therefore, are acceptable per the elastic-plastic ratcheting criteria for an operating pressure cycle between $0\ psig$ and $380\ psig$.

PART 6

FABRICATION REQUIREMENTS

PART CONTENTS

6.1 Example E6.1 – Postweld Heat Treatment of a Pressure Vessel

Establish the postweld heat treatment (PWHT) requirements for a process tower considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-537,\ Class\ 1$
- Design Conditions = $1650\ psig\ @600°F$
- Liquid Head = $60\ ft$
- Liquid Specific Gravity = 0.89
- Inside Diameter = $96.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $25700\ psi$
- P Number and Group = $P-No.\ 1,\ Group\ 2$
- Weld Joint Efficiency = 1.0
- Tangent-to-Tangent Vessel Length = $80\ ft$
- Top and Bottom Heads = $Hemispherical$

Evaluate the requirements of PWHT per paragraph 6.4.

The design pressure used to establish the wall thickness for the bottom head and cylindrical shell section must be adjusted for the liquid head in accordance with Part 4, paragraph 4.1.5.2.a.

Adjusted pressure for the cylindrical shell:

$$Design\ Pressure = Specified\ Design\ Pressure + \gamma gh$$

$$Design\ Pressure = 1650 + \frac{0.89(62.4)(56)}{144} = 1671.597\ psig$$

Adjusted pressure for the bottom hemispherical head:

$$Design\ Pressure = Specified\ Design\ Pressure + \gamma gh$$

$$Design\ Pressure = 1650 + \frac{0.89(62.4)(60)}{144} = 1673.140\ psig$$

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the top head.

$$D = 96.0 + 2(Corrosion\ \ Allowance) = 96.0 + 2(0.125) = 96.25\ in$$

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right]-1\right) = \frac{96.25}{2}\left(\exp\left[\frac{0.5(1650)}{25700(1.0)}\right]-1\right) = 1.5699\ in$$

$$t = 1.5699 + Corrosion\ \ Allowance = 1.5699 + 0.125 = 1.6949\ in$$

The required thickness of the top head is $1.6949\ in$.

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the bottom head, including the liquid head static pressure.

$$D = 96.0 + 2(Corrosion\ \ Allowance) = 96.0 + 2(0.125) = 96.25\ in$$

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right]-1\right) = \frac{96.25}{2}\left(\exp\left[\frac{0.5(1673.140)}{25700(1.0)}\right]-1\right) = 1.5923\ in$$

$$t = 1.5923 + Corrosion\ \ Allowance = 1.5923 + 0.125 = 1.7173\ in$$

The required thickness of the bottom head is $1.7173\ in$.

In accordance with Part 4, paragraph 4.3.3, determine the required thickness of the cylindrical shell, including the liquid head static pressure.

$$D = 96.0 + 2(Corrosion\ \ Allowance) = 96.0 + 2(0.125) = 96.25\ in$$

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{96.25}{2}\left(\exp\left[\frac{1671.597}{25700(1.0)}\right]-1\right) = 3.2342\ in$$

$$t = 3.2373 + Corrosion\ \ Allowance = 3.2342 + 0.125 = 3.3592\ in$$

The required thickness of the cylindrical shell is $3.3592\ in$.

Required Thickness Summary:

Top Head = $1.6949\ in$

Bottom Head = $1.7173\ in$

Cylindrical Shell = $3.3592\ in$

The requirements for postweld heat treatment are found in paragraph 6.4.2. Material specification SA-537 Class 1 is a P-No. 1, Group No. 2, material. Therefore, in accordance with paragraph 6.4.2.2.e, the PWHT requirements are provided in Table 6.8. The definition of nominal thickness governing PWHT is provided in paragraph 6.4.2.7. For pressure vessels or parts of pressure vessels being postweld heat treated in a furnace charge, the nominal thickness is the greatest weld thickness in any vessel or vessel part which has not previously been postweld heat treated. Therefore, the governing nominal thickness is that of the cylindrical shell, $3.3592\ in$.

The procedures for postweld heat treatment are found in paragraph 6.4.3. PWHT of the vessel will be performed by heating the vessel as a whole in a closed furnace in accordance with paragraph 6.4.3.2.

Based on Table 6.8, see Table E6.1 of this example, PWHT is mandatory due to the governing nominal thickness of $3.3592\ in$. The holding temperature and time based on the nominal thickness within the range of $2\ in < t_n < 5\ in$ is 1100°F for 2 hours plus 15 minutes for each additional inch over 2 inches, respectively. For the vessel in question, the holding time is calculated as follows:

$$Holding\ time = 120\ min + \left(\frac{15\ min}{in}\right)(3.3592\ in - 2\ in) = 140\ min$$

The requirements for operation of PWHT are provided in paragraph 6.4.4. The operation of postweld heat treatment shall be carried out by one of the procedures given in paragraph 6.4.3 in accordance with the following requirements.

a) When post weld heat treatment is performed in a furnace (see paragraph 6.4.3.2), the temperature of the furnace shall not exceed 800°F at the time the vessel or part is placed in it.
b) Above 800°F, the rate of heating shall be not more than 400°F/hr divided by the maximum metal thickness of the shell or head plate in inches, but in no case more than 400°F/hr, and in no case need it be less than 100°F/hr. During the heating period there shall not be a greater variation in temperature throughout the portion of the vessel being heated than 250°F within any 15 ft interval of length.

$$Maximum\ Heating\ Rate = \frac{400°F/hr}{3.3592\ in} = 119°F/hr$$

c) The vessel or vessel part shall be held at or above the temperature specified in paragraph 6.4.2 for the period of time specified in this paragraph. During the holding period, there shall not be a difference greater than 150°F between the highest and lowest temperatures throughout the portion of the vessel being heated, except where the range is further limited in paragraph 6.4.2.
d) When post weld heat treatment is performed in a furnace (see paragraph 6.4.3.2), during the heating and holding periods, the furnace atmosphere shall be so controlled as to avoid excessive oxidation of the surface of the vessel. The furnace shall be of such design as to prevent direct impingement of the flame on the vessel.
e) Above 800°F, cooling shall be done at a rate not greater than 500°F/hr divided by the maximum metal thickness of the shell or head plate in inches, but in no case need it be less than 100°F/hr. From 800°F, the vessel may be cooled in still air.

$$Maximum\ Cooling\ Rate = \frac{500°F/hr}{3.3592\ in} = 149°F/hr$$

Table E6.1 - Design Loads and Load Combinations from VIII-2

Table 6.8 – Requirements For Post Weld Heat Treatment (PWHT) Of Pressure Parts And Attachments For Material: P-No. 1, Group 1, 2, 3

PWHT Requirements	Holding Temperature and Time Based On The Nominal Thickness
a) PWHT is mandatory for the following conditions: 1) For welded joints over 38mm (1 1/2 in.) nominal thickness. 2) For welded joints over 32 mm (1 1/4 in.) through 38 mm (1 1/2 in.) nominal thickness unless a 95°C (200°F) minimum preheat is applied during welding b) When it is impractical to perform PWHT at the temperatures specified in this table, it is permissible to carry out PWHT at lower temperatures for longer periods of time in accordance with Table 6.16.	SI Units • For $t_n \le 50\ mm$: 595°C, 0.04 hr/mm, 15 minutes minimum • For: $50\ mm < t_n \le 125\ mm$ 595°C, 2 hr plus 0.6 minutes for each additional mm over 50 mm • For $t_n > 125\ mm$: 595°C, 2 hr plus 0.6minutes for each additional mm over 50 mm US Customary Units • For $t_n \le 2\ in$: 1100°F, 1 hr/in, 15 minutes minimum • For $2\ in < t_n \le 5\ in$: 1100°F, 2 hr plus 15 minutes for each additional inch over 2 in. • For $t_n > 5\ in$: 1100°F, 2 hr plus 15 minutes for each additional inch over 2 in.

6.2 Example E6.2 – Out-of-Roundness of a Cylindrical Forged Vessel

A vessel is being fabricated using forged cylindrical shell segments. During fabrication, tolerances where checked and it was noted that out-of-roundness of one of the cylindrical shell segments is present that exceed tolerance limits specified in paragraph 6.1.2.7. In order to establish a plan of action, it was decided to use the provisions in Part 6 that permit a reduced permissible operating pressure be determined for cylindrical shells with general out-of-roundness characterized by a major and minor diameter. Establish the reduced permissible operating pressure requirements considering the following design conditions.

Vessel Data:

- Material = $SA-372,\ Grade\ C$
- Design Conditions = $2800\ psig @ 400°F$
- Inside Diameter = $112.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $27600\ psi$
- Weld Joint Efficiency = 1.0
- Modulus of Elasticity = $27.9E+06\ psi$

Evaluate the special requirements for forged fabrication per paragraph 6.7.4

In accordance with Part 4, paragraph 4.3.3, determine the required thickness of the cylindrical shell.

$$D = 96.0 + 2(Corrosion\ \ Allowance) = 112.0 + 2(0.125) = 112.25\ in$$

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{112.25}{2}\left(\exp\left[\frac{2800}{27600(1.0)}\right]-1\right) = 5.9927\ in$$

$$t = 5.9927 + Corrosion\ \ Allowance = 5.9927 + 0.125 = 6.1177\ in$$

The required thickness of the cylindrical shell is $6.1177\ in$; therefore a forging with a wall thickness of $6.25\ in$ will be used.

During fabrication of a section of the cylindrical shell with a nominal inside diameter of $112.0\ in$, the following tolerance readings were taken.

Maximum Inside Diameter = $113.0\ in$

Minimum Inside Diameter = $110.0\ in$

The shell tolerance limits provided in paragraph 6.1.2.7.a and paragraph 4.3.2, state that the difference between the maximum and minimum inside diameters at any cross section shall not exceed 1% of the nominal diameter at the cross section under consideration.

$$\frac{Max\ \ Diameter - Min\ \ Diameter}{Nominal\ \ Diameter} = \frac{113.0 - 110.0}{112.0} = 2.7\%$$

In accordance with paragraph 6.7.4.2.b, if the out-of-roundness exceeds the limit in paragraph 6.1.2.7.a and the condition cannot be corrected, then the forging shall be rejected, except that if the out-of-roundness does not exceed 3%, the forging may be certified for a reduced pressure, P^*, calculated using Equations (6.1) and (6.2). The measurements used in these equations shall be corrected for the specified corrosion allowance.

With,

$$\{S_b = 14961.4\ psi\} \geq \{0.25S = 0.25(27600) = 6900\ psi\}$$

$$P^* = P\left(\frac{1.25}{S_b/S+1}\right) = 2800\left(\frac{1.25}{(14961.4/27600)+1}\right) = 2269.7\ psi$$

Where,

$$S_b = \frac{1.5PR_1t(D_1-D_2)}{t^3+3\left(\dfrac{P}{E_y}\right)(R_1R_a^2)} = \frac{1.5(2800)(55.875)(6.125)(113.25-110.25)}{(6.125)^3+3\left(\dfrac{2800}{27.9E+06}\right)(55.875)(58.9375)^2}$$

$$S_b = 14961.4\ psi$$

And,

$$D_1 = 113.0 + 2(Corrosion\ \ Allowance) = 113.0 + 2(0.125) = 113.25\ in$$

$$D_2 = 110.0 + 2(Corrosion\ \ Allowance) = 110.0 + 2(0.125) = 110.25\ in$$

$$t = 6.25 - Corrosion\ \ Allowance = 6.25 - 0.125 = 6.125\ in$$

$$R_1 = \frac{D_1+D_2}{4} = \frac{113.25+110.25}{4} = 55.875\ in$$

$$R_a = R_1 + \frac{t}{2} = 55.875 + \frac{6.125}{2} = 58.9375\ in$$

Therefore, with the current out-of-roundness in place, the maximum operating pressure of the vessel would be limited to $2269.7\ psi$, which represents a 19% reduction in pressure. This is unacceptable for the planned operation of the vessel. It is determined that the condition cannot be corrected; therefore, the forging shall be rejected and a new cylindrical forging shall be fabricated.

PART 7

INSPECTION AND EXAMINATION REQUIREMENTS

PART CONTENTS

The subject of Examination Groups is covered in Part 7, paragraph 7.4.2. The assignment of a welded joint to a particular Examination Group is dependent on the manufacturing complexity of the material, the maximum thickness, the welding process, and the selected weld joint efficiency. The Examination Groups are defined in Table 7.1. There are three Examination Groups defined within VIII-2, which are then further subdivided in sub-groups “a” and “b” to reflect the crack sensitivity of the material. The required method and extent of nondestructive examination is defined Table 7.2 based on the Examination Group, the joint category, and the corresponding permissible joint type.

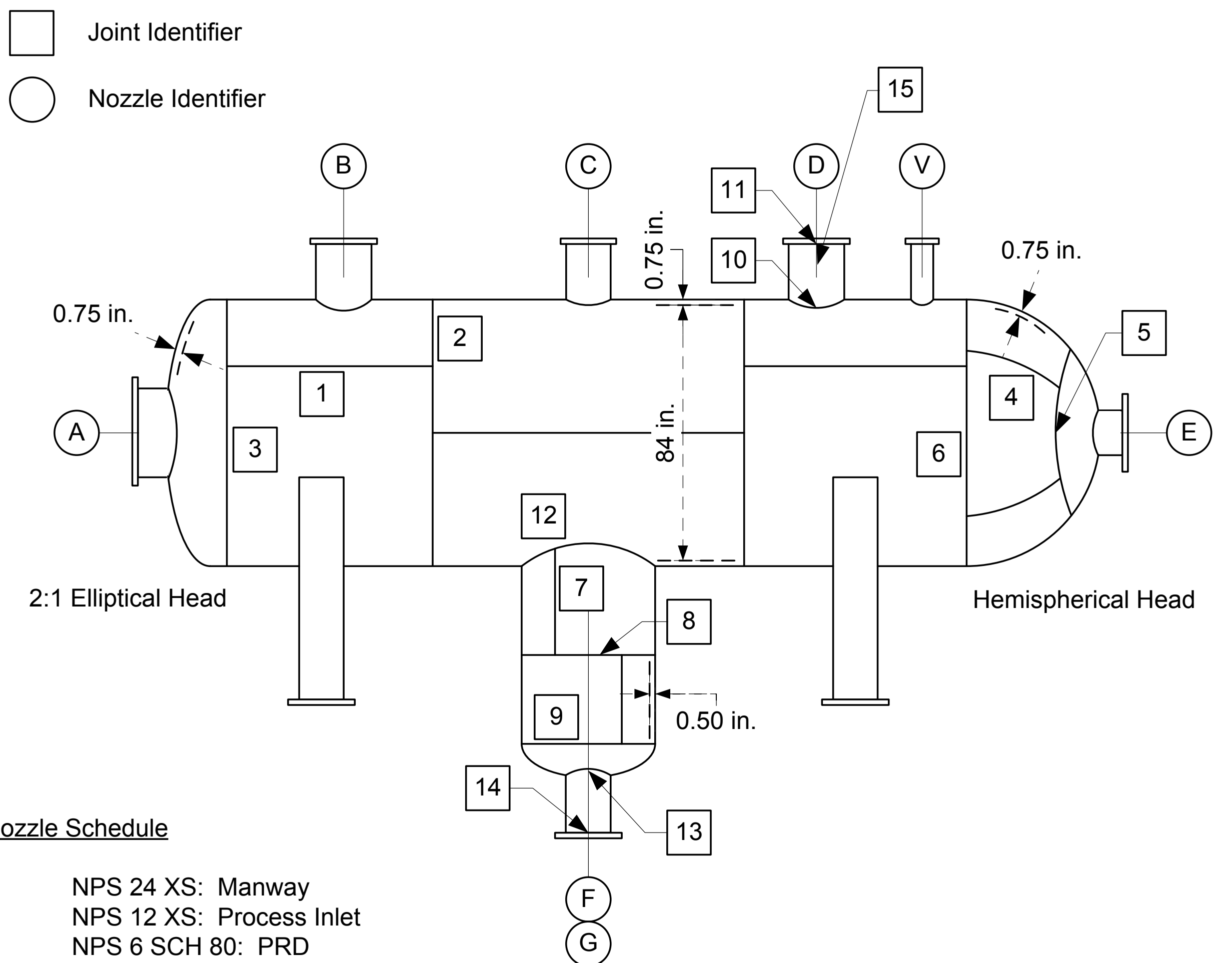
Also introduced in Part 7, paragraph 7.4.2 is the concept of the governing welded joint, which is defined as that welded joint within a given vessel section (such as a shell course or vessel head) that, as a result of the selected joint efficiency, determines the thickness of that vessel section. For example, in a given shell course, the longitudinal weld seam would control the thickness of that shell course in most cases and would be the governing welded joint. However, if the component was subject to significant longitudinal stress from wind, seismic, or other external loadings such that the circumferential seam dictated the thickness of the shell course, then it would be the governing welded joint.

Since it is possible for a pressure vessel to have more than one governing welded joint, it is also possible to have a pressure vessel with multiple Examination Groups. The requirements for the case of a single vessel containing a combination of Examination Groups are covered in Part 7, paragraph 7.4.2.2.b. In each vessel section, the Examination Group of the governing welded joint shall be applied to all welds within that vessel section, including any nozzle attachment welds. A weld that joins two welded vessel sections assigned to different Examination Groups shall be assigned to that Group that requires the greater level of examination. Finally, a weld that joins a welded section to a seamless section, or a weld connecting two seamless sections, is assigned to an Examination Group based on the available thickness (the available thickness is defined as the thickness at the weld, less tolerances and corrosion allowance). If the ratio of available thickness to the minimum required thickness in a given vessel section is greater than 1.18, then Examination Group 3 may be used for that section. Otherwise, the Examination Group is assigned in accordance with the criteria in Table 7.1. The significance of the 1.18 value is that it represents a ratio of 1/0.85, the ratio of the joint efficiencies between Examination Groups 2 and 3.

7.1 Example E7.1 – NDE Requirements: Vessel with One Examination Group Designation

A plant engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with VIII-2. Based on the anticipated design data, materials of construction, and welding process, the engineer selects Examination Group 1a to set the joint efficiency and NDE requirements for the entire vessel. A sketch of the vessel showing nozzle sizes, orientation, and weld seams is shown in Figure E7.1.

To assist with fabrication and inspection of the vessel, the engineer developed a table to summarize the NDE requirements applicable to each welded joint of the vessel based on the Examination Group selected. Table E7.1 is a sub-set of the original table and only addresses the weld joint identifiers referenced on the vessel sketch in Figure E7.1.



Nozzle Schedule

A	NPS 24 XS: Manway
B	NPS 12 XS: Process Inlet
C	NPS 6 SCH 80: PRD
D	NPS 10 SCH 80: Process Outlet
E	NPS 4 SCH 80: Process Inlet
F	NPS 36: Boot
G	NPS 6 SCH 80: Process Outlet
V	NPS 2 XXS: Vent

Figure E7.1 - Vessel Sketch

Table E7.1 - Weld Joint Requirements

Examination Group			1a				
P Number/Group Number			P-1,Gr-2	Governing Weld Joint Thickness			2.75 in
Welding Process			Unrestricted	Weld Joint Efficiency			1.0
Joint Identifier	Joint Category	Joint Type	Table - Detail	Non Destructive Examination			
				Volumetric		Surface	
				Type	Extent	Type	Extent
1	A	1	4.2.4 - 1	RT	100%	PT	10%
2	B	1	4.2.4 - 1	RT	100%	PT	10%
3	B	1	4.2.4 - 1	RT	100%	PT	10%
4	A	1	4.2.4 - 1	UT	100%	PT	10%
5	A	1	4.2.4 - 1	UT	100%	PT	10%
6	A	1	4.2.4 - 1	RT	100%	PT	10%
7	A	1	4.2.4 - 1	RT	100%	MT	10%
8	B	1	4.2.4 - 1	RT	100%	MT	10%
9	B	1	4.2.4 - 1	RT	100%	MT	10%
10	D	7	4.2.10 - 4	UT	100%	PT	10%
11	C	3	4.2.9 - 6	RT	100%	PT	10%
12	D	7	4.2.10 - 4	UT	100%	PT	10%
13	D	7	4.2.10 - 4	---	---	MT	100%
14	C	3	4.2.9 - 6	---	---	MT	10%
15	E	10	---	UT	25%	PT	100%

COMMENTARY:

Tables 7.1, 7.2, and 7.3 from Part 7 of VIII-2, and Figure 4.1 and Tables 4.1 and 4.2 from Part 4 of VIII-2 have been included at the end of the examples for reference.

a) Based on Table 7.1, Examination Groups 1a and 1b are the only groups that have no restrictions on maximum thickness governing weld joints. All other Examination Groups have limitations on welded joint thickness.

b) Other Examination Groups could have been specified for the elliptical and torispherical heads or the boot (communicating chamber), due to the different governing weld joint thicknesses. If multiple Examination Groups were specified for a single vessel, Table E7.1 would need to be repeated for each individual Examination Group.

c) If one or multiple vessel sections were assigned alternate Examination Groups, requirements of the common welded joint between Examination Groups must be followed as noted in paragraph 7.4.2.2.b.

d) All Examination Groups require 100% visual examination to the maximum extent possible.

e) The type of volumetric NDE used in fabrication is a function of the shell thickness and is shown in Table 7.3. For the governing thickness of this Examination Group (1a), the volumetric NDE can either be RT or UT with the same 100% extent of examination. The extent of examination is a percentage of the total length of the welded joint under consideration. The selection of RT or UT in this example (see Table E7.1) was for illustration only.

f) The type of surface NDE can either be MT or PT. The extent of examination varies based on the Joint Category under consideration. The percentage of surface examination refers to the percentage of length of the welds both on the inside and outside surfaces. The selection of MT or PT in this example (see Table E7.1) was for illustration only.

g) Joint Categories and Type of Weld Joints:

   1) Joint Category A:

      i) All welds shall be full penetration butt joints of Type 1 as defined in Table 4.2.2.

      ii) All welds require volumetric (RT or UT) and surface examination (MT or PT).

   2) Joint Category B:

      i) Welds shall be full penetration butt joints of Type 1, 2, or 3; or angle joints of Type 8 as defined in Table 4.2.2.

      ii) All welds in the main shell require volumetric (RT or UT) and surface examination (MT or PT).

      iii) Welds in nozzles may require volumetric (RT or UT) and surface examination (MT or PT) or only surface examination, depending on diameter and thickness.

   3) Joint Category C:

      i) For the assembly of flat heads or tubesheets with cylindrical shells or the assembly of a flange or collar with a shell:

         1. Welds can be full penetration butt joints of Type 1, 2, or 3; or corner joints of Type 7 as defined in Table 4.2.2. These types of welds require volumetric (UT) and surface examination (MT or PT).
         2. Welds can be partial penetration joints of Type 9 or fillet welds of Type 10 as defined in Table 4.2.2, only when Examination Group 3a or 3b is specified. These types of welds require volumetric (UT) and surface examination (MT or PT).

      ii) For the assembly of a flange or a collar with a nozzle:

         1. Welds can be full penetration butt joints of Type 1, 2, or 3; or corner joints of Type 7 as defined in Table 4.2.2. These welds require volumetric (RT or UT) and surface examination (MT or PT).
         2. Welds can be partial penetration joints of Type 9 or fillet welds of Type 10 as defined in Table 4.2.2, only when Examination Group 3a or 3b is specified. These types of welds only require surface examination (MT or PT).
         3. For full or partial penetration welds with a diameter less than or equal to NPS 6 and a thickness less than or equal to 0.625 in, only surface examination (MT or PT) is required.

   4) Joint Category D:

      i) Welds can be full penetration butt joints of Type 1, 2, or 3; or corner joints of Type 7 as defined in Table 4.2.2.

         1. For welds with a nozzle diameter greater than or equal to NPS 6 or a thickness greater than or equal to 0.625 in, volumetric (RT or UT) and surface examination (MT or PT) is required.
         2. For welds with a nozzle diameter less than or equal to NPS 6 and a thickness greater than or equal to 0.625 in., only surface examination (MT or PT) is required.

ii) Welds can be partial penetration joints of Type 9 or fillet welds of Type 10 as defined in Table 4.2.2., provided:

1. For welds with any nozzle diameter and a thickness greater than or equal to 0.625 in., volumetric (UT) and surface examination (MT or PT) is required.
2. For welds with nozzle diameters greater than NPS 6 and a thickness less than or equal to 0.625 in., with Examination Group 3a or 3b specified, only surface examination (MT or PT) is required.
3. For welds with nozzle diameters less than NPS 6 and a thickness less than or equal to 0.625 in., only surface examination (MT or PT) is required.

iii) For tube-to-tubesheet welds in accordance with Figure 4.18.13 and Table 4.C.1, only surface examination (MT or PT) is required.

5) Joint Category E:

i) Welds can be full penetration butt joints of Type 1 or Type 7 as defined in Table 4.2.2. These types of welds requires volumetric (RT or UT) and surface examination (MT or PT). The RT designation is only applicable to the Type 1 weld joint.

ii) Welds can be partial penetration joints of Type 9 or fillet welds of Type 10 as defined in Table 4.2.2. These types of welds require volumetric (UT) and surface examination (MT or PT).

7.2 Example E7.2 – NDE Requirements: Vessel with Two Examination Group Designations

A plant engineer is tasked with developing a design specification for a new pressure vessel that is to be constructed in accordance with VIII-2. Based on the anticipated design data, materials of construction, and welding process, the engineer selects Examination Group 1a to set the joint efficiency and NDE requirements for the main vessel, but selects Examination Group 3b to set the joint efficiency and NDE requirements for the boot (communicating chamber). Refer to Figure E7.1.

Two important limitations require comment for this example problem.

1) The Category D weld joint attaching the boot (communicating chamber) to the main cylindrical shell (Joint Identifier 12) falls under the Examination Group 1a requirements, see paragraph 7.4.2.2.b.
2) All components assigned to Examination Group 3b are required to be designed in accordance with Part 4, Design-by-Rule. If any components in the boot (communicating chamber) required a design-by-analysis in accordance with Part 5, Examination Group 3b would not be permitted.

Similar to Example E7.1, the engineer develops a table to summarize the NDE requirements to assist with fabrication and inspection of the vessel. However, Table E7.1 is reduced by eliminating Joint Identifiers 7, 8, 9, 13, and 14 from Examination Group 1a, as shown in Table E7.2.1; and an additional table is developed to summarize the NDE requirements applicable to each welded joint of the boot (communicating chamber), based on Examination Group 3b, as shown in Table E7.2.2.

Table E7.2.1 - Weld Joint Requirements

Examination Group			1a				
P Number/Group Number			P-1,Gr-2	Governing Weld Joint Thickness			2.75 in
Welding Process			Unrestricted	Weld Joint Efficiency			1.0
Joint Identifier	Joint Category	Joint Type	Table - Detail	Non Destructive Examination			
				Volumetric		Surface	
				Type	Extent	Type	Extent
1	A	1	4.2.4 - 1	RT	100%	PT	10%
2	B	1	4.2.4 - 1	RT	100%	PT	10%
3	B	1	4.2.4 - 1	RT	100%	PT	10%
4	A	1	4.2.4 - 1	UT	100%	PT	10%
5	A	1	4.2.4 - 1	UT	100%	PT	10%
6	A	1	4.2.4 - 1	RT	100%	PT	10%
10	D	7	4.2.10 - 4	UT	100%	PT	10%
11	C	3	4.2.9 - 6	RT	100%	PT	10%
12	D	7	4.2.10 - 4	UT	100%	PT	10%
15	E	10	---	UT	25%	PT	100%

Table E7.2.2 - Weld Joint Requirements

Examination Group			3b				
P Number/Group Number			P-1,Gr-2	Governing Weld Joint Thickness			1.125 in
Welding Process			Unrestricted	Weld Joint Efficiency			0.85
Joint Identifier	Joint Category	Joint Type	Table - Detail	Non Destructive Examination			
				Volumetric		Surface	
				Type	Extent	Type	Extent
7	A	1	4.2.4 - 1	RT	10%	MT	10%
8	B	1	4.2.4 - 1	RT	10%	MT	10%
9	B	1	4.2.4 - 1	RT	10%	MT	10%
13	D	7	4.2.10 - 4	---	---	MT	10%
14	C	3	4.2.9 - 6	---	---	MT	10%

COMMENTARY:

Tables 7.1, 7.2, and 7.3 from Part 7 of VIII-2, and Figure 4.1 and Tables 4.1 and 4.2 from Part 4 of VIII-2 have been included at the end of the examples for reference.

a) The extent of volumetric examination per Examination Group 3b for the Category A and B full penetration weld seams, Joint Identifiers 7, 8, and 9, is only 10% compared to the Examination Group 1a requirement of 100%.

b) The extent of surface examination per Examination Group 3b for the Category D full penetration corner joint, Joint Identifier 13, is only 10% compared to the Examination Group 1a requirement of 100%. Due to limitations on diameter and thickness of the nozzle, volumetric examination was not required.

c) Similar to Examination Group 1a, the extent of surface examination per Examination Group 3b for the Category C full penetration circumferential joint, Joint Identifier 14, is only 10%. Due to limitations on diameter and thickness of the nozzle, volumetric examination was not required.

Table 7.1
Examination Groups For Pressure Vessels

Parameter	Examination Group(1)					
	1a	1b	2a	2b	3a	3b
Permitted Material (1)(2)	All Materials in Annex 3-A	P-No.1 Gr 1 and 2, P-No. 8 Gr 1	P-No. 8 Gr 2 P-No 9A Gr 1 P-No 9B Gr 1 P-No 11A Gr 1 P-No. 11A Gr 2 P-No. 10H Gr 1	P-No.1 Gr 1 and 2, P-No. 8 Gr 1	P-No. 8 Gr 2, P-No 9A Gr 1, P-No 9B Gr 1, P-No. 10H Gr 1	P-No.1 Gr 1 and 2, P-No. 8 Gr 1
Maximum thickness of governing welded joints	Unlimited(4)		30 mm (1-3/16 in.) for P-No 9A Gr 1 and P-No 9B Gr 1; 16 mm (5/8 in.) for P-No. 8,Gr 2(5) P-No. 11A Gr 1 P-No. 11A Gr 2 P-No. 10H Gr 1	50 mm (2 in.) for P-No.1 Gr 1 and P-No. 8 Gr 1; 30 mm (1-3/16 in.) for P-No.1 Gr 2	30 mm (1-3/16 in.) for P-No. 9A Gr 1 and P-No. 9B Gr 1; 16 mm (5/8 in.) for P-No.8, Gr 2(5) P-No. 10H Gr 1	50 mm (2 in.) for P-No.1 Gr 1 and P-No. 8 Gr 1; 30 mm (1-3/16 in.) for P-No.1 Gr 2
Welding process	Unrestricted(4)		Mechanized Welding Only(3)		Unrestricted(4)	
Design Basis(6)	Part 4 or Part 5of this Division		Part 4 or Part 5of this Division		Part 4 of this Division	

NOTES:
(1) All Examination Groups require 100% visual examination to the maximum extent possible.
(2) See Part 3 for permitted material.
(3) Mechanized means machine and/or automatic welding methods.
(4) Unrestricted with respect to weld application modes as set forth in this Table.
(5) See Table 7.2 for NDE, joint category, and permissible weld joint details that differ between Examination Groups 1a and 1b.
(6) The design basis is the analysis method used to establish the wall thickness.

Table 7.2
Nondestructive Examination

Examination Group					1a	1b	2a	2b	3a	3b
Permitted Materials					All Materials in Annex 3-A (18)	P-No. 1 Gr 1 & 2 P-No. 8 Gr 1	P-No. 8 Gr 2 P-No. 9A Gr 1 P-No. 9B Gr 1 P-No. 11A Gr 1 P-No. 11A Gr 2 P-No. 10H Gr 1	P-No. 1 Gr 1 & 2 P-No. 8 Gr 1	P-No. 8 Gr 2 P-No. 9A Gr 1 P-No. 9B Gr 1 P-No. 10H Gr 1	P-No. 1 Gr 1 & 2 P-No. 8 Gr 1
Weld Joint Efficiency					1.0	1.0	1.0	1.0	0.85	0.85
Joint Category	Type of Weld.(1)			Type of NDE.(2)	Extent of NDE (10)(11)(12)					
A	Full penetration butt weld	1	...	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	25% 10%	10% 10%(4)
B		1	...	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	10% 10%	10%(3) 10%(4)
B		2,3	...	RT or UT MT or PT	NA NA	100% 10%	NA NA	25% 10%	NA NA	25% 10%
B		1	...	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	10% 10%	10%(3) 10%(4)
B		2,3	...	RT or UT MT or PT	NA NA	100% 10%	NA NA	25% 10%	NA NA	25% 10%
B		1	...	MT or PT	100%	10%	100%	10%	10%	10%
A		1	...	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	25% 10%	10% 10%(4)
B		1	...	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	10% 10%	10% 10%(4)
B		8	...	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	25% 10%	10% 10%(4)

Table 7.2
Nondestructive Examination

Examination Group					1a	1b	2a	2b	3a	3b
Permitted Materials					All Materials in Annex 3-A (18)	P-No. 1 Gr 1 & 2 P-No. 8 Gr 1	P-No. 8 Gr 2 P-No. 9A Gr 1 P-No. 9B Gr 1 P-No. 11A Gr 1 P-No. 11A Gr 2 P-No. 10H Gr 1	P-No. 1 Gr 1 & 2 P-No. 8 Gr 1	P-No. 8 Gr 2 P-No. 9A Gr 1 P-No. 9B Gr 1 P-No. 10H Gr 1	P-No. 1 Gr 1 & 2 P-No. 8 Gr 1
Weld Joint Efficiency					1.0	1.0	1.0	1.0	0.85	0.85
Joint Category	Type of Weld.(1)			Type of NDE.(2)	Extent of NDE (10)(11)(12)					
A	Full penetration butt weld	1	...	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	25% 10%	10% 10%(4)
B		1	...	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	10% 10%	10%(3) 10%(4)
B		2,3	...	RT or UT MT or PT	NA NA	100% 10%	NA NA	25% 10%	NA NA	25% 10%
B		1	...	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	10% 10%	10%(3) 10%(4)
B		2,3	...	RT or UT MT or PT	NA NA	100% 10%	NA NA	25% 10%	NA NA	25% 10%
B		1	...	MT or PT	100%	10%	100%	10%	10%	10%
A		1	...	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	25% 10%	10% 10%(4)
B		1	...	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	10% 10%	10% 10%(4)
B		8	...	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	25% 10%	10% 10%(4)

Table 7.2
Nondestructive Examination (Cont'd)

Examination Group					1a	1b	2a	2b	3a	3b
Permitted Materials					All Materials in Annex 3-A (18)	P-No. 1 Gr 1 & 2 P-No. 8 Gr 1	P-No. 8 Gr 2 P-No. 9A Gr 1 P-No. 9B Gr 1 P-No. 11A Gr 1 P-No. 11A Gr 2 P-No. 10H Gr 1	P-No. 1 Gr 1 & 2 P-No. 8 Gr 1	P-No. 8 Gr 2 P-No. 9A Gr 1 P-No. 9B Gr 1 P-No. 10H Gr 1	P-No. 1 Gr 1 & 2 P-No. 8 Gr 1
Weld Joint Efficiency					1.0	1.0	1.0	1.0	0.85	0.85
Joint Category	Type of Weld.(1)			Type of NDE.(2)	Extent of NDE (10)(11)(12)					
D	Nozzle or branch (5)	1, 2, 3, 7	With full penetration $d>$ 150 mm (6 in.) or $t>$ 16 mm (5/8 in.)	RT or UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	25% 10%	10% 10%(4)
D		1, 2, 3, 7	With full penetration $d \le$ 150 mm (6 in.) and $t \le$ 16 mm (5/8 in.)	MT or PT	100%	10%	100%	10%	10%	10%
D		9, 10	With partial penetration for any d a>16 mm (5/8 in.) (17)	UT MT or PT	100% 10%	100% 10%(4)	100% 10%	100% 10%(4)	25% 10%	10% 10%(4)
D		9, 10	With partial penetration d > 150 mm (6 in.) $a \le$ 16 mm (5/8 in.)(17)	MT or PT	NA	NA	NA	NA	10%	10%
D		9, 10	With partial penetration $d \le$ 150 mm (6 in.) $a \le$ 16 mm (5/8 in.)	MT or PT	100%	10%	100%	10%	10%	10%
D	Tube-to-Tubesheet Welds			MT or PT	100%	100%	100%	100%	25%	10%
E	Permanent attachments (6)	1, 7, 9, 10	With full penetration or partial penetration(15)	RT or UT MT or PT	25%(7) 100%	10%(4) 10%	10% 100%	10%(4) 10%	10% 100%	10%(4) 10%(4)
NA	Pressure retaining areas after removal of attachments	NA	...	MT or PT	100%	100%	100%	100%	100%	100%

Table 7.2
Nondestructive Examination (Cont'd)

Joint Category	Type of Weld.(1)			Type of NDE.(2)	1a	1b	2a	2b	3a	3b
Examination Group					1a	1b	2a	2b	3a	3b
Permitted Materials					All Materials in Annex 3-A (18)	P-No. 1 Gr 1 & 2 P-No. 8 Gr 1	P-No. 8 Gr 2 P-No. 9A Gr 1 P-No. 9B Gr 1 P-No. 11A Gr 1 P-No. 11A Gr 2 P-No. 10H Gr 1	P-No. 1 Gr 1 & 2 P-No. 8 Gr 1	P-No. 8 Gr 2 P-No. 9A Gr 1 P-No. 9B Gr 1 P-No. 10H Gr 1	P-No. 1 Gr 1 & 2 P-No. 8 Gr 1
Weld Joint Efficiency					1.0	1.0	1.0	1.0	0.85	0.85
Joint Category	Type of Weld.(1)			Type of NDE.(2)	Extent of NDE (10)(11)(12)					
...	Cladding by welding	...	...	RT or UT	(13)	(13)	(13)	(13)	(13)	(13)
				MT or PT	100%	100%	100%	100%	100%	100%
...	Repairs (14)	...	...	RT or UT	100%	100%	100%	100%	100%	100%
				MT or PT	100%	100%	100%	100%	100%	100%

NOTES:
(1) See paragraph 4.2
(2) RT = Radiographic Examination, UT = Ultrasonic Examination, MT = Magnetic Particle Examination, PT = Liquid Penetrant Examination.
(3) 2% if $t \leq 30$ mm (1-3/16 in.) and same weld procedure specification as longitudinal, for steel of P-No. 1 Gr 1 and P-No. 8 Gr 1
(4) 10% if $t > 30$ mm (1-3/16 in.), 0% if $t \leq 30$ mm (1-3/16 in.)
(5) Percentage in the table refers to the aggregate weld length of all the nozzles, see paragraph 7.4.3.5(b).
(6) RT or UT is not required for weld thicknesses ≤ 16 mm (5/8 in.)
(7) 10% for steel of P-No. 8 Gr 2, P-No. 9A Gr 1, P-No. 9B Gr 1, P-No. 11A Gr 1, P-No. 11A Gr 2, P-No. 10H Gr 1
(8) (Currently not used.)
(9) For limitations of application see paragraph 4.2.
(10) The percentage of surface examination refers to the percentage of length of the welds both on the inside and the outside.
(11) RT and UT are volumetric examination methods, and MT and PT are surface examination methods. Both volumetric and surface examinations are required to be applied the extent shown.
(12) NA means "not applicable". All Examination Groups require 100% visual examination to the maximum extent possible.
(13) See paragraph 7.4.8.1 for detailed examination requirements.
(14) The percentage of examination refers only to the repair weld and the original examination methods, see paragraph 6.2.7.3.
(15) RT is applicable only to Type 1, full penetration welds.
(16) The term "α" as defined in Figure 7.16.
(17) The term "α" as defined in Figure 7.17.
(18) For SAW welds in $2^1/_4$ Cr–1Mo–$^1/_4$V vessels, ultrasonic examination in accordance with 7.5.4.1(e) is required.

Table 7.3
Selection of Nondestructive Testing Method For Full Penetration Joints

Type of Joint	Shell thickness – t	
	$t < 13\ mm\ (1/2\ in.)$	$t \ge 13\ mm\ (1/2\ in.)$
1, 2, 3	RT	RT or UT per 7.5.5
7, 8	N/A	UT per 7.5.4 or 7.5.5

Figure 4.2.1
Weld Joint Locations Typical Of categories A, B, C, D, and E

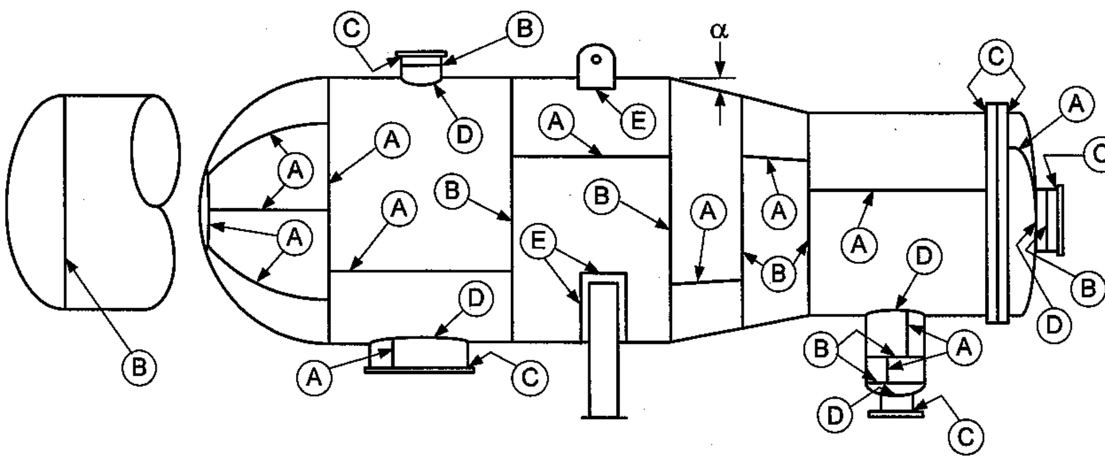


Table 4.2.1
Definition Of Weld Categories

Weld Category	Description
A	• Longitudinal and spiral welded joints within the main shell, communicating chambers (1), transitions in diameter, or nozzles • Any welded joint within a sphere, within a formed or flat head, or within the side plates (2) of a flat-sided vessel • Circumferential welded joints connecting hemispherical heads to main shells, to transitions in diameter, to nozzles, or to communicating chambers.
B	• Circumferential welded joints within the main shell, communicating chambers (1), nozzles or transitions in diameter including joints between the transition and a cylinder at either the large or small end • Circumferential welded joints connecting formed heads other than hemispherical to main shells, to transitions in diameter, to nozzles, or to communicating chambers.
C	• Welded joints connecting flanges, Van Stone laps, tubesheets or flat heads to main shell, to formed heads, to transitions in diameter, to nozzles, or to communicating chambers (1) • Any welded joint connecting one side plate (2) to another side plate of a flat-sided vessel.
D	• Welded joints connecting communicating chambers (1) or nozzles to main shells, to spheres, to transitions in diameter, to heads, or to flat-sided vessels • Welded joints connecting nozzles to communicating chambers (1) (for nozzles at the small end of a transition in diameter see Category B).
E	• Welded joints attaching nonpressure parts and stiffeners

NOTES:
(1) Communicating chambers are defined as appurtenances to the vessel that intersect the shell or heads of a vessel and form an integral part of the pressure containing enclosure, e.g., sumps.
(2) Side plates of a flat-sided vessel are defined as any of the flat plates forming an integral part of the pressure containing enclosure.

Table 4.2.2
Definition Of Weld Joint Types

Weld Joint Type	Description
1	Butt joints and angle joints where the cone half-apex angle is less than or equal to 30 deg produced by double welding or by other means which produce the same quality of deposited weld metal on both inside and outside weld surfaces. Welds using backing strips which remain in place do not qualify as Type No.1 butt joints.
2	Butt joints produced by welding from one side with a backing strip that remains in place.
3	Butt joints produced by welding from one side without a backing strip.
7	Corner joints made with full penetration welds with or without cover fillet welds
8	Angle joints made with a full penetration weld where the cone half-apex angle is greater than 30 deg
9	Corner joints made with partial penetration welds with or without cover fillet welds
10	Fillet welds

PART 8

PRESSURE TESTING REQUIREMENTS

PART CONTENTS

8.1 Example E8.1 – Determination of a Hydrostatic Test Pressure

Establish the hydrostatic test pressure for a process tower considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-537,\ Class\ 1$
- Design Conditions = $1650\ psig @ 600°F$
- Liquid Head = $60\ ft$
- Liquid Specific Gravity = 0.89
- Inside Diameter = $96.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $25700\ psi$
- Allowable Stress at Ambient Conditions = $29200\ psi$
- Yield Stress at Ambient Conditions = $50000\ psi$
- Weld Joint Efficiency = 1.0
- Tangent-to-Tangent Vessel Length = $80\ ft$
- Top and Bottom Heads = $Hemispherical$

Evaluate the requirements of hydrostatic testing per paragraph 8.2.

The design pressure used to establish the wall thickness for the bottom head and cylindrical shell section must be adjusted for the liquid head in accordance with Part 4, paragraph 4.1.5.2.a.

Adjusted pressure for the cylindrical shell:

$$Design\ \ Pressure = Specified\ \ Design\ \ Pressure + \gamma gh$$

$$Design\ \ Pressure = 1650 + \frac{0.89(62.4)(56)}{144} = 1671.597\ \ psig$$

Adjusted pressure for the bottom hemispherical head:

$$Design\ \ Pressure = Specified\ \ Design\ \ Pressure + \gamma gh$$

$$Design\ \ Pressure = 1650 + \frac{0.89(62.4)(60)}{144} = 1673.140\ \ psig$$

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the top head.

$$D = 96.0 + 2(Corrosion\ \ Allowance) = 96.0 + 2(0.125) = 96.25\ in$$

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right]-1\right) = \frac{96.25}{2}\left(\exp\left[\frac{0.5(1650)}{25700(1.0)}\right]-1\right) = 1.5699\ in$$

$$t = 1.5699 + Corrosion\ \ Allowance = 1.5699 + 0.125 = 1.6949\ in$$

The required thickness of the top head is $1.6949\ in$.

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the bottom head, including the liquid head static pressure.

$$D = 96.0 + 2(Corrosion\ \ Allowance) = 96.0 + 2(0.125) = 96.25\ in$$

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right]-1\right) = \frac{96.25}{2}\left(\exp\left[\frac{0.5(1673.140)}{25700(1.0)}\right]-1\right) = 1.5923\ in$$

$$t = 1.5923 + Corrosion\ \ Allowance = 1.5923 + 0.125 = 1.7173\ in$$

The required thickness of the bottom head is $1.7173\ in$.

In accordance with Part 4, paragraph 4.3.3, determine the required thickness of the cylindrical shell, including the liquid head static pressure.

$$D = 96.0 + 2(Corrosion\ \ Allowance) = 96.0 + 2(0.125) = 96.25\ in$$

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right]-1\right) = \frac{96.25}{2}\left(\exp\left[\frac{1671.597}{25700(1.0)}\right]-1\right) = 3.2342\ in$$

$$t = 3.2373 + Corrosion\ \ Allowance = 3.2342 + 0.125 = 3.3592\ in$$

The required thickness of the cylindrical shell is $3.3592\ in$.

Required Thickness Summary:

Top Head = $1.6949\ in$

Bottom Head = $1.7173\ in$

Cylindrical Shell = $3.3592\ in$

Per paragraph 8.2.1,

a) The minimum hydrostatic test pressure shall be the greater of the following. In this example problem, the MAWP is taken as the design pressure and the ratio of S_T/S is based on the shell material.

$$P_T = \max\left[\{1.43 \cdot MAWP\},\ \left\{1.25 \cdot MAWP \cdot \left(\frac{S_T}{S}\right)\right\}\right]$$

$$P_T = \max\left[\{1.43(1650) = 2359.5\},\ \left\{1.25(1650)\left(\frac{29200}{25700}\right) = 2343.4\right\}\right] = 2360\ psi$$

b) The ratio S_T/S shall be the lowest ratio for the pressure-boundary materials, excluding bolting materials, of which the vessel is constructed.

c) The test pressure is the pressure to be applied at the top of the vessel during the test. The vessel is to be pressure tested in the horizontal position; therefore, the additional pressure from hydrostatic head is negligible. This pressure is used in the applicable design equations to check the vessel under test conditions, see Part 4, paragraph 4.1.6.2.a.

Per paragraph 4.1.6.2.a, when a hydrostatic test is performed in accordance with Part 8, the hydrostatic test pressure of a completed vessel shall not exceed that value which results in the following equivalent stress limit.

$$P_m \le 0.95 S_y$$

The general primary membrane stress in the hemispherical heads is computed using the ASME membrane equations from VIII-1. For the hemispherical head, note that it is only necessary to check the thinner head.

$$R = D_i/2 = 96.0/2 = 48.0\ in$$

$$P_m = \frac{P}{2E}\left(\frac{R}{t} + 0.2\right) = \frac{2360}{2(1.0)}\left(\frac{48.0}{1.6949} + 0.2\right) = 34163\ psi$$

$$\{P_m = 34163\ psi\} \le \{0.95 S_y = 0.95(50000) = 47500\ psi\}$$

The general primary membrane stress in the cylindrical shell is computed using the ASME membrane equations from VIII-1.

$$R = D_i/2 = 96.0/2 = 48.0\ in$$

$$P_m = \frac{P}{E}\left(\frac{R}{t} + 0.6\right) = \frac{2360}{1.0}\left(\frac{48.0}{3.3592} + 0.6\right) = 35138\ psi$$

$$\{P_m = 35138\ psi\} \le \{0.95 S_y = 0.95(50000) = 47500\ psi\}$$

Therefore, the proposed design is acceptable for a hydrostatic test pressure of 2360 psig.

d) The requirement of paragraph 8.2.1.a represents the minimum required hydrostatic test pressure. The upper limit of the test pressure shall be determined using the method in paragraph 4.1.6.2.a. Any intermediate value or pressure may be used.

e) A hydrostatic test based on a calculated pressure may be used by agreement between the user and the Manufacturer.

8.2 Example E8.2 – Determination of a Pneumatic Test Pressure

Establish the pneumatic test pressure for a vessel considering the following design conditions. All Category A and B joints are Type 1 butt welds and have been 100% radiographically examined.

Vessel Data:

- Material = $SA-516,\ Grade\ 70$
- Design Conditions = $150\ psig @ 300°F$
- Inside Diameter = $240.0\ in$
- Corrosion Allowance = $0.125\ in$
- Allowable Stress = $22400\ psi$
- Allowable Stress at Ambient Conditions = $25300\ psi$
- Yield Stress at Ambient Conditions = $38000\ psi$
- Weld Joint Efficiency = 1.0
- Tangent-to-Tangent Vessel Length = $80\ ft$
- Top and Bottom Heads = $Hemispherical$

Evaluate the requirements of pneumatic testing per paragraph 8.3.

In accordance with Part 4, paragraph 4.3.5, determine the required thickness of the top and bottom hemispherical heads.

$$D = 240.0 + 2(Corrosion\ \ Allowance) = 240.0 + 2(0.125) = 240.25\ in$$

$$t = \frac{D}{2}\left(\exp\left[\frac{0.5P}{SE}\right] - 1\right) = \frac{240.25}{2}\left(\exp\left[\frac{0.5(150)}{22400(1.0)}\right] - 1\right) = 0.4029\ in$$

$$t = 0.4029 + Corrosion\ \ Allowance = 0.4029 + 0.125 = 0.5279\ in$$

The required thickness of the heads is $0.5279\ in$.

In accordance with Part 4, paragraph 4.3.3, determine the required thickness of the cylindrical shell.

$$D = 240.0 + 2(Corrosion\ \ Allowance) = 240.0 + 2(0.125) = 240.25\ in$$

$$t = \frac{D}{2}\left(\exp\left[\frac{P}{SE}\right] - 1\right) = \frac{240.25}{2}\left(\exp\left[\frac{150}{22400(1.0)}\right] - 1\right) = 0.8071\ in$$

$$t = 0.8071 + Corrosion\ \ Allowance = 0.8071 + 0.125 = 0.9321\ in$$

The required thickness of the cylindrical shell is $0.9321\ in$.

Required Thickness Summary:

Top/Bottom Head = $0.5279\ in$

Bottom Cylindrical Shell = $0.9321\ in$

Per paragraph 8.2.1,

a) The minimum pneumatic test pressure shall be computed as follows. In this example problem, the MAWP is taken as the design pressure and the ratio of S_T/S is based on the shell material.

$$P_T = 1.15 \cdot MAWP \cdot \left(\frac{S_T}{S}\right)$$

$$P_T = 1.15(150.0)\left(\frac{25300}{22400}\right) = 195\ psi$$

b) The ratio S_T/S shall be the lowest ratio for the pressure-boundary materials, excluding bolting materials, of which the vessel is constructed.

c) The requirement of paragraph 8.3.1.a represents the minimum required pneumatic test pressure. The upper limit of the test pressure shall be determined using the method in paragraph 4.1.6.2.b. Any intermediate value or pressure may be used.

d) The test pressure is the pressure to be applied at the top of the vessel during the test. This pressure is used in the applicable design equations to check the vessel under test conditions, see Part 4, paragraph 4.1.6.2.b.

Per paragraph 4.1.6.2.b, when a pneumatic test is performed in accordance with Part 8, the pneumatic test pressure of a completed vessel shall not exceed that value which results in the following equivalent stress limit.

$$P_m \le 0.80 S_y$$

The general primary membrane stress in the hemispherical heads is computed using the ASME membrane equations from VIII-1.

$$R = D_i/2 = 240.0/2 = 120.0\ in$$

$$P_m = \frac{P}{2E}\left(\frac{R}{t} + 0.2\right) = \frac{195}{2(1.0)}\left(\frac{120.0}{0.5279} + 0.2\right) = 22183\ psi$$

$$\{P_m = 22183\ psi\} \le \{0.8S_y = 0.8(38000) = 30400\ psi\}$$

The general primary membrane stress in the cylindrical shell is computed using the ASME membrane equations from VIII-1.

$$R = D_i/2 = 240.0/2 = 120.0\ in$$

$$P_m = \frac{P}{E}\left(\frac{R}{t} + 0.6\right) = \frac{195}{1.0}\left(\frac{120.0}{0.9321} + 0.6\right) = 25222\ psi$$

$$\{P_m = 25222\ psi\} \le \{0.8S_y = 0.8(38000) = 30400\ psi\}$$

Therefore, the proposed design is acceptable for a pneumatic test pressure of $195\ psi$.

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